Title: Self force review

Speakers: Maarten van de Meent

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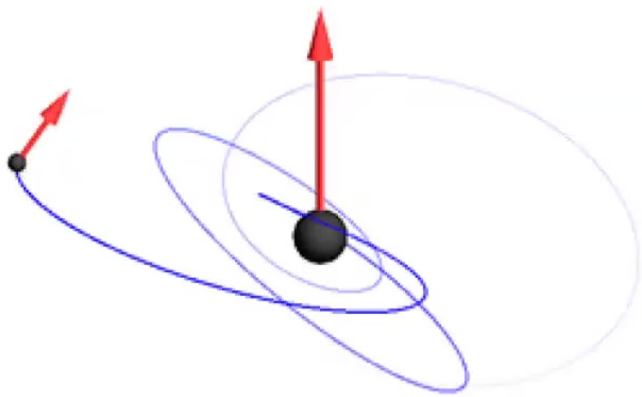
Gravitational Self-Force & You

An Introduction to Everybody's Capra talk

Maarten van de Meent
Max Planck Institute for Gravitational Physics, Potsdam



Capra 24, Perimeter Institute (online), 7 June 2021

First principle approaches to modelling relativistic binaries

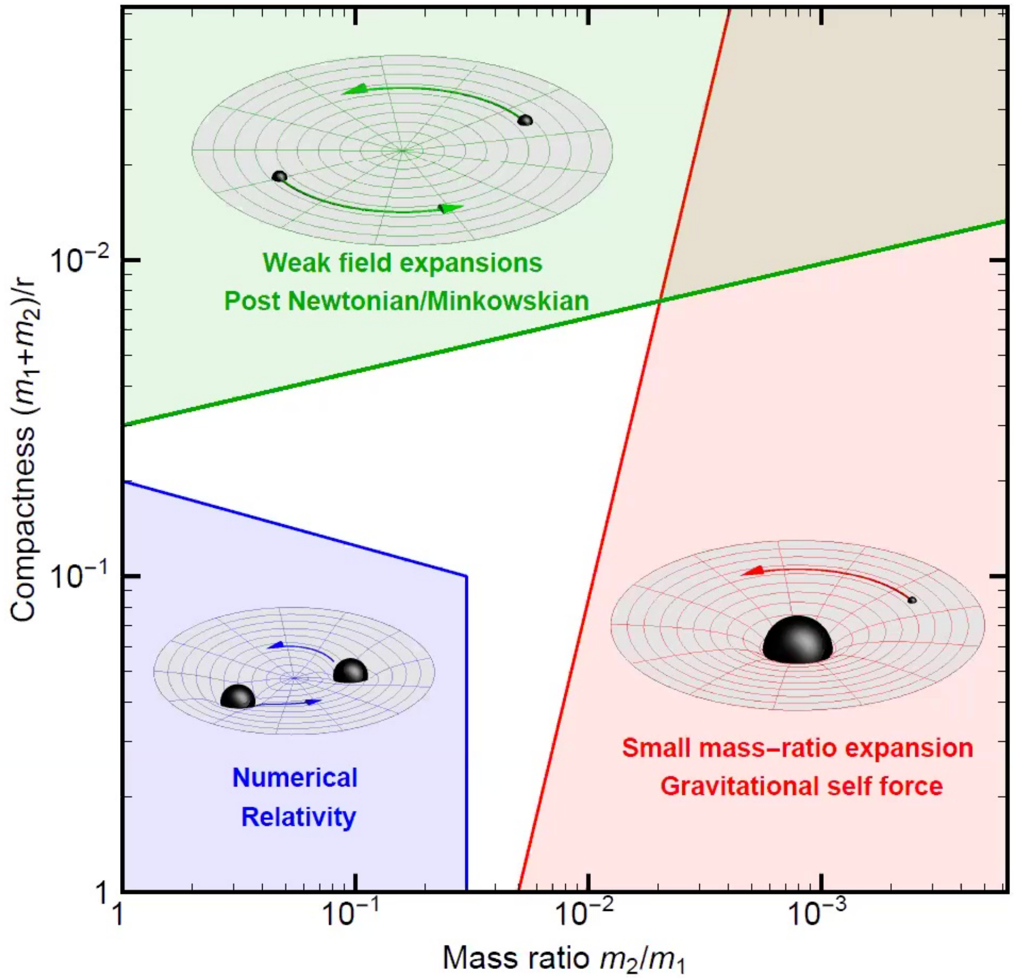
Weak Field

- Analytical
- LO: Newtonian dynamics
- Does not converge in strong field

Numerical Relativity

- Numerical
- "GR on a grid"
