Title: TBA

Speakers: Mikhail Solon

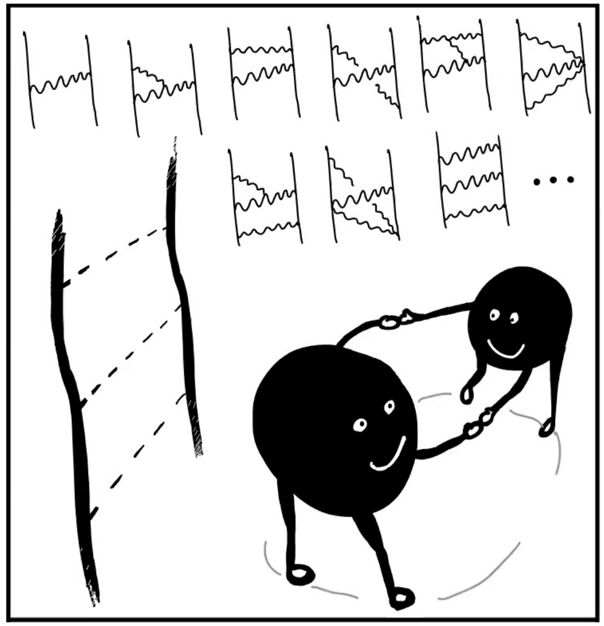
Series: Particle Physics

Date: June 15, 2021 - 1:00 PM

URL: http://pirsa.org/21060000

Abstract: Abstract: TBD

Zoom Link: https://pitp.zoom.us/j/96516977019?pwd=WVlpZG5WTTUwbFJVZ2wvcXdNWUR5Zz09



Binary Black Holes and Scattering Amplitudes

Mikhail Solon

Bhaumik Institute for Theoretical Physics, UCLA

Quantum Field Theory Tools

↓

Precision Gravitational Wave Science

Quantum Field Theory Tools

↓

Precision Gravitational Wave Science

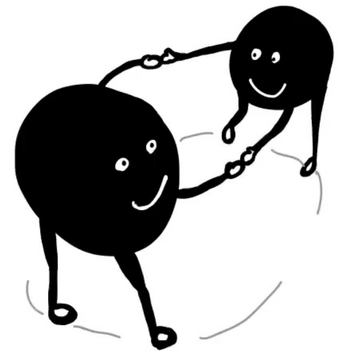
Cheung, Rothstein, MS (PRL 18)
Bern, Cheung, Roiban, Shen, MS, Zeng (PRL 19, JHEP 19)
Cheung, MS (JHEP 20, PRL 20)
Cheung, Shah, MS (PRD 20)
Bern, Parra-Martinez, Roiban, Ruf, Shen, MS, Zeng (PRL 21)
...

Gravitational Wave Science

2015	**2020**	**2025**	**2030+**
LIGO	LIGO/Virgo	+ KAGRA, INDIGO, …	LISA Einstein Telescope Cosmic Explorer …

Gravitational Wave Science

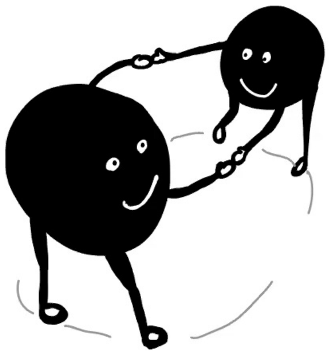
$$G_{ab} = 8\pi T_{ab} \longrightarrow$$



Decades of heroic effort.

Gravitational Wave Science

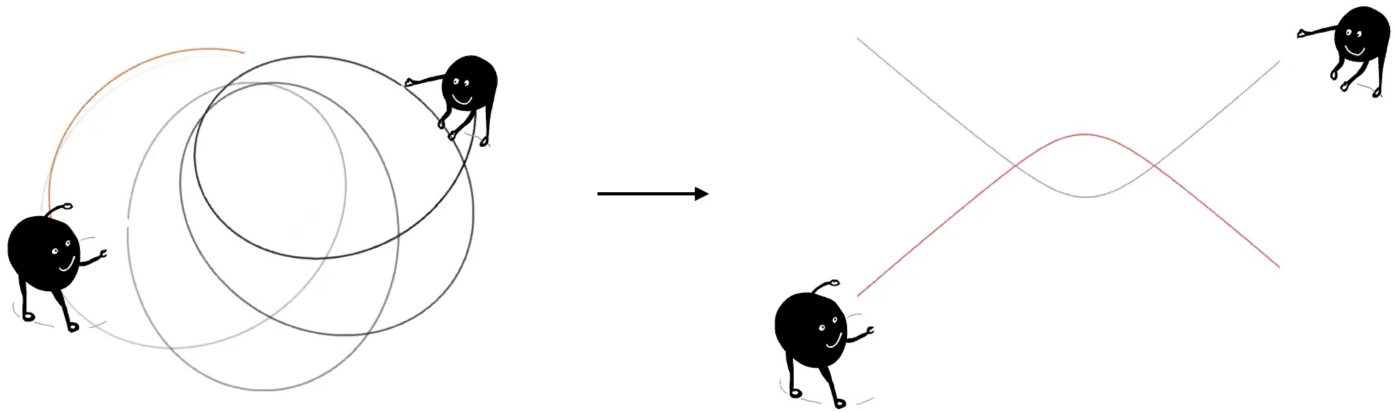
$$G_{ab} = 8\pi T_{ab} \rightarrow$$



Decades of heroic effort.

