Title: Progress in Green Function Methods for Extreme Mass Ratio Inspirals

Speakers: Conor O'Toole

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

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Abstract: We present an update on Green function methods for modelling Extreme Mass Ratio Inspirals. In particular, we present an accurate, efficient and robust procedure for computing the Green functions of the Regge-Wheeler and Zerilli equations, and show the application to the computation of self-force and energy flux results, considering a range of sample orbits from circular geodesics to unbound encounters. In addition, we demonstrate further possible applications and improvements to the method, including progress in extending it beyond the Regge-Wheeler-Zerilli formalism.

Progress in Green Function Methods for Extreme Mass Ratio Inspirals

Conor O'Toole
Adrian Ottewill, Barry Wardell

Green Functions in Black Hole Spacetimes

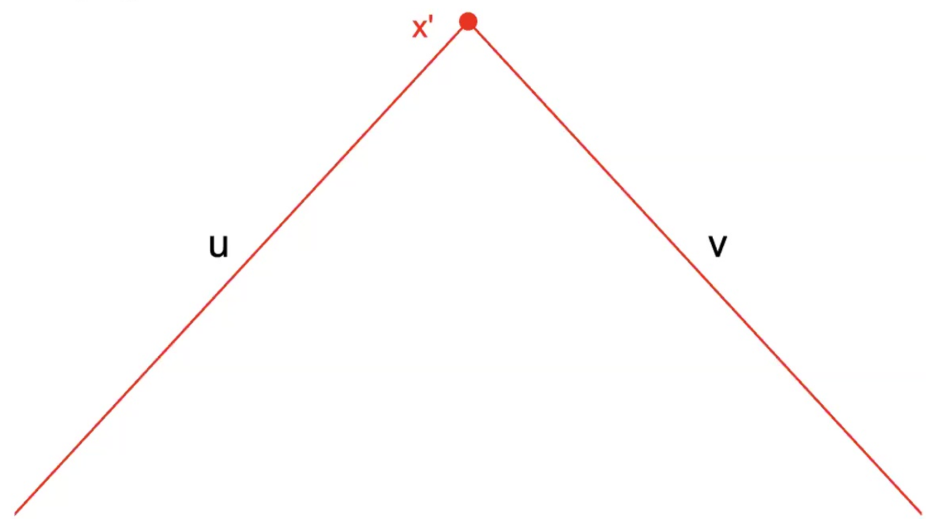
$$F^{\alpha'} = q^2 g^{\alpha'\beta'} \lim_{\epsilon \to 0^+} \int_{-\infty}^{\tau' - \epsilon} \nabla_{\beta'} G(x(\tau'), x(\tau)) d\tau$$

Mino, Sasaki, Tanaka, *1997, Phys. Rev. D* **55**
Quinn, Wald, *1997, Phys. Rev. D* **56**
Detweiler, Whiting, *2003, Phys. Rev. D* **67**

Capra 22

(arxiv:2010.15818)

$$(\partial_{uv}^2 + P)G_\ell^{\mathrm{ret}} = 0$$



Barack, Sago, *2007, Phys. Rev. D* **75**

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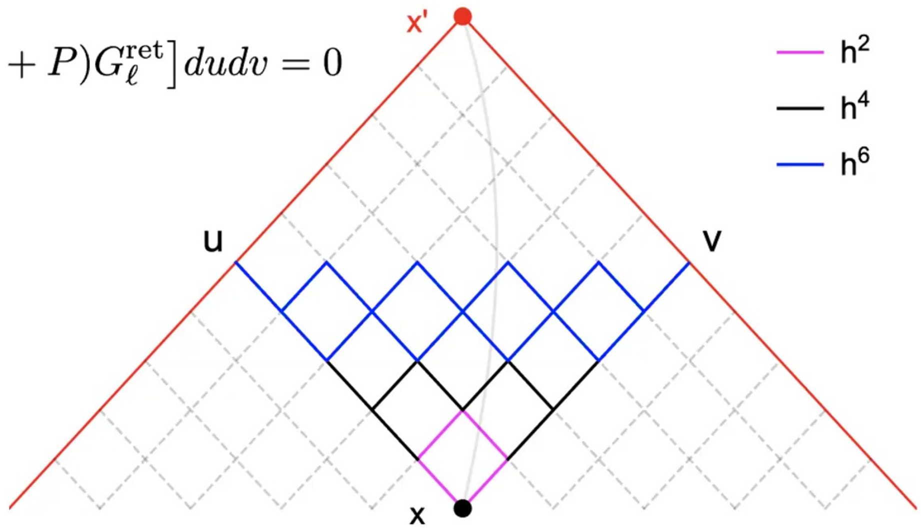
Capra 22

