Title: Comparison of eccentric numerical relativity simulations to small mass-ratio perturbation theory

Speakers: Antoni Ramos-Buades

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Abstract: In this work we compare two approaches to modeling binary black holes (BBHs): 1) small mass-ratio (SMR) perturbation theory, and 2) numerical relativity (NR). We extend recent work on combining information from quasicircular nonspinning NR simulations of BBHs with results from SMR perturbation theory to nonspinning eccentric BBHs. We produce a dataset of long and accurate eccentric nonspinning NR simulations with the Spectral Einstein Code (SpEC) from mass ratios 1 to 10, and eccentricities up to 0.7. We analyze these NR simulations, compute gauge invariant quantities from the gravitational radiation, and develop tools to map points in parameter space between eccentric NR and SMR waveforms. Finally, we discuss discrepancies between SMR and NR predictions for the energy and angular momentum fluxes due to eccentricity, and limitations of such comparisons due to the limited parameter space in mass ratio covered by the NR simulations.

Comparison of eccentric numerical relativity simulations to small mass-ratio perturbation theory

Antoni Ramos-Buades, Maarten van de Meent and Harald Pfeiffer

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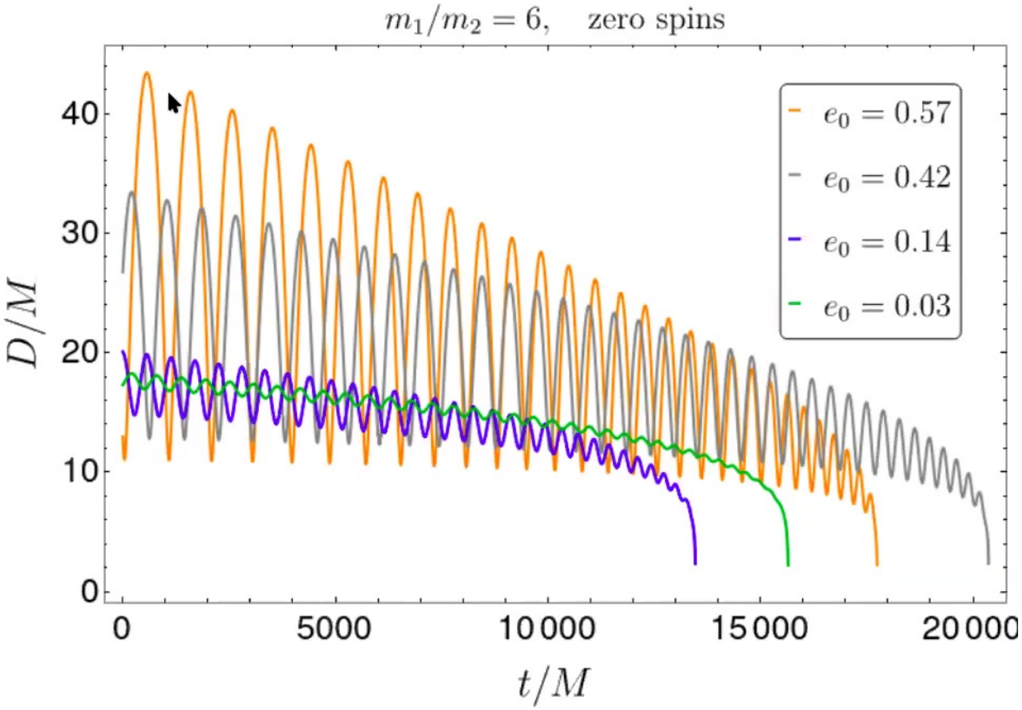
MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut
für Gravitationsphysik
Albert-Einstein-Institut

Introduction

- Spectral Einstein Code (SpEC), numerical relativity (NR) code → Accurate catalogs [Mroué+2013, Boyle+2020] of quasicircular (QC) waveforms from binary black holes (BBH).
- Small subset of simulations from Hinder+2017 with e<0.3, and recently first surrogate model for e<0.2 [Islam+2021].
- Generate a catalog of eccentric NR simulations for the upcoming LIGO observing run, O4 (~2022), increasing rate of events, likely to detect an eccentric binary.
- Recently VanDeMeent+ proposed a method to combine information from quasicircular nonspinning NR simulations of BBHs with small mass ratio (SMR) perturbation theory to recover known leading order and obtain a prediction for the next-to-leading order term.
- This work → extension of VanDeMeent+ study to the non-spinning eccentric case.



