Title: Fast Self-Forced Inspirals into a Rotating Black Hole

Speakers: Philip Lynch

Collection: The 24th Capra meeting on Radiation Reaction in General Relativity

Date: June 07, 2021 - 10:15 AM

URL: http://pirsa.org/21060010

Abstract: Analysing the data for the upcoming LISA mission will require extreme mass ratio inpsiral (EMRI) waveforms waveforms that are not only accurate but also fast to compute and extensive in the parameter space. To this end, we present a method for rapidly calculating the inspiral trajectory of EMRIs with a spinning primary. We extend the work of van de Meent and Warburton (2018) by applying the technique of near-identity (averaging) transformations (NITs) to the osculating geodesic equations for a rotating (Kerr) black hole, resulting in equations of motion that do not explicitly depend upon the orbital phases. This allows us accurately to calculate the evolving constants of motion, orbital phases and waveform phase to within subradian accuracy, while dramatically reducing computational cost. We have implemented this scheme with an interpolated gravitational self-force model in both the equatorial and the spherical cases as a proof of concept, and present the first inspirals in Kerr spacetime to include all first order self-force effects.

Fast Self-Forced Inspirals into a Rotating Black Hole

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Capra 24

Perimeter Institute

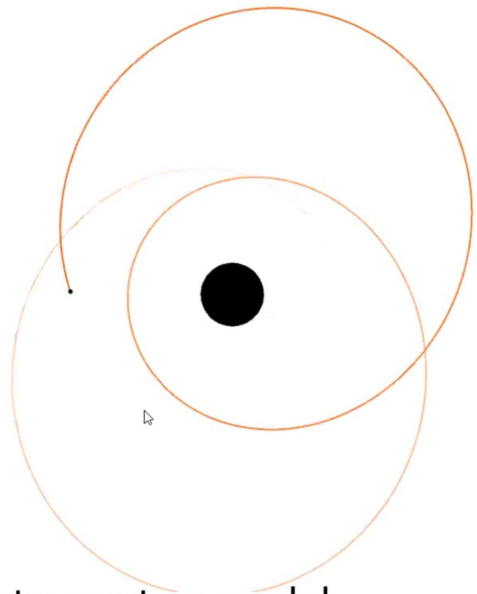
UCD RELATIVITY GROUP

UCD DUBLIN UCD School of Mathematics and Statistics

IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

Motivation

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = \epsilon f^\alpha_{(1)} + \epsilon^2 f^\alpha_{(2)} + \mathcal{O}(\epsilon^3)$$



- Use Kerr GSF data to create a model
- Use this to make Fast AND Accurate inspirals
- Proof of Concept: Equatorial Kerr Inspirals

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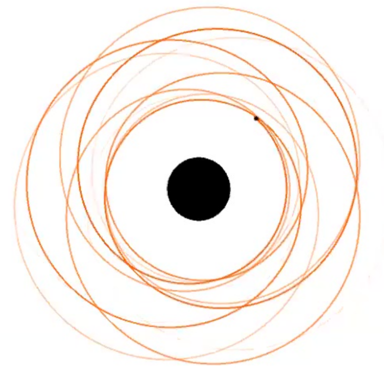

- **Adiabatic Inspiral**
  - $\left\langle f^{(1)}_{Dis} \right\rangle$
- **Post -Adiabatic Inspiral**
  - $\widetilde{f^{(1)}_{Dis}}$
  - $f^{(1)}_{Cons}$
  - $\cancel{\left\langle f^{(2)}_{Dis} \right\rangle}$

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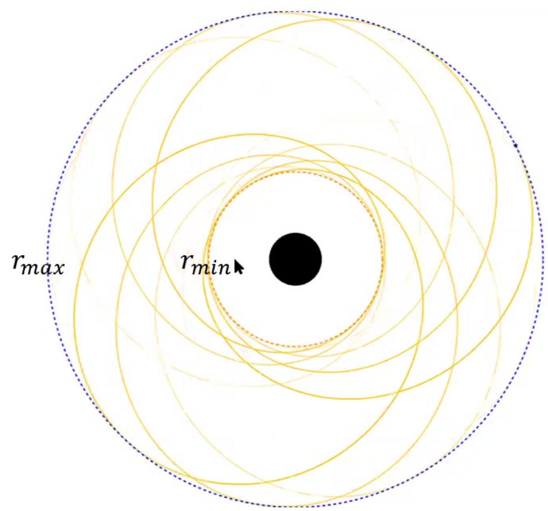
Equatorial Geodesic Motion in Kerr Spacetime

