Title: Redshift factor from numerical relativity simulations

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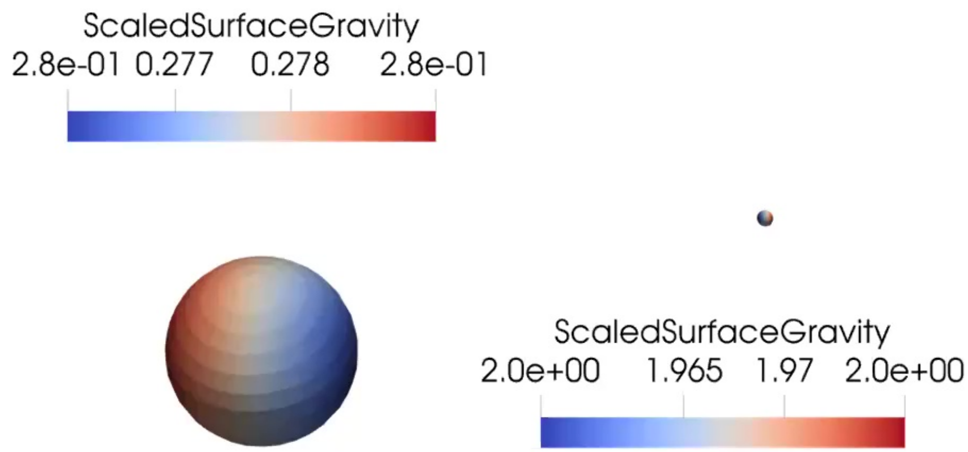
Abstract: We present a method to extract the redshift factor in numerical relativity simulations by means of its connection to the surface gravity. We proceed to analyze the small mass ratio limit and extract 2nd and higher orders in the case of non-spinning, quasi-circular binaries. We also compare our results to analytic post-newtonian and self-force calculations.

Redshift factor from numerical relativity in the SMR limit

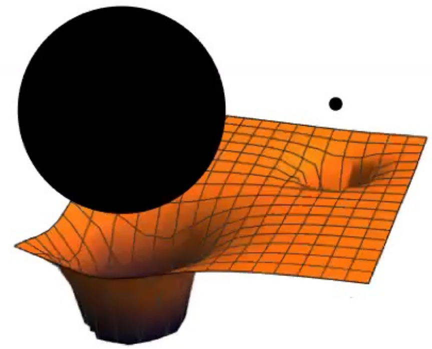


SpEC simulation: BBH, q=4, non-spinning, quasi-circular

Sergi N. Albalat & Aaron Zimmerman

24th Capra Meeting, June 8th, 2021.

❖ Self-force perturbation theory



$$g_{\mu\nu}^{\text{physical}} = g_{\mu\nu}^{\text{background}} + \epsilon h_{\mu\nu}^{1} + \epsilon^2 h_{\mu\nu}^{2} + \mathcal{O}(\epsilon^3)$$

