Title: Gravitational wave flux for compact binaries through second-order in the mass-ratio

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Abstract: Within the framework of self force theory we compute the gravitational wave flux through second-order in the mass ratio for quasi-circular compact binaries. Our results are consistent with post-Newtonian calculations in the weak field and we find they agree remarkably well with numerical relativity simulations of comparable mass binaries in the strong field. We also find good agreement for binaries with a spinning secondary or a slowly spinning primary.

Gravitational wave flux for compact binaries through second-order in the mass-ratio

$$\mathbf{g}_{\alpha\beta} = g_{\alpha\beta} + \epsilon h^1_{\alpha\beta} + \epsilon^2 h^2_{\alpha\beta}$$

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Capra 24 @ Perimeter Institute (virtually)
9th June 2021

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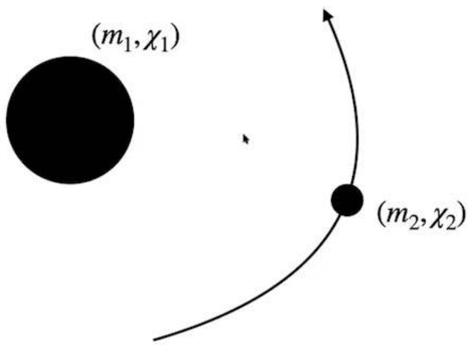
Description of the binary and method



$$m_1 > m_2$$
$$M = m_1 + m_2$$
$$\epsilon = m_2/m_1$$
$$q = m_1/m_2 = 1/\epsilon$$
$$\nu = m_1 m_2/M^2$$
$$\chi_i = S_i/m_i^2$$

Quasi-circular, spin-aligned binary with orbital frequency Ω and $S_i/m_i \lesssim m_2$

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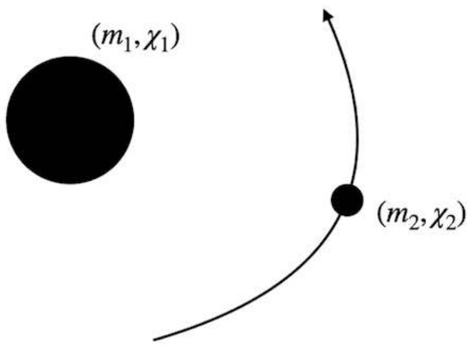

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Two-timescale expansion:

$$\mathsf{g}_{\alpha\beta} = g_{\alpha\beta} + \sum_m \left[\epsilon h^{1,m}_{\alpha\beta}(\Omega) + \epsilon^2 h^{2,m}_{\alpha\beta}(\Omega)\right] e^{-im\phi_p} + \mathcal{O}(\epsilon^3)$$

Tensor spherical harmonic decomposition:

$$\bar{h}^n_{\alpha\beta} = \sum_{ilm} \frac{a_{il}}{r} R^n_{ilm} e^{-im\phi_p} Y^{ilm}_{\alpha\beta}$$

Field equations in Lorenz gauge:

$$E^0_{ijlm} R^{1R}_{jlm} = -\, E^0_{ijlm} R^{1P}_{jlm}$$
$$E^0_{ijlm} R^{2R}_{jlm} = 2\delta^2 G^0_{ilm} - E^0_{ijlm} R^{2P}_{jlm} - E^1_{ijlm} R^1_{jlm}$$

See Miller and Pound, arXiv:2006.11263

Gravitational wave flux for quasi-circular binaries

Decompose into spherical harmonics: $h(t) = h_+(t) + ih_\times(t) = r^{-1} \sum_{lm} h_{lm}(t)\,{}_{-2}Y_{lm}(\theta, \phi)$

Split into amplitude and phase: $h_{lm}(t) = A_{lm}(t) e^{i\Phi(t)}$ $\quad A, \Phi \in \mathbb{R}$

Define the flux and frequency: $\mathscr{F}_{lm}(t) = \frac{1}{16\pi} |\dot{A}_{lm}(t)|^2, \quad \varpi(t) = \dot{\Phi}_{22}(t)/2$