(arxiv:2010.15818)

$$(\partial_{uv}^2 + P)G_\ell^{\rm ret} = 0$$

$$\int\int \left[(\partial_{uv}^2 + P)G_\ell^{\rm ret}\right] du dv = 0$$



Barack, Sago, *2007, Phys. Rev. D* **75**

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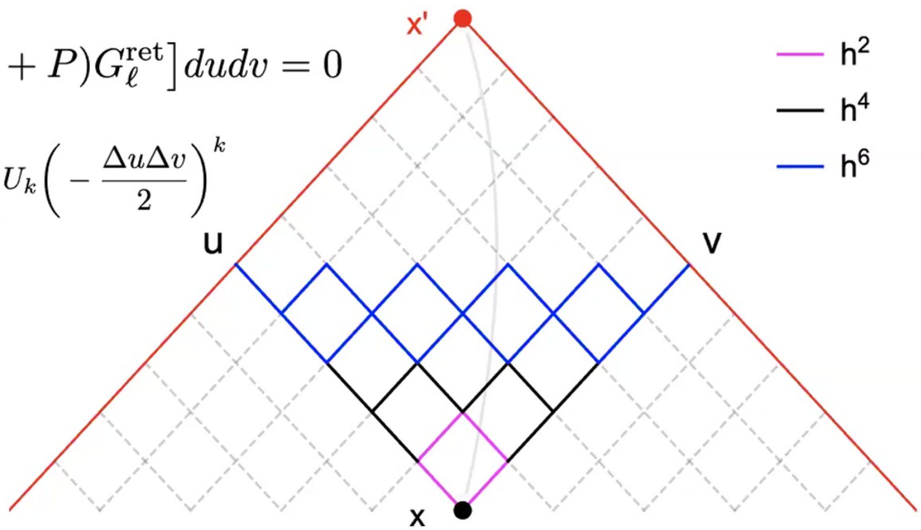
Capra 22

(arxiv:2010.15818)

$$(\partial^2_{uv} + P)G^{\rm ret}_\ell = 0$$

$$\int\int \left[(\partial^2_{uv} + P)G^{\rm ret}_\ell\right] du dv = 0$$

$$G^{\rm ret} = \sum_{k=0}^{\infty} U_k \left(- \frac{\Delta u \Delta v}{2} \right)^k$$



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Capra 22
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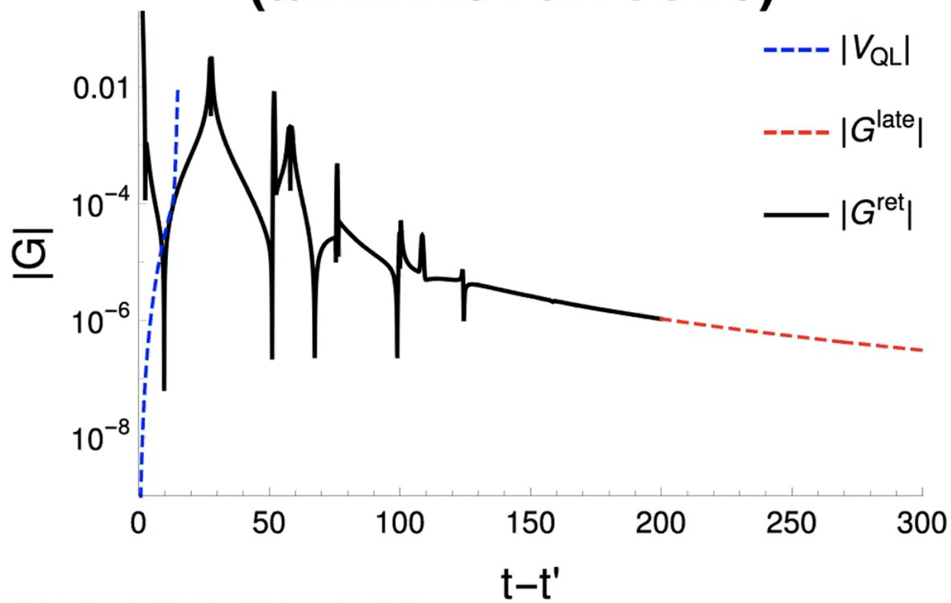