Simulations properties

- We present 32 non-spinning simulations :

$$q = m_1/m_2 = [1-10], \quad e_{\omega_{22}} = [0.01 - 0.7]$$

- Initial eccentricity measured from the frequency of the 22-mode,

$$e_0 = \frac{\omega_p^{1/2} - \omega_a^{1/2}}{\omega_p^{1/2} + \omega_a^{1/2}},$$

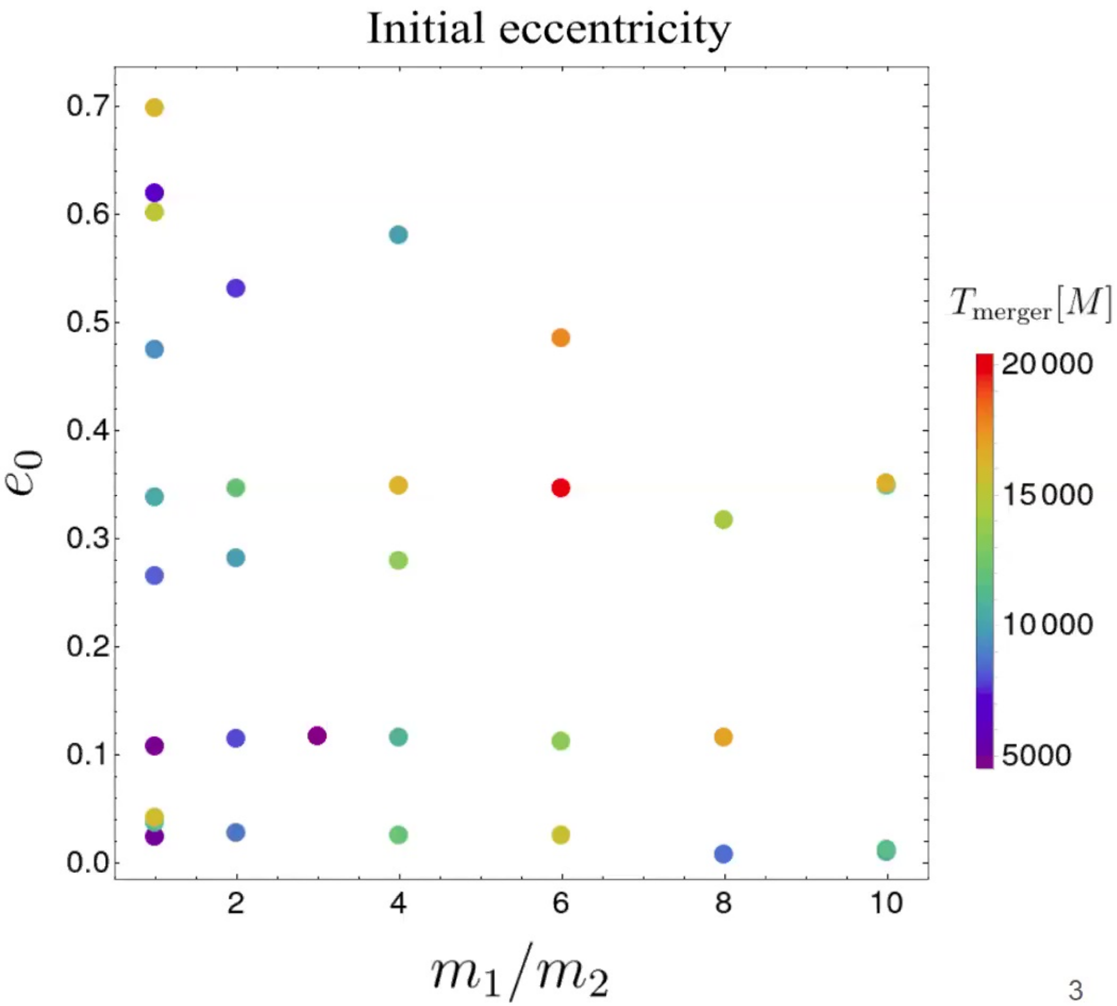
ω_p: ω at periastron

ω_a: ω at apastron

- Long simulations, [20-50] GW cycles.

- 3 different resolutions for each simulation.

- Typical wall clock times:
  - ○ q =1 : ~ 5 days
  - ○ q=10 : ~ 2 months.



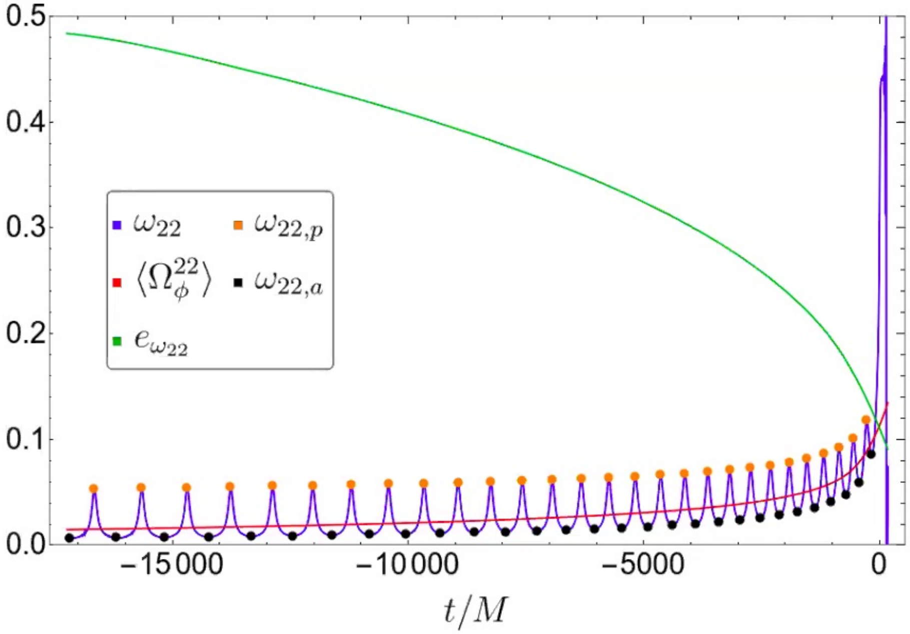
NR measurements: eccentricity, fluxes, frequencies

- Measure eccentricity from the frequency of the 22-mode. $\omega_{22} = \frac{d}{dt}\mathrm{Arg}[h_{22}(t)], \ e_{\omega_{22}} = \frac{\omega_{22,p}^{1/2} - \omega_{22,a}^{1/2}}{\omega_{22,p}^{1/2} + \omega_{22,a}^{1/2}},$ $\omega_{22,p}$: ω_{22} at periastron. $\omega_{22,a}$: ω_{22} at apastron.

- Use orbit average procedure to extract frequencies and fluxes from simulations, very similar to Lewis+2017.

$$\langle \Omega_r^{22} \rangle_i = \frac{2\pi}{t_{max}^{i+1} - t_{max}^{i}}, \quad \langle \Omega_\phi^{22} \rangle_i = \frac{1}{t_{max}^{i+1} - t_{max}^{i}} \int_{t_{max}^{i}}^{t_{max}^{i+1}} \omega_{22}(t)dt, \qquad \langle \dot{E}_{GW} \rangle_i = \frac{1}{t_{max}^{i+1} - t_{max}^{i}} \int_{t_{max}^{i}}^{t_{max}^{i+1}} \dot{E}_{GW}(t)dt, \quad \langle \dot{J}_{GW} \rangle_i = \frac{1}{t_{max}^{i+1} - t_{max}^{i}} \int_{t_{max}^{i}}^{t_{max}^{i+1}} \dot{J}_{GW}(t)dt.$$