post-Newtonian	self-force	numerical relativity
$G \sim v^2 \ll 1$	$m/M \ll 1$	
4PN(2014) → 6PN	1SF(1990) → 2SF	

Turn it into an easier problem



Relativity

Tools of Theoretical High Energy Physics

onshell methods, advanced multiloop integration, effective field theory

Factorize states from underlying dynamics

$$\langle \Psi | \, \bar{u} \, \mathcal{O} \, u \, | \Psi \rangle$$

Does it work?

$$G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$

Does it work?

$$G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
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$$G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$

Cheung, Rothstein, MS PRL18

Does it work?

$$
\begin{aligned}
&G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\\
&G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)
\end{aligned}
$$

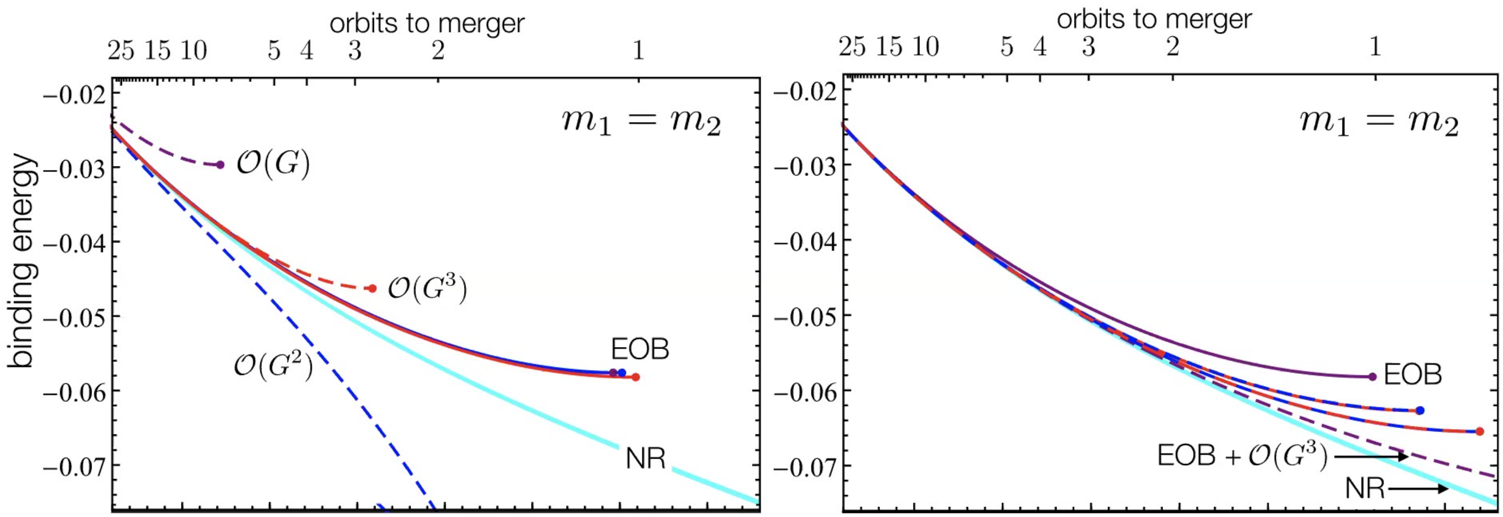
Cheung, Rothstein, MS PRL18

Bern, Cheung, Roiban, Shen, MS, Zeng PRL19

Bern, Parra-Martinez, Roiban, Ruf, Shen, MS, Zeng PRL 21

Theorists at LIGO are interested

Antonelli, Buonanno, Steinhoff, van de Meent, Vines 2019



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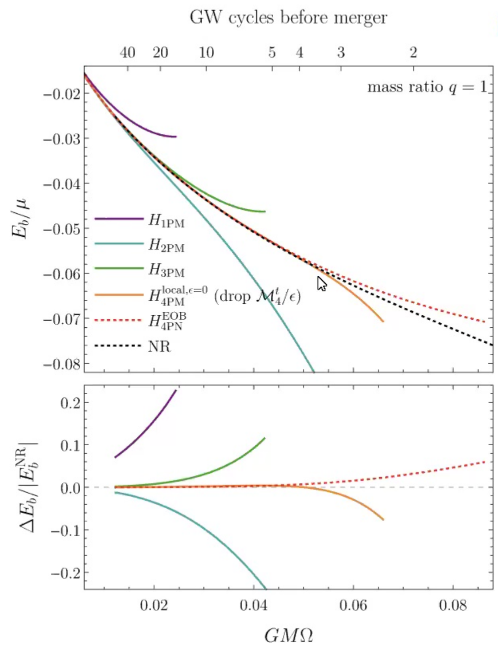
NEW! from Alessandra Buonanno's recent talk at GGI

Crucial to **push PM** calculations at **higher order,** and **resume them** in EOB formalism.