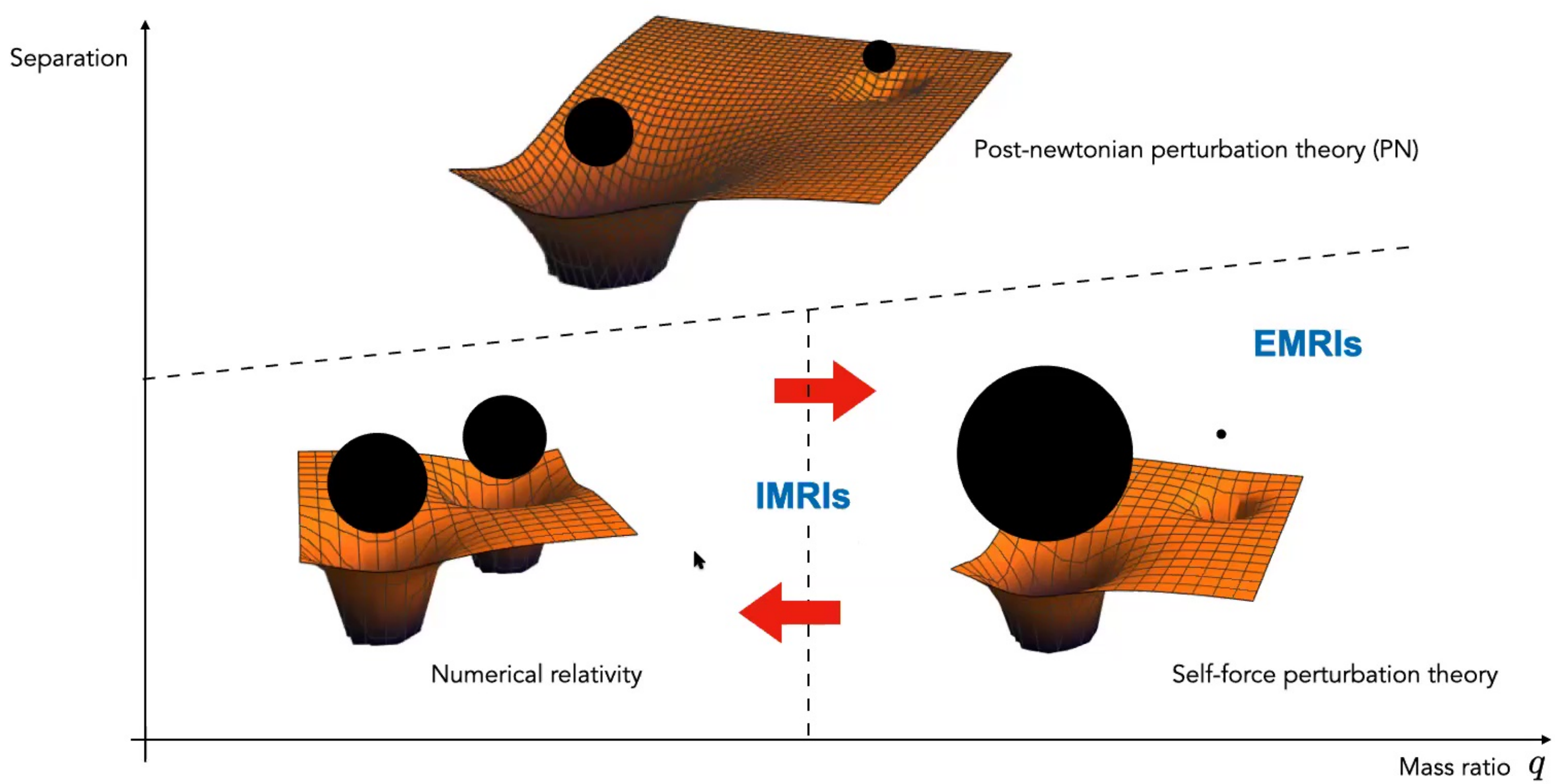
$$\epsilon = \frac{m}{M}$$

Small mass trajectory satisfies: $\dfrac{Du^\mu}{d\tau} = \epsilon F_1^\mu + \epsilon^2 F_2^\mu + \mathcal{O}(\epsilon^3)$

Gauge invariant* redshift factor (Detweiler)

$$z = \frac{1}{\tilde{u}^t} \qquad \tilde{u}^t = \left[1 - 3(M\Omega)^{2/3}\right]^{-1/2} \left(1 + \frac{1}{2} h_{uu}^R\right) \qquad \tilde{g}_{\mu\nu} = g_{\mu\nu}^{\text{background}} + h_{\mu\nu}^R$$

Synergies between numerical relativity (NR) and self-force perturbation theory (GSF)



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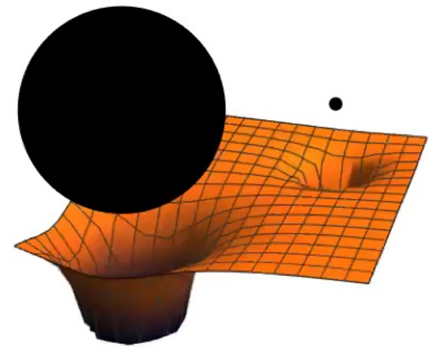
Self-force perturbation theory



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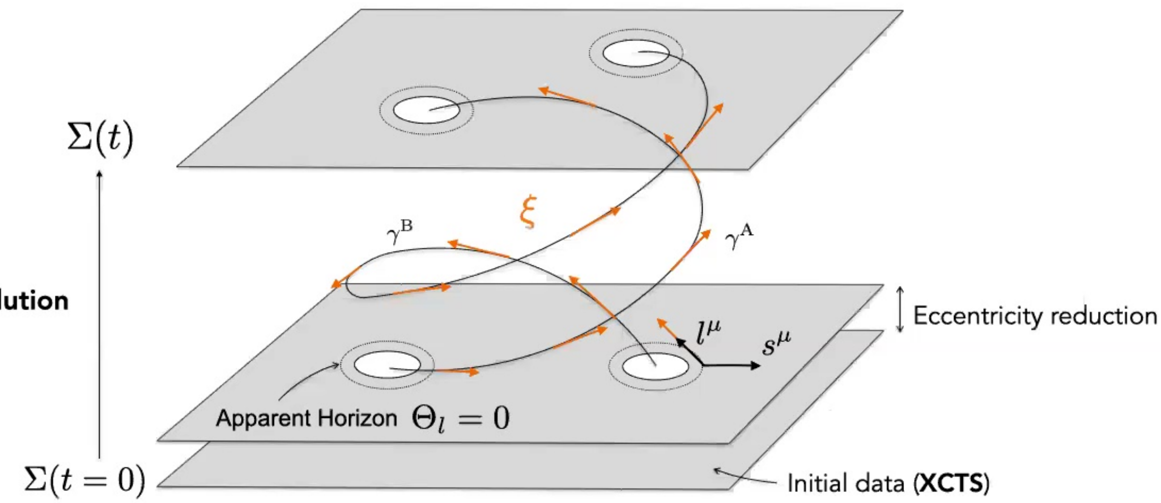
❖ Redshift factor in numerical relativity

