Title: Separable electromagnetic perturbations of rotating black holes

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Abstract: We identify a set of Hertz potentials for solutions to the vector wave equation on black hole spacetimes. The Hertz potentials yield Lorenz gauge electromagnetic vector potentials that represent physical solutions to the Maxwell equations, satisfy the Teukolsky equation, and are related to the Maxwell scalars by straightforward and separable inversion relations. Our construction, based on the GHP formalism, avoids the need for a mode ansatz and leads to potentials that represent both static and non-static solutions. As an explicit example, we specialise the procedure to mode-decomposed perturbations of Kerr spacetime and in the process make connections with previous results.

Separable electromagnetic perturbations of rotating black holes

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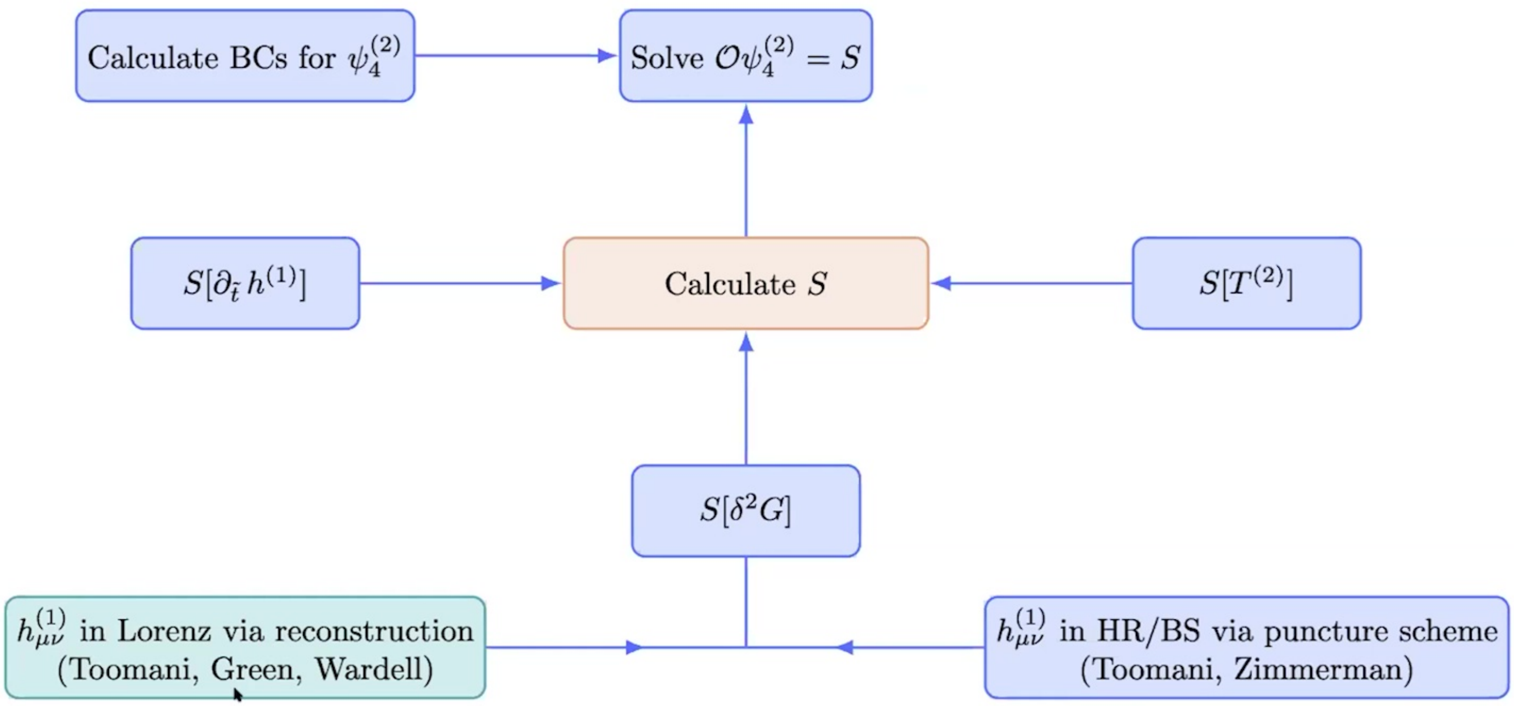
24th Capra Meeting on Radiation Reaction in General Relativity