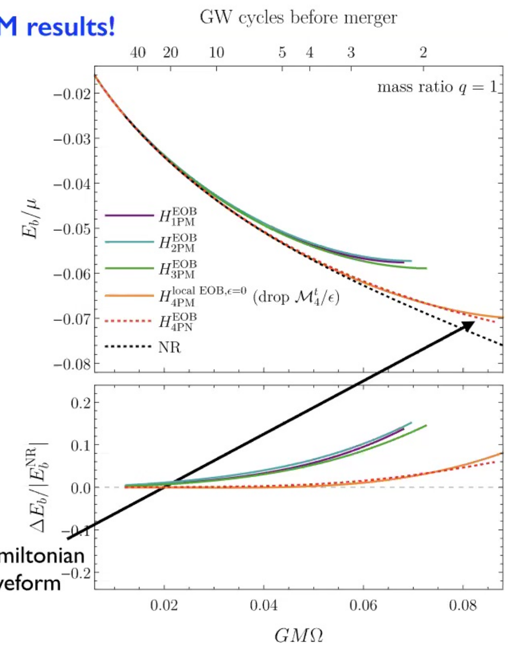
(Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)

(Khalil, AB, Steinhoff & Vines in prep 21)



binding energy

Encouraging (local-in-time) **4PM results!**



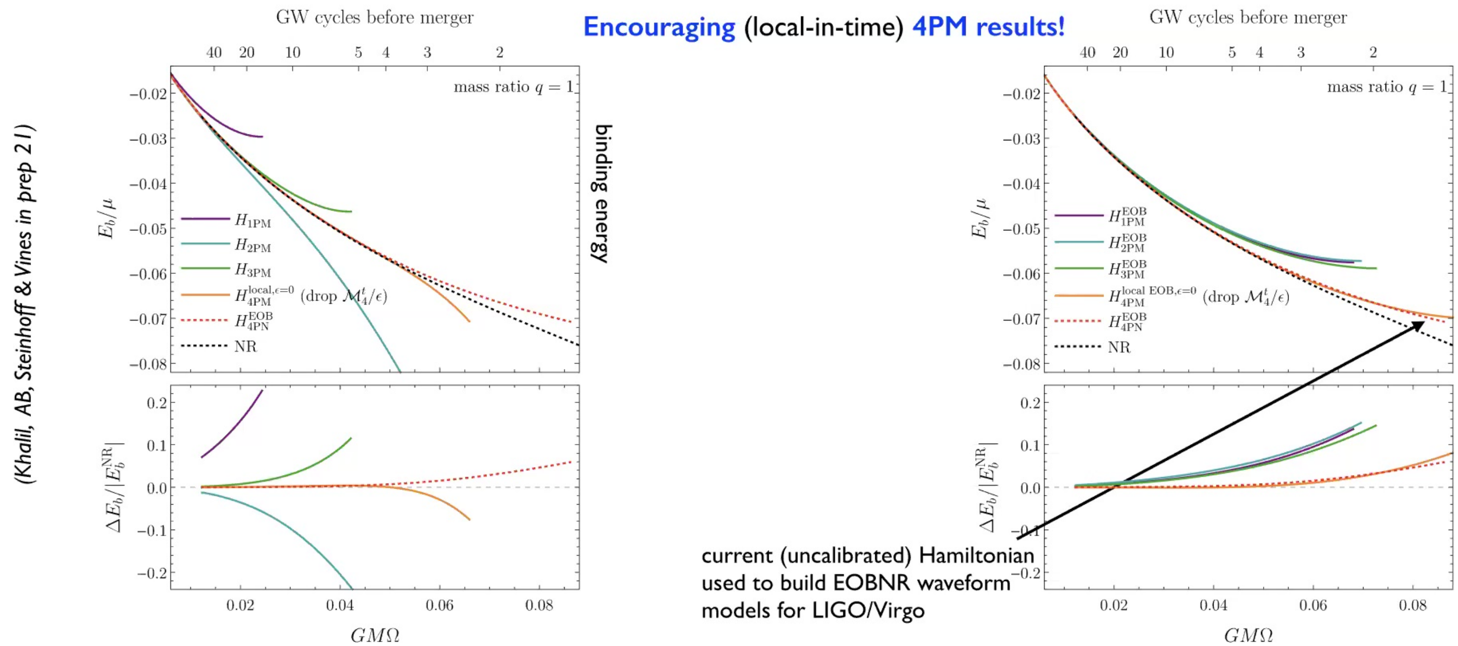
binding energy

current (uncalibrated) Hamiltonian used to build EOBNR waveform models for LIGO/Virgo

NEW! from Alessandra Buonanno's recent talk at GGI

Crucial to **push PM** calculations at **higher order,** and **resume them** in EOB formalism.

(Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)

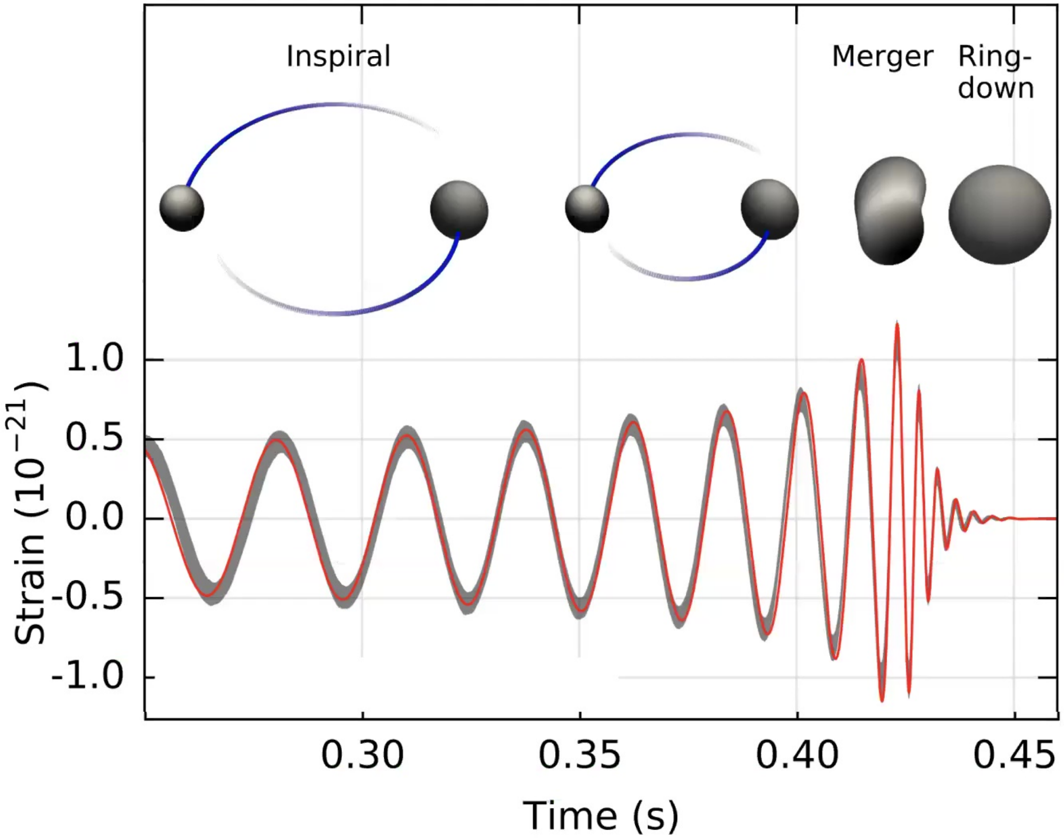


Interesting exchanges between GR and HEP

Bini, Damour, Geralico 2019, 2020, … Blümlein, Maier, Marquard, Schäfer 2021

Damour 2019 Damour 2020