Define the separation: $\bar{x} = (M\varpi)^{2/3}$

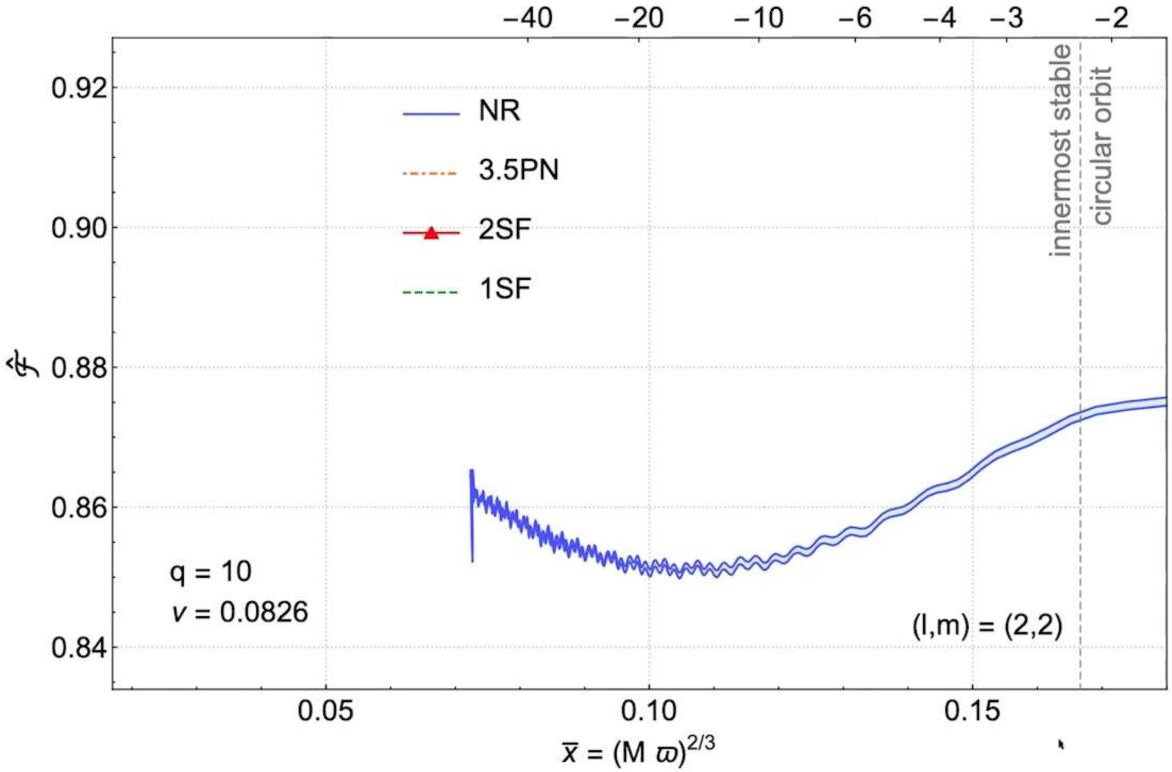
In the weak field we have $\varpi \simeq \Omega$ and $\bar{x} \simeq x = (M\Omega)^{2/3}$

Define Newtonian-normalised flux: $\hat{\mathscr{F}}_{lm}(\bar{x}) = \mathscr{F}_{lm}(\bar{x}) / \mathscr{F}^N_{lm}(\bar{x})$

where, e.g., $\mathscr{F}^N_{22}(x) = \frac{32}{5} x^5 \nu^2$ $\quad \lim_{\bar{x}\to 0} \hat{\mathscr{F}}(\bar{x}) = 1$

