Title: A framework for the quasinormal mode shifts of arbitrary spin beyond-kerr black holes.

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Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 07, 2021 - 12:00 PM

URL: http://pirsa.org/21060021

Abstract: """Gravitational wave detectors and their increasing precision have enabled more specific tests of general relativity, including spectroscopic tests of black holes by measuring the quasinormal modes within the ringdown signal. These tests ideally compare the QNM frequencies to predictions from theories beyond GR, where black holes may be described by deformations to the Kerr metric. I will present a framework to compute the first order QNM shifts of these deformed Kerr Black Holes at arbitrary spin and present some initial results for the spin-0 case. In addition, I will lay out some of the technical issues that come up when computing the shifts for the spin-2 modes, and explain how they are surmountable."""

A framework for the quasinormal mode shifts of arbitrary spin beyond-kerr black holes

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June 7, 2021

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Ringdown

- Anatomy of a black hole merger signal on the right
- Our emphasis will be on the tail end of the signal
- A key component of the ringdown signal are quasinormal modes.

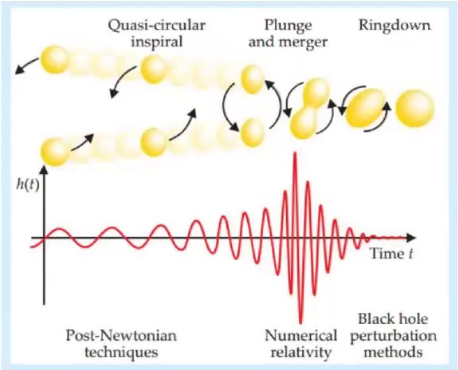


Figure: Baumgarte and Shapiro, 2011

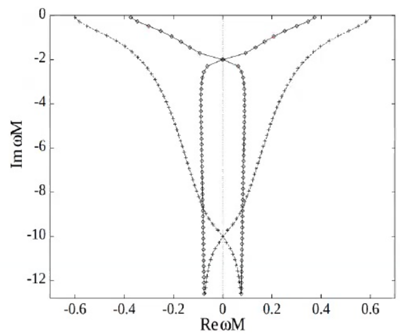
Quasinormal Modes

- A **discrete set of decaying modes** which are solutions to the Tuekolsky equation.
- Finding these modes is essentially an eigenvalue problem
- Very efficient solvers exist to compute these modes (e.g. Cook-Zalutskiy method arXiv:1410.7698 [gr-qc])

$$\hat{T}[\ {}_s\psi(t,r,\theta,\phi)] = 0$$

$${}_s\psi = \sum_{nlm} \mathrm{e}^{-\mathrm{i}\omega_n t + \mathrm{i}m\phi}\, {}_s\psi_{\mathrm{lmn}}(r,\theta)$$

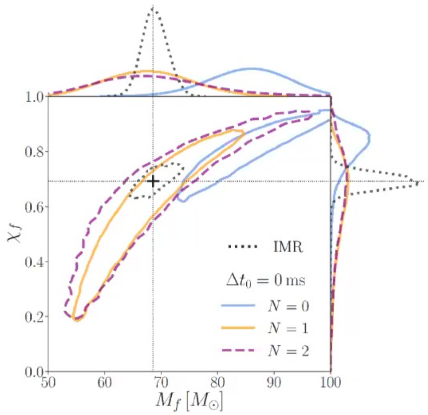
$${}_s\hat{T}_{lm\omega}[\ {}_s\psi_{lm\omega}(r,\theta)] = 0$$

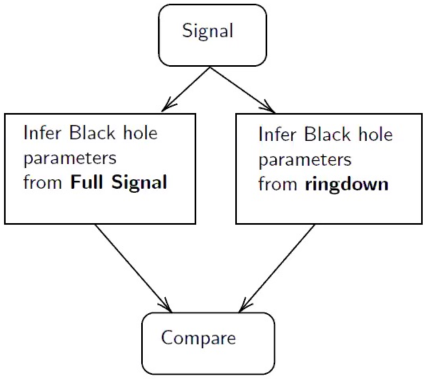


Kokkotas & Schmidt 1999, arXiv:gr-qc/9909058

Consistency Tests of General Relativity

Testing the no-hair theorem with GW150914 - Maximiliano Isi, Matthew Giesler, Will M. Farr, Mark A. Scheel, and Saul A. Teukolsky

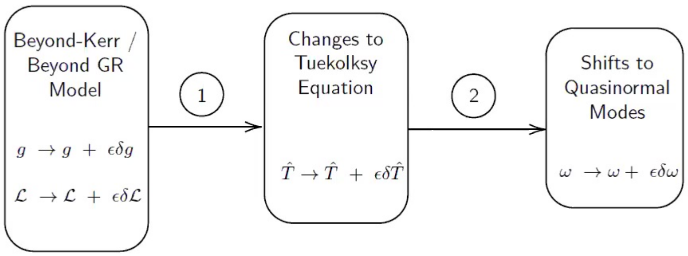




Good agreement when overtones taken into account

Specific changes to Kerr

- If we find disagreement $\rightarrow$ what change to GR / Kerr would this imply?
- We will need to test each proposed beyond-Kerr model on a case by case basis.
- We need a general framework to do this



