Title: Flux-balance laws in the Kerr spacetime

Speakers: Alexander Grant

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Abstract: The motion of a radiating point particle in the Kerr spacetime can be represented by a series of geodesics whose constants of motion change slowly over its motion. In the case of energy and axial angular momentum, there are conserved currents, defined for the field, whose fluxes at infinity and the horizon directly determine the evolution of these constants of motion. This relationship between the properties of point-particle motion and fluxes of conserved currents is known as a "flux-balance law". Despite the flux-balance laws for energy and axial angular momentum, the third constant of motion in Kerr, the Carter constant, has no known flux-balance law. While there are conserved currents that can be defined for the field that are, in certain senses, "associated with the Carter constant", the fluxes of these currents are not clearly related to the Carter constant for the particle. In this talk, we present our recent efforts to find such flux-balance laws, in the case of a point particle in Kerr that is coupled to a scalar field.

Flux-balance laws in the Kerr spacetime

Alexander Grant*, Jordan Moxon†

*University of Virginia, †California Institute of Technology

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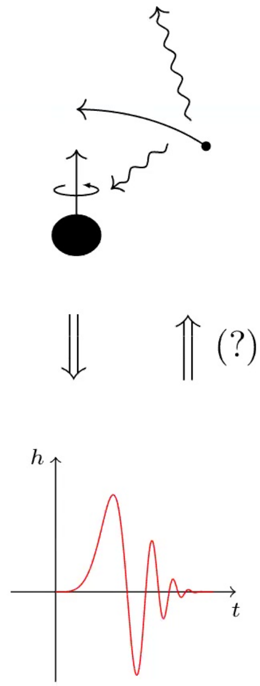
Introduction

- Solution to EMRI problem: radiation during inspiral
- BH perturbation theory:

$$\text{Motion} \implies \text{radiation}$$

- Computing motion (self force) is hard
- Can any portion be computed from radiation?
- Yes, by *flux-balance laws*:

$$\text{Conserved currents} \implies \text{dissipative self force}$$

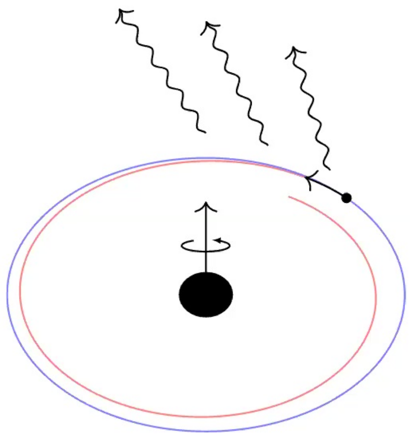


Osculating orbits

- Geodesic motion: three conserved quantities Q
- Non-geodesic motion: can be approximated by "osculating" geodesics, with varying "conserved quantities":

$$\underbrace{\frac{\mathrm{d}Q}{\mathrm{d}\tau}}_{\text{instantaneous}} \implies \underbrace{\Delta Q}_{\substack{\text{averaged over orbit} \\ \text{(adiabatic approx.)}}}$$

- How to find $Q(\tau)$?



The origin of flux-balance

- Conserved quantities from isometries:

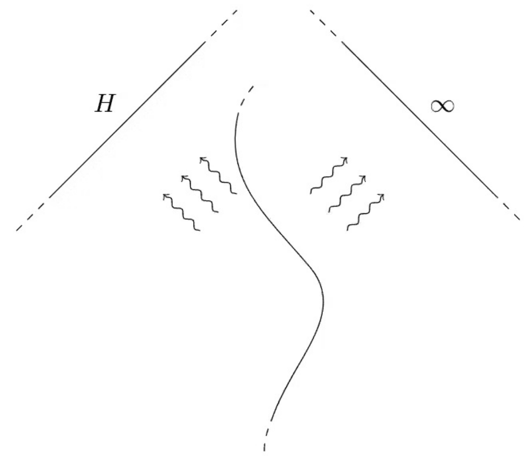
$$\underbrace{Q = \xi^a p_a}_{\text{conserved quantity}} \iff j^a_{\text{part.}} \equiv \underbrace{T^{ab}_{\text{part.}} \xi_b}_{\text{conserved current}}$$

- Particle & field contribute to T^{ab}:

$$T^{ab} = T^{ab}_{\text{part.}} + T^{ab}_{\text{field}} \implies \nabla_b T^{ab} = 0$$

- Flux-balance law:

$$\Delta Q = -\left(\int_H j^a_{\text{field}} \mathrm{d}\Sigma_a + \int_\infty j^a_{\text{field}} \mathrm{d}\Sigma_a \right)$$