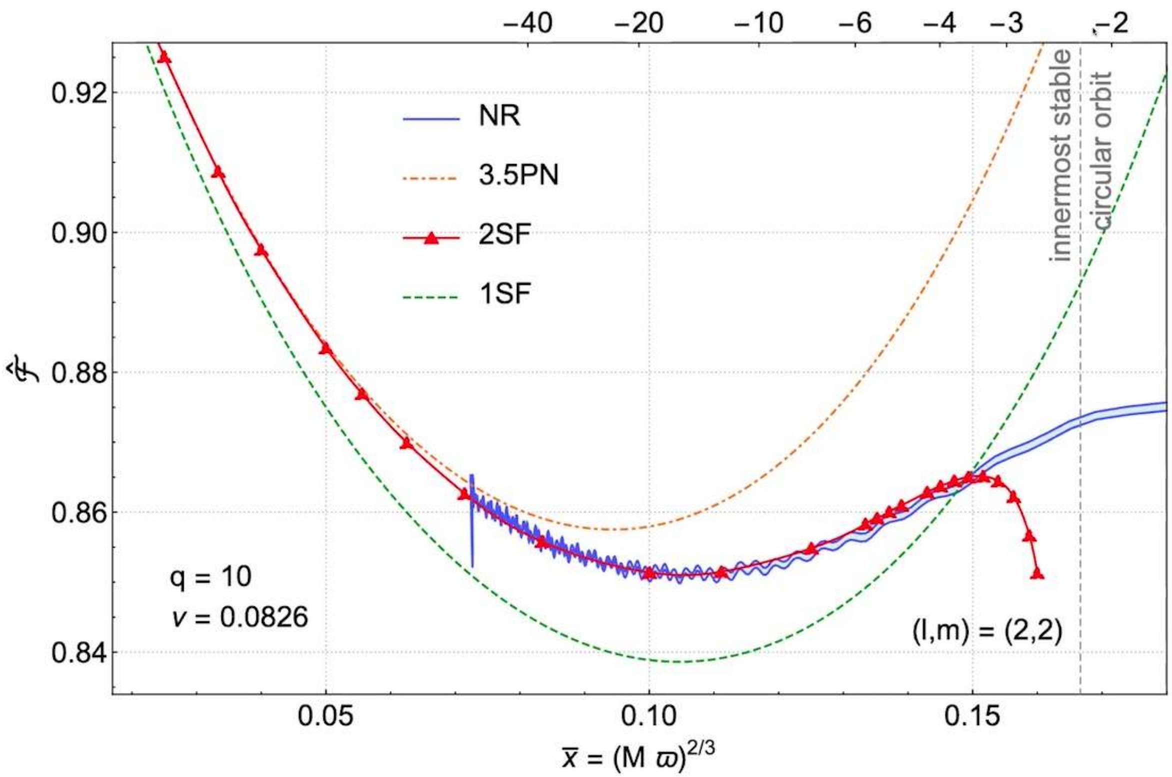
Write SF flux as: $\mathscr{F}^{SF}(\bar{x})_{lm} = \nu^2 \mathscr{F}^{1,SF}_{lm}(\bar{x}) + \nu^3 \mathscr{F}^{2,SF}_{lm}(\bar{x}) + \mathcal{O}(\nu^4)$