Hybrid Waveforms



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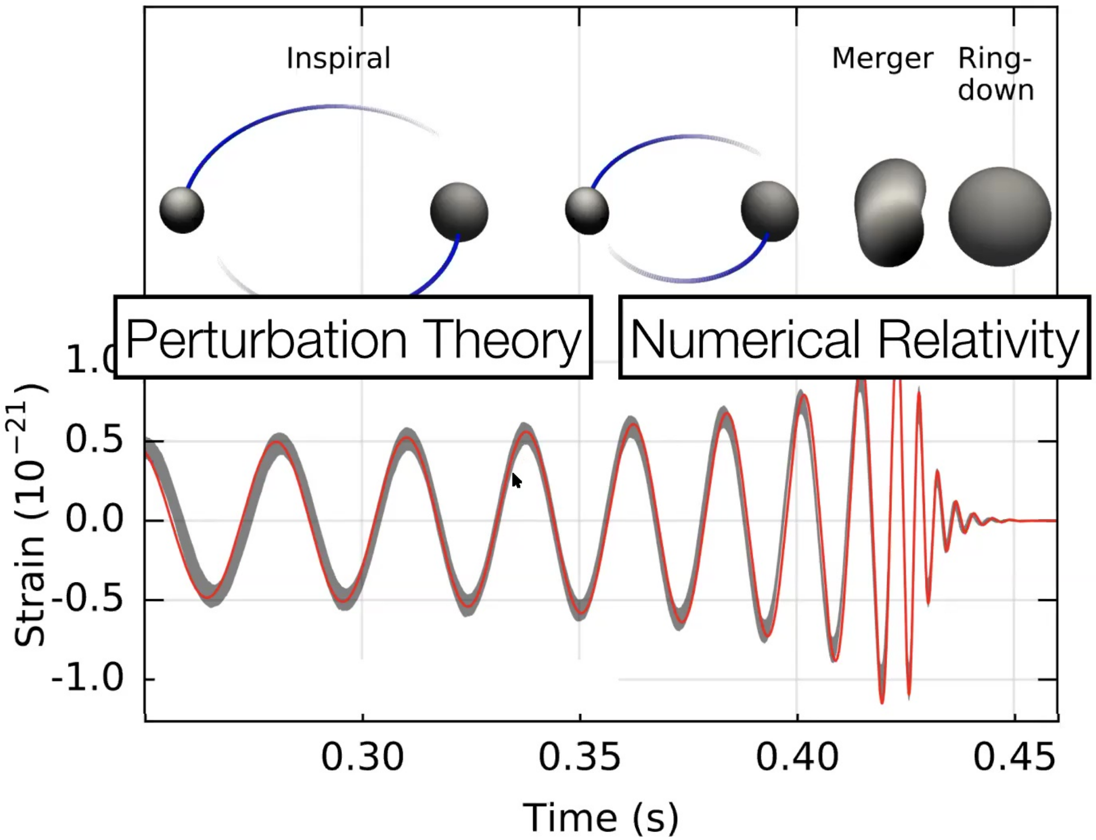
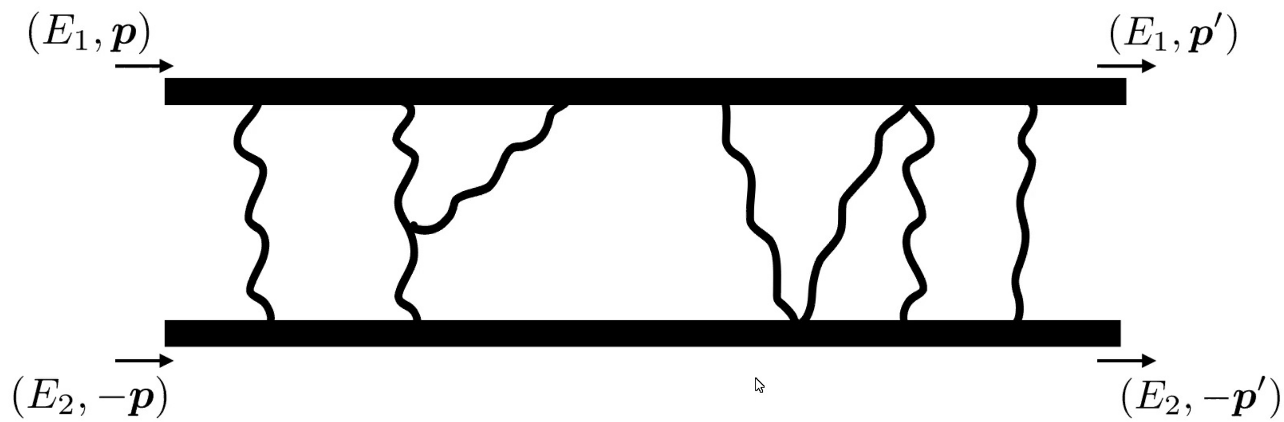
Hybrid Waveforms

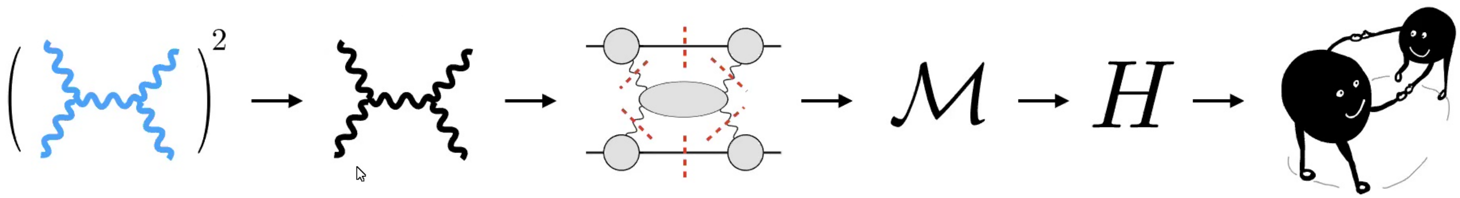


Scalable pipeline using tools from QFT ?