$m_1/m_2 = 6, \quad e^0_{\omega_{22}} = 0.48$



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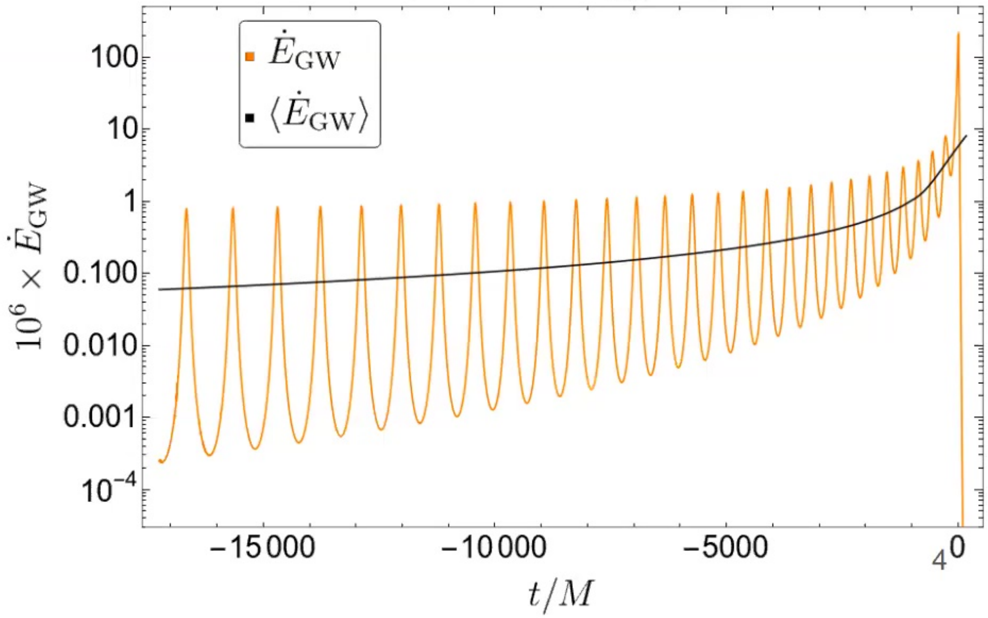


Small mass ratio calculations

- Solve equations of motion at adiabatic order:
  - Geodesic frequencies are known analytically.
  - Fluxes determined numerically at any (p,e) value using frequency domain Teukolsky code.
  - Quantities computed on grid of Chebyshev nodes in $u = 1 - \left(1 - \frac{6+2e}{p}\right)^{1/3}$ and e, and interpolated using Chebyshev polynomials. Relative interpolation error 10^{-5} .

$$\frac{dp}{d\lambda} = \eta F_p(p, e, q_r) + \mathcal{O}(\eta^2)$$

$$\frac{de}{d\lambda} = \eta F_e(p, e, q_r) + \mathcal{O}(\eta^2)$$

$$\frac{dq_r}{d\lambda} = \Upsilon_r(p, e) + \eta f_r(p, e, q_r) + \mathcal{O}(\eta^2)$$

$$\frac{dq_t}{d\lambda} = \Upsilon_t(p, e) + \eta f_t(p, e, q_r) + \mathcal{O}(\eta^2)$$

