Title: Improving Semi-Analytic Spin Precession with NITs

Speakers: Michael LaHaye

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 07, 2021 - 10:30 AM

URL: http://pirsa.org/21060011

Abstract: Semi-analytic solutions are useful because they are much faster than full numerical evolutions by virtue of the fact that they do not have to use as many points to achieve similar levels of accuracy. Currently there exists a semi-analytic solution for spin precessing binaries, which is implemented in LIGO and used to generate waveforms for comparison with gravitational waves. This solution comes with a caveat: it was calculated using precession averaged equations and thus has an oscillating error associated with the unaccounted for precession. This error is large enough that it canâ€™t be overlooked for second and third generation detectors. By using near identity transforms (NITs) to reintroduce the effects of precession to the evolution, we can lower the error significantly. When implementing the NIT we found that the phase of oscillations we introduce (which coincides with the precession phase) does not line up with the precession in the numerical evolution, this causes the NIT to be less effective at reducing the error than it could be. To fix this issue we also introduce corrections to the evolution of the precession phase that were previously overlooked.

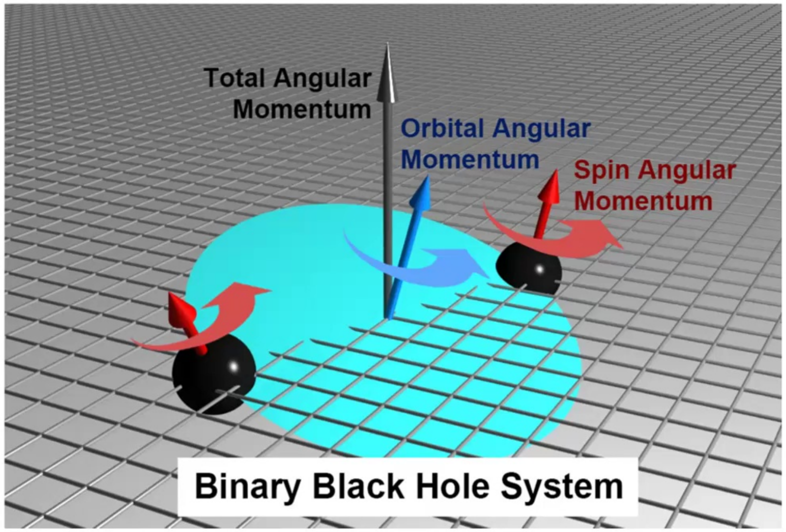
Improving Semi-Analytic Spin Precession with NITs

Michael LaHaye,
Béatrice Bonga,
Huan Yang

University of Guelph

Capra, June 2021

General Idea



Previous Work

Chatziioannou, Klein, Yunes, and Cornish came up with a semi-analytic solution for the spin-precession equations.

The caveat is that this solution is only to the orbit and spin-precession averaged equations.

Because they were not accounting for the spin-precession there is an oscillating error.

NIT

The idea of a near identity transform (NIT) is that we transform the quantities by small, oscillating terms

$$\tilde{L} = L$$
$$\tilde{J} = J + \sum_n \Delta J_n e^{i\Omega_P n t}$$
$$\tilde{S} = S + \sum_n \Delta S_n e^{i\Omega_P n t}$$
$$\tilde{\Phi} = \Phi + \sum_n \Delta \Phi_n e^{i\Omega_P n t}$$

NIT

The idea of a near identity transform (NIT) is that we transform the quantities by small, oscillating terms

$\tilde{L} = L$
$\tilde{J} = J + \sum_n \Delta J_n e^{i\Omega_P n t}$
$\tilde{S} = S + \sum_n \Delta S_n e^{i\Omega_P n t}$
$\tilde{\Phi} = \Phi + \sum_n \Delta \Phi_n e^{i\Omega_P n t}$

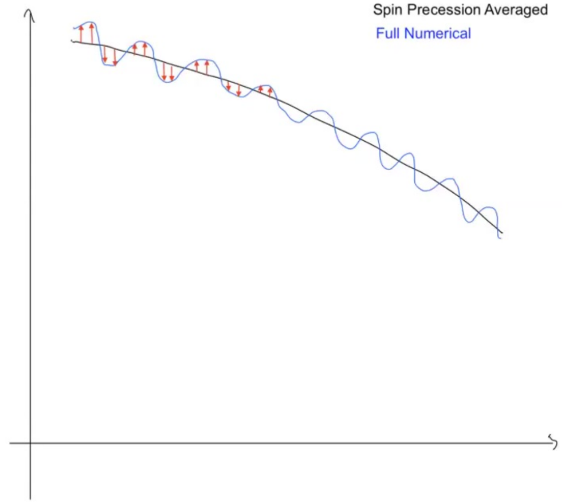
When we want to return, the inverse transformation is simply

