Title: Adiabatic waveform for extreme mass-ratio inspirals: an analytical approach

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Abstract: We will discuss an adiabatic waveform model for generic (eccentric, inclined) EMRI orbits in Kerr spacetime, based on a high-order PN expansion as well as an expansion in eccentricity to the (frequency-domain) Teukolsky equations.

# Adiabatic waveform for extreme mass-ratio inspirals (EMRIs): an analytical approach

Soichiro Isoyama
+ Chua, Pound and JPN school

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# EMRI waveform models

EMRI waveform model for LISA needs to be accurate, efficient, and **extensive** (over a timescale of ~months).

✔ **Extensiveness:**

Generic (ie, eccentric and inclined) orbits in Kerr spacetime (+ secondary spin etc + detector response, ultimately)

Even **at first perturbative `adiabatic' order**, actual calculation of self-force waveforms is computationally (very) expensive.

"few" examples in Fujita & Shibata: 2008.13554, Hughes+ 2102.02713.

This is why "Kludge" models have to be used, trading accuracy for extensiveness (and efficiency).

Barack & Cutler (2004); Babak+ (2007); Chua+(2015, 2017)

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# Computational challenge

In the standard multipole decomposition, the challenge is the need for ~$10^5$ of modes per point in EMRI parameter space.

✔ **Wall time of numerical Teukolsky solver:** e.g. Fujita, Hikida, Tagoshi: 0904.3810

hrs – day for given (Kerr spin, separation, eccentricity and inclination.)

➡ Too expensive to cover ~ $10^{3\text{-}4}$ points for evolution...

Recent efforts for acceleration:
Chua+: 2008.06071, Katz+2104.04582.

**This work:**

**Analytically** compute harmonic modes (and hence waveform), solving Teukolsky eqs. in a "post-Newtonian" expansion.

Sasaki and Tagoshi 0306120 (LRR), Black hole perturbation Club.

# "Multivoice" adiabatic waveform

Pound and Wardell 2101.04592

$$h(t) = -\frac{2\mu}{r} \sum_{\ell mkn} \frac{Z^{\infty}_{\ell mkn}}{\omega^2_{mkn}} \frac{{}_{-2}S^{a\omega_{mkn}}_{\ell m}}{\sqrt{2\pi}} e^{-i\Phi_{mkn}+im\varphi} \qquad \frac{d\Phi_{mkn}}{dt} = \eta\, \omega_{mkn}$$

Mode amplitudes ***Z***, spheroidal harmonics ***S*** and frequency ***ω*** are all functions of slowly-evolving orbital parameters: ***I(t)*** := [*p(t)*, *e(t)*, *ι(t)*].

At first-perturbative adiabatic order, *I(t)* is just driven by fluxes *F(I)*.

$$dI/dt = \eta \mathcal{F}(I)$$

We then **analytically** compute waveform inputs [*Z(I)*, *S(I; θ)*, *F(I)*] through 5PN while also expanding (only) in eccentricity through $e^{10}$.

No expansion in inclination, using Gantz+:0702054, Fujita & Hikida 0906.1420

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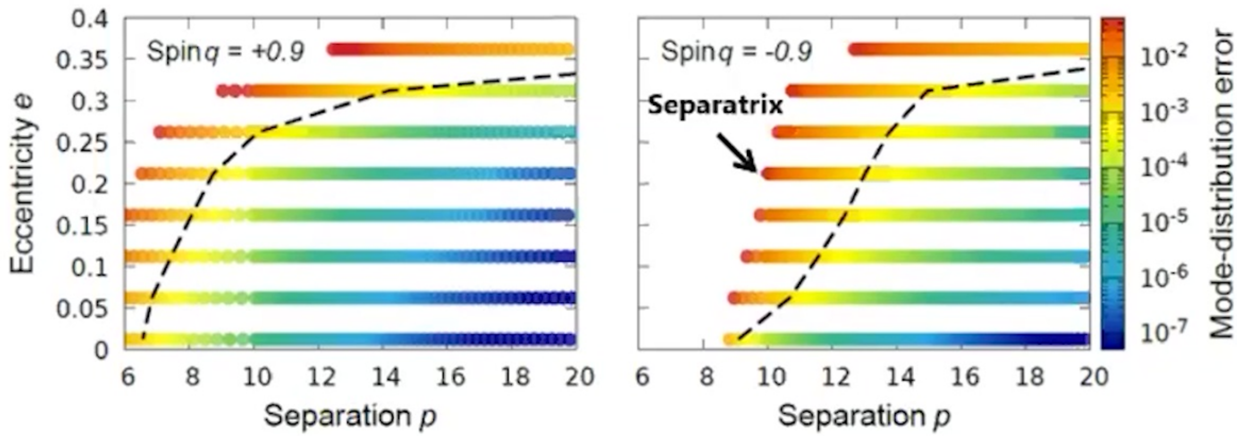
# Accuracy of 5PN-e10 calculations

