Title: The second-order Teukolsky source in Schwarzschild.

Speakers: Andrew Spiers

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

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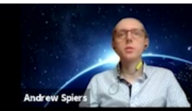
Abstract: Precise parameter extraction from EMRI signals requires, among other things, the dissipative piece of the second-order self-force in a Kerr background. We have shown how a new form of the second-order Teukolsky equation has a well-defined source in a highly regular gauge, and how to construct gauge invariant second-order quantities using a gauge fixing method. For the current prospective second-order self-force methods in Kerr solving the second-order Teukolsky equation will be a crucial step. In this talk, I show our progress in calculating the source in the second-order Teukolsky equation for quasi-circular orbits in Schwarzschild, and discuss how the source can be made more regular at future null infinity by transforming to the Bondi-Sachs gauge.



The second-order Teukolsky source in Schwarzschild

Andrew Spiers

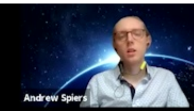
Supervisor: Adam Pound
Collaborators: Moxon, Wardell, Durkan, Leather, Upton and Warburton

9th June 2021

THE ROYAL SOCIETY

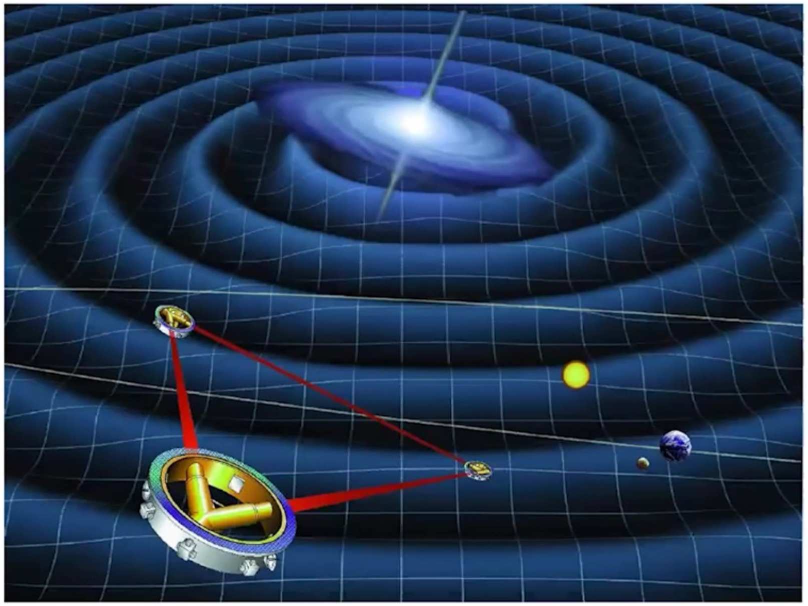
UNIVERSITY OF Southampton

Overview



1. **Review:** Second-order self-force in Kerr and the *reduced* second-order Teukolsky equation
2. **Outline:** The **group** effort working on second-order calculations
3. **Progress:** Calculating the second-order source in Schwarzschild
4. **Improvement:** Regularising the source by transforming to the Bondi–Sachs gauge

Motivation

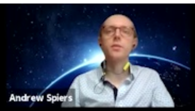


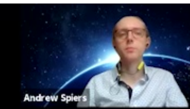
[Source: NASA,

http://lisa.jpl.nasa.gov/gallery/lisa-waves.html.]

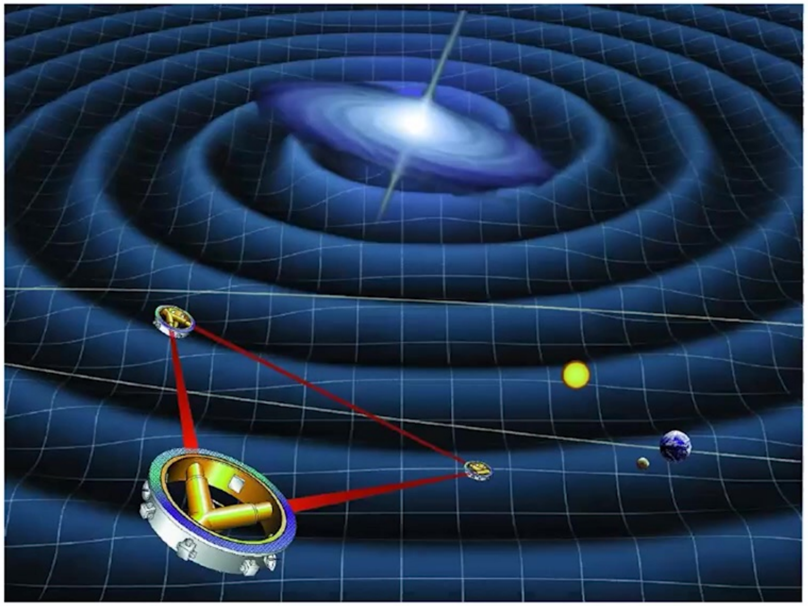
Astrophysical supermassive black holes spin