Motivation

Second order perturbations of Kerr spacetime



Motivation

- Two paths to separable Lorenz gauge metric perturbations:
  - **Direct:** Electromagnetic case provides a good intermediate step to develop ideas which can then be applied to find similar approaches for metric perturbations.
  - **Indirect:** Gauge vector required to transform from radiation gauge to Lorenz gauge satisfies the vector wave equation.
- Electromagnetic Lorenz-gauge self-force in Kerr.
- Advantages of Lorenz gauge:
  - Self-force equation of motion and regularization well understood.
  - Avoid string-like singularities that appear in the radiation gauge.
  - Manifestly separable forms.

EM Perturbations

Faraday tensor

$$F_{\alpha\beta} = 2\nabla_{[\alpha}A_{\beta]}$$

Maxwell equations

$$\nabla_\alpha F^{\alpha\beta} = J^\beta \leftrightarrow (\mathscr{E}A)^\beta := 2\nabla_\alpha \nabla^{[\alpha}A^{\beta]} = J^\beta$$

Lorenz gauge	Ingoing radiation gauge
$\nabla^\alpha A_\alpha = 0$	$l^\alpha A_\alpha = 0 \quad l^\alpha(\mathscr{E}A)_\alpha = 0$
$(\mathscr{L}A)^\alpha = J^\alpha$	Outgoing radiation gauge
$(\mathscr{L}A)^\alpha := \Box A^\alpha - R^\alpha{}_\beta A^\beta$	$n^\alpha A_\alpha = 0 \quad n^\alpha(\mathscr{E}A)_\alpha = 0$

EM Perturbations of Kerr

1972 — [Teukolsky, Phys. Rev. Lett. 29, 1114]

Separable equations for Weyl scalars

1975 — [Chrzanowski, Phys. Rev. D11, 2042]

1979 — [Kegeles and Cohen, Phys. Rev. D19, 1641]

Reconstruction of radiation gauge vector potential in vacuum

2017 — [Lunin, JHEP 12, 138]

Lorenz gauge vector potential in vacuum from separable functions

2018 — [Frolov, Krtouš and Kubizňňák Phys. Rev. D 97, 101701; Nucl. Phys. B 934, 7] [Frolov, Krtouš, Kubizňňák and Santos, Phys. Rev. Lett. 120, 231103]

$$A_\alpha = \underbrace{B_{\alpha\beta}}_{\text{“Polarization tensor”}} \underbrace{\nabla^n Z}_{\text{Not Teukolsky, but related}} \qquad B^{\alpha\rho}(g_{\rho\beta} + i\mu h_{\rho\beta}) = \delta^\alpha_\beta$$

2019 — [Dolan, Phys. Rev. D 98, 104006] [Dolan, Phys. Rev. D 100, 044044]

Non-static Lorenz gauge vector potential in vacuum from Teukolsky functions

Teukolsky Formalism

Null tetrad

$$(l, n, m, \bar{m}) \qquad l^\alpha n_\alpha = -1 \qquad m^\alpha \bar{m}_\alpha = 1 \qquad g_{\alpha\beta} = -2 l_{(\alpha} n_{\beta)} + 2 m_{(\alpha} \bar{m}_{\beta)}$$