$L = \tilde{L}$
$J = \tilde{J} - \sum_n \Delta J_n e^{i\Omega_P n t}$
$S = \tilde{S} - \sum_n \Delta S_n e^{i\Omega_P n t}$
$\Phi = \tilde{\Phi} - \sum_n \Delta \Phi_n e^{i\Omega_P n t}$

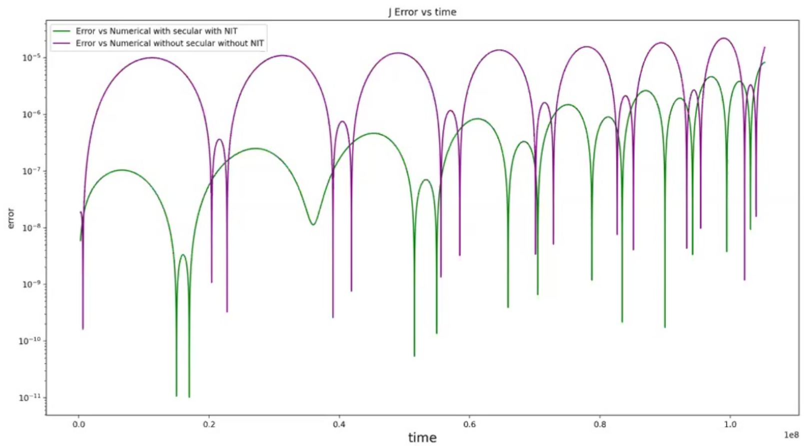
We use these oscillating terms to cancel terms in the equations, making the equations that govern $\tilde{L}, \tilde{J}, \tilde{S}$ and $\tilde{\Phi}$ the spin-averaged equations. There is three main steps to the process

NIT

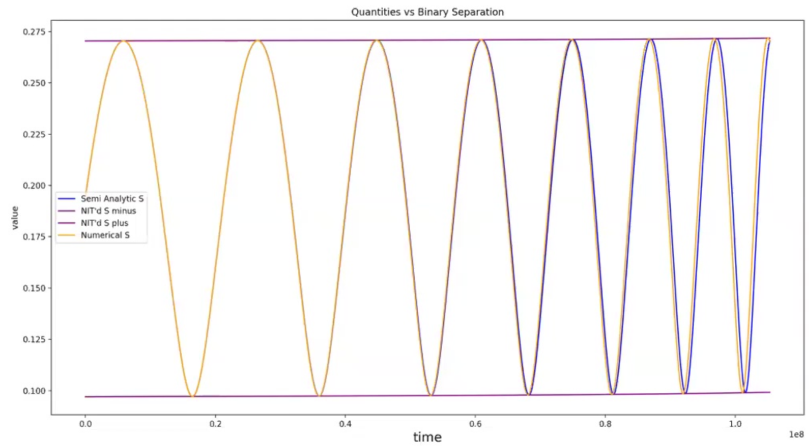
3. Use the inverse NIT to get L, J, S and Φ



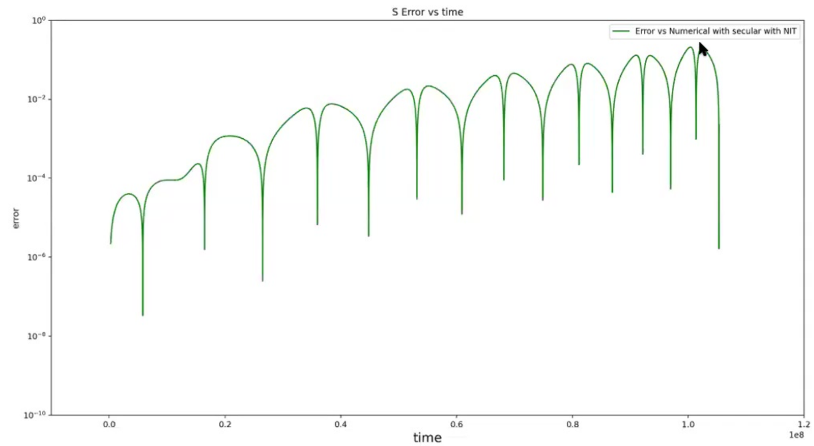
The NIT Doesn't Correct Everything



Characterizing The Error



Characterizing The Error



Correcting the Precession Phase

What is causing this phase error?

To see why we need to understand how Chatziioannou, Klein, Yunes, and Cornish's semi analytic solution for S works.

For reference the equation for S looks like:
$(dS^2/dt^2)^2 = -A^2(S^6 + BS^4 + CS^2 + D)$.