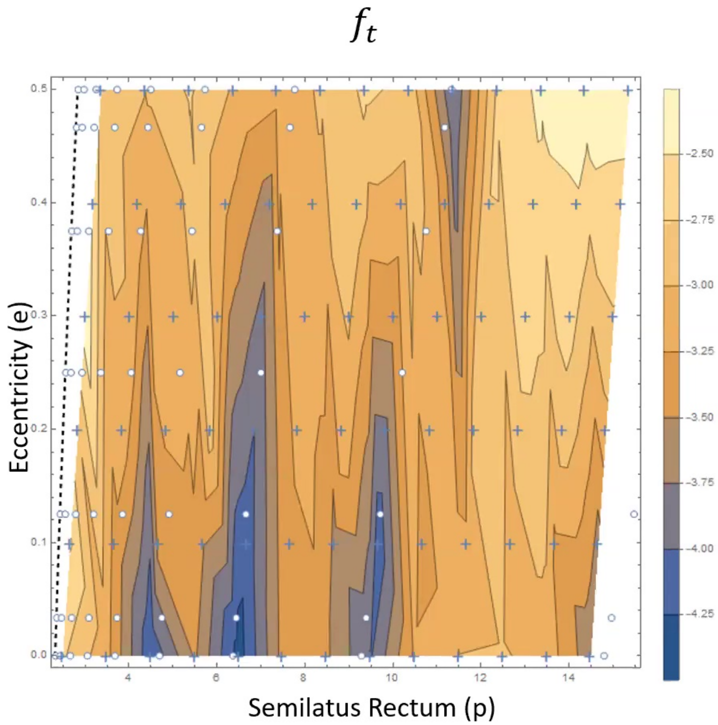


$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = 0$$

- Spin Parameter: $a = \frac{J}{M} = 0.9$
- Semilatus Rectum: $p = \frac{2r_{max}r_{min}}{r_{max}+r_{min}}$
- Eccentricity: $e = \frac{r_{max}-r_{min}}{r_{max}+r_{min}}$
- Radial Action Angle: q_r

Kerr GSF Model
(Outgoing Radiation Gauge)

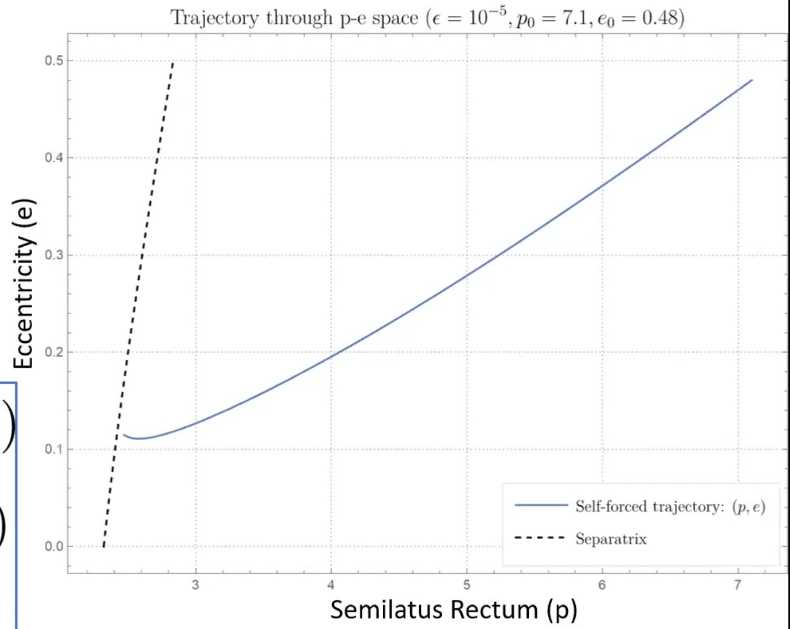
- $a = 0.9,\ 0 \le e \le 0.5$
- $y = \sqrt{\frac{p_{sep}}{p}},\ \ 0 < y < 1$
- $15 \times 7 = 105$ Chebyshev Nodes
- $f_\alpha = \sum_{n=0}^{15} A_\alpha^n(y,e)\cos(n\,q_r) + B_\alpha^n(y,e)\ \sin(n\,q_r)$
- $A_\alpha^n = \sum_{i=0}^{14}\sum_{j=0}^{6} A_{\alpha,i,j}^n\ T_i(2\,y\ -1)T_j(4e\ -1)$
- $B_\alpha^n = \sum_{i=0}^{14}\sum_{j=0}^{6} B_{\alpha,i,j}^n\ T_i(2\,y\ -1)T_j(4e\ -1)$
- Relative Error $< 5\ \times 10^{-3}$