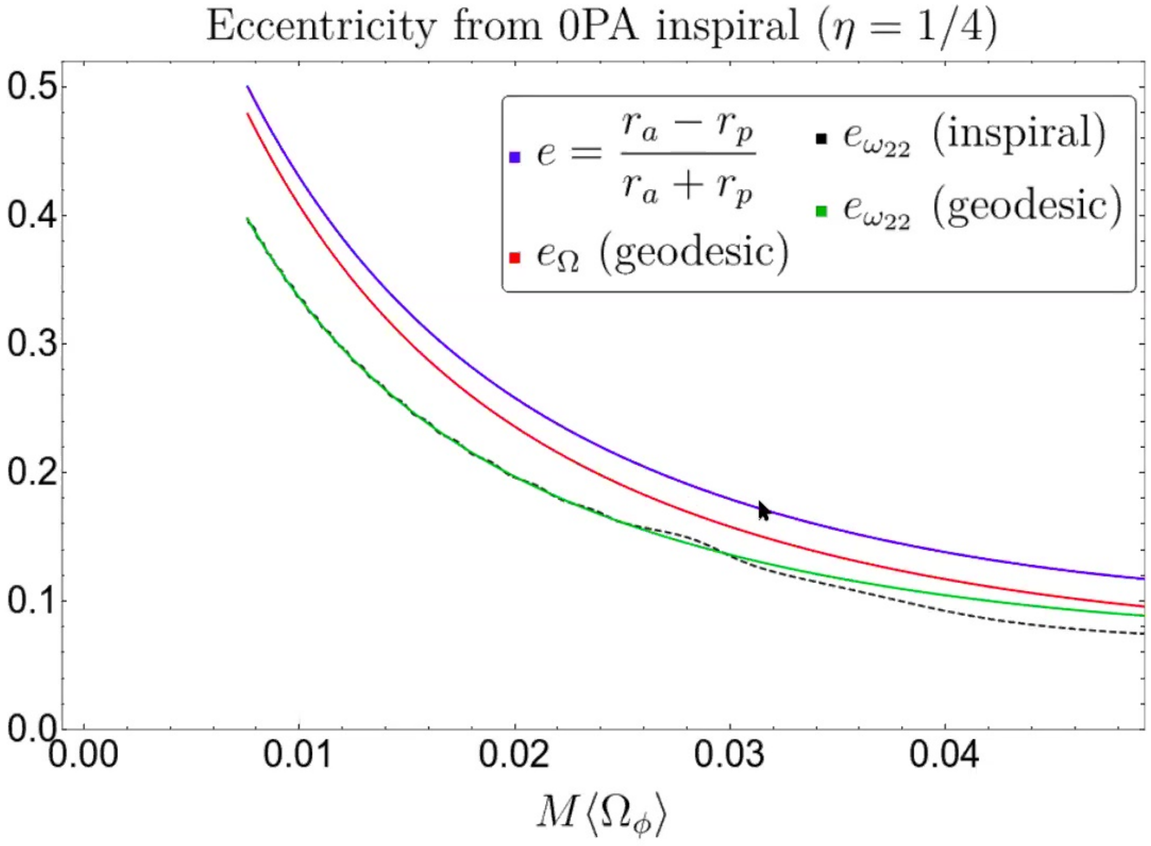
$$\frac{dq_\phi}{d\lambda} = \Upsilon_\phi(p, e) + \eta f_\phi(p, e, q_r) + \mathcal{O}(\eta^2)$$

$$\eta = \frac{m_1 m_2}{(m1+m2)^2}.$$

- We need 2 quantities to determine the state of the inspiral that must satisfy,
  - The 2 variables uniquely characterize instantaneous state of the inspiral.
  - The variables can be unambiguously computed from NR data.
  - The variables can be unambiguously computed from SMR data.

Small mass ratio waveforms: eccentricity measurements

- 'Geodesic eccentricity' (e_Ω from geodesics snapshot) differs from the one obtained from the separations and ω_{22}
- Good agreement during inspiral between $e_{\omega_{22}}$ from maxima and minima of ω_{22} and $e_{\omega_{22}}$ from geodesic data and evaluated at (p,e) from evolution.



