Title: Progress toward post-adiabatic waveforms

Speakers: Adam Pound

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Abstract: LISA science will require EMRI waveforms that are accurate to first-post-adiabatic order, which in turn requires the calculation of second-order self-force effects. In this talk I describe a post-adiabatic waveform-generation framework and progress toward its implementation. This lays the groundwork for talks by Durkan, Warburton, Spiers, Leathers, Upton, and others.

Progress toward post-adiabatic waveforms

Adam Pound
with Durkan, Flanagan, Green, Hinderer, Hollands, Leather, Miller, Moxon, Spiers, Toomani, Upton, van de Meent, Warburton, Wardell, and Zimmerman

24th Capra Meeting, Perimeter Institute

9 June 2021

What's what? [Hinderer and Flanagan 2008]

- asymptotic expansion in $\epsilon \sim m/M$

$$\mathrm{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^{1}_{\mu\nu} + \epsilon^2 h^{2}_{\mu\nu} + \dots$$

- on inspiral timescale $t \sim 1/\epsilon$, the gravitational wave phase has an expansion

$$\varphi = \frac{1}{\epsilon}\varphi_0 + \varphi_1 + O(\epsilon)$$

What's what? [Hinderer and Flanagan 2008]

Adiabatic order

determined by

- averaged dissipative piece of F_1^μ

Post-adiabatic order

determined by

- averaged dissipative piece of F_2^μ
- conservative piece of F_1^μ
- oscillatory dissipative piece of F_1^μ

$$\mathrm{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \ldots$$

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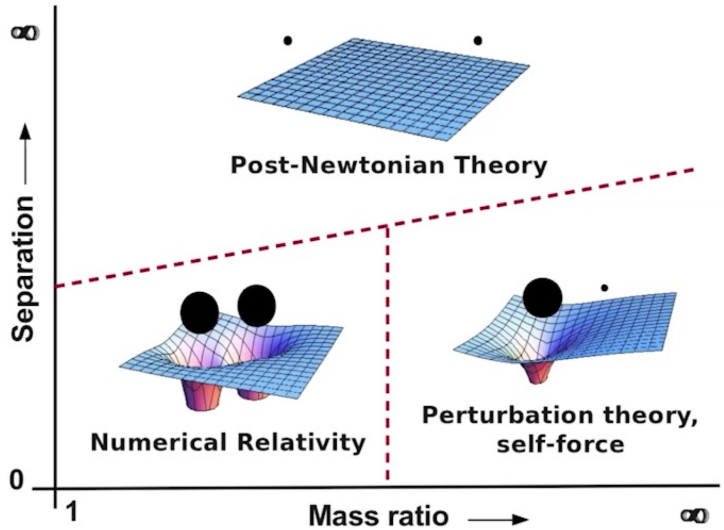
$$\varphi = \frac{1}{\epsilon}\boxed{\varphi_0} + \boxed{\varphi_1} + O(\epsilon)$$

Modelling IMRIs and similar-mass binaries

- 2SF results would fully fix 5PN dynamics and 6PM dynamics [Bini, Damour, Geralico 2019]
- also can use SF to *directly* model IMRIs (at least in some regions of parameter space)

Binary parameter space



[Image credit: Leor Barack]

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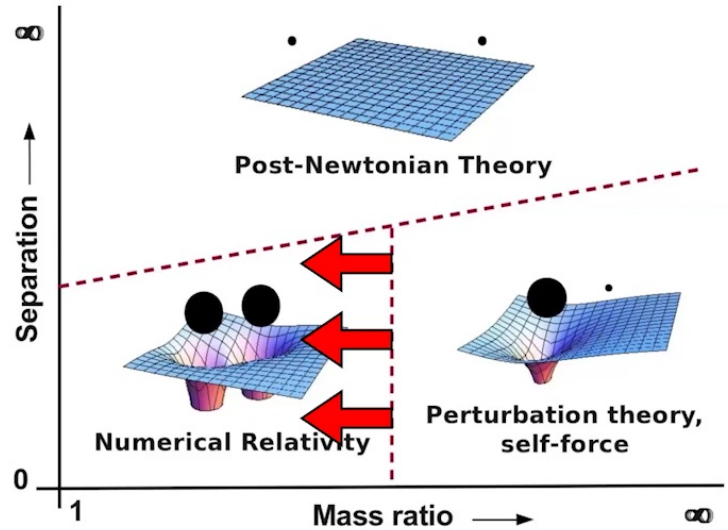

[Image credit: Leor Barack]