Shifts for arbitrary spin (a)

- I want to outline a framework for computing shifts for arbitrary spin.
- Get inspired by perturbation theory in quantum mechanics

$$\hat{H} \to \hat{H^{(0)}} + \epsilon \hat{H^{(1)}}; \omega \to \omega^{(0)} + \epsilon \delta\omega$$

$$\delta\omega = \frac{\langle n^{(0)} | H^{(1)} | n^{(0)} \rangle}{\langle n^{(0)} | n^{(0)} \rangle}$$

- We need an inner product that:
  - is finite (of course)
  - follows $\langle n^{(0)} | \hat{H^{(0)}} - \omega^{(0)} | n^{(1)} \rangle = 0. \implies$ makes $\hat{H^{(0)}}$ self adjoint

Eigenvalue Perturbation Method - shifts for arbitrary a

- Start with the tuekolsky equation for a particular l, m and ω mode.

$$ {}_s\hat{\tilde{T}}_{lm\omega'}[\ {}_s\psi_{lm\omega'}(r,\theta)] = 0 $$

- Expand it to first order

$$ \left(\tilde{T}^{(0)}_{slm\omega} + \epsilon\delta\omega \frac{\partial \tilde{T}^{(0)}_{slm\omega}}{\partial\omega} + \epsilon \tilde{T}^{(1)}_{slm\omega} \right) \left[\psi^{(0)}_{\mathrm{slm}\,\omega} + \epsilon \psi^{(1)}_{\mathrm{slm}\,\omega} \right] \approx 0 $$

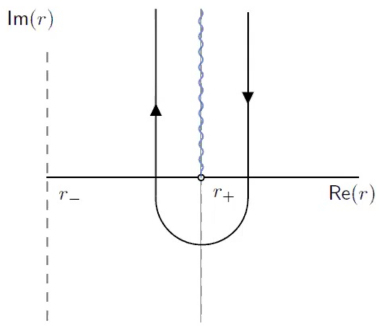
- Come up with a norm that is finite and makes $\tilde{T}^{(0)}_{slm\omega}$ self-adjoint (this has been done), then rearrange.

$$ \delta\omega_{slmn} = - \frac{\langle {}_s\psi^{(0)}_{lm\omega} | \ \tilde{T}^{(1)}_{lm\omega} \ | {}_s\psi^{(0)}_{lm\omega} \rangle}{\langle {}_s\psi^{(0)}_{lm\omega} | \partial_\omega \tilde{T}^{(0)}_{lm\omega} | {}_s\psi^{(0)}_{lm\omega} \rangle} $$

Inner product on wavefunctions

The inner product that makes the Kerr Tuekolsky operator self-adjoint is defined as

$$\langle \psi \mid \chi \rangle = \int_0^{\pi} \sin\theta d\theta \int_{\mathcal{C}} dr \Delta^s \psi(r,\theta)\chi(r,\theta)$$



The radial contour goes around the branch cut at the horizon

Eigenvalue Perturbation Method - Previous Work

- Kerr-Newmann qnm shifts (arXiv:1409.5800) "The Quasinormal Modes of Weakly Charged Kerr-Newman Spacetimes" - Zachary Mark, Huan Yang, Aaron Zimmerman, and Yanbei Chen
  - arbitrary a
  - first order in expansion parameter $\epsilon = \frac{Q^2}{M^2}$
- Parametric instability of black holes (arXiv:1402.4859) "Turbulent Black Holes" - Huan Yang, Aaron Zimmerman, Luis Lehner
  - Looks at qnm shifts due to a background of Kerr + Qnms.
  - At high a, found that higher frequency modes can excite lower frequency ones $\implies$ hints of turbulence.

Quasinormal Mode spectrum of a shifted Laplacian

- For $s = 0$, the Tuekolsky equation is just the laplacian, and the shifts in the laplacian can be computed from the new metric.

$$\Box_{g_{Kerr}+\epsilon\delta g} = \Box_{g_{Kerr}} + \epsilon\delta\Box_{(g,\delta g)}$$

- By considering the extra term to be the shift in the Tuekolsky operator, gives the shifts in the quasi-normal modes for the new metric.