Flux for a non-spinning binary with $q = 10$



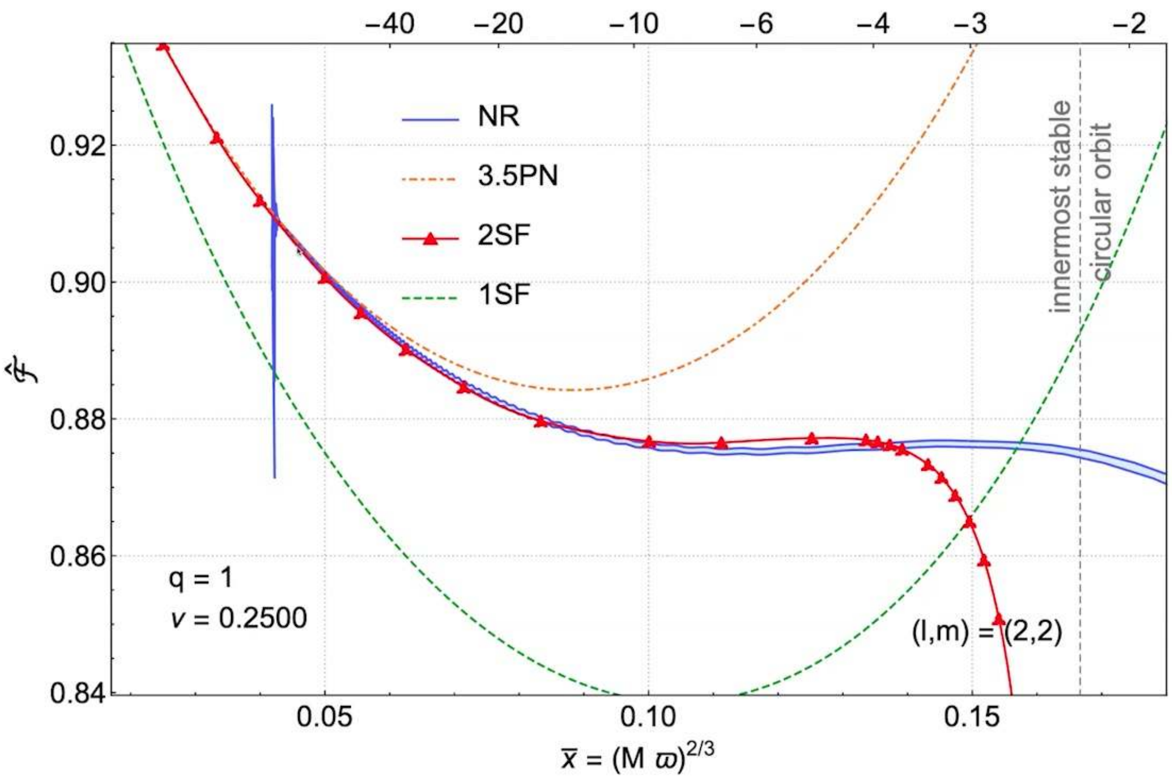
NR waveform: SXS:BBH:1107

Flux for a non-spinning binary with $q = 10$



NR waveform: SXS:BBH:1107

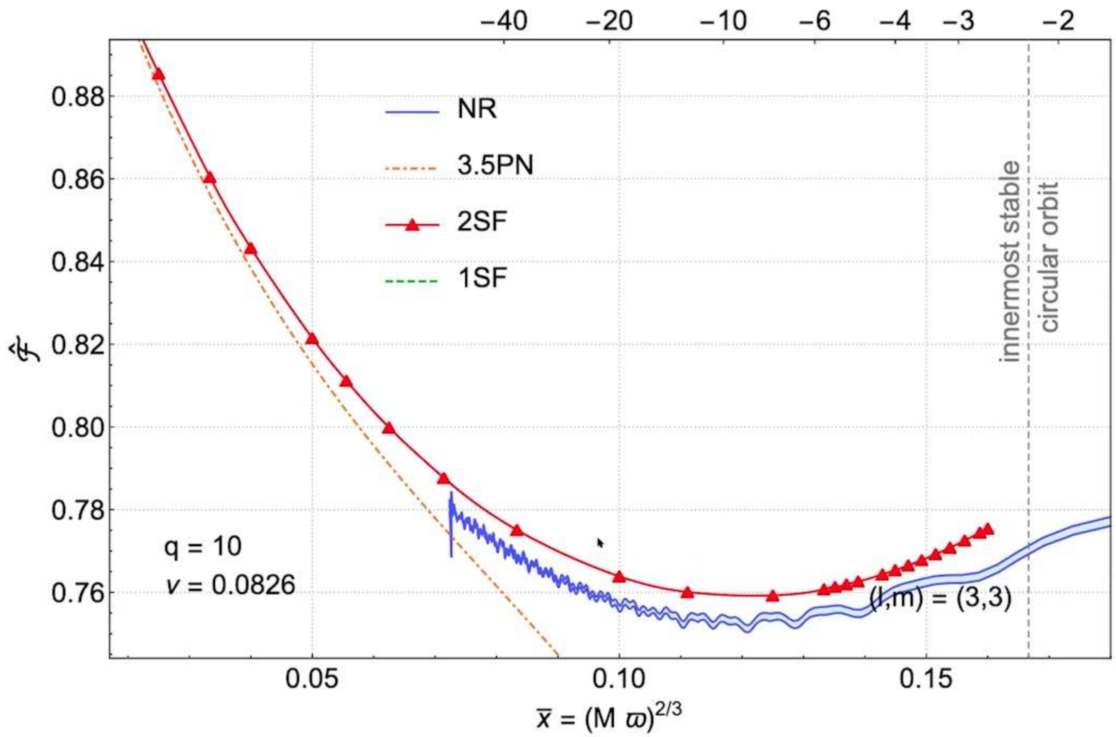
Flux for a non-spinning binary with $q = 1$