Goal: calculating the dissipative piece of the second-order self-force in **Kerr**.





Motivation



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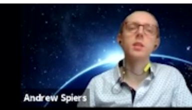
Astrophysical supermassive black holes spin

Goal: calculating the dissipative piece of the second-order self-force in **Kerr**.

But, the linearised Einstein field equations are **non-separable** in Kerr.

The reduced second-order Teukolsky equation



First order:

$$\mathcal{O}\psi_4^{(1)} = \mathcal{S}[T_{ab}^{(1)}]$$

Second order:

$$\mathcal{O}\psi_4^{(2)} = \mathcal{S}[T_{ab}^{(2)}] + S_{CL}[T_{ab}^{(1)}, h_{ab}^{(1)}].$$

or,

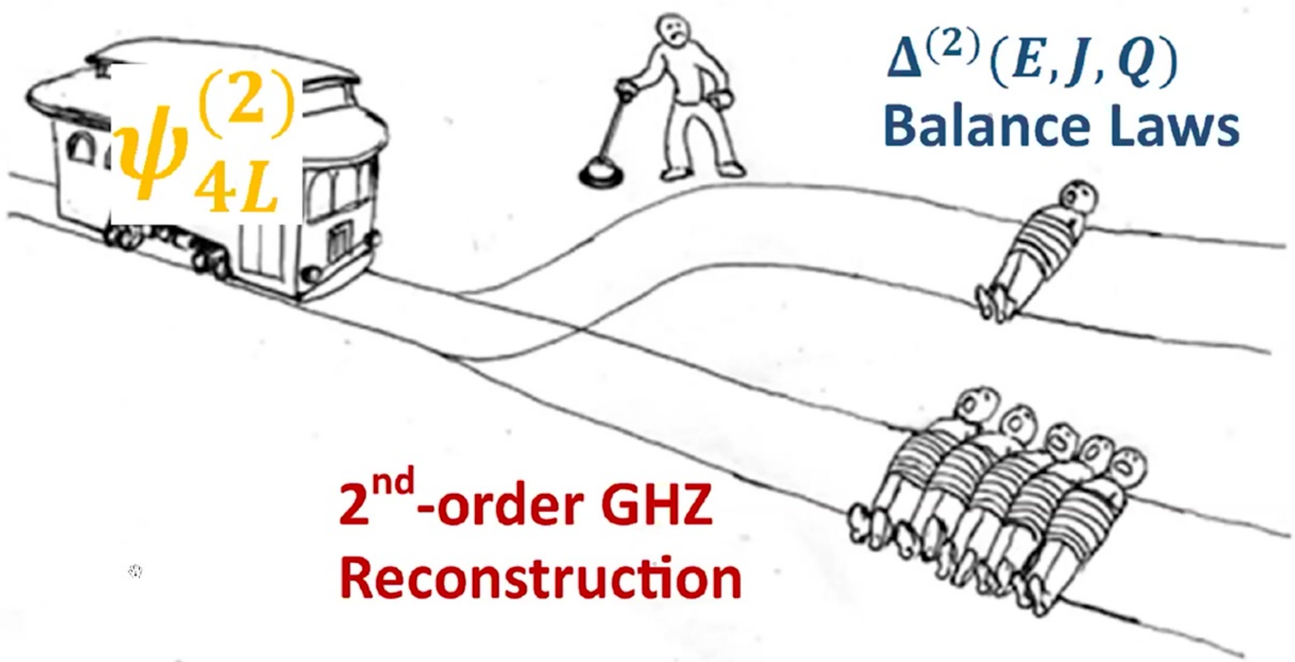
$$\mathcal{O}\psi_{4L}^{(2)} = \mathcal{S}\Big[T_{ab}^{(2)} - \delta^2 G[h_{ab}^{(1)}]\Big]$$

where $\psi_4^{(2)} = \psi_{4L}^{(2)}[h_{ab}^{(2)}] + \psi_{4Q}^{(2)}[h_{ab}^{(1)}]$