- Limited in length and mass-ratio

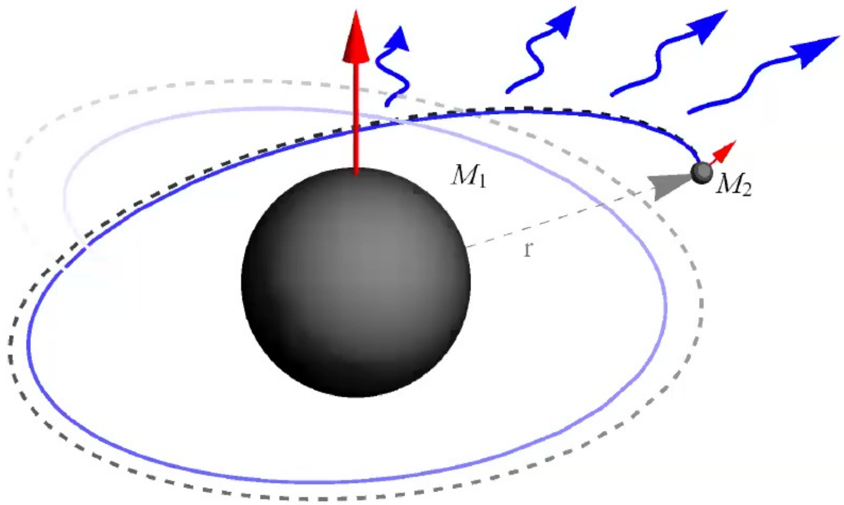
See: Rüter's talk



Small mass-ratio

- Numerical/Analytical
- Black hole perturbation theory
- LO: geodesics
- Works for complete Inspiral-Merger-Ringdown

The canonical setup: Extreme Mass Ratio Inspiral



Primary object

- Kerr black hole
- mass M_1 (typical: $10^6 M_\odot$)
- spin $a_1 = S_1/M_1$

Secondary object

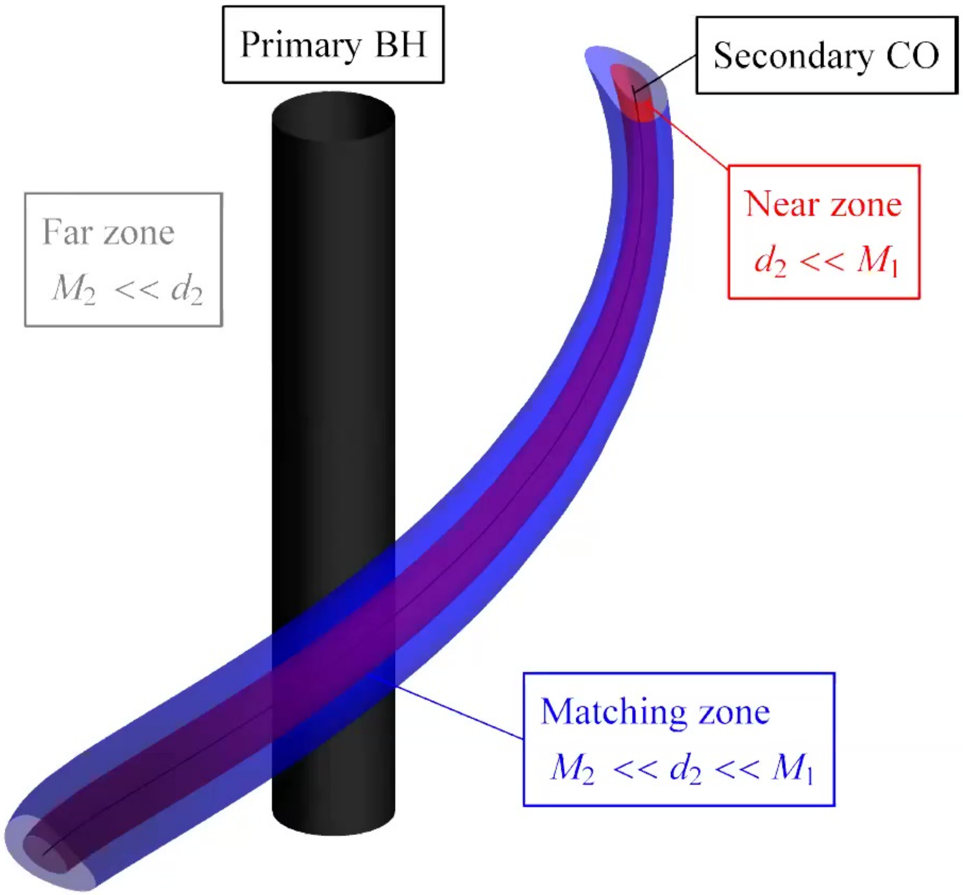
- Compact object (black hole/neutron star/white dwarf/ECO)
- mass M_2 (typical: $1 - 30 M_\odot$)
- size $\mathcal{O}(M_2)$
- spin $a_2 = S_2/M_2 = \mathcal{O}(M_2)$