Shifts of Generic Beyond Kerr models

- There is a generic parametrization of beyond kerr models of black holes,due to Johannsen and Psaltis, called the JP metric. (Phys. Rev. D 83, 124015)

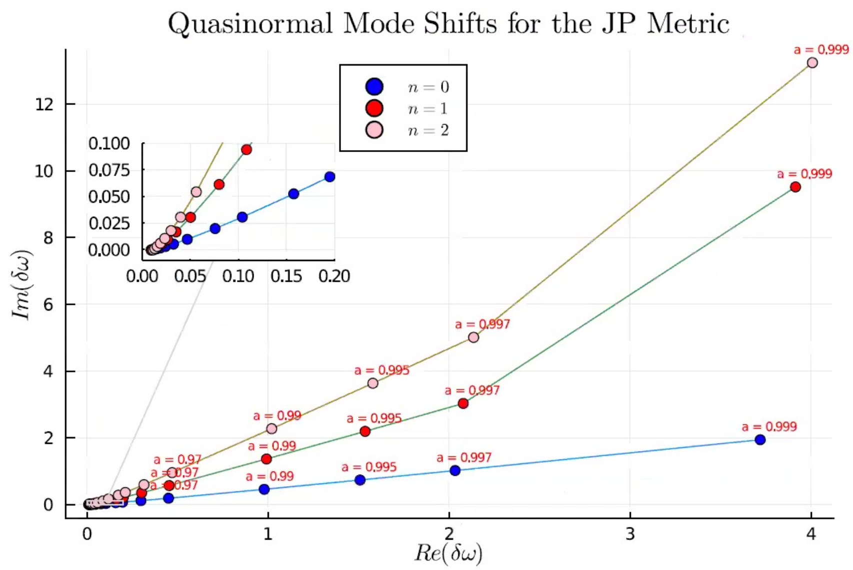
$$\begin{aligned}
\Delta(r) &\to \Delta(r) + a^2 \sin^2(\theta) h(r,\theta) \\
g_{tt} &\to g_{tt}(1 + h(r,\theta)) \\
g_{t\phi} &\to g_{t\phi}(1 + h(r,\theta)) \\
g_{rr} &\to g_{rr}(1 + h(r,\theta)) \\
g_{\phi\phi} &\to g_{\phi\phi} + h(r,\theta)\left(\frac{a^2(\Sigma + 2Mr)\sin^4\theta}{\Sigma}\right)
\end{aligned}$$

- An infinite tower of parameters ϵ_i

$$h(r,\theta) \equiv \sum_{k=0}^{\infty}\left(\epsilon_{2k} + \epsilon_{2k+1}\frac{Mr}{\Sigma(r,\theta)}\right)\left(\frac{M^2}{\Sigma(r,\theta)}\right)^k$$

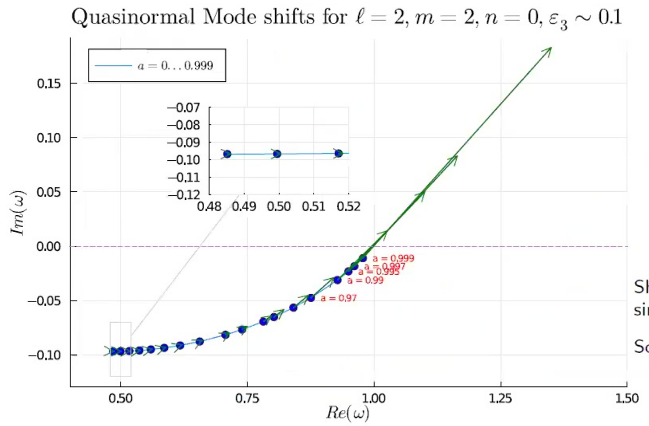
Preliminary Results

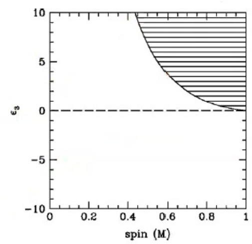
Results for $s = 0, l = 2, m = 2$



Preliminary Results

Instability for large a (results only for the $s = 0$ perturbations)

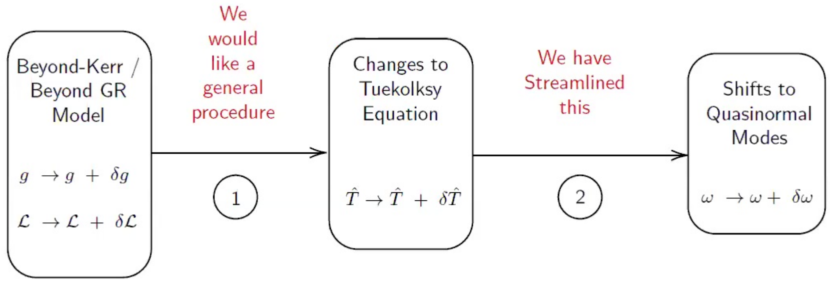




Shaded: Values for ϵ_3 with a naked singularity

Source: Johannsen and Psaltis 2011

What needs to be done



- In general we would like a process that inputs equations of motions for any theory and its metric BH solution, and just outputs the shifts.