Casals, Dolan, Ottewill, Wardell, *2009, Phys. Rev. D* **79**

Casals, Ottewill, *2015, Phys. Rev. D* **92**

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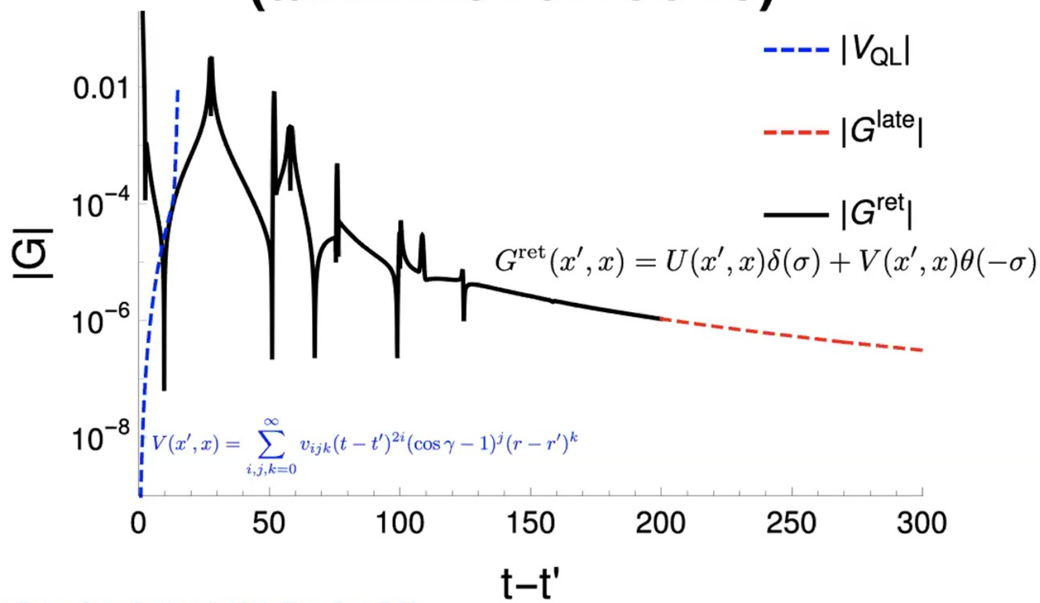
Capra 22

(arxiv:2010.15818)



Casals, Dolan, Ottewill, Wardell, *2009, Phys. Rev. D* **79**

Casals, Ottewill, *2015, Phys. Rev. D* **92**

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Capra 22

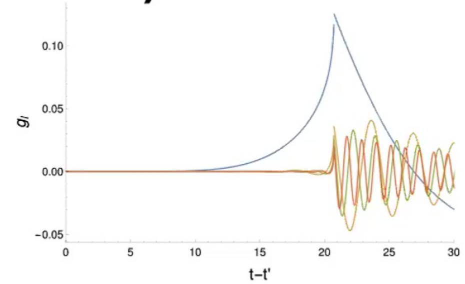
(arxiv:2010.15818)

	Reference	Num. Result	Rel. Err. (%)
Φ	-0.00545483	-0.00545526	0.0079
F_t	0.00036091	0.000360997	0.0249
F_r	0.00016773	0.000167698	0.0179
F_ϕ	-0.00530423	-0.00530738	0.0593

r=6M
100 modes
QL matching

Casals, Dolan, Ottewill, Wardell, *2009, Phys. Rev. D* **79**
Casals, Ottewill, *2015, Phys. Rev. D* **92**

Improvements
(arxiv:2010.15818)



- QL Matching -> Direct mode subtraction
- Circular Orbits
- RWZ formalism
- C Implementation

Casals, Nolan, Ottewill, Wardell, *2019, Phys. Rev. D* **100**
Hopper, Evans, *2010, Phys. Rev. D* **82**