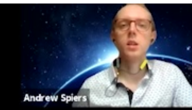
[Campanelli & Lousto. *Phys. Rev. D*, 59(12):124022, 1999.] [Wald. *PRL*, 41(4):243, 1978.]

[Green, Hollands & Zimmerman. *CQG*, 37(7):075001, 2020.]

The Track to the Dissipative Piece of the Second-Order Self-Force



$\mathcal{S}\left[\delta^2 G\left[h_{ab}^{(1)}\right]\right]$ in Schwarzschild

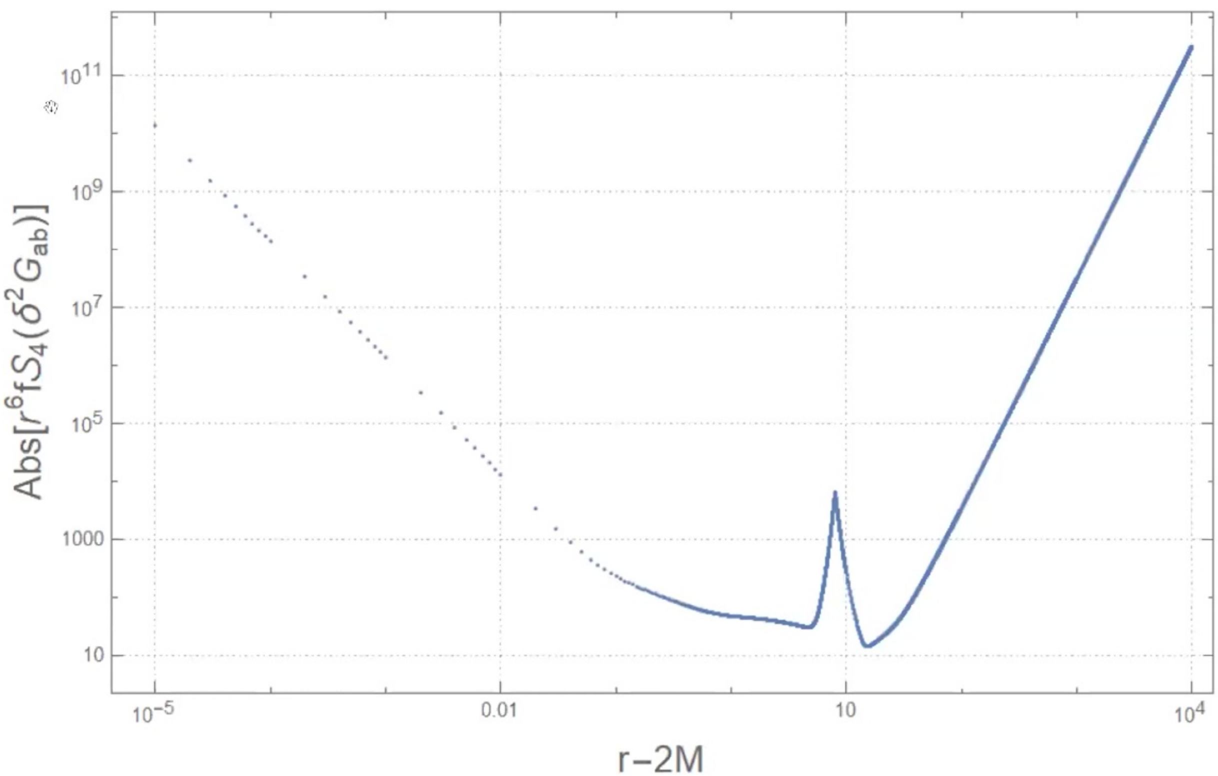


$$\mathcal{S}[\delta^2 G_{ab}] = \eth' \left((\text{Þ}' + 2\rho)\,\delta^2 G_{n\bar{m}} - \eth' \delta^2 G_{nn} \right) + (\text{Þ}' - 3\rho)\left(\eth' \delta^2 G_{n\bar{m}} - (\text{Þ}' + \rho)\,\delta^2 G_{\bar{m}\bar{m}} \right).$$