Geroch-Held-Penrose formalism

Graded algebra
Covariant version of Newman-Penrose

GHP Type $\{p, q\} \Longleftrightarrow$ Spin-weight and Boost-weight

$$s = \frac{p-q}{2} \qquad b = \frac{p+q}{2}$$

Symmetries and discrete transformations

$$' : l^\alpha \leftrightarrow n^\alpha, m^\alpha \leftrightarrow \bar{m}^\alpha, \{p, q\} \to \{-p, -q\} \qquad \bar{} : m^\alpha \leftrightarrow \bar{m}^\alpha, \{p, q\} \to \{q, p\}$$

Spin coefficients

Components of the connection for a non-coordinate basis

$$\overset{\{3,1\}}{\kappa} = -l^\mu m^\nu \nabla_\mu l_\nu, \quad \overset{\{3,-1\}}{\sigma} = -m^\mu m^\nu \nabla_\mu l_\nu, \quad \overset{\{1,1\}}{\rho} = -\bar{m}^\mu m^\nu \nabla_\mu l_\nu, \quad \overset{\{1,-1\}}{\tau} = -n^\mu m^\nu \nabla_\mu l_\nu,$$

GHP directional covariant derivatives

Spin/boost raising/lowering operators

$$\overset{\{1,1\}}{Þ} = (l^\alpha \nabla_\alpha - p\epsilon - q\bar{\epsilon}), \quad \overset{\{-1,-1\}}{Þ'} = (n^\alpha \nabla_\alpha + p\epsilon' + q\bar{\epsilon}'), \quad \overset{\{1,-1\}}{ð} = (m^\alpha \nabla_\alpha - p\beta + q\bar{\beta}'), \quad \overset{\{-1,1\}}{ð'} = (\bar{m}^\alpha \nabla_\alpha + p\beta' - q\bar{\beta})$$

Teukolsky Equation

Maxwell scalars

Null tetrad components of Faraday tensor

$$\overset{\{2,0\}}{\phi_0} := F_{lm} = \mathscr{T}_0 A, \quad \overset{\{0,0\}}{\phi_1} := \frac{1}{2}\left(F_{ln} - F_{m\bar{m}}\right) = \mathscr{T}_1 A, \quad \overset{\{-2,0\}}{\phi_2} := F_{\bar{m}n} = \mathscr{T}_2 A$$

Decoupling operators

$$\mathcal{S}_0 J = \tfrac{1}{2}\left[(\eth - 2\tau - \bar{\tau}')J_l - (\text{Þ} - 2\rho - \bar{\rho})J_m\right], \quad \mathcal{S}_2 J = \tfrac{1}{2}\left[-(\eth' - 2\tau' - \bar{\tau})J_n + (\text{Þ}' - 2\rho' - \bar{\rho}')J_{\bar{m}}\right]$$

Wald identities

Decoupled

$$\mathscr{O}\mathscr{T}_0 = \mathscr{S}_0\mathscr{E}, \quad \mathscr{O}_1\mathscr{T}_1 = \mathscr{S}_1\mathscr{E}, \quad \mathscr{O}'\mathscr{T}_2 = \mathscr{S}_2\mathscr{E}$$

Teukolsky equations

Separable

$$\mathscr{O}\phi_0 = \mathscr{S}_0 J, \quad \mathscr{O}'\phi_2 = \mathscr{S}_2 J$$

$$\mathcal{O} := \left(\text{Þ} - 2\,s\,\rho - \bar{\rho}\right)\left(\text{Þ}' - \rho'\right) - \left(\eth - 2\,s\,\tau - \bar{\tau}'\right)\left(\eth' - \tau'\right) + \tfrac{1}{2}\left[\left(6s - 2\right) - 4s^2\right]\psi_2 = (\zeta\bar{\zeta})^{-1}(\mathscr{R} - \mathcal{S})$$

Teukolsky-Starobinsky identities

Killing spinor coefficient [$\zeta = r - ia\cos\theta$ in Kerr]

$$\text{Þ}^2\zeta^2\phi_2 = \eth'^2\zeta^2\phi_0, \quad \text{Þ}'^2\zeta^2\phi_0 = \eth^2\zeta^2\phi_2, \quad [\text{Þ}'\eth' + \bar{\tau}\text{Þ}']\zeta^2\phi_0 = [\text{Þ}\eth + \bar{\tau}'\text{Þ}]\zeta^2\phi_2$$