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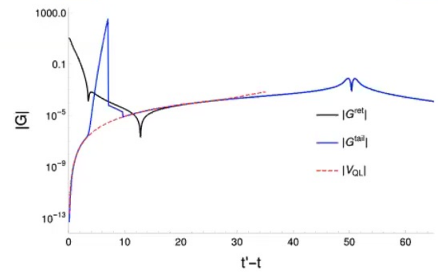
Improvements
(arxiv:2010.15818)

- QL Matching + Direct mode subtraction



- Circular Orbits

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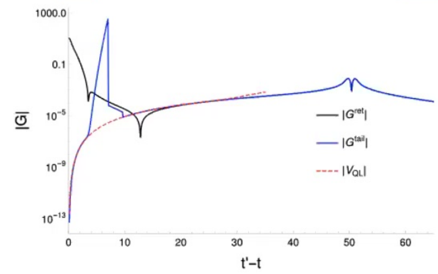
- C Implementation

Casals, Nolan, Ottewill, Wardell, *2019, Phys. Rev. D* **100**
Hopper, Evans, *2010, Phys. Rev. D* **82**

Improvements

(arxiv:2010.15818)

- QL Matching + Direct mode subtraction



- Circular Orbits —> Generic Orbits

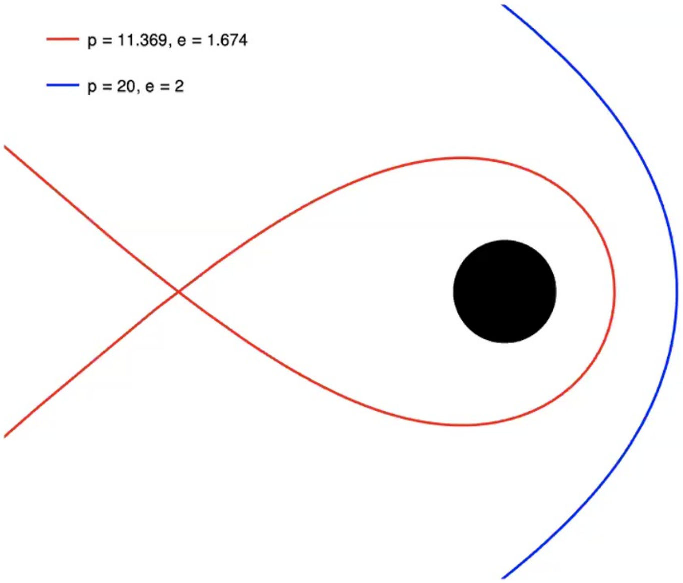
- RWZ formalism —> Fluxes

	Reference	Num. Result	Rel. Err. (%)
$\dot{E}_{21}$	9.719×10^{-8}	9.833×10^{-8}	1.17
$\dot{E}_{22}$	2.685×10^{-5}	2.753×10^{-5}	2.54

- C Implementation

Casals, Nolan, Ottewill, Wardell, *2019, Phys. Rev. D* **100**
Hopper, Evans, *2010, Phys. Rev. D* **82**

Applications: Hyperbolic Encounters



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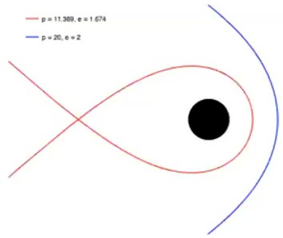
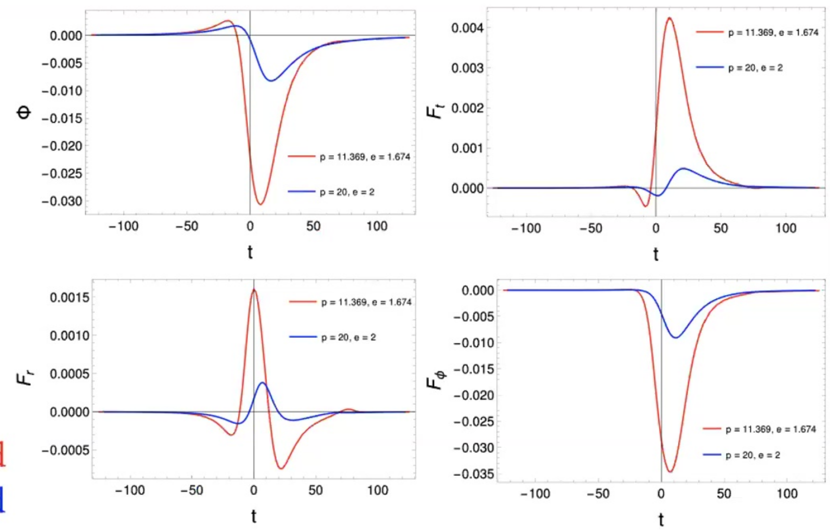
Applications: Hyperbolic Encounters