When the small mass is a black hole we have* (A.Pound): $z = \frac{\kappa}{\bar{\kappa}}$ $\quad \bar{\kappa} = \frac{\sqrt{m^2 - a^2}}{2m(m + \sqrt{m^2 - a^2})}$

3+1 split: $ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^i dt)$



* The invariance property of the redshift factor and surface gravity depends on the existence of a helical killing vector field: $\xi = \partial_t + \Omega \partial_\phi$

Normalization $\xi^\mu_\infty = (1, 0, 0, \Omega)$

We calculate the surface gravity from the properly normalized tangents to the horizon generators:

$$\kappa_{(\xi)} = s^i D_i N - N K_{ij} s^i s^j + \xi^\mu \nabla_\mu \ln N$$

❖ Data analysis (10 non-spinning, quasicircular SpEC simulations)

1) Move from time to frequency space

$$z(t) \to z(m\Omega) \qquad \Omega = \frac{|\vec{r} \times \dot{\vec{r}}|}{r^2}$$

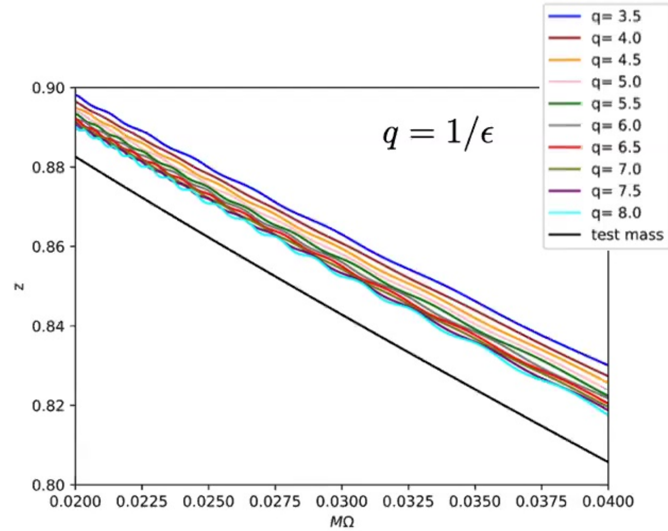
2) Center of mass motion produces oscillations in the data.

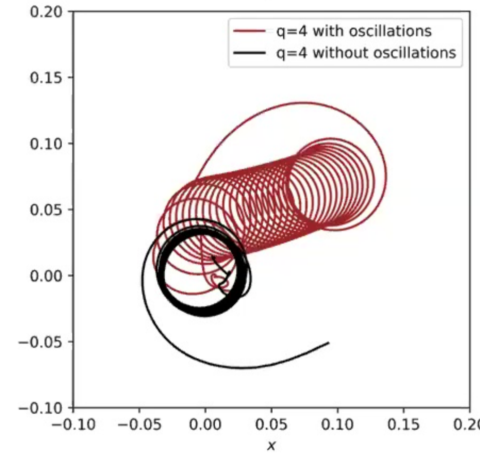
$$\xi^t \to \xi'^t = \gamma(\xi^t - v_i^{\mathrm{CoM}} \xi^i)$$

We take as our NR data the mid-line between the upper and lower envelopes.

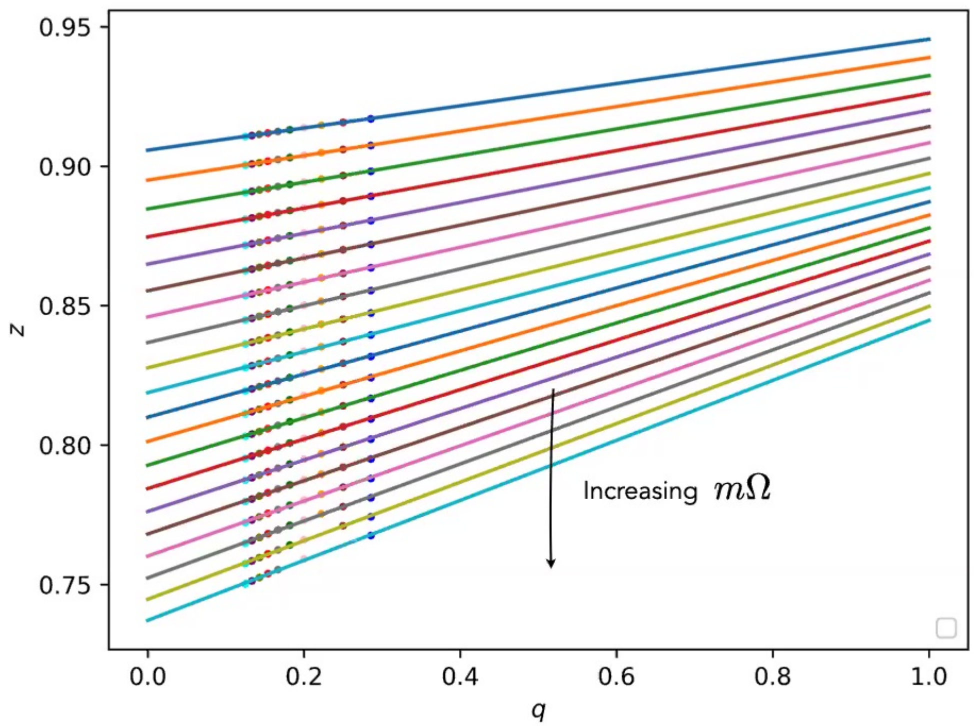
3) At each frequency, fit for the coefficients of the mass ratio power series