Radiation Gauge

Reconstruct the radiation gauge vector potential from the Maxwell scalars (CCK)

Ingoing Radiation Gauge

$$l^\alpha A_\alpha = 0 \quad l^\alpha (\mathscr{E}A)_\alpha = 0$$

$$A_\alpha^{\mathrm{IRG}} = 2\Re[(\mathscr{S}_0^\dagger \Phi^{\mathrm{IRG}})_\alpha]$$

Separable

$$\phi_0 = \frac{1}{2} \text{Þ}^2 \overline{\Phi^{\mathrm{IRG}}}$$

$$\phi_1 = \frac{1}{2}[\text{Þ}\eth' + \tau' \text{Þ}]\overline{\Phi^{\mathrm{IRG}}}$$

$$\phi_2 = \frac{1}{2}\eth'^2 \overline{\Phi^{\mathrm{IRG}}} - \frac{1}{2}\eta^{\mathrm{IRG}}$$

Outgoing Radiation Gauge

$$n^\alpha A_\alpha = 0 \quad n^\alpha (\mathscr{E}A)_\alpha = 0$$

$$A_\alpha^{\mathrm{ORG}} = 2\Re[(\mathscr{S}_2^\dagger \Phi^{\mathrm{ORG}})_\alpha]$$

$$\phi_0 = -\frac{1}{2}\eth^2 \overline{\Phi^{\mathrm{ORG}}} + \frac{1}{2}\eta^{\mathrm{ORG}}$$

$$\phi_1 = \frac{1}{2}[\text{Þ}'\eth + \tau \text{Þ}']\overline{\Phi^{\mathrm{ORG}}}$$

$$\phi_2 = -\frac{1}{2}\text{Þ}'^2 \overline{\Phi^{\mathrm{ORG}}}$$

Lorenz gauge

- Aim to find a vector potential that:

  1. Is the desired physical solution of the Maxwell equations (i.e. it produces the expected ϕ_0 and ϕ_2)

  2. Satisfies the Lorenz gauge condition

  3. Is a solution of the Lorenz gauge equations

  4. Is constructed from a first order differential operator acting on spin-weight ±1 scalars (i.e. GHP type {2,0}, {-2,0}, {0,2} and {0,-2})

Lorenz gauge

Maximum-spin-weight two-form satisfying Teukolsky & T-S

$$H_{\alpha\beta} = H_{[\alpha\beta]} \quad H_{lm} = \Phi_0 \quad H_{\bar{m}n} = \Phi_2$$

Generic ansatz

$$A_l = (c_{l_1} \eth' + c_{l_2} \tau' + c_{l_3} \bar{\tau})\Phi_0,$$

$$A_n = (c_{n_1} \eth + c_{n_2} \tau + c_{n_3} \bar{\tau}')\Phi_2,$$

$$A_m = (c_{m_1} \text{Þ}' + c_{m_2} \rho' + c_{m_3} \bar{\rho}')\Phi_0,$$

$$A_{\bar{m}} = (c_{\bar{m}_1} \text{Þ} + c_{\bar{m}_2} \rho + c_{\bar{m}_3} \bar{\rho})\Phi_2,$$

Coefficients are type $\{0,0\}$: numeric constants and functions of ζ, $\bar{\zeta}$

Lorenz gauge

Maximum-spin-weight two-form satisfying Teukolsky & T-S

$$H_{\alpha\beta} = H_{[\alpha\beta]} \qquad H_{lm} = \Phi_0 \qquad H_{\bar{m}n} = \Phi_2$$