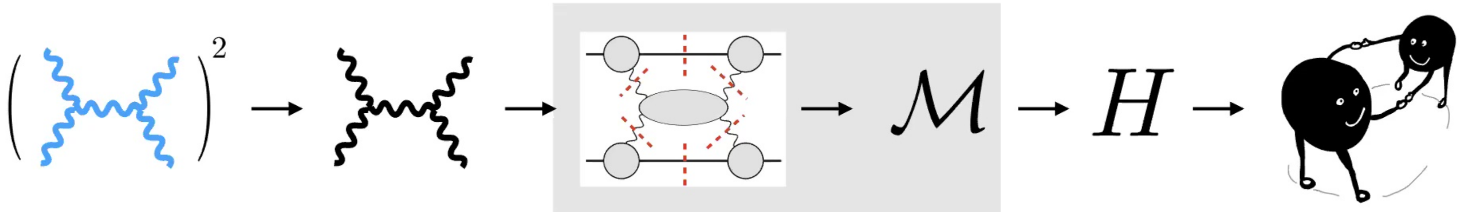
$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} \sum_{i=1,2} \left(\nabla\phi_i \nabla\phi_i - m_i^2 \phi_i^2 \right) \right]$$



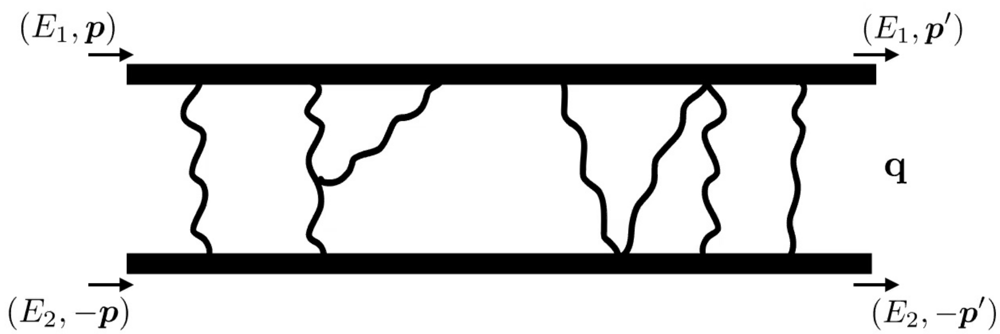
Scalable pipeline using tools from QFT:



Classical limit is taken at the earliest stages.



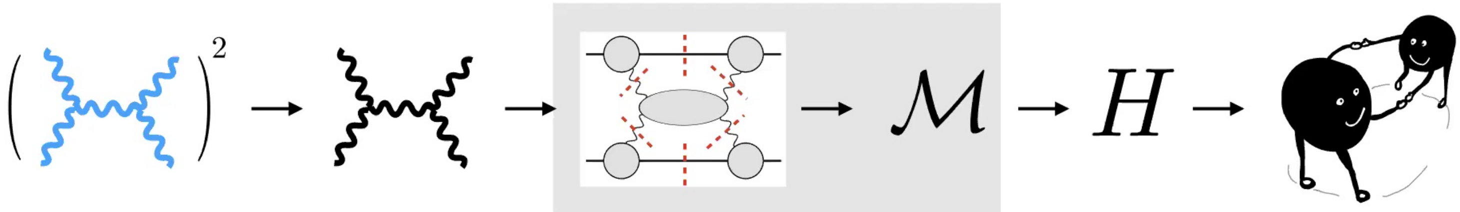
Full integration of quantum loops won't scale



$$q \ll m$$

"In reality classical effects are smaller than quantum." - Aneesh Manohar

$$\mathcal{M}_{\rm QED} \sim \left(\frac{Q^2}{Q_{\rm pl}^2}\frac{q}{m}\right)^{\mathcal{C}} \left(\frac{q}{m}\right)^{\mathcal{Q}} \qquad e \sim 10^{-1} Q_{\rm pl}$$



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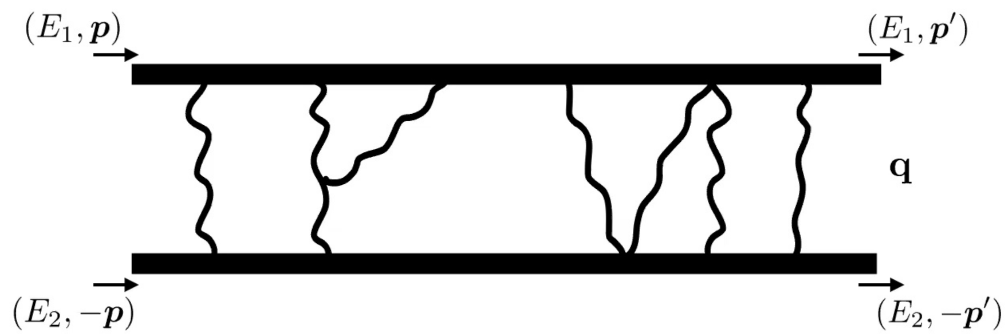