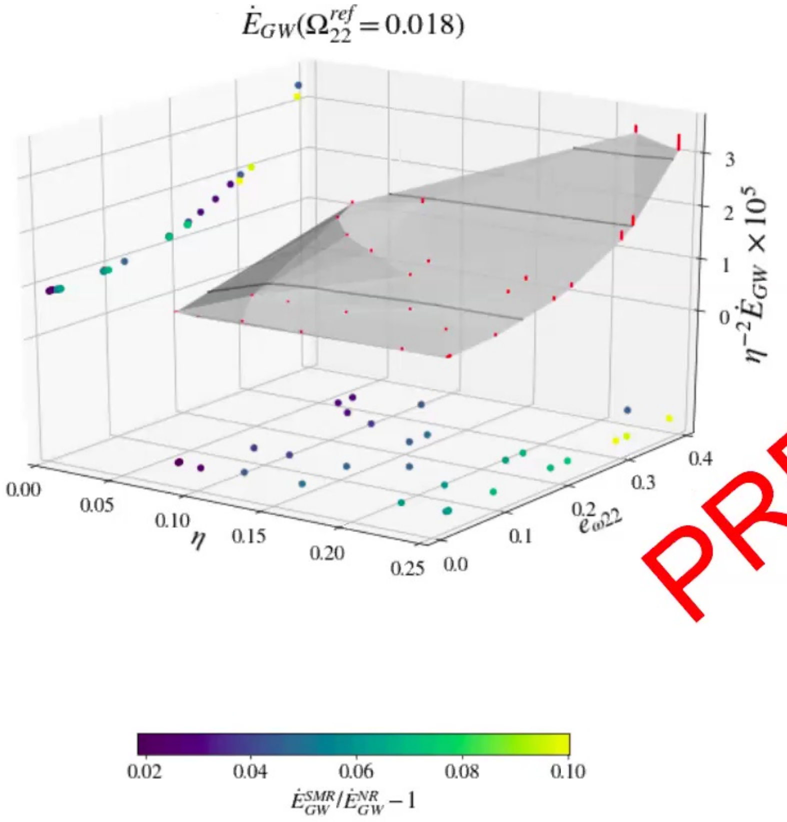
- Agreement lost in the last few cycles prior to last stable orbit (LSO).

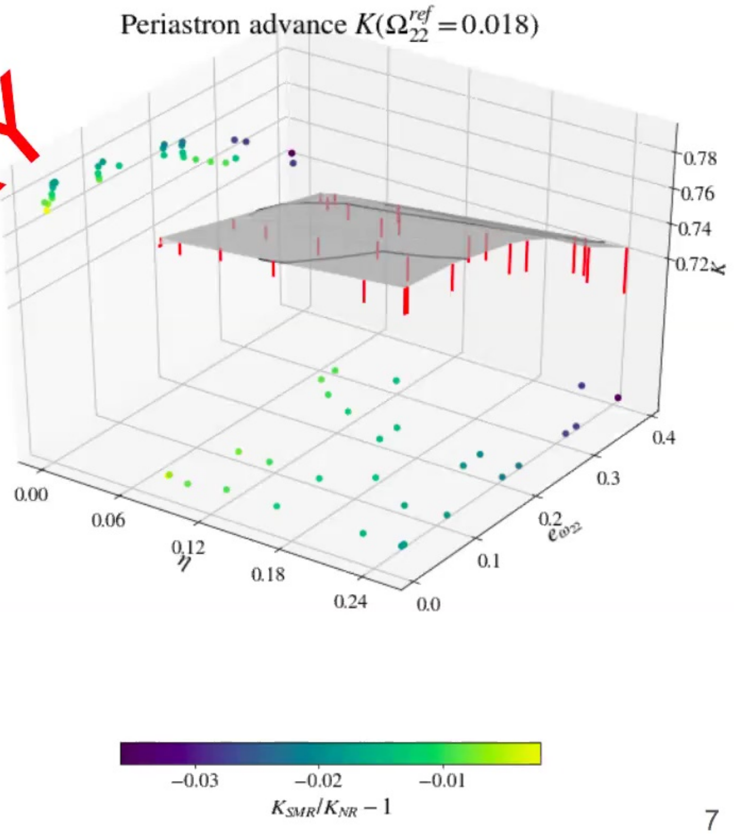
- Given a waveform $e_{\omega_{22}}$ unambiguously determined from SMR and NR.