$$\delta^2 G_{nn} = \frac{n^a n^b}{2}\left[h^{cd}\left(h_{ab;cd} + h_{cd;ab} - 2h_{c(a;b)d} \right) + h_a{}^{c;d} h_{b[c;d]} + \frac{1}{2} h^{cd}{}_{;a} h_{cd;b} + \left(\frac{1}{2} h^d{}_{d;c} - h_c{}^d{}_{;d} \right)\left(2h^c{}_{(a;b)} - h_{ab}{}^{;c} \right) \right],$$

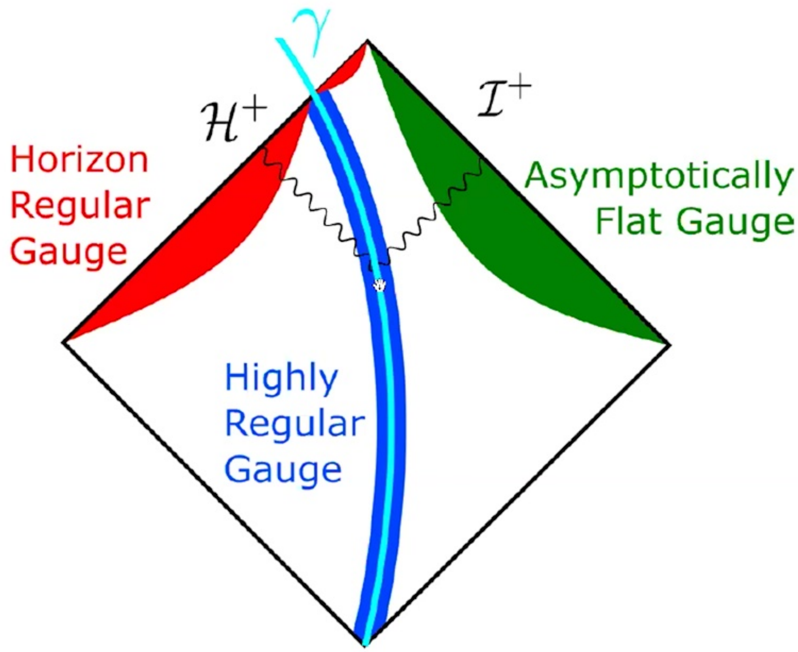
$\Rightarrow \mathcal{S}[\delta^2 G[h_{ab}^{(1)}]]$ has ~ 1600 terms

Preliminary results: $l = 2$, $m = 2$ source mode



Gauge Fixing Region by Region

- For each physically significant region we want a **fully fixed** *good* gauge:



Transforming to the Bondi–Sachs gauge

$\psi_{4L}^{(2)}$ is **not** gauge invariant, but it only depends on first-order gauge transforms:

Can solve $h'^{(1)}_{ab} = h^{(1)}_{ab} + 2\nabla_{(a}\xi^{(1)}_{b)}$ for $\xi^{(1)}_b$ such that $h'^{(1)}_{ab}$ satisfies the Bondi–Sachs gauge conditions and constrains the BMS freedoms.

$\Rightarrow$ Gauge transform is much simpler than previously: e.g., for $l = 0$

$$\xi^l = -\sqrt{2}M^{(1)}\ln r, \quad \xi^n = -\sqrt{2}M^{(1)}\ln r + 2\sqrt{2}M^{(1)},$$

Summary

- **I am calculating the reduced second-order Teukolsky source** in Schwarzschild
- Transforming to the Bondi–Sachs gauge will improve the behaviour of the source near $\mathcal{I}^+$ by two orders in r
- After I correct my source Ben Leather will use it to calculate $\psi_{4L}^{(2)}$
- Extending to Kerr requires a non-singular first-order metric perturbation; **see talks by Toomani, Zimmerman and Green** later today!

Thank you for listening

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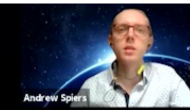
Transforming to the Bondi–Sachs gauge



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Second-Order Teukolsky Source 9th June 2021 10 / 11