Notations/Conventions

- $G = c = 1$ (What else would they be?)
- $q = M_2/M_1 \ll 1$ mass-ratio (typical: 10^{-5})
- $\nu = M_1 M_2/(M_1 + M_2)^2 = \mathcal{O}(q)$ symmetric mass-ratio

Equations of Motion: Matched asymptotic expansions

[Mino, Sasaki & Tanaka, 1997] [Poisson, 2003][Pound, 2008-]



Near zone

Geometry of secondary (in rest frame) plus perturbations generated by primary. General solution as expansion in $\delta = d_2/M_1$:

$$g_{\mu\nu}^{\text{near}} = g_{\mu\nu}^{\text{CO}} + \delta h_{\mu\nu}^{\text{near},1} + \delta^2 h_{\mu\nu}^{\text{near},2} + \mathcal{O}(\delta^3)$$

Far zone

Kerr geometry of primary plus perturbations generated by secondary. General solution as expansion in q:

$$g_{\mu\nu}^{\text{far}} = g_{\mu\nu}^{\text{Kerr}} + q h_{\mu\nu}^{\text{far},1} + q^2 h_{\mu\nu}^{\text{far},2} + \mathcal{O}(q^3)$$

Matching

In the matching zone both approximations are valid, and the two expansions can be matched.

Equations of Motion: Matched asymptotic expansions II

[Mino, Sasaki & Tanaka, 1997] [Poisson, 2003][Pound, 2008-]

Results

Coupled equations for metric:

$$g^{\text{far}}_{\mu\nu} = g^{\text{Kerr}}_{\mu\nu} + q h^1_{\mu\nu}[z^\mu] + q^2 h^2_{\mu\nu}[z^\mu] + \mathcal{O}(q^3)$$

and secondary is parallel transported along an effective worldline in the background metric

$$\frac{D}{d\tau} z^\mu(\tau) = q F^\mu_1[h^1] + q^2 F^\mu_2[h^1, h^2] + \mathcal{O}(q^3)$$

Leading order

- Metric is given by the Kerr metric of the primary.
- The effective worldline of the secondary is given by a geodesic.

First order

- The first order metric perturbation $h^1_{\mu\nu}$ is sourced by a point mass M_2.
- Split into a singular $h^{1,S}_{\mu\nu}$ and regular part $h^{1,R}_{\mu\nu}$ [Detweiler&Whiting, 2003]
- Effective worldline is a geodesic in the effective spacetime $g^{\text{Kerr}}_{\mu\nu} + q h^{1,R}_{\mu\nu}$,
- corrected by the Mathisson-Papapetrou-Dixon force due to a_2.

Second order See: Pound's talk

- $h^2_{\mu\nu}$ is sourced by $h^1_{\mu\nu}$, M_2, and a_2.
- Can't use point source, solve directly for $h^{2,R}_{\mu\nu}$.
- Effective worldline is a geodesic in the effective spacetime $g^{\text{Kerr}}_{\mu\nu} + q h^{1,R}_{\mu\nu} + q^2 h^{2,R}_{\mu\nu}$,
- corrections due to spin and quadrupole coupling to background.

Evolution and waveforms: two-timescale expansion [Hinderer&Flanagan, 2008][MvdM&Warburton, 2018] [Pound&Wardell, 2021][Miller&Pound, 2021]

Two approaches:

1. Self-consistent evolution

Simultaneous solve for both worldline and metric in time domain. Issues:

- Computationally expensive (=> slow!)
- Tricky numerical stability over $\mathcal{O}(10^5)$ orbital cycles.

2. Two-timescale expansion

Split dynamics in processes that happen on the orbital timescale ($\mathcal{O}(M_1)$) and processes that happen on the much longer inspiral timescale ($\mathcal{O}(M_1^2/M_2)$).
Advantages:

- Strain and gravitational self-force can be calculated as “snapshots” starting from a geodesic.
- Given precomputed strain and self-force data covering the parameter space, inspirals and waveforms can be generated efficiently and robustly.

Evolution and waveforms: Post-Adiabatic expansion [Hinderer&Flanagan, 2008][MvdM&Warburton, 2018] [Pound&Wardell, 2021][Miller&Pound, 2021]

When expanding the waveform in powers of q, the separation of timescales leads to reordering of the terms with resprect to the equations of motion. Secular dissipative terms are promoted relative to their conservative and oscillatory counterparts.