Osculating Geodesics

- Orbital Elements: $P_j = \{p, e\}$
- Orbital Phases $q_i = \{q_r\}$
- Extrinsic Quantities: $S_K = \{t, \phi\}$
- (Carter-) Mino-Time: λ

$$\boxed{\begin{aligned} \dot{P}_j &= 0 &&+ \epsilon F_j^{(1)}(\vec{P}, \vec{q} : a^\alpha) \\ \dot{q}_i &= \Upsilon_i(\vec{P}) &&+ \epsilon f_i^{(1)}(\vec{P}, \vec{q} : a^\alpha) \\ \dot{S}_k &= s_k(\vec{P}, \vec{q}) \end{aligned}}$$



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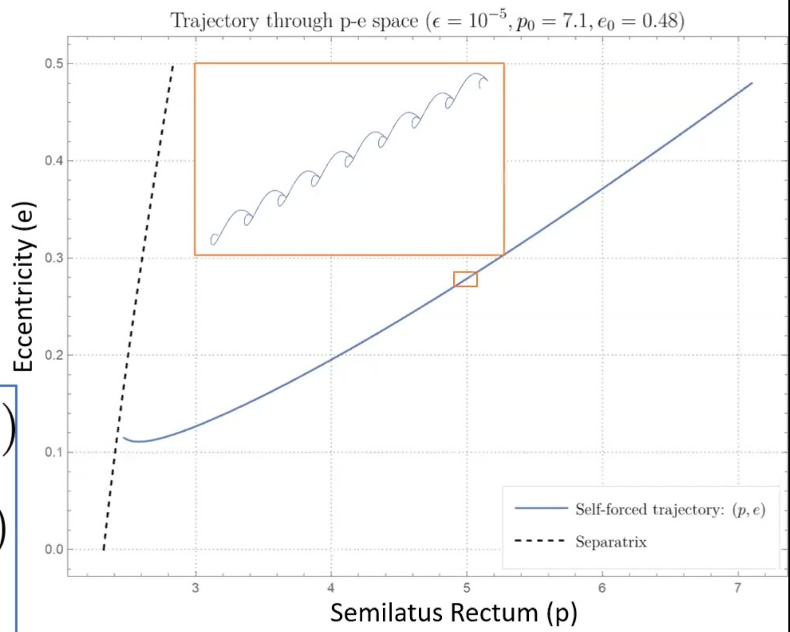

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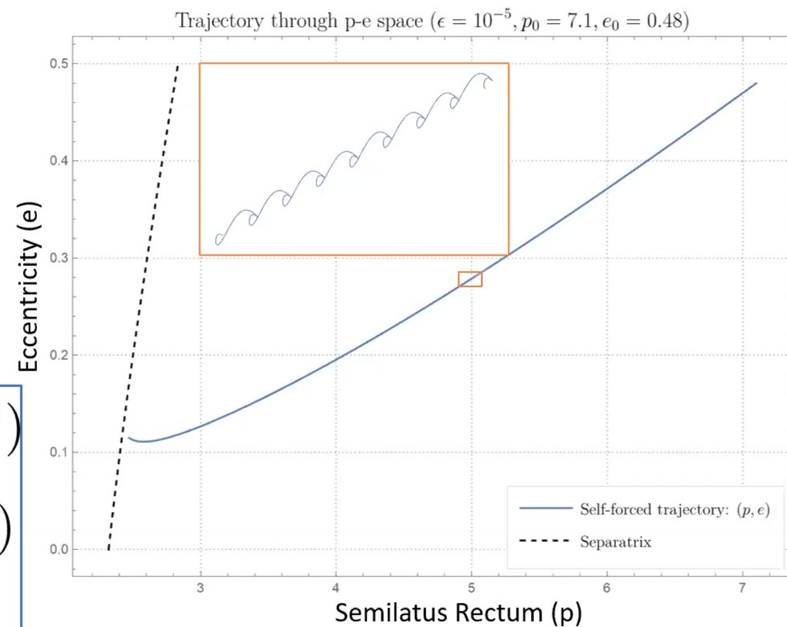