Assume polynomial in $\zeta, \bar{\zeta}$ → Two Lorenz-gauge vector potentials (others obtained as linear combinations and/or compositions)

conformal Killing-Yano tensor

$$h_{\alpha\beta} = (\zeta + \bar{\zeta}) n_{[\alpha} l_{\beta]} - (\zeta - \bar{\zeta}) \bar{m}_{[\alpha} m_{\beta]}$$

$A_\alpha^{\mathscr{L}1} = \nabla^\beta (\zeta H_{\beta\alpha}^{\mathscr{L}1})$	$A_\alpha^{\mathscr{L}2} = h^{\beta\gamma} \nabla_\beta (\zeta H_{\alpha\gamma}^{\mathscr{L}2})$
$\mathscr{L}_\xi \Phi_0^{\mathscr{L}1} = \phi_0$	$\frac{1}{2}(\mathscr{R} + \mathscr{S}) \Phi_0^{\mathscr{L}2} = \phi_0$
$\mathscr{L}_\xi \Phi_2^{\mathscr{L}1} = -\phi_2$	$\frac{1}{2}(\mathscr{R} + \mathscr{S}) \Phi_2^{\mathscr{L}2} = -\phi_2$

[Dolan]

Lorenz gauge modes

$$A_l^{\mathscr{L}1} = \frac{1}{\sqrt{2\Delta\Sigma}}\left[ia\sin\theta\,\partial_t - \partial_\theta + i\csc\theta\,\partial_\varphi - \cot\theta\right]\left(\zeta\sqrt{\Delta}\Phi_0^{\mathscr{L}1}\right)$$

$$A_n^{\mathscr{L}1} = \frac{1}{\sqrt{2\Delta\Sigma}}\left[ia\sin\theta\,\partial_t + \partial_\theta + i\csc\theta\,\partial_\varphi + \cot\theta\right]\left(\zeta\sqrt{\Delta}\Phi_2^{\mathscr{L}1}\right)$$

$$A_m^{\mathscr{L}1} = \frac{-1}{\Delta\sqrt{2\Sigma}}\left[(a^2 + r^2)\,\partial_t - \Delta\partial_r + a\,\partial_\varphi\right]\left(\zeta\sqrt{\Delta}\Phi_0^{\mathscr{L}1}\right)$$

$$A_{\bar{m}}^{\mathscr{L}1} = \frac{1}{\Delta\sqrt{2\Sigma}}\left[(a^2 + r^2)\,\partial_t + \Delta\partial_r + a\,\partial_\varphi\right]\left(\zeta\sqrt{\Delta}\Phi_2^{\mathscr{L}1}\right)$$

$${}_1\Phi_{\ell m\omega}^{\mathscr{L}1} = -\frac{{}_1\phi_{\ell m\omega}}{i\omega} \qquad {}_{-1}\Phi_{\ell m\omega}^{\mathscr{L}1} = \frac{{}_{-1}\phi_{\ell m\omega}}{i\omega}$$

$$\Delta = r^2 - 2Mr + a^2 \qquad \Sigma = r^2 + a^2\cos^2\theta \qquad \zeta = r - ia\cos\theta$$

Gravitational Case

- Initial investigations suggest that there is **no second order operator** that would act on the Weyl scalars to get a Lorenz gauge metric potential.
- We are already doing something very similar in the Schwarzschild gravitational case using Berndtson's metric reconstruction from Regge-Wheeler-Zerilli master functions [Durkan].
- Many of the identities (including a Teukolsky equation and Teukolsky-Starobinsky identities) have analogues in the gravitational case. One possible complication is that the Teukolsky-Starobinsky identities for the perturbed Weyl scalars mix not only the scalars ψ_0 and ψ_4, but also their complex conjugates.

Conclusions

- Systematic approach to identifying Hertz potentials for the vector wave equation.
- Identified a set of solutions for the vector wave equation in the form of separable operators acting on solutions of the Teukolsky equation.
- No mode ansatz, works for arbitrary frequency (including zero).
- Restricted to homogeneous case (see talk by Stephen Green).
- Extension to gravity not yet clear, but reason for optimism.