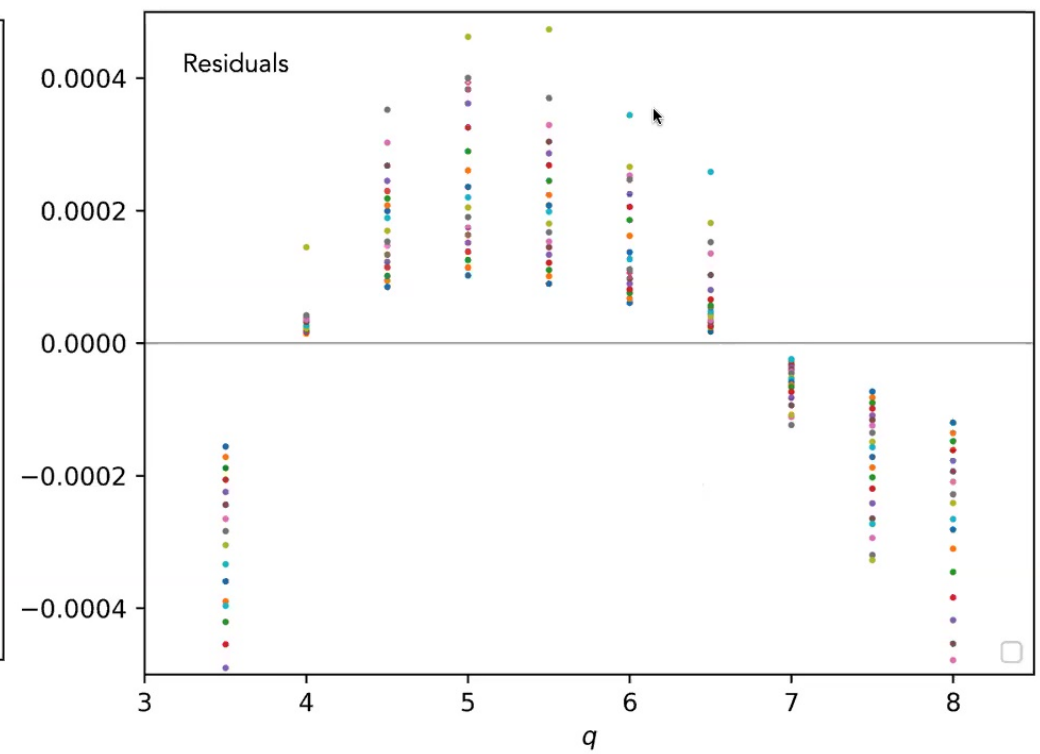
$$z^{\mathrm{fit}}_{m\Omega} = \sum_{n=0}^{N} \epsilon^n z_n$$





Results for the fit $z^{\text{fit}}_{m\Omega} = z_{0\,m\Omega} + \epsilon z_{1\,m\Omega}$