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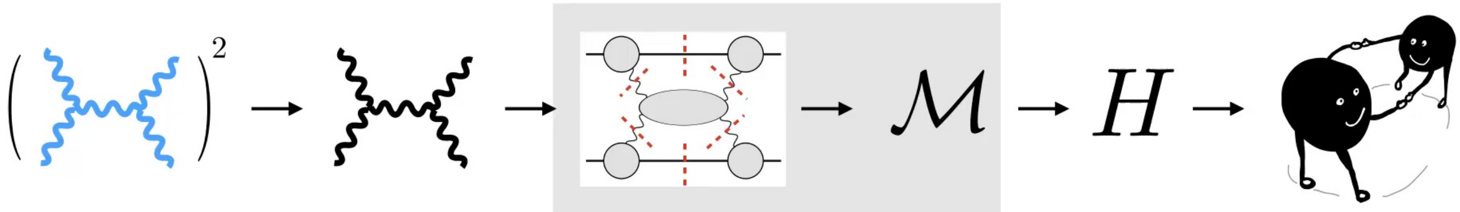
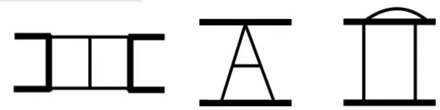
$$\mathcal{M}_{\text{QED}} \sim \left(\frac{Q^2}{Q_{\text{pl}}^2}\frac{q}{m}\right)^{\mathcal{C}} \left(\frac{q}{m}\right)^{\mathcal{Q}} \qquad e \sim 10^{-1} Q_{\text{pl}}$$

$$\mathcal{M}_{\text{GR}} \sim \left(\frac{m^2}{m_{\text{pl}}^2}\frac{q}{m}\right)^{\mathcal{C}} \left(\frac{q}{m}\right)^{\mathcal{Q}} \qquad M_{\odot} \sim 10^{38} m_{\text{pl}}$$



Classical limit is simple.



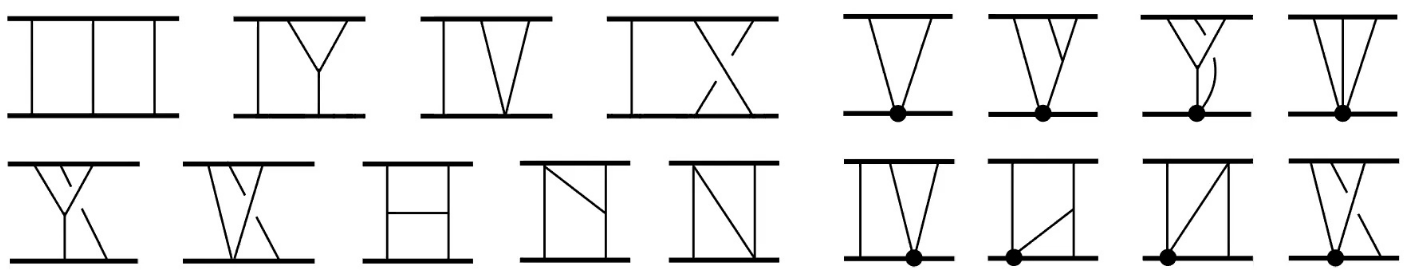
$$S = \int d^D x \sqrt{-g}\left[-\frac{1}{16\pi G}R + \frac{1}{2}\sum_{i=1,2}\left(\nabla\phi_i\nabla\phi_i - m_i^2\phi_i^2\right)\right] \quad S_{\rm GF} = -\frac{1}{32\pi G}\int d^D x\, F_\mu F^\mu$$

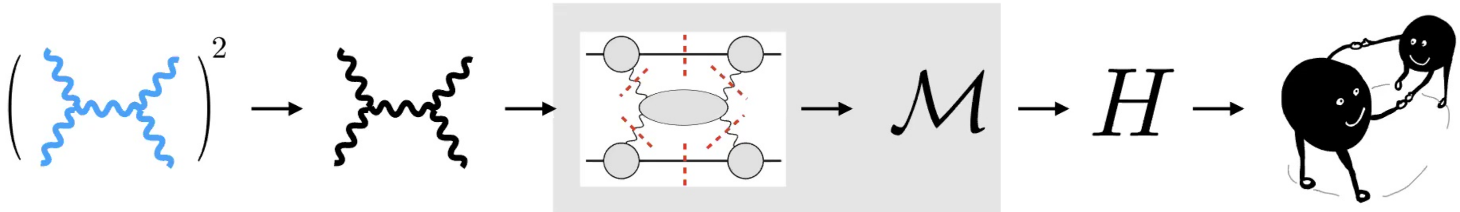
$$F_\mu = \partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \partial^\nu h^{\rho\sigma}(\zeta_1 h_{\mu\nu}\eta_{\rho\sigma} + \zeta_2 h_{\mu\rho}\eta_{\nu\sigma} + \zeta_3 h_{\rho\sigma}\eta_{\mu\nu} + \zeta_4 h_{\nu\sigma}\eta_{\mu\rho} + \zeta_5\eta_{\mu\nu}\eta_{\rho\sigma}h + \zeta_6\eta_{\mu\rho}\eta_{\nu\sigma}h)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \xi_1 h_{\mu\rho}h_\nu{}^\rho + \xi_2 h_{\mu\nu}h + \xi_3\eta_{\mu\nu}h_{\rho\sigma}h^{\rho\sigma} + \xi_4\eta_{\mu\nu}h^2$$