Second-order self-force theory [AP, 2012–] [AP and Miller, 2014]

- skeletonization: small object $\rightarrow$ moving puncture:

$$\delta G_{\mu\nu}[h^{\mathcal{R}1}] = -\delta G_{\mu\nu}[h^{\mathcal{P}1}]$$
$$\delta G_{\mu\nu}[h^{\mathcal{R}2}] = \delta^2 G_{\mu\nu}[h^1, h^1] - \delta G_{\mu\nu}[h^{\mathcal{P}2}]$$
$$\frac{D^2 z^\mu}{d\tau^2} = -\frac{1}{2}(g^{\mu\nu} + u^\mu u^\nu)(g_\nu{}^\delta - h_\nu^{\mathcal{R}\delta})(2h^{\mathcal{R}}_{\delta\beta;\gamma} - h^{\mathcal{R}}_{\beta\gamma;\delta})u^\beta u^\gamma$$

- puncture diverges at worldline z^μ:

$$h^{\mathcal{P}1}_{\mu\nu} \sim \frac{m}{|x-z|} + \ldots$$
$$h^{\mathcal{P}2}_{\mu\nu} \sim \frac{m^2}{|x-z|^2} + \frac{m h^{\mathrm{R}1}}{|x-z|} + \ldots$$

- solve for residual fields $h^{\mathcal{R}n}_{\mu\nu} = h^n_{\mu\nu} - h^{\mathcal{P}n}_{\mu\nu}$

Typical approach at first order

- geodesic orbit. System parameters $J_A = \{\mathcal{E}, \mathcal{L}, \mathcal{C}\} \qquad (+\{M, S\})$
- discrete Fourier series

$$h^1_{\mu\nu} = \sum_{k^A} h^{1,\omega_k}_{\mu\nu}(J_A, r, \theta, \phi) e^{-i(k^r\Omega_r + k^\theta\Omega_\theta + k^\phi\Omega_\phi)t}$$

- evolution

$$\frac{d\varphi_A}{dt} = \Omega_A(J_B) \qquad \text{and} \qquad \frac{dJ_A}{dt} = 0$$

$\varphi_A = \Omega_A t$ are action angles

Two-timescale expansion

[AP and Wardell, 2021] [Flanagan, Hinderer, Moxon, AP (in prep)]

- system parameters $J_A = \{\mathcal{E}, \mathcal{L}, \mathcal{C}, \delta M, \delta S\} \qquad (+\{M, S\})$
- two-timescale expansion

$$h^n_{\mu\nu} = \sum_{k^A} h^{n,\omega_k}_{\mu\nu}(J_A, r, \theta, \phi) e^{-i(k^r \varphi_r + k^\theta \varphi_\theta + k^\phi \varphi_\phi)}$$

- evolution

$$\frac{d\varphi_A}{dt} = \Omega_A(J_B) \quad \text{and} \quad \frac{dJ_A}{dt} = \epsilon f^{(1)}_A(J_B) + \epsilon^2 f^{(2)}_A(J_B) + O(\epsilon^3)$$

Field equations

- idea: $\partial_t h^n \to (\omega_k + \epsilon J^B \partial_B) h^{n,\omega_k}_{\mu\nu}(J_A, r, \theta, \phi)$
- phases factor out of equations

$$\delta G[h^1] = T^1$$

$$\delta G[h^{\mathcal{R}2}] = \delta^2 G[h^1, h^1] - \delta G[h^{\mathcal{P}2}]$$

- we need NITs *before* solving 2SF field equations

Field equations

- idea: $\partial_t h^n \to (\omega_k + \epsilon \dot{J}^B \partial_B) h^{n,\omega_k}_{\mu\nu}(J_A, r, \theta, \phi)$
- phases factor out of equations

$$\delta G_{\omega_k}[h^1_{\omega_k}] = T^1_{\omega_k}$$

$$\delta G_{\omega_k}[h^{\mathcal{R}2}_{\omega_k}] = \delta^2 G_{\omega_k}[h^1_{\omega_{k'}}, h^1_{\omega_{k''}}] - \delta G_{\omega_k}[h^{\mathcal{P}2}_{\omega_k}] + \dot{J}^B_{(0)} \partial_B h^1_{\omega_k}$$