Comparison between SMR and NR results

- Energy flux differences between SMR and NR small at leading order.

$$K = \Omega_r / \Omega_\phi .$$

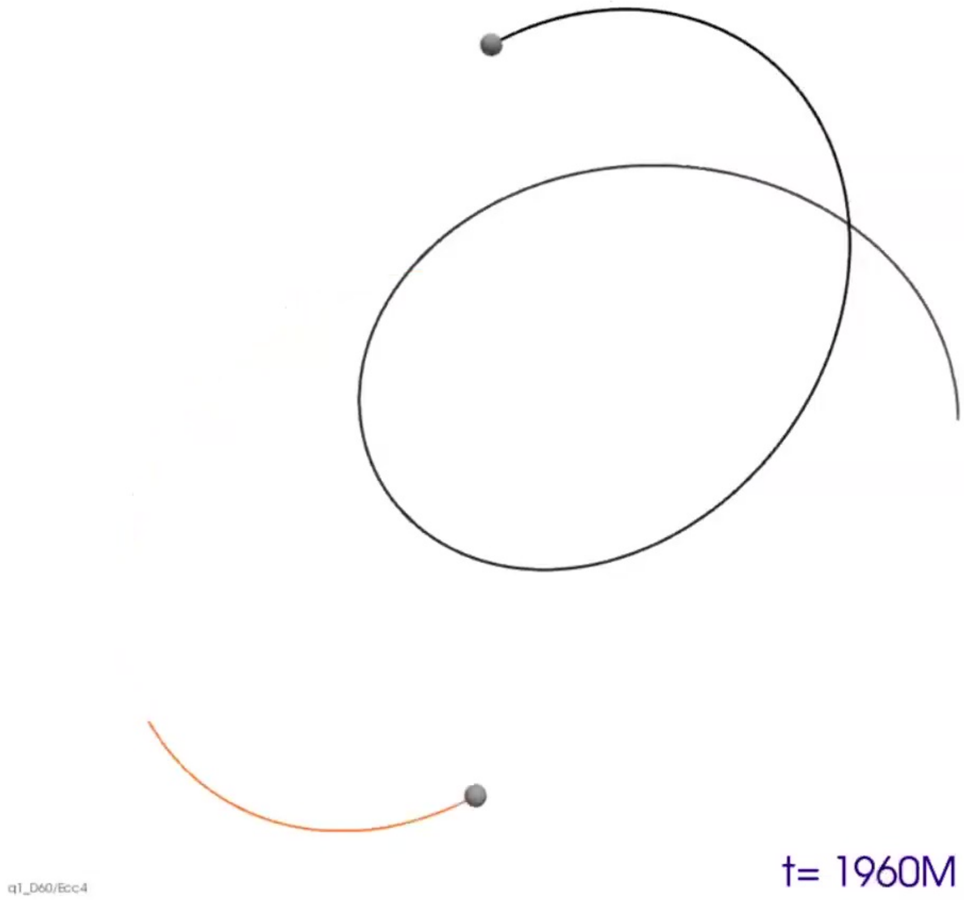

$\dot{E}_{GW}(\Omega_{22}^{ref}=0.018)$


Periastron advance $K(\Omega_{22}^{ref}=0.018)$

PRELIMINARY

Conclusions and future work

- Presented 32 long (≥ 20 orbits) non-spinning simulations with $e_{22}\leq 0.7$ and q=[1-10] .
- Developed tools to compare eccentric SMR and NR simulations.
- Preliminary comparisons between SMR and NR results indicate small mass ratio dependence at next mass ratio order.
- Ongoing work:
    - Fit that dependence from the data.
    - Increase the NR dataset of simulations to better compare to SMR results.



Visualization by H. Pfeiffer.