A useful error metric is the mode-distribution error. Chua+: 2008.06071.

$$\mathrm{error} = 1 - \frac{\mathrm{Re}(H_{\mathrm{PN}},\, H_{\mathrm{num}})}{\sqrt{|H_{\mathrm{PN}}||H_{\mathrm{num}}|}} \qquad H \equiv \mathrm{vec}(Z^{\infty}_{\ell mkn}\ {}_{-2}S_{\ell m}/\omega^2_{mkn})$$

**(viewing angle: $\theta$ = 45 deg, inclination = 80 deg)**




Err. < 1e-3 is adequate for LISA inference studies. Katz+: 2104.04582.

**Validity domain**: $p > 6$, $e < 0.3$

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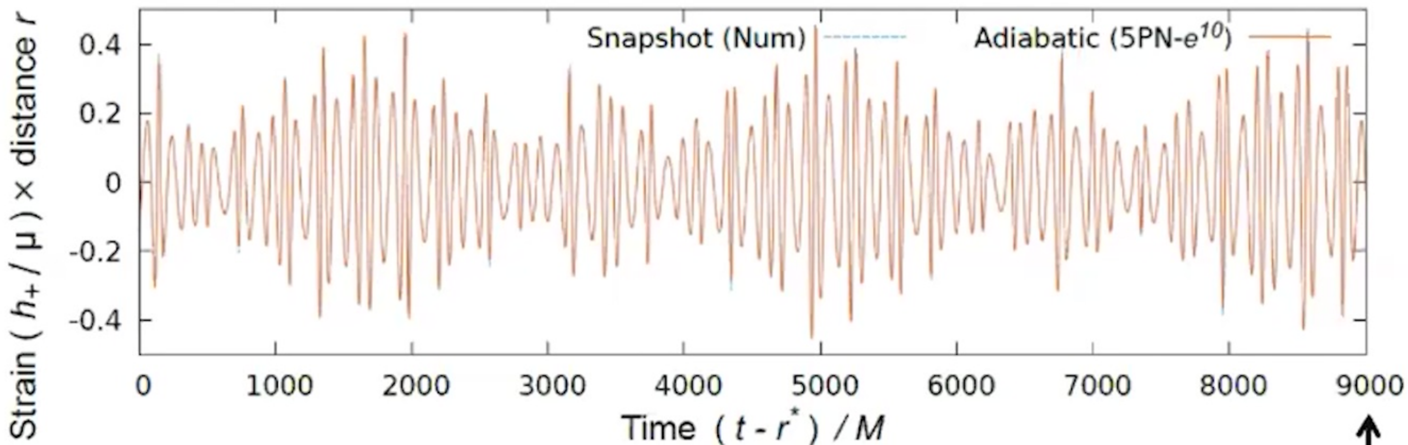
# A sample EMRI waveform

Special thanks to Hughes and van de Meent for verification.

($\mu$ = 10 $M_\odot$, $M = 10^6$ $M_\odot$ $a/M = 0.9$; $e0 = 0.21$, $i0 = 80$ deg, $p0 = 9.6$; $\theta = 45$ deg.)




~ 0.5 day

**Snapshot**: reference numerical waveform of geodesic "snapshot" at $t = 0$.

Rich structures are due to, e.g.,. periastron and Lense-Thirring precessions

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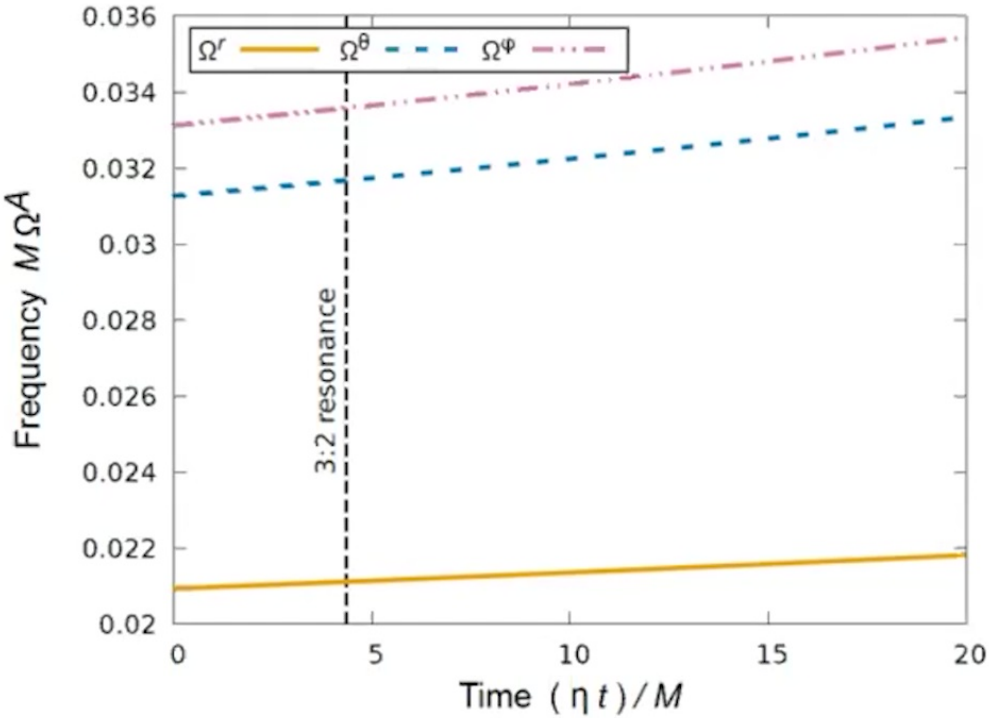