Adiabatic order (0PA)

- Accumulated phase error over inspiral $\mathcal{O}(q^{-1})$.
- Needs: strain generated by geodesic.
- Needs: orbited averaged dissipative part of F_1 ("Fluxes").

1-post adiabatic order (1PA)

- Accumulated phase error over inspiral $\mathcal{O}(q^0)$.
- Needs: full F_1
- Needs: orbited averaged dissipative part of F_2.
- Needs: MPD corrections due to a_2

2-post adiabatic order (2PA)

- Accumulated phase error over inspiral $\mathcal{O}(q)$.
- Needs: second order correction to the snapshot strain.
- Needs: full F_2 and orbited averaged dissipative part of F_3.
- Needs: corrections due to a_2 and secondary quadrupole.

Evolution and waveforms: Orbital Resonances

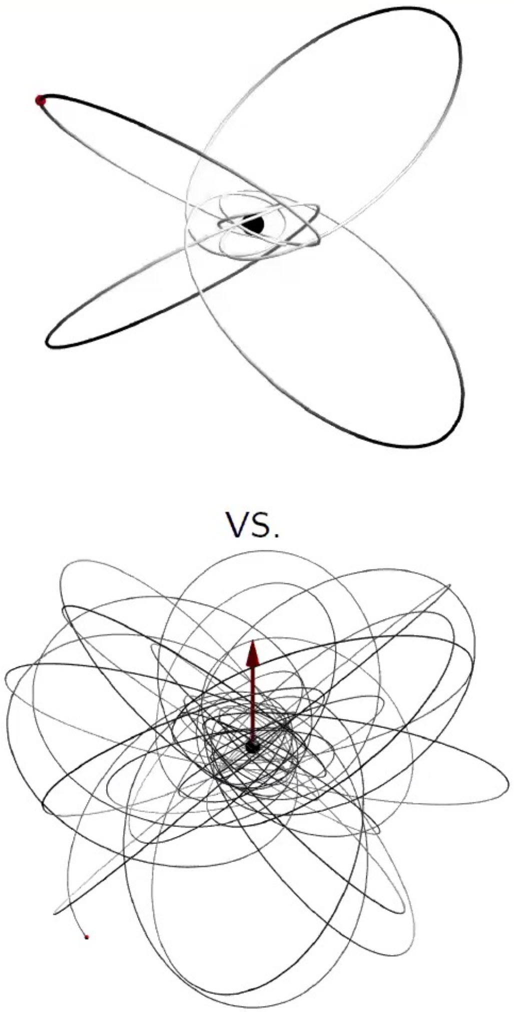
- Occur when integer combination $n\Omega_r + k\Omega_\theta + m\Omega_\phi$ vanishes.
- Temporarily transforms oscillatory effect into secular effect.
- Introduces a third time-scale $\mathcal{O}(q^{-1/2})$ in the dynamics.

Types:

- r-θ [Flanagan&Hinderer, 2012]
- r-ϕ [MvdM, 2014]
- ϕ-θ [Hirata, 2011]

Drivers:

- Gravitational self-force [Flanagan&Hinderer, 2012]
- Astrophysical perturbations e.g. [Yang&Casals, 2017][Bonga et al., 2019]
- Deviations from Kerr e.g. [Lukes-Gerakopoulos et al., 2010]



Duration:

- Transient
- Sustained/Locked [MvdM, 2013]

Effects:[Flanagan&Hinderer, 2012]

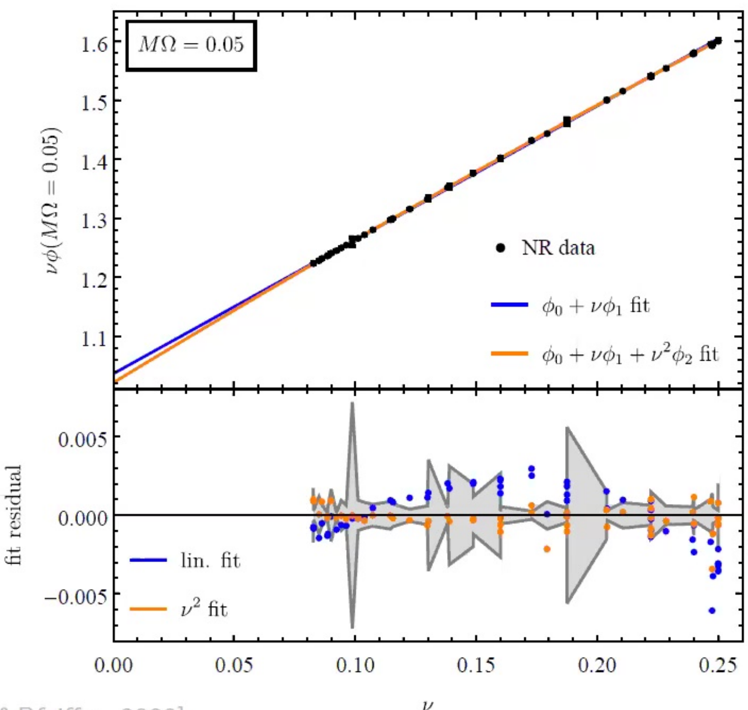
- Resonant episode causes $\mathcal{O}(\epsilon q^{-1/2})$ "kick" to orbital parameters.
- Grows to a $\mathcal{O}(\epsilon q^{-3/2})$ phase correction over the remaining inspiral
- Size of kick depends on orbital phases. ("Phase microscope")