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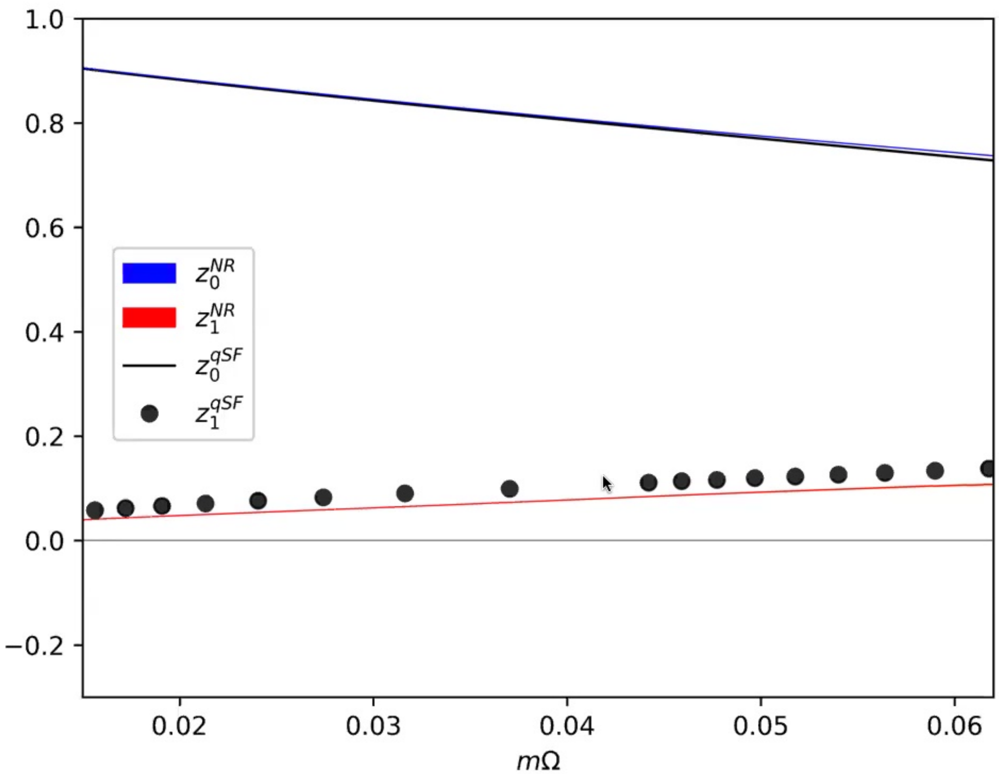