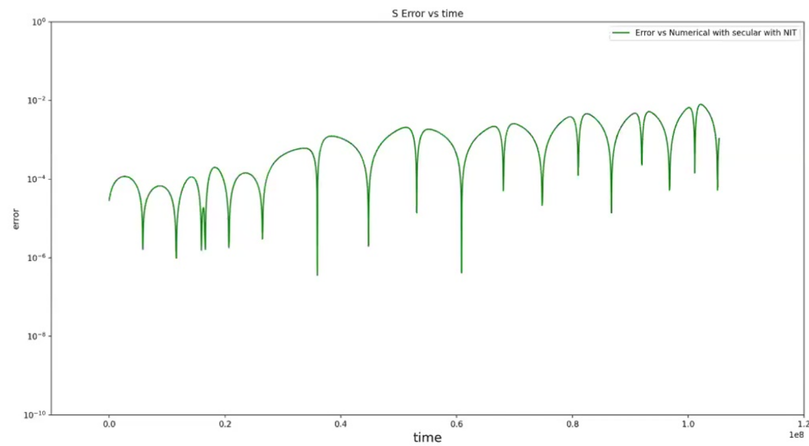
The exact solution to this (when J and L are constant) is
$S^2 = S_+^2 - (S_+^2 - S_-^2)sn^2(\psi, m)$, with $m = (S_+^2 - S_-^2)/(S_+^2 - S_3^2)$ and
$d\psi/dt = A/2\sqrt{S_+^2 - S_3^2}$.

but it really should be

$$\frac{d\psi}{dt} = \frac{A}{2}\sqrt{S_+^2 - S_3^2} - \frac{1}{2m}\frac{dm}{dt}\left(1 - \frac{1}{1-m}\frac{E(m)}{K(m)}\right)\psi$$

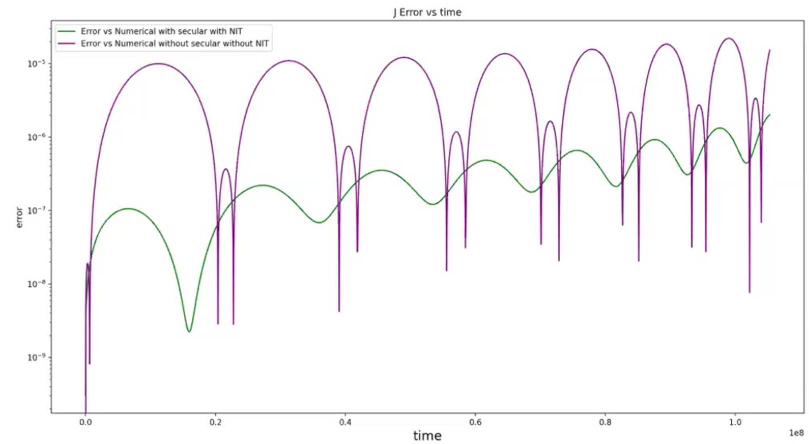
Results So Far

After correcting the phase issue the error in S looks significantly better, it is improved by 3 orders of magnitude in places.

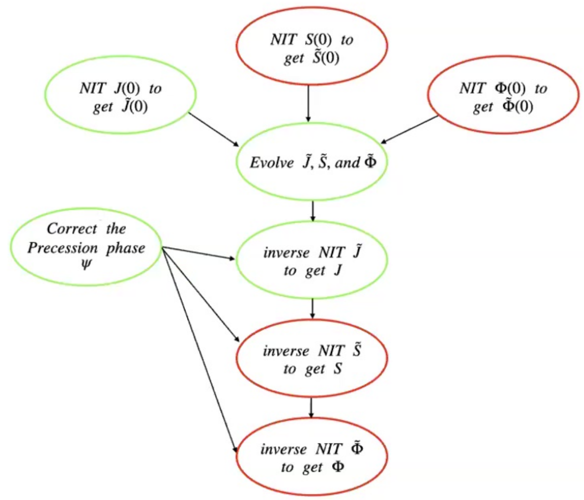


Results So Far

the NIT performs better as well, as a result of the precession phase lining up better



Roadmap



NIT $S(0)$ to get $\tilde{S}(0)$
NIT $J(0)$ to get $\tilde{J}(0)$
NIT $\Phi(0)$ to get $\tilde{\Phi}(0)$
Evolve $\tilde{J}$, $\tilde{S}$, and $\tilde{\Phi}$
Correct the Precession phase ψ
inverse NIT $\tilde{J}$ to get J
inverse NIT $\tilde{S}$ to get S
inverse NIT $\tilde{\Phi}$ to get Φ

Thanks for listening!

Questions?