The Carter constant

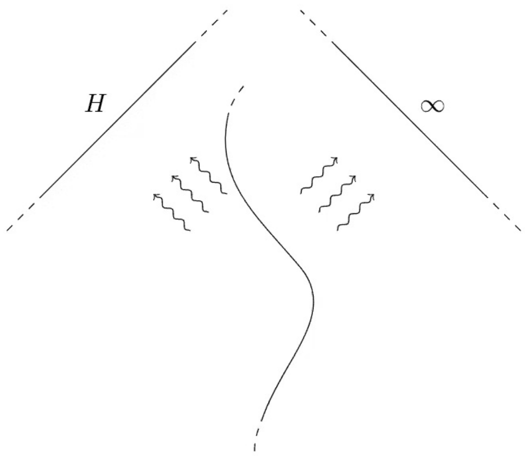
- Kerr only has *two* isometries, t^a & ϕ^a:

$$E \equiv -t^a p_a, \qquad L_z \equiv \phi^a p_a$$

- Third conserved quantity, the "Carter constant":

$$K \equiv K_{ab} p^a p^b \qquad (K_{ab} \text{ a "Killing tensor"})$$

- *No* conserved current constructable from K_{ab} & T^{ab}, for arbitrary theories [G & Flanagan, 2015]
- Note: ΔK can be computed from radiation, despite *no known flux-balance law* (e.g., [Mino, 2003] & [Isoyama et al., 2018])
- Goal of this work: why?



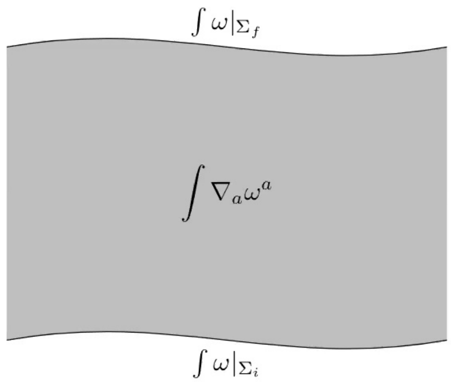
Non-stress-energy currents

- Linear perturbations to any field theory Φ possess conserved, "symplectic" current $\omega^a(\delta_1\Phi, \delta_2\Phi)$
- (Linear) scalar fields: ω^a is the Klein-Gordon current:

$$\omega_a(\phi_1, \phi_2) = \frac{1}{2}\left(\phi_1 \nabla_a \phi_2 - \phi_2 \nabla_a \phi_1\right)$$

- Conserved for "vacuum" solutions ϕ_1, ϕ_2:

$$\nabla_a \omega^a = \frac{1}{2}\left(\phi_1 \Box \phi_2 - \phi_2 \Box \phi_1\right)$$



Symmetry operators

- Symmetry operator $\mathcal{D}$:

$$\Box \mathcal{D}\phi = \widetilde{\mathcal{D}} \Box \phi \implies \mathcal{D} \text{ maps b/w solutions of } \Box\phi = 0$$

- Quadratic conserved current $\omega^a(\phi, \mathcal{D}\phi)$
- Examples:
  - For isometry ξ,

$$\mathcal{D}_\xi : \phi \mapsto \xi^a \nabla_a \phi$$

  - For Killing tensor K_{ab},

$$\mathcal{D}_K : \phi \mapsto \nabla_a (K^{ab} \nabla_b \phi) \qquad \text{[Carter, 1977]}$$

Symplectic currents and worldline quantities

- Worldline source $\rho = \Box\phi$:

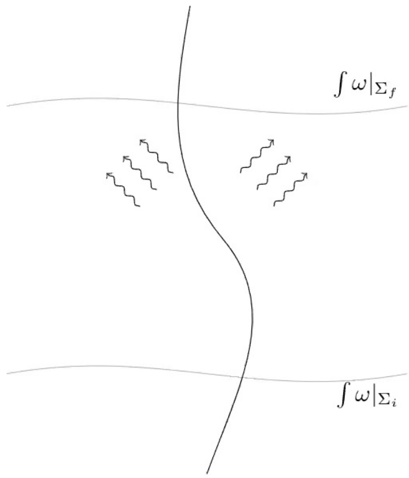
$$\phi(x) = \int G(x, x')\rho(x')\mathrm{d}V'$$

- Radiative solution:

$$\phi^{\mathrm{rad}}(x) = \int \underbrace{\frac{1}{2}[G(x, x') - G(x', x)]}_{\equiv G^{\mathrm{rad}}(x,x')} \rho(x')\mathrm{d}V'$$

- Flux-balance law?

$$\Delta \int \omega^a(\phi, \mathcal{D}\phi)\mathrm{d}\Sigma_a = \int \rho\mathcal{D}\phi^{\mathrm{rad}}\mathrm{d}V + \int (\dots)\mathrm{d}\Sigma$$



Relation to conserved quantities

- Force on a scalar charge: $\dot{p}^a = q\nabla^a\phi^{\mathrm{rad}}$
- Change in E (L_z similar):

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -t^a\dot{p}_a = -qt^a\nabla_a\phi^{\mathrm{rad}} = -q\mathcal{D}_t\phi^{\mathrm{rad}}$$