~ 4 months

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# Self-force resonance in EMRIs

More comes in later sessions (Destounis, Gupta, Nasipak, Speri...)

A salient feature of generic EMRI is a (self-force) resonance:

$$\beta^r \Omega^r(t_{\rm res}) - \beta^\theta \Omega^\theta(t_{\rm res}) = 0$$

due to the **Killing symmetry** of Kerr background.

At resonance, the fluxes := the stationary piece of the (dissipative) self-force in Fourier domain can be **enhanced / diminished**.

Flanagan+: 1208.3906, van de Meent: 1311.4457, SI+: 1809.11118

$$\frac{dI}{dt} \sim \mathcal{F} + \sum_{\beta^r, \beta^\theta \neq 0} G_{\beta^r \beta^\theta}\, e^{i(\beta^r \Phi^r - \beta^\theta \Phi^\theta)}$$

**Standard, "(00)"-mode of the self-force.**

**This "oscillatory" mode also becomes constant at resonance.**

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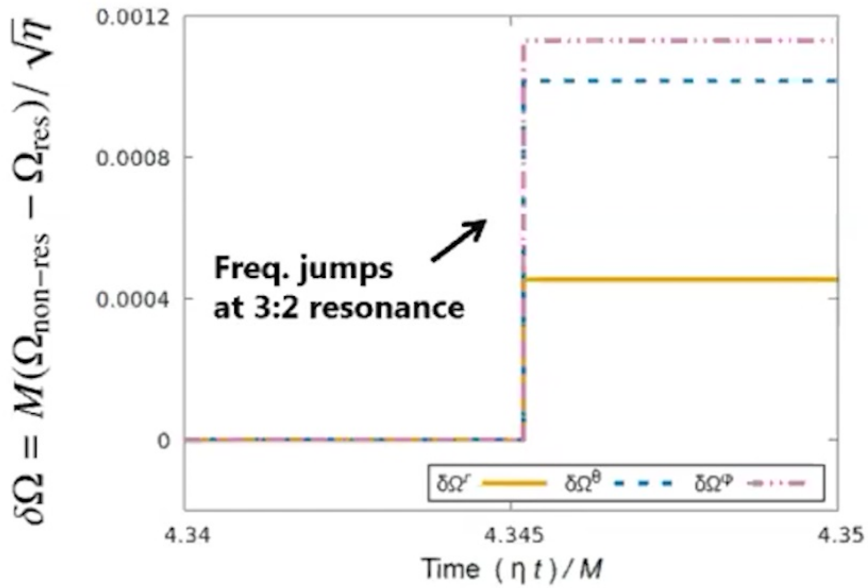
# Bonus: approx. resonant jump

More comes in later sessions (Destounis, Gupta, Nasipak, Speri...)

A salient feature of generic EMRI for is a (self-force) resonance:

**(mass ratio: $\eta$ = $10^{-5}$ ; $a/M$ = 0.9; e_res ~ 0.20, i_res ~ 80 deg, p_res ~ 9.5)**




Can induce ***O(1)*** phase shift ~ months after the resonance.

Berry+: 1608.08951, Speri & Gair: 2103.06306

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# Summary and outlook

Computed **adiabatic EMRI waveforms for generic orbits**, using analytical "PN framework" for the Teukolsky equations.

- usable for $p > 6$, $e < 0.3$ but with any Kerr spins and inclinations;
- can include self-force resonance in evolution (approximately).

Near-term outlook includes…

- higher-order calculations;
- to have more efficiency;
- f-domain waveforms.

through "FastEMRIWaveforms (FEW)", Black hole perturbation Club etc