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Wave generation

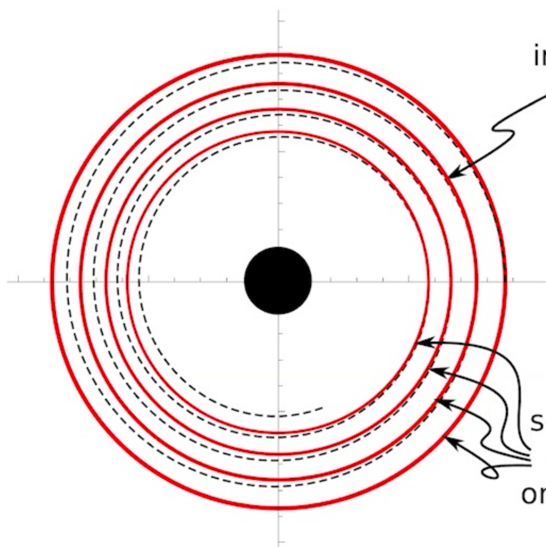
- compute amplitudes $h^{1,\omega_k}(J_A)$, frequencies $\omega_A(J_B)$, and driving forces $f_A(J_B)$ across J_A space
- generate waveform $\sum_{k^A} h^{1,\omega_k}(J_A) e^{-ik^A\varphi_A}$ by solving

$$\frac{d\varphi_A}{d\tilde{t}} = \frac{1}{\epsilon}\Omega_A(J_B)$$
$$\frac{dJ_A}{d\tilde{t}} = f_A^{(0)}(J_B) + \epsilon f_A^{(1)}(J_B) + O(\epsilon^2)$$

Example: quasicircular orbits [AP, Wardell, Warburton, Miller, 2019]

- parameters: $J_A = \{\Omega, \delta M, \delta S\}$
- field: $h \sim \sum_{nlm} \epsilon^n h^n_{\omega_m lm}(J_A, r) e^{-im\phi_p} Y_{lm}$
- phase: $\phi_p = \int \Omega\, dt = \frac{1}{\epsilon} \int \Omega d\tilde{t}$
- frequencies: $\omega_m = m\Omega$

- the simple test case
- that will never die



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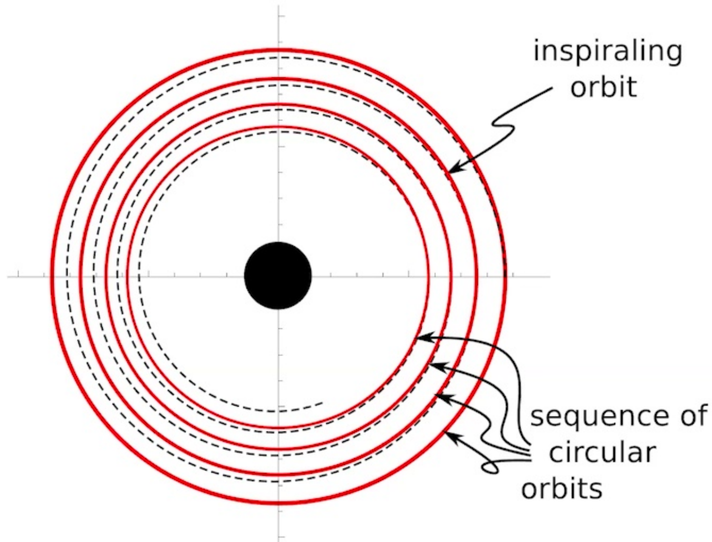
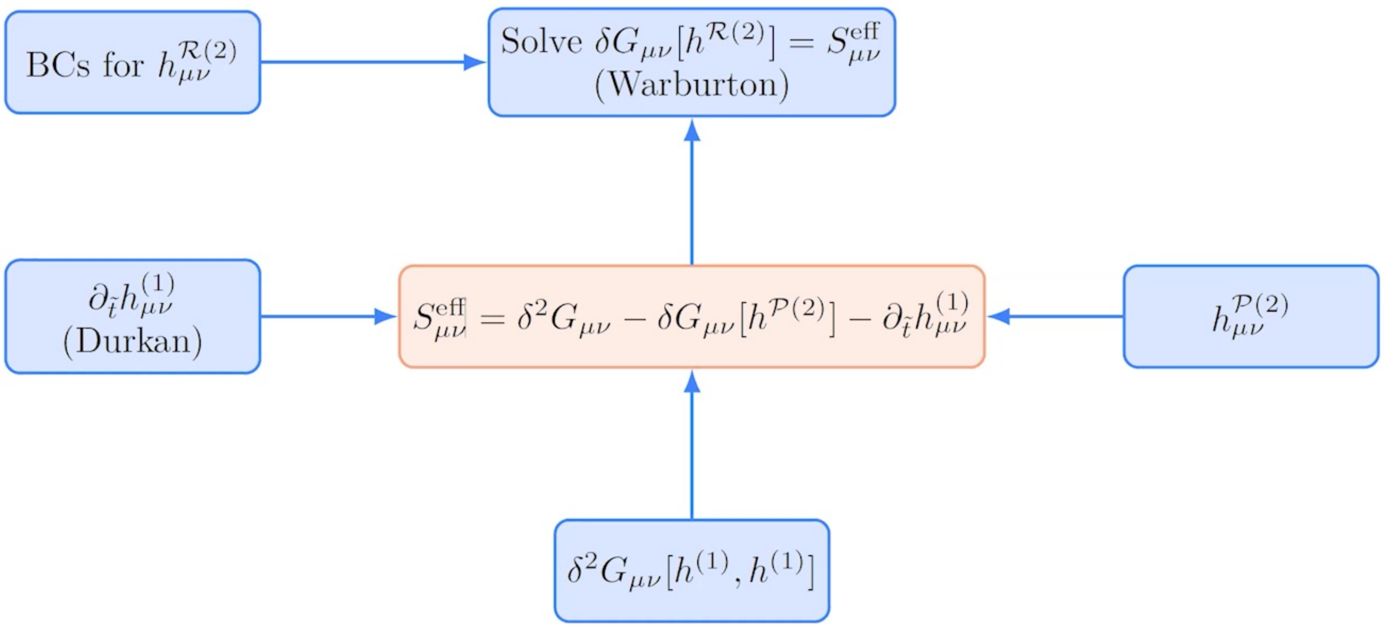