This Capra...

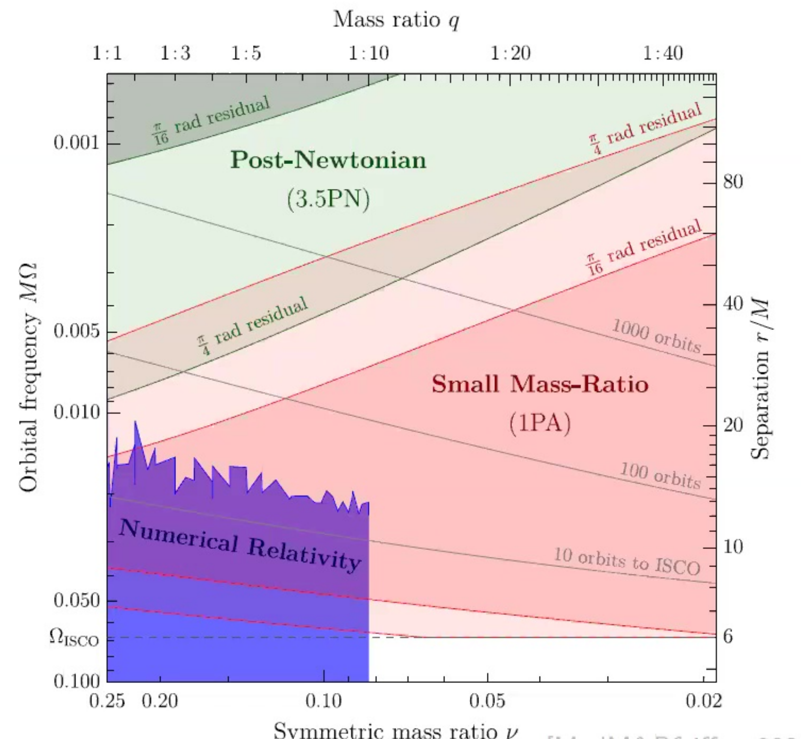
See: Nasipak's talk
See: Speri's talk
See: Gupta's talk

How many PA orders are needed? [MvdM&Pfeiffer,2020]

- How far do we have to go in the PA expansion to obtain result that are accurate enough to for use in gravitational wave observations? How small does ν have to be? Is $\nu = 1/4$ (equal mass) "small"?
- Can get an idea by looking at quasicircular non-spinning numerical relativity simulations.



[MvdM&Pfeiffer, 2020]



[MvdM&Pfeiffer, 2020]

Take home

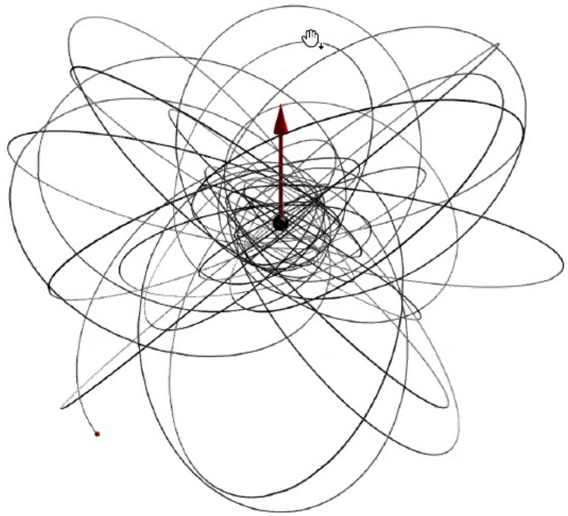
- 0PA is insufficient at any mass-ratio
- 2PA (and higher contributions) are small
- Effect of eccentricity? See: Ramos-Baudes's talk

Status of 0PA: Geodesics, Strains, and Fluxes

Geodesics

Closed form analytic expressions for bound geodesics available, including analytic expressions for orbital frequencies etc.[Carter, 1968] [Schmidt, 2002] [Fujita&Hikida, 2009] [Hackmann et al., 2008-] [MvdM, 2019]

See: Hackmann's talk



Strain

The gravitiational wave strain for a distant observer can be obtained by solving the Teukolsky equation with a point mass source following a geodesic.
(This sufficient for 1PA waveforms.)