Near-Identity (Averaging) Transformations

Transform to new variables

$$\tilde{P}_j = P_j + \epsilon Y_j^{(1)}(\vec{P},\vec{q}) + \epsilon^2 Y_j^{(2)}(\vec{P},\vec{q}) + \mathcal{O}(\epsilon^3)$$

$$\tilde{q}_i = q_i + \epsilon X_i^{(1)}(\vec{P},\vec{q}) + \epsilon^2 X_i^{(2)}(\vec{P},\vec{q}) + \mathcal{O}(\epsilon^3)$$

$$\tilde{S}_k = S_k + Z_k^{(0)}(\vec{P},\vec{q}) + \epsilon Z_k^{(1)}(\vec{P},\vec{q}) + \mathcal{O}(\epsilon^2)$$

To obtain equations of motion independent of $\vec{q}$

$$\dot{\tilde{P}}_j = 0 + \epsilon \tilde{F}_j^{(1)}(\vec{\tilde{P}}) + \epsilon^2 \tilde{F}_j^{(2)}(\vec{\tilde{P}})$$

$$\dot{\tilde{q}}_i = \Omega(\vec{\tilde{P}}) + \epsilon \tilde{f}_i^{(1)}(\vec{\tilde{P}})$$

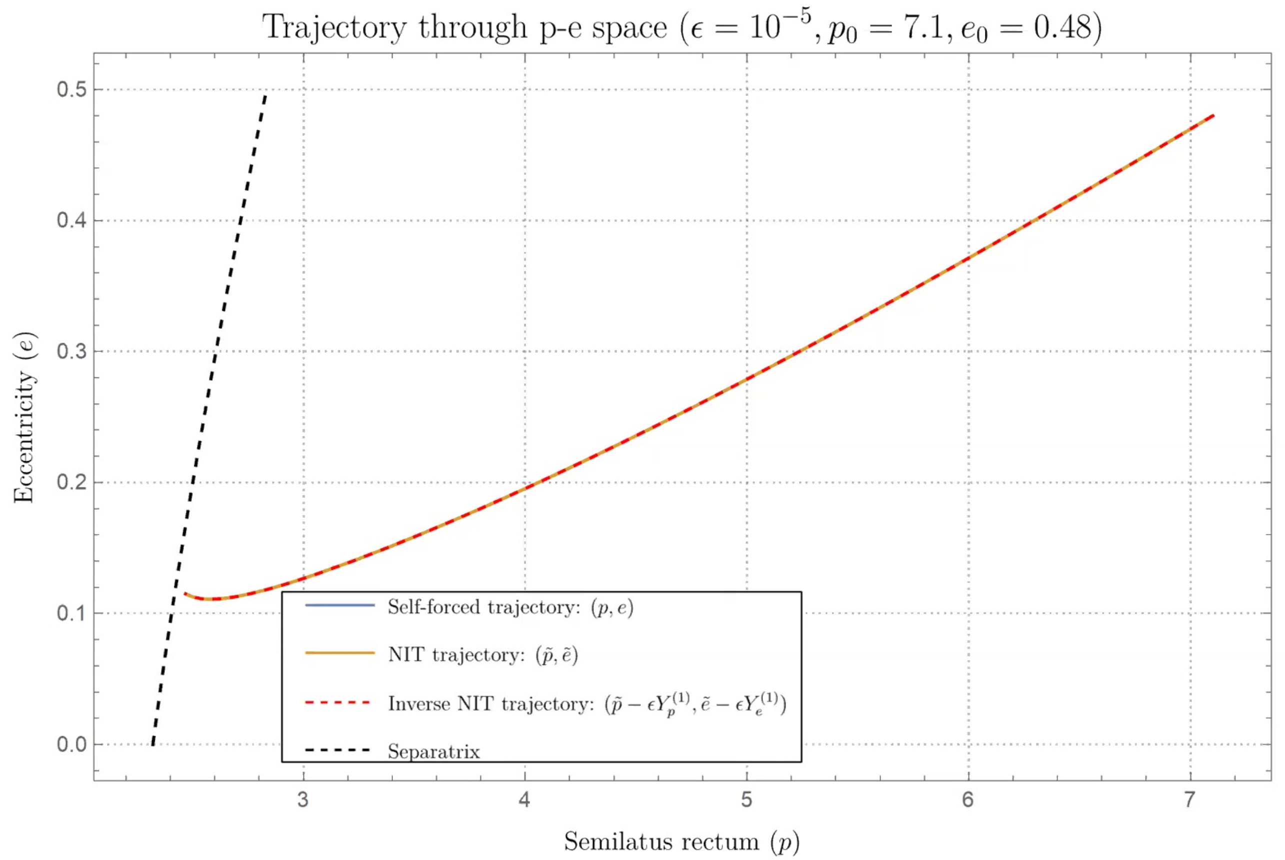
$$\dot{\tilde{S}}_k = \tilde{s}_k^{(0)}(\vec{\tilde{P}}) + \epsilon \tilde{s}_k^{(1)}(\vec{\tilde{P}})$$