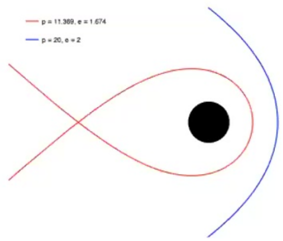




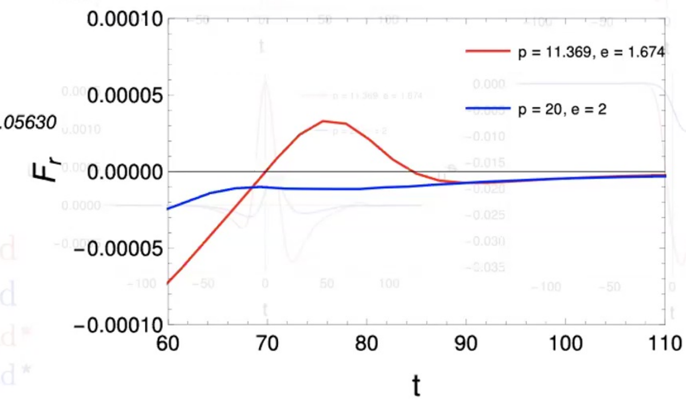
$\delta\phi^{(0)} \sim 4.678 \text{ rad}$

$\delta\phi^{(0)} \sim 2.2362 \text{ rad}$

Applications: Hyperbolic Encounters



Long, Barack, *2021, arxiv:2105.05630*



$\delta\phi^{(0)} \sim 4.678$ rad

$\delta\phi^{(0)} \sim 2.2362$ rad

$\delta\phi^{(1)} \sim -0.0514$ rad*

$\delta\phi^{(1)} \sim -0.0013$ rad*

* - Preliminary

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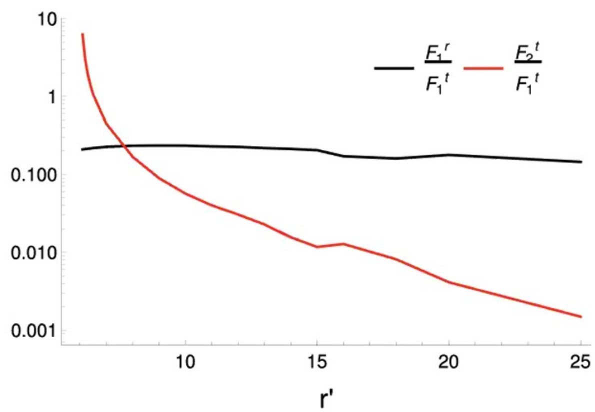
Applications: Beyond First Order
(With Adam Pound)

$$x(\tau) = x_0(\tau) + \epsilon x_1(\tau) + \ldots$$

$$\Rightarrow \mathcal{F}_2 \sim \int_{-\infty}^{\tau^-} x^{\mu}_{1\perp} G_{,\mu'\mu} d\tau' + \int_{-\infty}^{\tau^-} u^{\rho} G_{,\mu\rho} d\tau'$$



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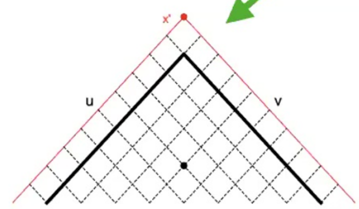
Applications: Beyond Schwarzschild

(With Leor Barack)

$$\psi^{s\ell}_{,uv} + U\psi^{s\ell}_{,u} + V\psi^{s\ell}_{,v} + W\psi^{s\ell}$$
$$-a^2 C_0^{s\ell} K \psi^{s\ell}_{,tt} - a^2 K \left[C^{s\ell}_{++} \psi^{s\ \ell+2}_{,tt} + C^{s\ell}_{+} \psi^{s\ \ell+1}_{,tt} + C^{s\ell}_{-} \psi^{s\ \ell-1}_{,tt} \right.$$
$$\left. + C^{s\ell}_{--} \psi^{s\ \ell-2}_{,tt} \right] + 2iasK \left[c^{s\ell}_{+} \psi^{s\ \ell+1}_{,t} + c^{s\ell}_{+} \psi^{s\ \ell-1}_{,t} \right] = 0$$