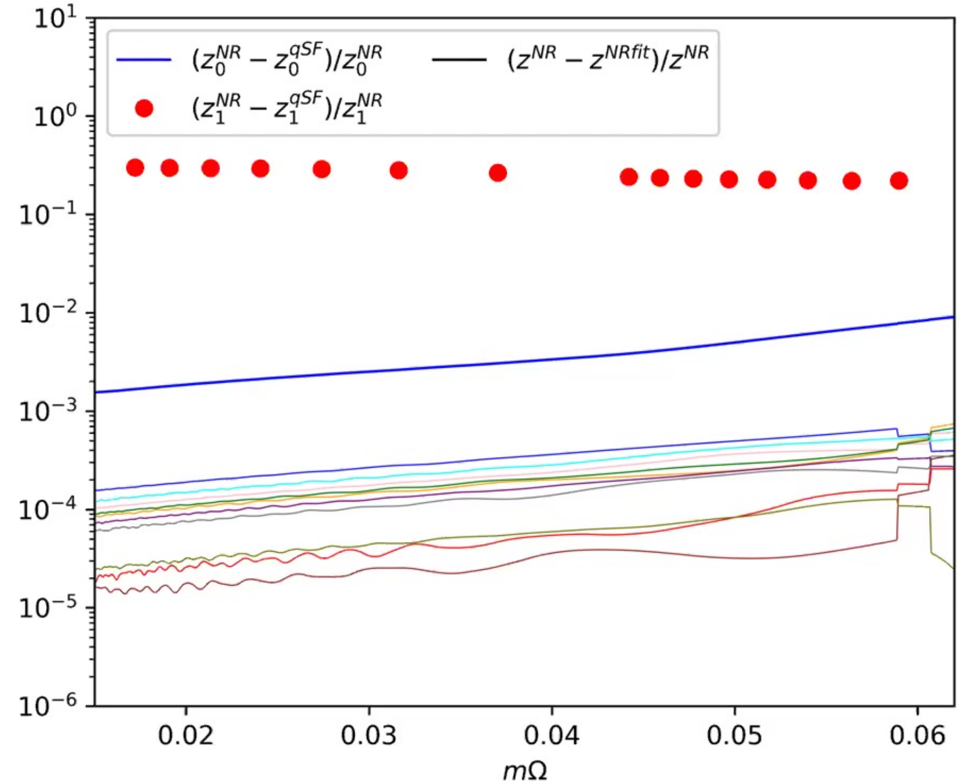
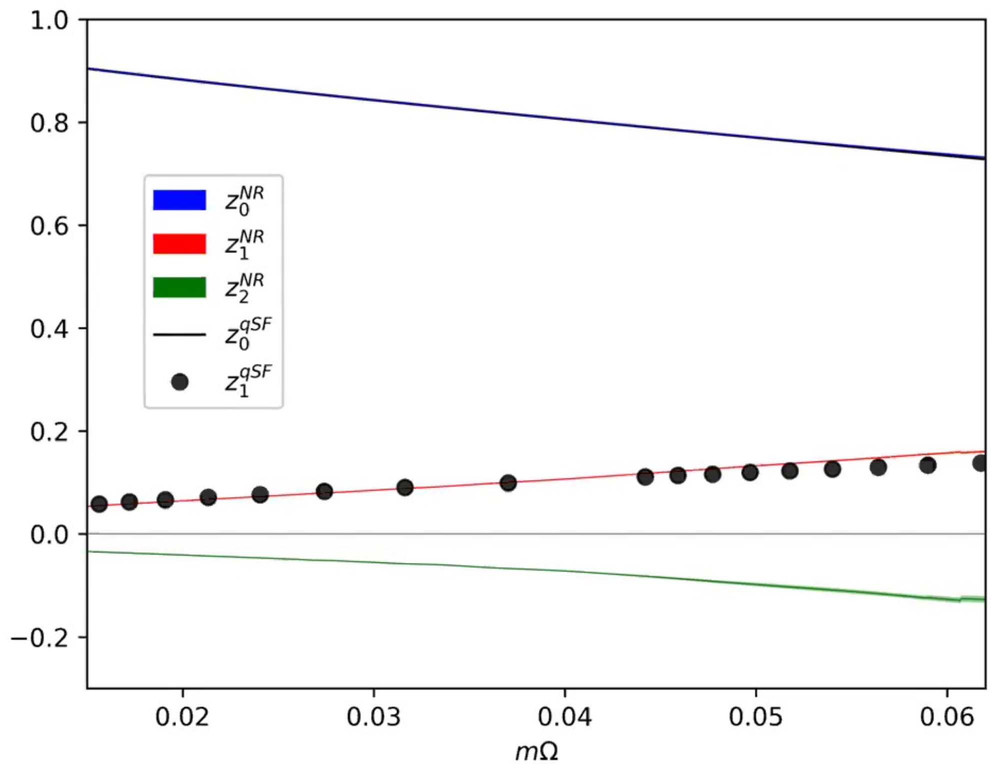
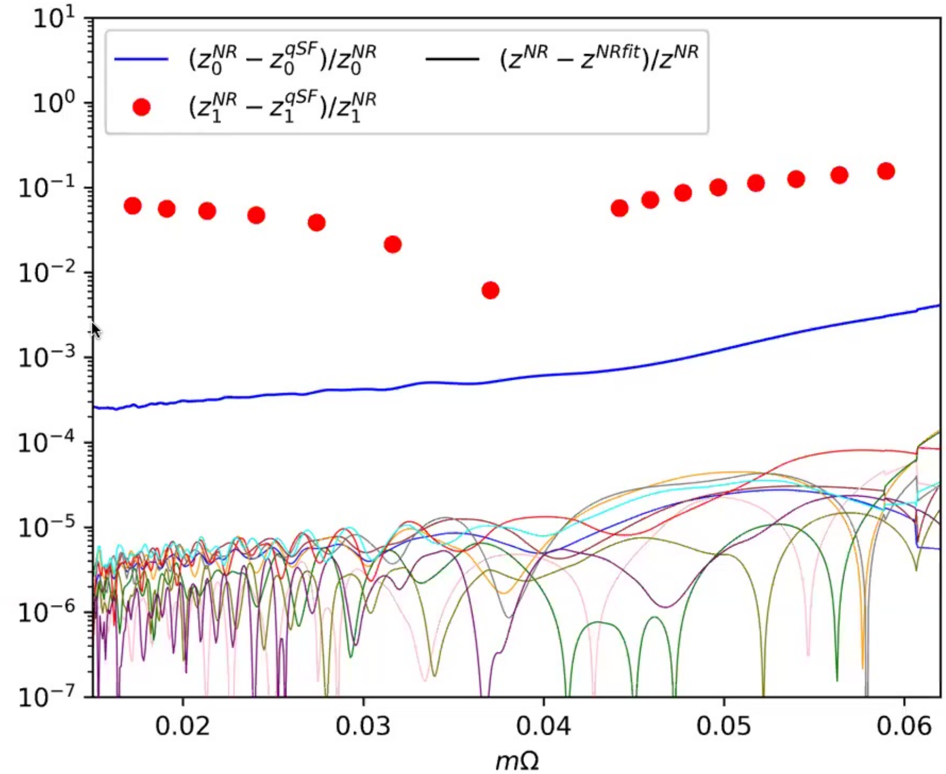