"Fluxes"

- Orbit averaged changes in E and L, can be obtained directly from solution to the Teukolsky equation [Teukolsky&Press, 1974].
- Changes to Q can also be obtained. [Sago et al., 2005][Isoyama et al., 2019]

See: Grant's talk

Available codes:

Currently three Teukolsky Frequency domain codes can handle generic orbits. (That I know of.)

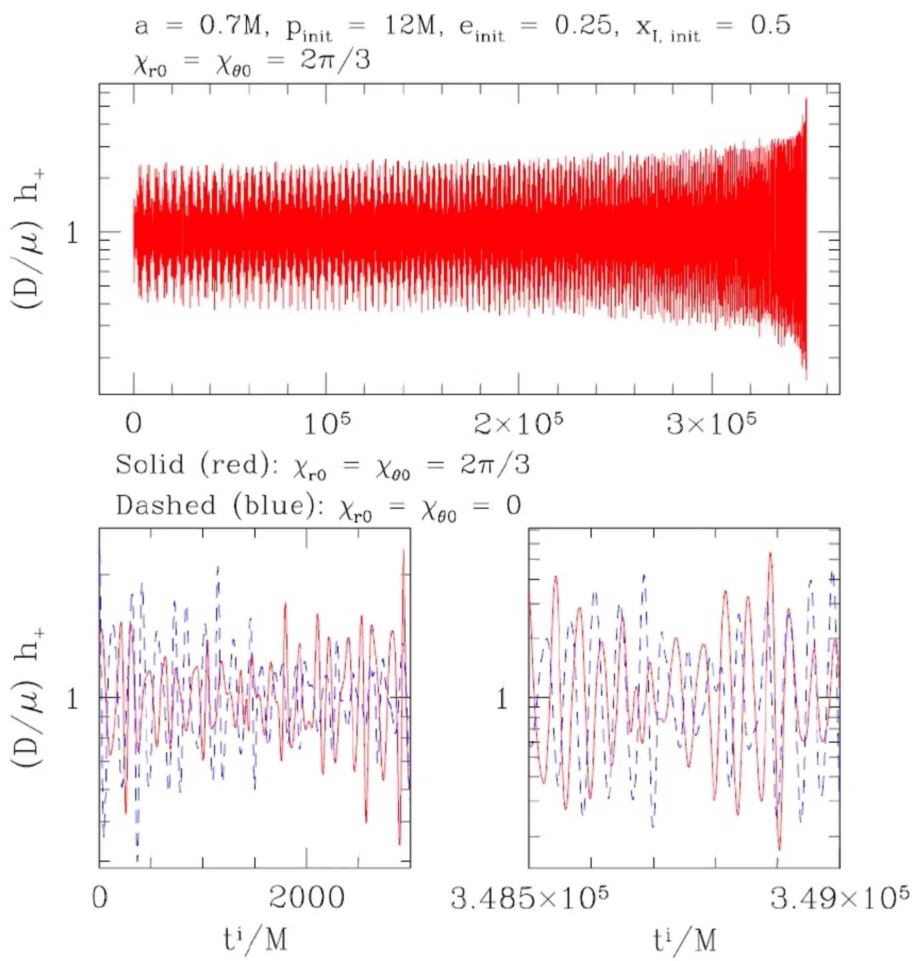
- GREMLIN [Drasco&Hughes, 2006]
- "Kyoto" code [Fujita et al., 2009]
- ModGEMS [MvdM, 2018]

Tasks:

- Cover parameter space with flux and strain data
- Interpolate
- Generate inspiral trajectory
- Generate waveform

Progress:

- "Fast EMRI Waveforms" (FEW) framework (currently non-spinning, eccentric) [Chua et al., 2020] [Katz et al., 2021]
- First generic adiabatic inspiral [Hughes et al., 2021]



[Hughes et al., 2021]

This Capra

See: Burke's talk
See: Islam's talk
See: Isoyama's talk
See: Küchler's talk

Status of 1PA: First-order gravitational self-force

"Regularization"

- Mode sum [Barack, 2001]
- Effective Source ("puncture") [Barack&Golbourn, 2007]
- Green's Function [Mino,Sasaki,Tanaka, 1997] See: Aruquipa's talk & See: O'Toole's talk