Barack, Giudice, *2017, Phys. Rev. D* **95**

Applications: Beyond Schwarzschild

(With Leor Barack)

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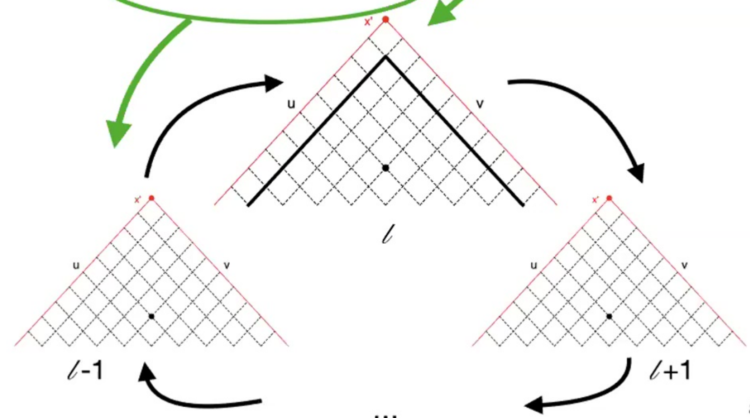
$$\left.+C^{s\ell}_{--}\psi^{s\;\ell-2}_{,tt}\right] + 2iasK\left[c^{s\ell}_{+}\psi^{s\;\ell+1}_{,t} + c^{s\ell}_{+}\psi^{s\;\ell-1}_{,t}\right] = 0$$



Barack, Giudice, *2017, Phys. Rev. D* **95**

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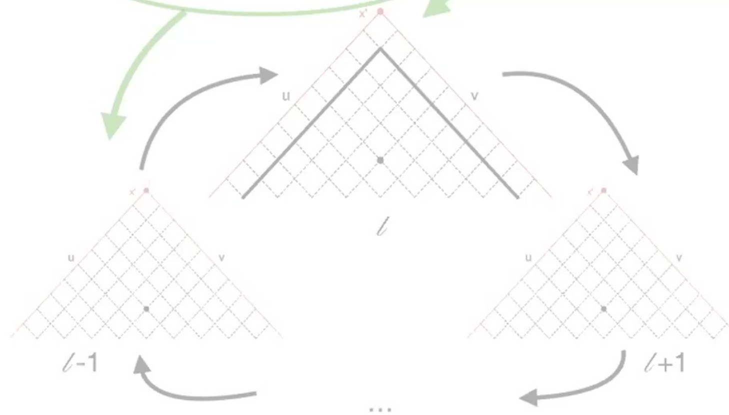
Applications: Beyond Schwarzschild

(With Leor Barack)

$$
\begin{aligned}
&\psi^{s\ell}_{,uv} + U\psi^{s\ell}_{,u} + V\psi^{s\ell}_{,v} + W\psi^{s\ell} \\
&- a^2 C_0^{s\ell} K \psi^{s\ell}_{,tt} - a^2 K \left[C^{s\ell}_{++} \psi^{s\ \ell+2}_{,tt} + C^{s\ell}_{+} \psi^{s\ \ell+1}_{,tt} + C^{s\ell}_{-} \psi^{s\ \ell-1}_{,tt} \right. \\
&\left. + C^{s\ell}_{--} \psi^{s\ \ell-2}_{,tt} \right] + 2iasK \left[c^{s\ell}_{+} \psi^{s\ \ell+1}_{,t} + c^{s\ell}_{+} \psi^{s\ \ell-1}_{,t} \right] = 0
\end{aligned}
$$



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Summary

- Scalar case starting to be effective *at scale*
- Already gravitational applications
- Generic orbits now possible
- Groundwork laid for a range of possible advancements (ie. Capra 25+)
    - GPU implementation
    - Surrogate models
    - RWZ gravitational self-force
    - Self-consistent evolution
    - Lorenz gauge