Results for the fit $z^{\rm fit}_{m\Omega} = z_{0\,m\Omega} + \epsilon z_{1\,m\Omega} + \epsilon^2 z_{2\,m\Omega}$





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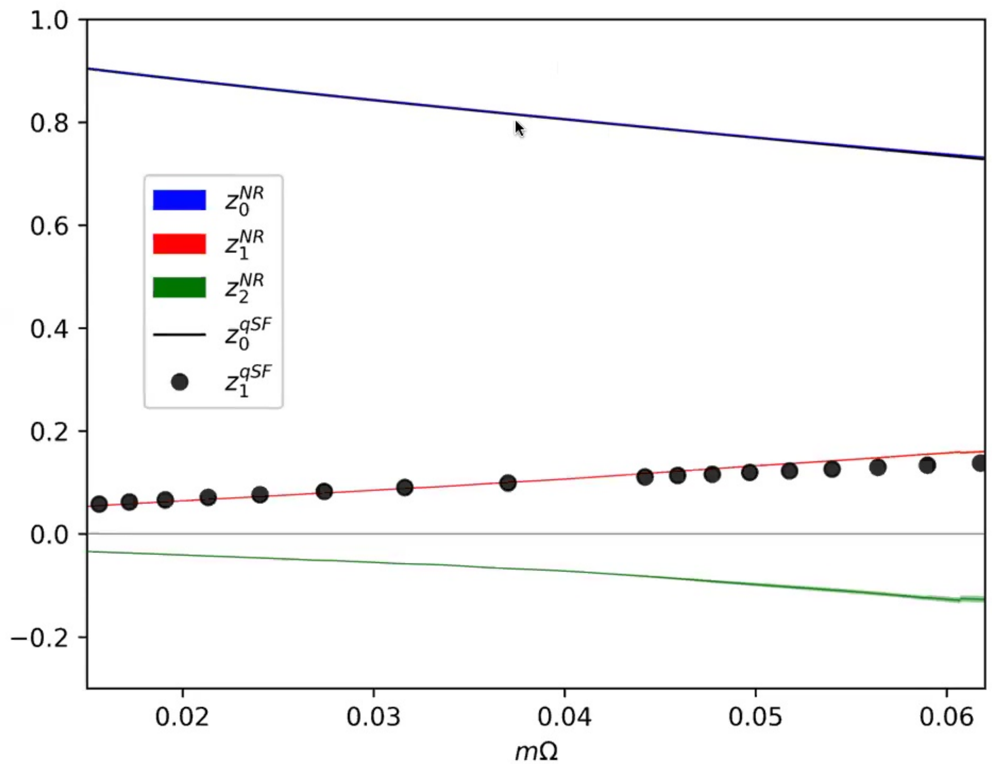
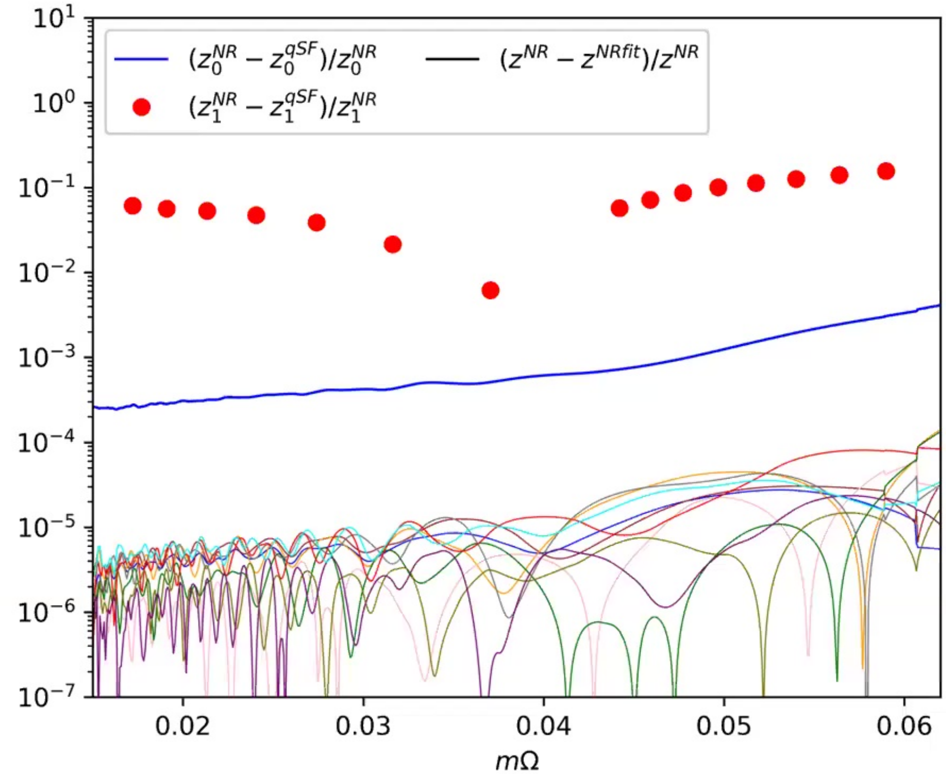
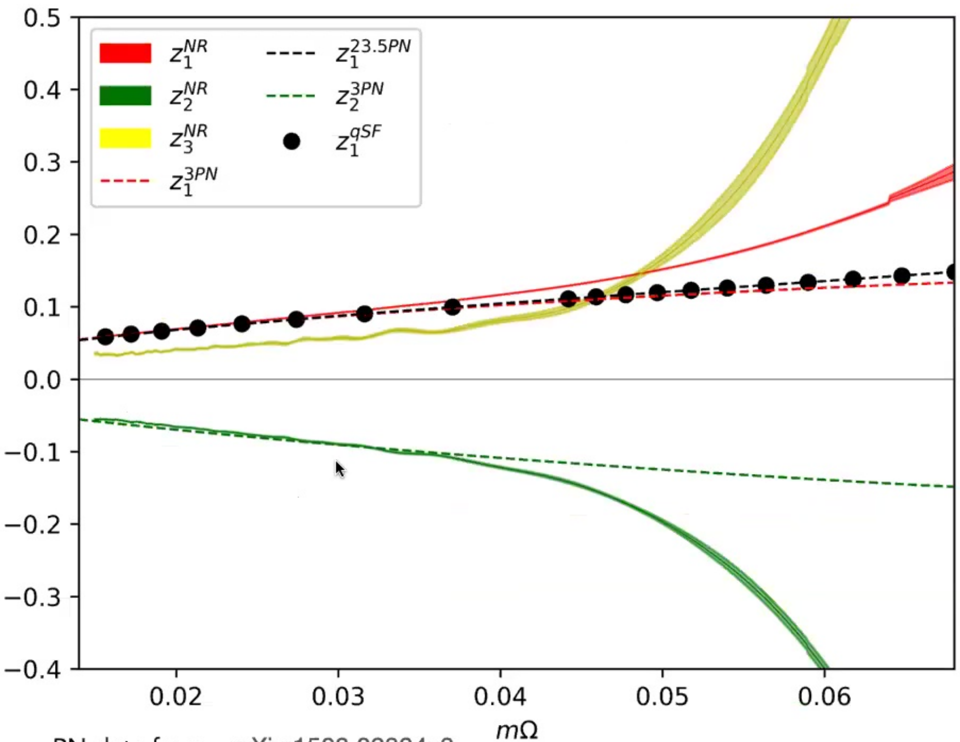