NR waveform: SXS:BBH:1132

Flux for a non-spinning binary with $q = 10$

Higher modes: $(l, m) = (3,3)$



NR waveform: SXS:BBH:1107

Flux for a non-spinning binary with $q = 10$

Higher modes: $(l, m) = (3,3)$

Why does the second-order flux not compare well against NR for the (3,3)-mode?

$$\mathcal{F}_{33}^{PN} = \frac{243}{28}\nu^2(1-4\nu) - \frac{243}{7}x^7(2\nu^2 - 9\nu^3 + 4\nu^4) + \mathcal{O}(x^{15/2})$$

Compare this with the PN series for the (2,2)-mode:

$$\mathcal{F}_{22}^{PN} = \frac{32\nu^2x^5}{5} + \frac{32}{105}\nu^2(55\nu - 107)x^6 + \frac{128}{5}\pi\nu^2x^{13/2} + \frac{8\left(19136\nu^2 - 87691\nu^3 + 23404\nu^4\right)x^7}{6615} + \mathcal{O}\left(x^{15/2}\right)$$

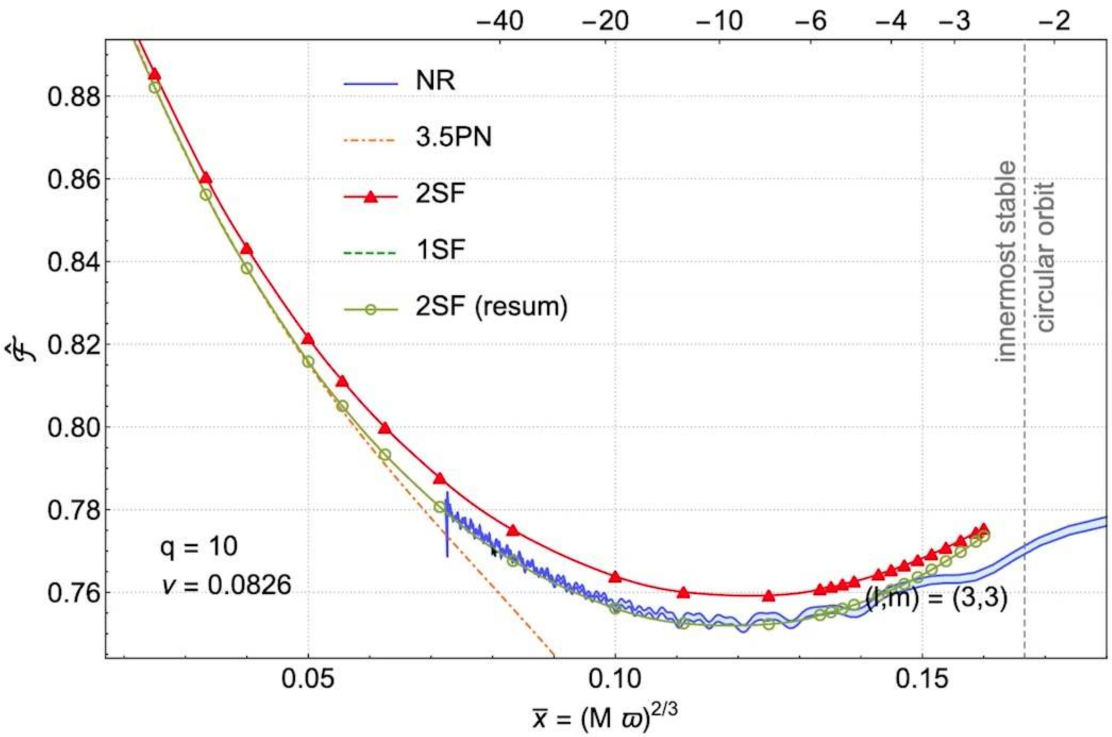
We can try a simple resummation to include some $\nu^{n\geq 4}$ information from the PN series