Mode decompositions

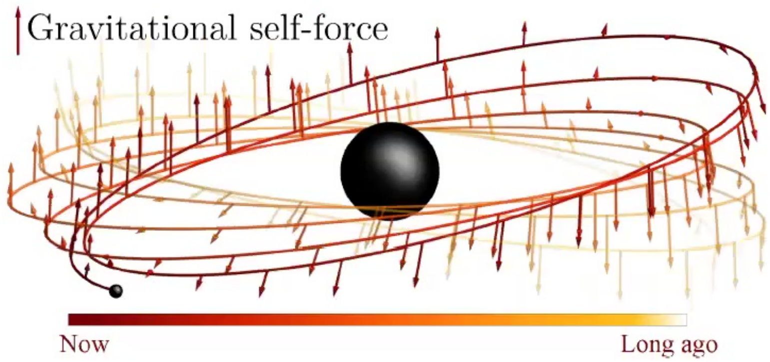
- Time domain [Barack&Lousto, 2002]
  See: Gomes Da Silva's talk & See: Markakis's talk & See: Long's talk
- Frequency domain [Detweiler, 2008]
  See: Nishimura's talk & See: Osburn's talk

Gauge

- Lorenz See: Nishimura's talk & See: Osburn's talk
- Regge-Wheeler/Zerilli See: Gomes Da Silva's talk & See: Markakis's talk
- Radiation See: Long's talk

History of progress

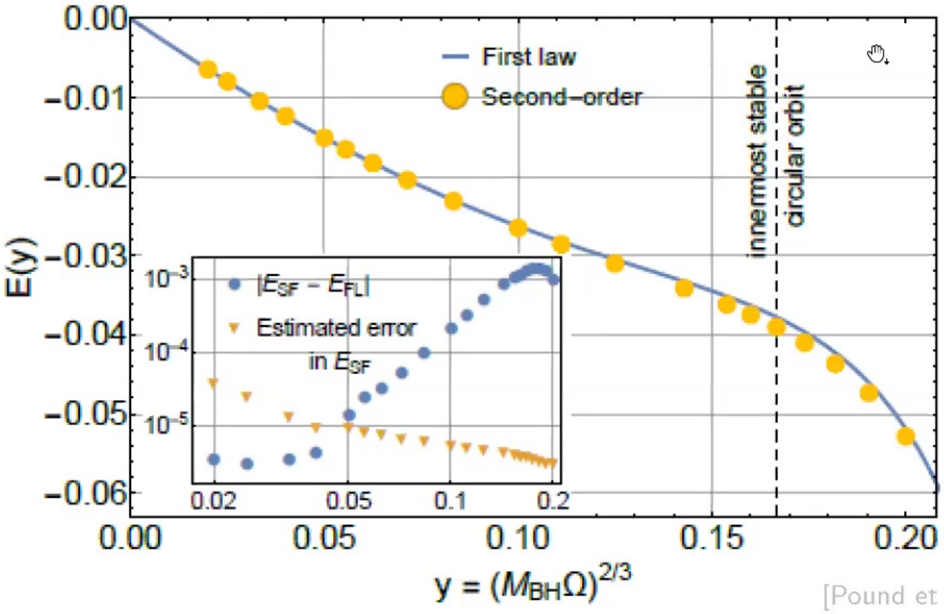
- 2007 quasicircular, non-spinning [Barack&Sago, 2007]
- 2011 Eccentric, non-spinning [Barack&Sago, 2010]
- 2012 Circular, spinning, non-precessing [Shah et al., 2012][Barack&Dolan, 2012]
- 2015 Eccentric, spinning, non-precessing [MvdM&Shah, 2015]
- 2018 Eccentric, spinning, and precessing [MvdM, 2018]



Status of 1PA: Second-order gravitational self-force See: Pound's talk

The Final Frontier

- The second order gravitational self-force is the last missing piece needed to obtain full 1PA waveforms.
- General framework in place
- First results for quasi-circular, non-spinning [Pound et al., 2020]
- See: Pound's talk



[Pound et al., 2020]

This Capra

See: Durkan's talk
See: Green's talk
See: Kavanagh's talk
See: Leather's talk
See: Pound's talk
See: Spiers's talk
See: Toomani's talk
See: Upton's talk
See: Warburton's talk
See: Wardell's talk
See: Zimmerman's talk

Status of 1PA: Effects of secondary spin

Parallel Transport

- Leading order equation of motion for secondary spin
- Closed form analytic solutions known.
  [Marck, 1983][Ruangsri, 2016][MvdM, 2019]

Self-Torque

- At first order in ν the spin is parallel transported in the effective metric, resulting in an effective gravitational self-torque.
- Gauge invariant correction to the spin precession, calculated for
  - circular orbits Schwarzschild [Dolan et al., 2013]
  - eccentric orbits Schwarzschild [Akcay et al., 2016]
  - circular equatorial orbits Kerr [Bini et al.&MvdM, 2019]
- To Do:
  - generic orbits in Kerr
  - full self-torque