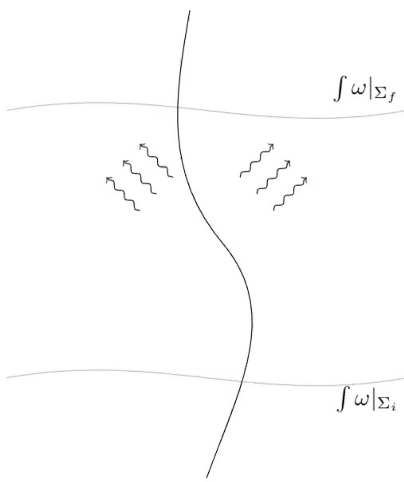
$$\Downarrow$$

$$\Delta E = -\int \rho\mathcal{D}_t\phi^{\mathrm{rad}}\mathrm{d}V = -\Delta\int \omega^a(\phi, \mathcal{D}_t\phi)\mathrm{d}\Sigma_a$$

- Similar result *does not hold* for K:

$$\Delta K = 2\int \rho\, p_a K^{ab}\nabla_b\phi^{\mathrm{rad}}\,\mathrm{d}V$$

$$\Delta\int \omega^a(\phi, \mathcal{D}_K\phi)\mathrm{d}\Sigma_a = \int \rho\,\nabla_a(K^{ab}\nabla_b\phi^{\mathrm{rad}})\mathrm{d}V$$



A new type of symmetry operator

- Field ϕ comes from *worldline source*:

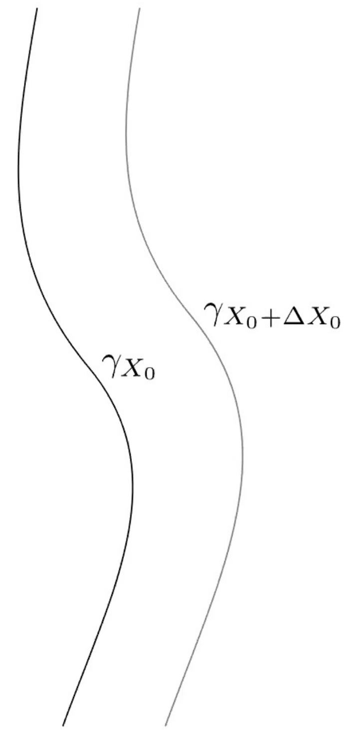
$$\gamma_{X_0}(\tau) \implies \rho_{X_0}(x) \implies \phi_{X_0}(x),$$

  where X_0 is the initial data for the worldline
- $\partial/\partial X_0$ is a symmetry operator:

$$\Box \frac{\partial \phi}{\partial X_0} = \frac{\partial(\Box\phi)}{\partial X_0}$$

- Similar flux-balance-like expression:

$$\Delta \int \omega^a \left(\phi_{X_0}, \frac{\partial \phi}{\partial X_0}\right) \mathrm{d}\Sigma_a = \int \rho_{X_0} \frac{\partial \phi^{\mathrm{rad}}}{\partial X_0} \mathrm{d}V$$



Action-angle variables

- Action-angle variables $(q^A,\, J_A)$ give conserved quantities for geodesics:

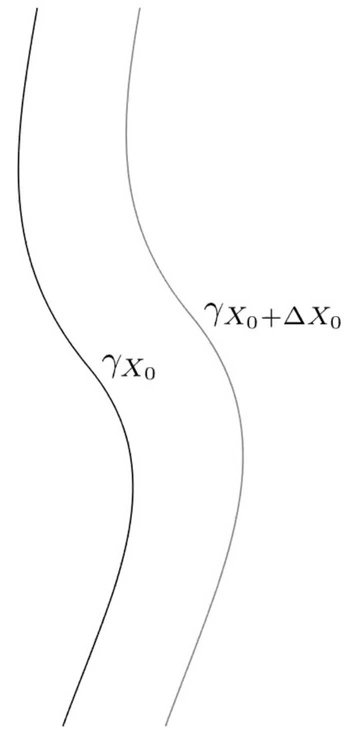
$$\frac{\mathrm{d}J_A}{\mathrm{d}\tau} = -\frac{\partial H}{\partial q^A} = 0$$

- When interaction with the field is added,

$$\Delta J_A = -\int \rho_{X_0} \frac{\partial \phi^{\mathrm{rad}}_{X_0}}{\partial q^A} \mathrm{d}V$$

- Using initial data $(q_0^A, J_{0,A})$ for worldline, this *may* give flux-balance law:

$$\Delta J_A \overset{?}{=} -\Delta \int \omega^a \left(\phi, \frac{\partial \phi}{\partial q_0^A}\right) \mathrm{d}\Sigma_a$$



Conclusions

- The flux-balance law that determines change in Carter constant still unclear
- More productive approach: action-angle variables, instead of conserved quantities?
- Note: this work straightforwardly generalizes to GR
- Approach applicable to all conserved quantities, may provide foundation for second-order flux-balance