$$\mathcal{F}_{lm}^{SF,resum} = \left[\frac{\nu^2\mathcal{F}_{lm}^{1SF} + \nu^3\mathcal{F}_{lm}^{2SF}}{\mathcal{F}_{lm}^{PN,leading}} + O(\nu^2)\right]\mathcal{F}_{lm}^{lm,leading}$$

This ensures that $\hat{\mathcal{F}}_{lm}^{SF,resum} = 1 + \dots$

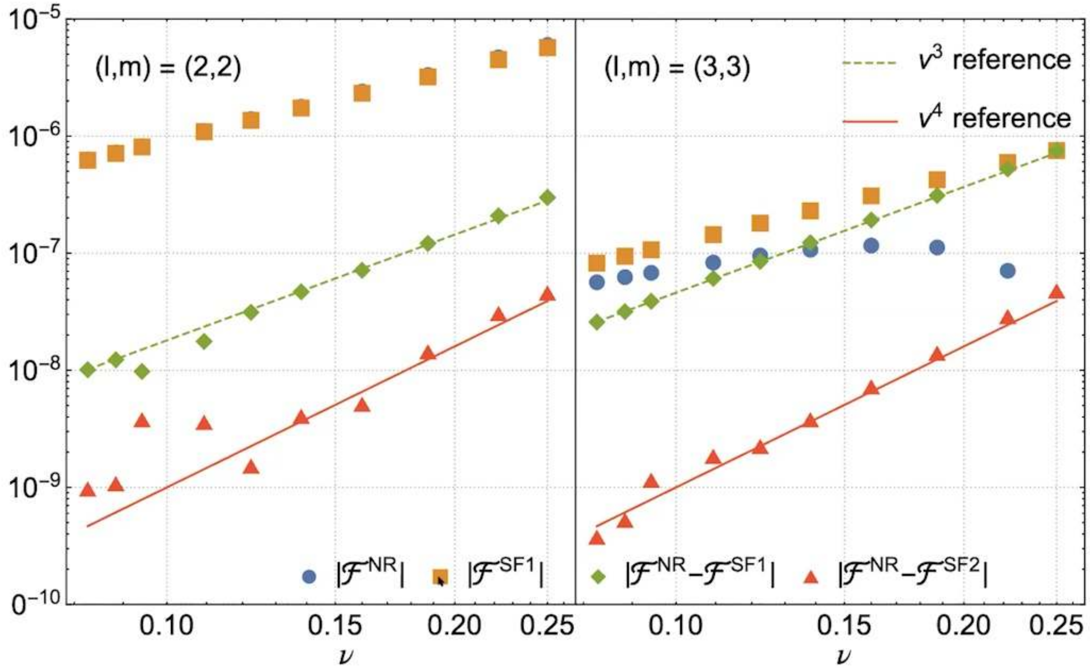
Flux for a non-spinning binary with $q = 10$

Higher modes: $(l, m) = (3,3)$

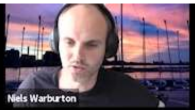


NR waveform: SXS:BBH:1107

Scaling with the mass ratio



Comparison made at $\bar{x} = 1/9$. Can estimate third-order fluxes.