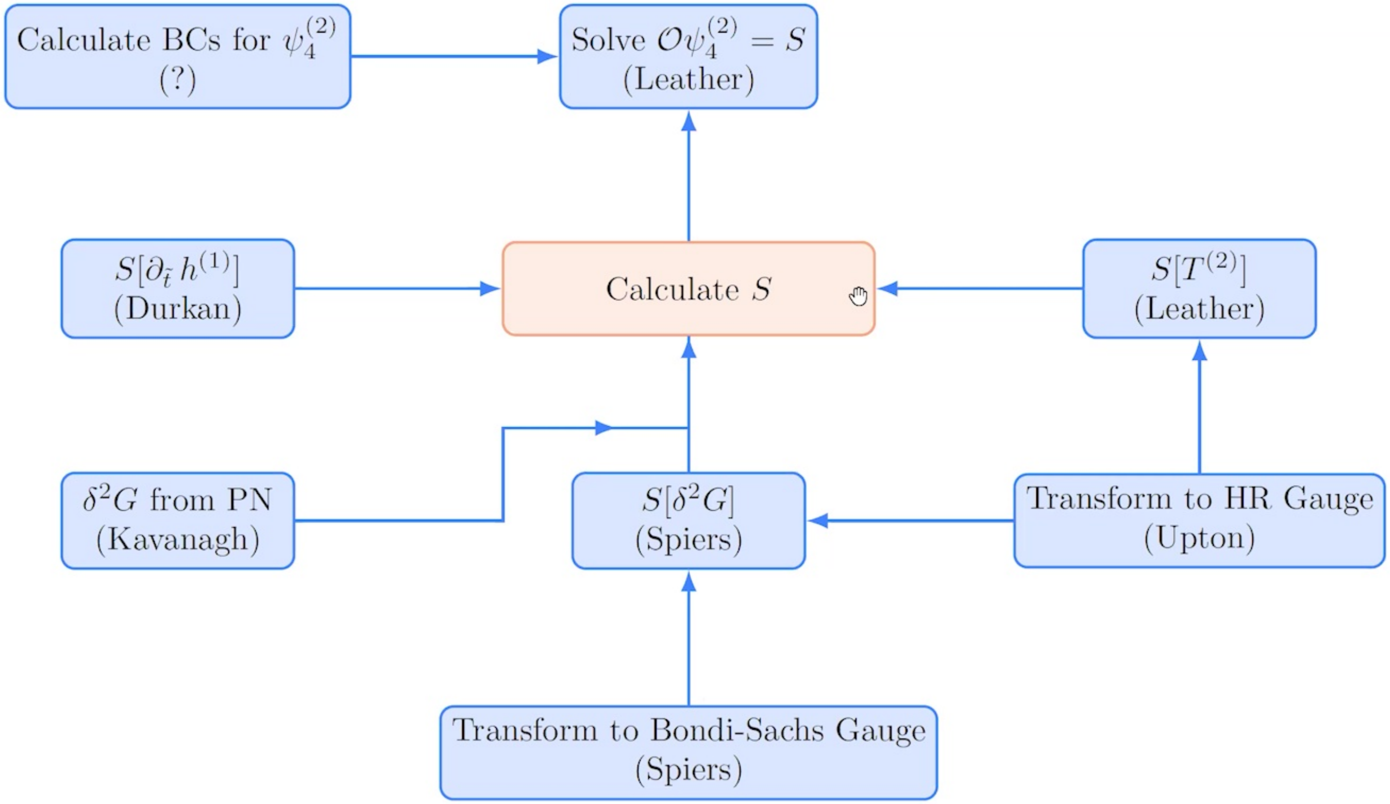
Road to Kerr (phase 0)