Geodesic | Adiabatic | Post-Adiabatic

Steps Involved in Applying the NIT

Offline Steps

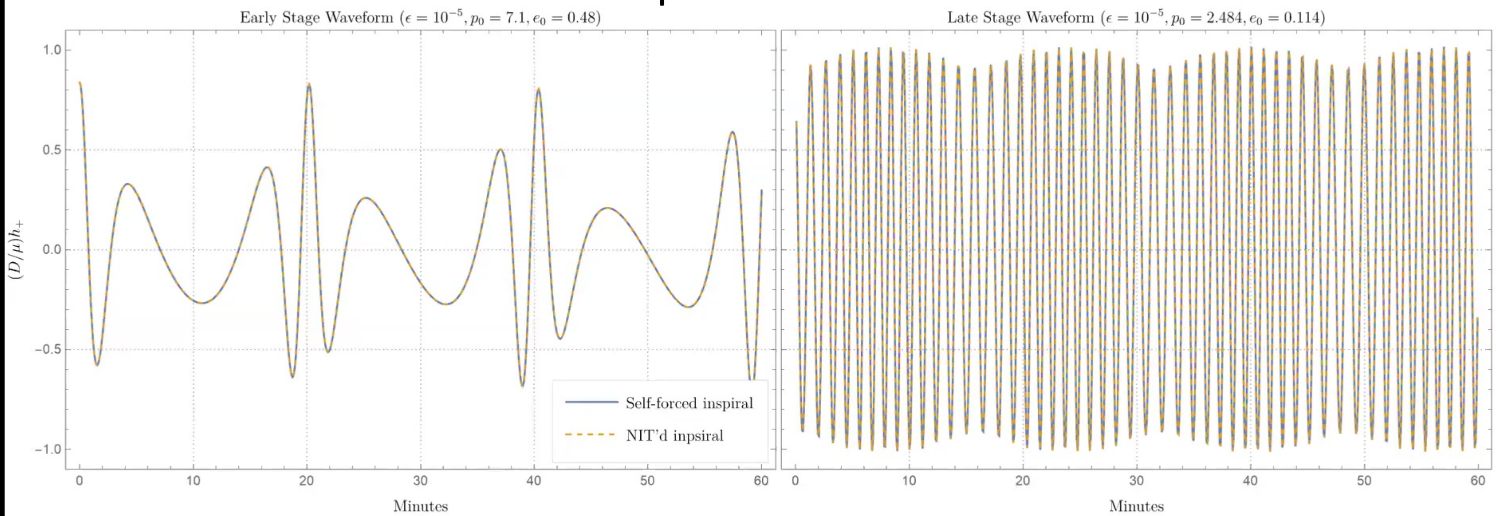
- Use fast Fourier transforms to find modes of $F_p, F_e, f_r, s_t, s_\phi$ + derivatives at a given (y,e)
- Combine these modes to find $\tilde{F}_j^{(1)}, \tilde{F}_j^{(2)}, \tilde{f}_i^{(1)}, \tilde{s}_k^{(0)}$ *and* $\tilde{s}_k^{(1)}$
- Repeat across the parameter space (320 x 500 points)
- Interpolate the points using cubic splines



Trajectory through p-e space ($\epsilon = 10^{-5}, p_0 = 7.1, e_0 = 0.48$)
Eccentricity (e)
Semilatus rectum (p)
0.5
0.4
0.3
0.2
0.1
0.0
3
4
5
6
7
Self-forced trajectory: (p, e)
NIT trajectory: $(\tilde{p}, \tilde{e})$
Inverse NIT trajectory: $(\tilde{p} - \epsilon Y_p^{(1)}, \tilde{e} - \epsilon Y_e^{(1)})$
Separatrix