Fluxes for spinning binaries

During the inspiral the primary's spin will evolve by $\mathcal{O}(\epsilon)$

We can consistently the flux due to the spin so long as $S_i/m_i \lesssim m_2$

To facilitate comparison between SF and NR fluxes we define

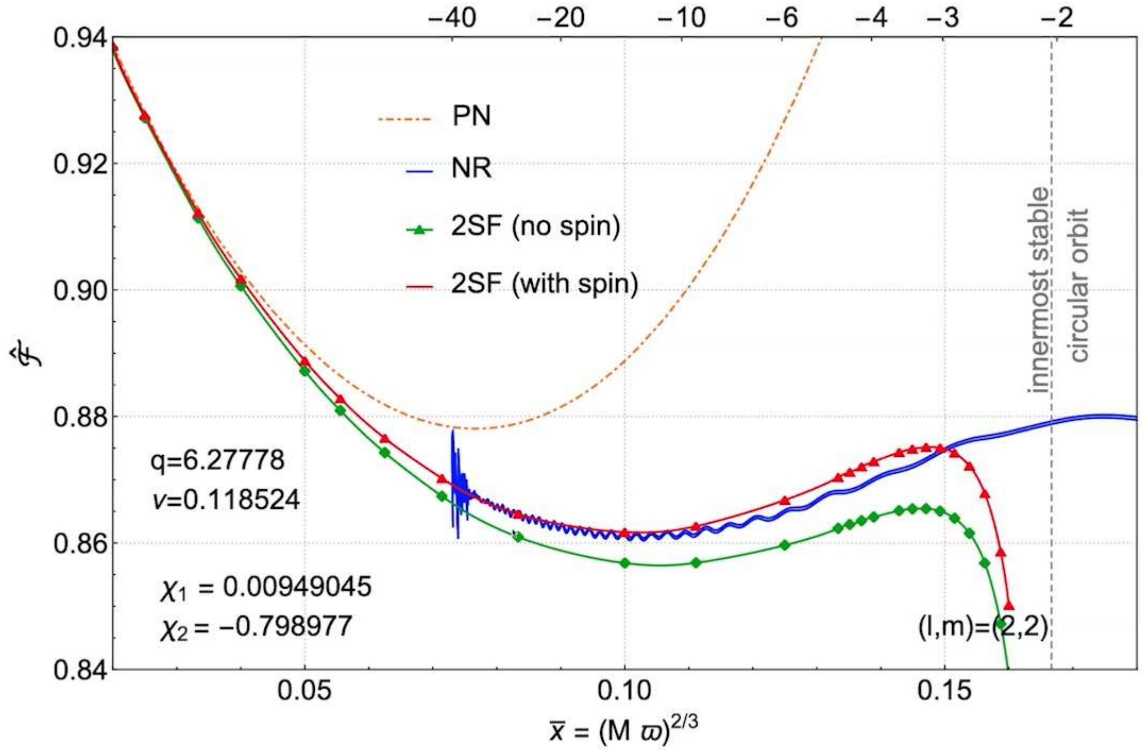
$$X_1 = (1 + \sqrt{1 - 4\nu})/2$$
$$X_2 = 1 - X_1$$
$$\tilde{a}_i = X_i \chi_i$$

$$\mathcal{F}^{SF,spin}_{lm}(x) = \mathcal{F}^{SF}_{lm}(x) + \sum_{i=1}^{2} \tilde{a}_i \nu^3 \mathcal{F}^{spin,i}_{lm}(x)$$

- We can compute $\mathcal{F}^{1,spin}_{lm}$ from our second-order calculation. It is also equal to the linear-in-spin Teukolsky flux
- The flux from a spinning secondary, $\mathcal{F}^{2,spin}_{lm}$, is computed in Akcay+ (arXiv:1912.09461 **)

** see also: arXiv:2004.02654, 2101.04533

Fluxes with a spinning secondary



NR waveform: SXS:BBH:1436

Conclusions and future work

- Waveforms
- Attach transition to plunge and ringdown
- Detailed comparisons with NR, PN and EOB
- Modelling IMRIs

- Extension to eccentric orbits
- Extension to Kerr: radiation gauge and Lorenz gauge