Quasicircular orbits in Schwarzschild, Lorenz gauge



Road to Kerr (phase 1)

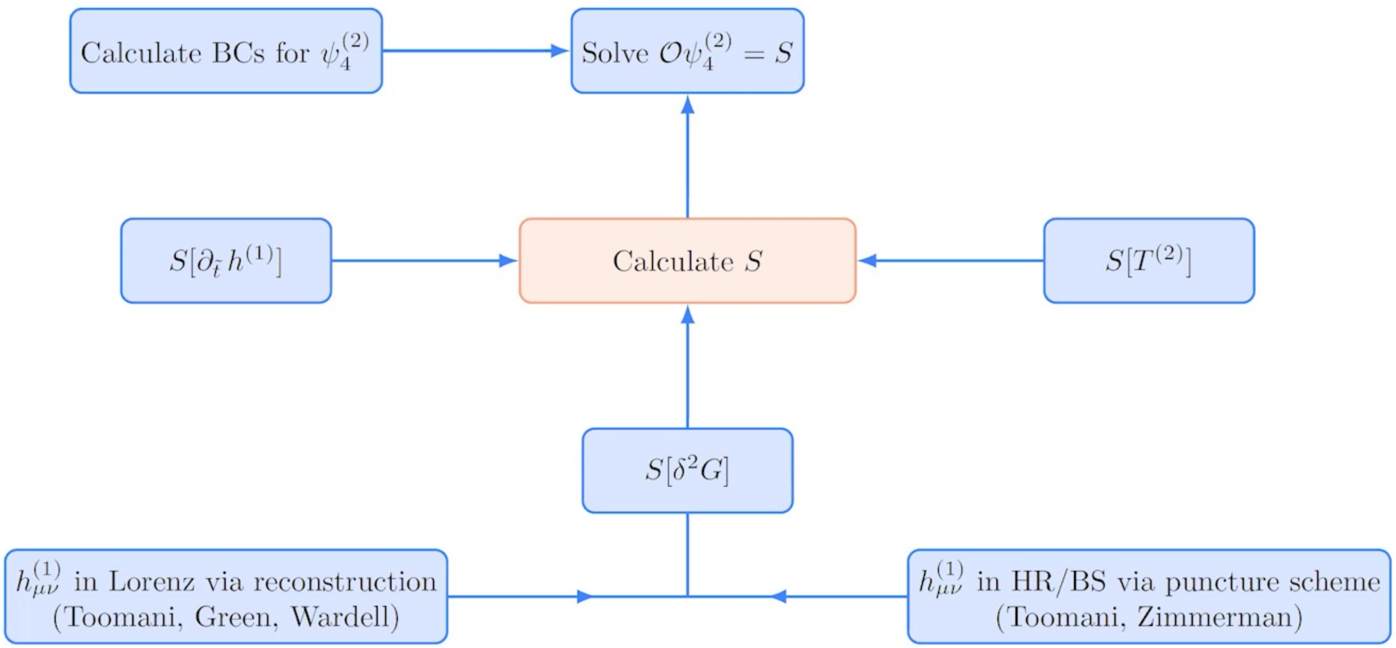
Quasicircular orbits in Schwarzschild, Teukolsky



Road to Kerr (phase 2)

Quasicircular orbits in Kerr, Teukolsky

New problem: $h^{(1)}_{\mu\nu} = h^{+}_{\mu\nu}\theta(r - r_p) + h^{-}_{\mu\nu}\theta(r_p - r) + h^{0}_{\mu\nu}\delta(r - r_p)$



Summary

- two-timescale expansion provides efficient framework for rapid waveform generation
- results slowly arriving for quasicircular orbits in Schwarzschild
- extension to eccentric orbits also looks feasible. See 2020 talk by B. Leathers
- we're on the bumpy road to Kerr