A sketch of the general procedure

- Assume a beyond-GR lagrangian, which has:
  - Einstein Hilbert Action
  - Kinetic term for (an) extra field(s)
  - ϵ - suppressed interaction term that couples some field current to curvature quantities

$$S = S_{EH} + \int dx \sqrt{-g}[\mathcal{L}_{\vartheta} + \epsilon \mathcal{L}_{int}] \tag{1}$$

- We can derive the equations of motion from this:

$$G_{ab}[g] = \epsilon^2 \, T^D_{ab}[g, \vartheta] \, , \tag{2}$$

Two Expansion Parameters

We start with the equation of motion above and expand our metric and our fields with **two** expansion parameters

- η: Wave Perturbations
- ϵ: Perturbations of the Background

$$g \to g^0 + \epsilon^2 g^1 + \eta(h^0 + \epsilon^2 h^1)$$
$$\vartheta \to 0 + \epsilon\vartheta^1 + \eta(0 + \epsilon^2 \varphi^1)$$

g^0 = Kerr Background

g^1 = Deformation to the Kerr Background

h^0 = Kerr QNM Waves

h^1 = Shifts to Kerr QNM Waves due to the deformation

Two Expansion Parameters

- Expanding as above and picking out the wave perturbation part, gives us the following:

$$\begin{pmatrix} [E_{ab} + \epsilon^2(\delta E_{ab} - \delta T^D_{ab})] & -\epsilon T^C_{ab} \\ \epsilon\,\delta\Box[\vartheta^1_A] & \Box_0 - \epsilon\,\delta\rho \end{pmatrix} \begin{pmatrix} h^0 + \epsilon^2 h^1 \\ 0 + \epsilon^2 \varphi^1 \end{pmatrix} = 0 \tag{3}$$

- $E_{ab} =$ The Linearized Einstein Equation for Kerr
- $\delta E_{ab} = \delta E_{ab}[g^1; \circ] =$ all terms in the expanded Einstein tensor linear in both g^1 and $\circ$ at the same time.
- $\delta T^D_{ab} = \delta T^D_{ab}[g^0, \vartheta^1; \circ]$ all expanded terms in the shifts Einstein equations linear in $\circ$ and containing linear and quadratic terms in ϑ^1

The Tuekolsky-like equation

- This essentially gives an equation for the shifts that is akin to the Tuekolsky equation we had earlier

$$E_{ab}[h^0 + \epsilon^2 h^1] + \epsilon^2(\delta E_{ab} - \delta T^D_{ab})[h^0] + \mathcal{O}(\epsilon^3) = 0 \tag{4}$$

- An interesting relation due to Wald is, that there exist Linear Differential Operators on Kerr, S^{ab} and K, such that:

$$S^{ab}[E_{ab}[h]] = \mathcal{T}[\ {}_s\psi]$$
$$K[h] = \ {}_s\psi$$

The Tuekolsky-like equation

- By applying S^{ab} to (4), we have

$$S^{ab}E_{ab}[h] + \epsilon^2 S^{ab}(\delta E_{ab} - \delta T^D_{ab})[h^0] + \mathcal{O}(\epsilon^3) = 0 \tag{5}$$

$$\mathcal{T}_{Kerr}[\,{}_s\psi] + \epsilon^2 S^{ab}(\delta E_{ab} - \delta T^D_{ab})[h^0] + \mathcal{O}(\epsilon^3) = 0 \tag{6}$$

$$\mathcal{T}_{Kerr}[\,{}_s\psi] + \epsilon^2 S^{ab}(\delta E_{ab} - \delta T^D_{ab})[K^{-1}[\,{}_s\psi^0]] + \mathcal{O}(\epsilon^3) = 0 \tag{7}$$

- We can pick out the shifted Tuekoslky operator above:

$$\delta\mathcal{T} = S^{ab}(\delta E_{ab} - \delta T^D_{ab}) \circ K^{-1}$$

- And to calculate the actual shifts of the quasinormal modes, the only missing ingredients were the following inner products:

$$\langle\,{}_s\psi^0_{lmn}|\delta\mathcal{T}|\,{}_s\psi^0_{lmn}\rangle$$

A Surmountable Problem - Metric Reconstruction

- This leads us to one thing we still haven't straightforwardly calculated:

$$h^0_{slmn} = K^{-1}[{}_s\psi^0_{lmn}]$$

- This is called metric reconstruction, and in our case we only have to "reconstruct" the Kerr Quasinormal Modes from their solutions in terms of Weyl scalars.
- This is not straightforward, but a lot of work has gone into metric reconstruction and though the process can include subtleties, these are definitely surmountable.

Thanks

Thank you for listening!