8

Quadrupole Waveforms



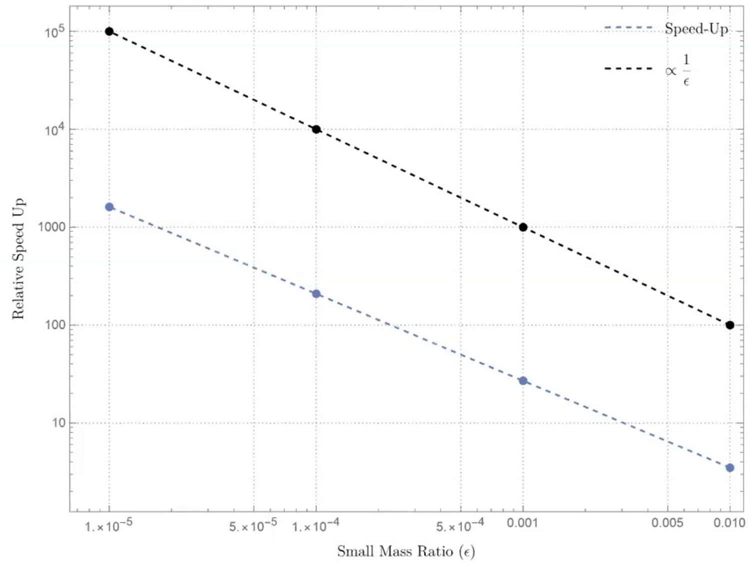
$$M = 10^6\ \mathrm{M}_{\odot}\,, \qquad \mu = 10\ M_{\odot}\,, \qquad \epsilon = 10^{-5}$$

$$Mismatch < 10^{-7}$$

Runtime (Wolfram Mathematica 12.2)

Mass Ratio	Osculating Geodesics	Relative Speed Up
10^{-2}	54s	$\sim \times 3.5$
10^{-3}	6mins 21s	$\sim \times 27$
10^{-4}	53mins	$\sim \times 209$
10^{-5}	6.9hrs	$\sim \times 1612$
10^{-6}	???	???

All NIT'd Inspirals all take ~ 15s



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10

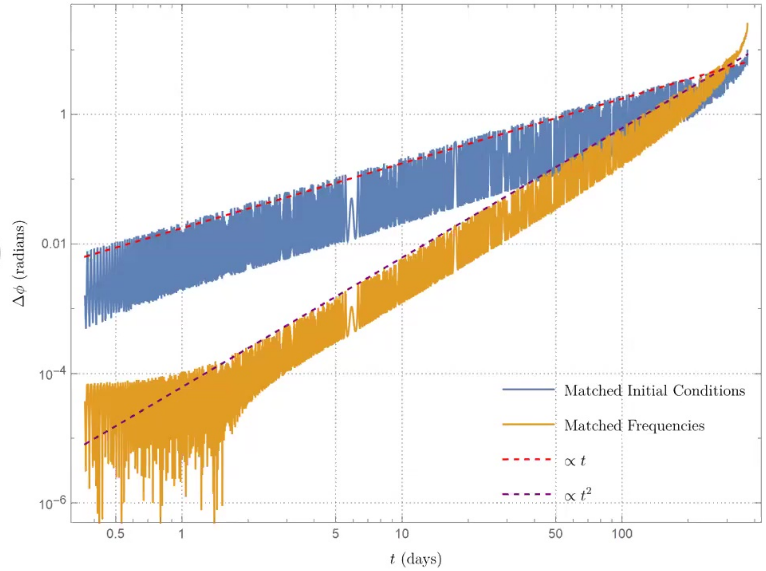
Comparing Adiabatic to PA Inspirals

$\varphi \equiv \phi$ or q_r

$$\Delta\varphi = \varphi^{Adb}(t) - \varphi^{PA}(t)$$

$$\Delta\varphi = (\varphi_0^{Adb} - \varphi_0^{PA}) + (\Omega_\varphi^{Adb} - \Omega_\varphi^{PA})t + \mathcal{O}(t^2)$$