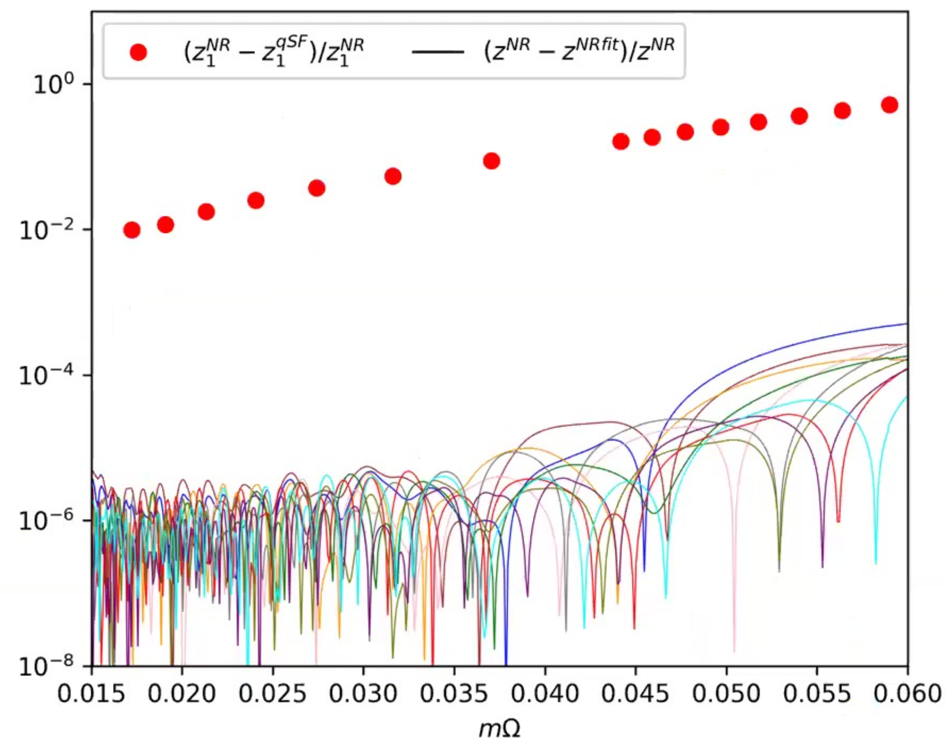


Results fixing the 0th order



PN data from arXiv:1503.02334v3

SF data from arXiv:1009.4876 arXiv:0804.3529



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❖ Conclusions of the analysis

A simple fit of the NR data shows a great agreement with the 0GSF, improving as we add higher orders. For the highest order fit relative differences are of order 10^{-4}

Agreement with the 1GSF redshift also improves as we add higher orders. Disagreement with 1GSF reduces towards the early inspiral as expected.

At second order NR results are consistent with the 2GSF from PN.

The symmetric mass ratio residuals, although small, show more structure than the qGSF residuals.

❖ Future directions

Extend the redshift extraction in NR to eccentric orbits in Kerr.

Possibly bringing NR and GSF to better agreement incorporating transition terms.