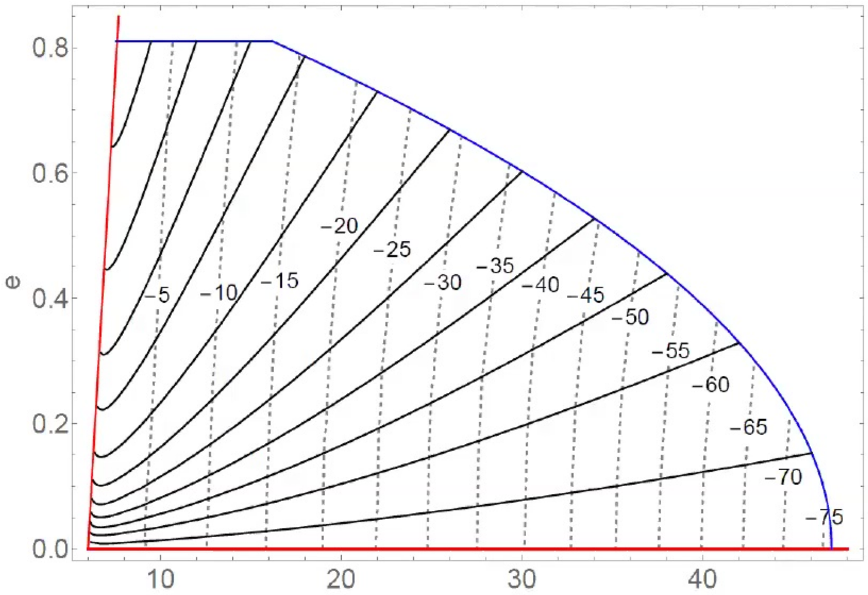
MPD force

- Conservative correction to EoMs due to secondary spin are known analytically.
- Many studies studying their impact. This Capra:
  - See: Druart's talk
  - See: Timogiannis's talk

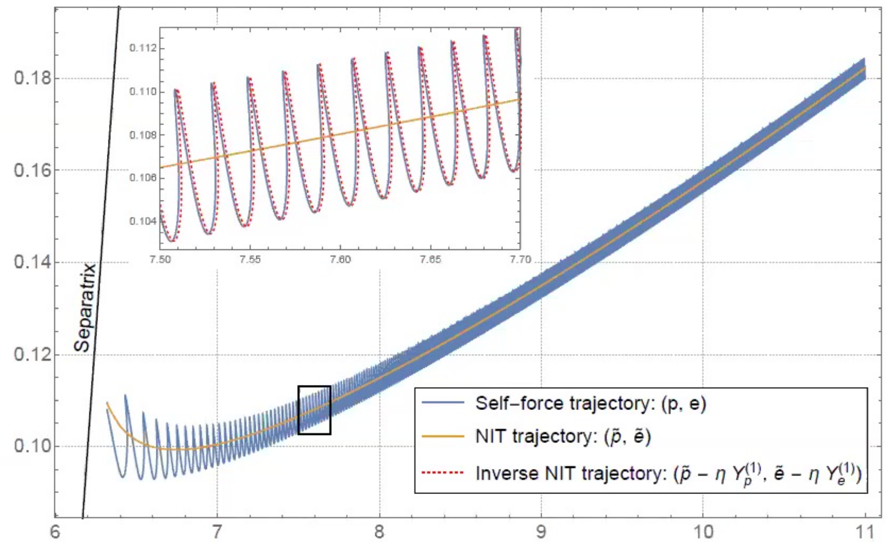
Spin corrections to GSF

- Since $a_2 = \mathcal{O}(m_2)$, the effective metric see the first secondary spin contributions at $\mathcal{O}\, q^2$.
- Corrections to fluxes have been calculated
  - Circular Schwarzschild [Akcay et al., 2020]
  - Eccentric equatorial Kerr
    [Skoupý&Lukes-Gerakopoulous, 2021] See: Skoupý's talk
- Conservative effects See: Mathews's talk

- Consistent 1PA evolution requires 2nd order GSF
- Evolution with just 1st order conservative GSF has been explored for non-spinning binaries. [Warburton et al., 2012][Osburn et al., 2015]
- Kerr? See: Lynch's talk



[Osburn et al., 2015]



[MvdM&Warburton, 2018]

Near-Identity-Transforms (NITs)

- Evolving inspirals with the full self-force is slow, because all $\mathcal{O}(10^5)$ cycles need to be resolved.
- Can be solved semi-analytically using NITs to obtain fast and robust evolution. [MvdM&Warburton, 2018]


That's all Folks!