- Matching $\tilde{p}_0$ & $\tilde{e}_0$: $\Delta\varphi \propto t$
- Matching Boyer Lindquist Frequencies: $\Delta\varphi \propto t^2$



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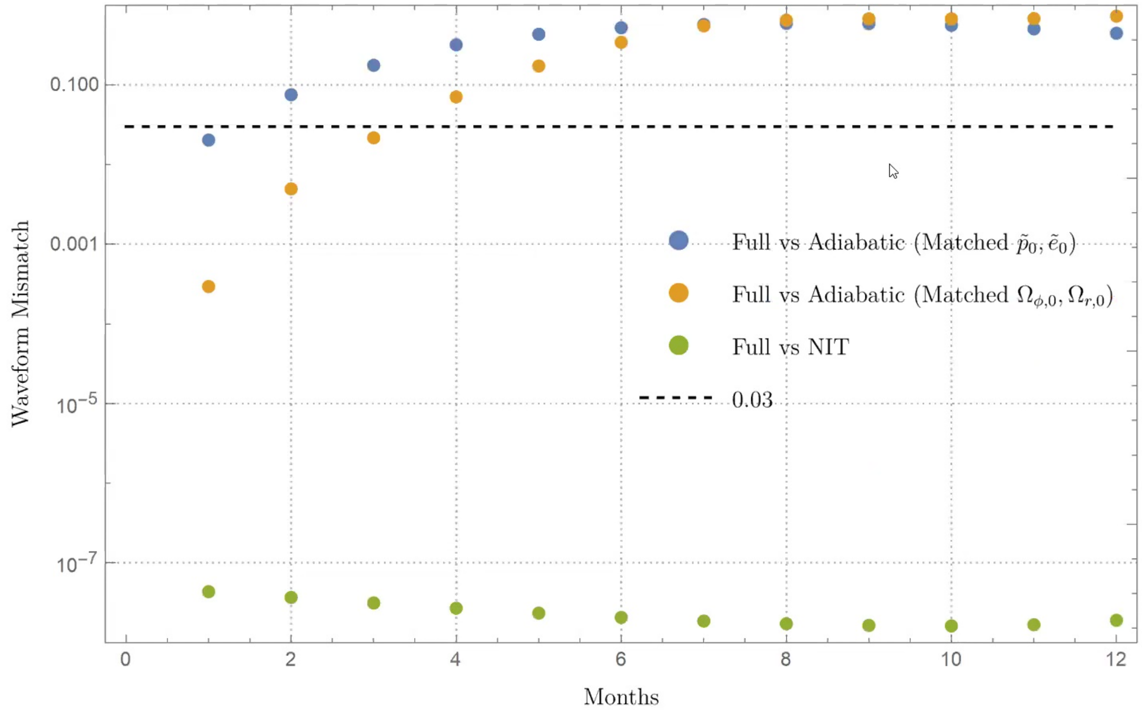
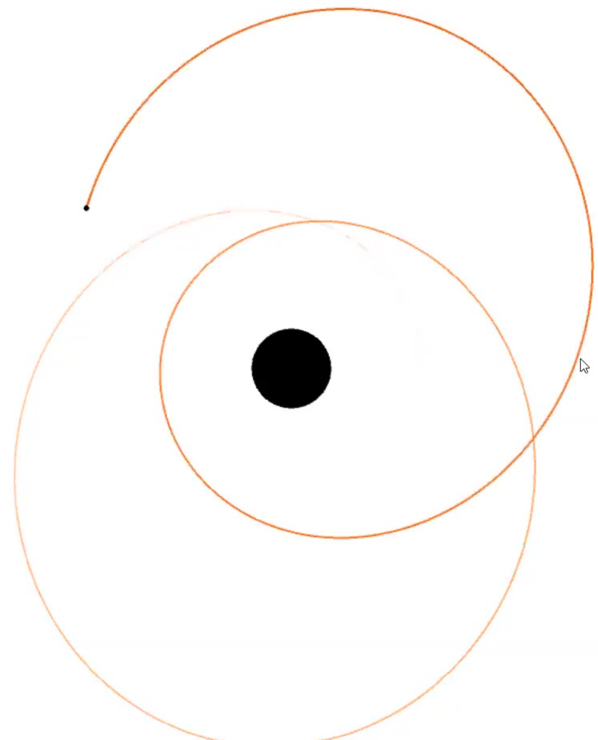
11

Waveform Mismatch



Questions



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14