$$\Delta S = \int d^4x\sqrt{-g}\,\frac{1}{4}C_{\mu\alpha\nu\beta}C^{\rho\alpha\sigma\beta}\sum_{i=1,2}\left(\lambda_i\phi_i^2\delta_\rho^\mu\delta_\sigma^\nu + \frac{\eta_i}{m_i^4}\nabla^\mu\nabla^\nu\phi_i\nabla_\rho\nabla_\sigma\phi_i\right)$$

Cheung, MS



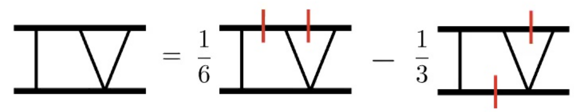


$$\text{soft}: \quad (\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}|, |\boldsymbol{q}|) \longrightarrow \begin{array}{ll} \text{potential}: & (\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}|) \\ \text{radiation}: & (\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}||\boldsymbol{v}|) \end{array}$$

NRGR Goldberger, Rothstein

Loop energies

$$\int \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \mathcal{I}(\omega, \omega') = \sum_{(i,j)} S_{ij} \underbrace{\operatorname*{Res}_{\omega_{ij}, \omega'_{ij}}}_{\text{matter poles}} \mathcal{I}(\omega, \omega')$$



Cheung, Rothstein, MS

$$1 + v^2 + v^4 + v^6 + v^8 + \cdots$$

Static limit boundary condition in the potential region

Obtain from differential equations.

Parra-Martinez, Ruf, Zeng

Amplitudes

$$\mathcal{M}_1 = 16\pi G\nu^2 M^4 |\boldsymbol{q}|^{-2}(2\sigma^2 - 1)$$

$$\mathcal{M}_2 = 6\pi^2 G^2\nu^2 M^5 |\boldsymbol{q}|^{-1}(5\sigma^2 - 1)$$

$$\mathcal{M}_3 = 2\pi G^3\nu^2 M^6 \log \boldsymbol{q}^2 \Bigg[\frac{(1+2\nu(\sigma-1))(5-60\sigma^2+120\sigma^4-64\sigma^2)}{3(\sigma^2-1)^2} + 8\nu\left(\frac{\sigma(14\sigma^2+25)}{3} + (4\sigma^4-12\sigma^2-3)\frac{\text{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\Bigg]$$

$$\mathcal{M}_4(\boldsymbol{q}) = G^4 M^7 \nu^2 |\boldsymbol{q}| \left(\frac{\boldsymbol{q}^2}{4^{\frac{1}{3}} \tilde{\mu}^2} \right)^{-3\epsilon} \pi^2 \left[\mathcal{M}_4^{\mathrm{p}} + \nu \left(\frac{\mathcal{M}_4^{\mathrm{t}}}{\epsilon} + \mathcal{M}_4^{\mathrm{f}} \right) \right]$$

$$\mathcal{M}_4^{\mathrm{p}} = -\frac{35\left(1 - 18\sigma^2 + 33\sigma^4\right)}{8\left(\sigma^2 - 1\right)} \qquad \mathcal{M}_4^{\mathrm{t}} = h_1 + h_2 \log\left(\tfrac{\sigma+1}{2}\right) + h_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}$$

$$\begin{aligned}
\mathcal{M}_4^{\mathrm{f}} = {} & h_4 + h_5 \log\left(\tfrac{\sigma+1}{2}\right) + h_6 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + h_7 \log(\sigma) - h_2 \frac{2\pi^2}{3} + h_8 \frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2 - 1} + h_9 \left[\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + \frac{1}{2}\log^2\left(\tfrac{\sigma+1}{2}\right) \right] \\
& + h_{10} \left[\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) - \frac{\pi^2}{6} \right] + h_{11} \left[\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) - \mathrm{Li}_2\left(\tfrac{\sigma-1}{\sigma+1}\right) + \frac{\pi^2}{3} \right] + h_2 \frac{2\sigma(2\sigma^2-3)}{(\sigma^2-1)^{3/2}} \left[\mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) \right] \\
& + \frac{2h_3}{\sqrt{\sigma^2-1}} \left[\mathrm{Li}_2\left(1-\sigma-\sqrt{\sigma^2-1}\right) - \mathrm{Li}_2\left(1-\sigma+\sqrt{\sigma^2-1}\right) + 5\mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - 5\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) + 2\log\left(\tfrac{\sigma+1}{2}\right)\operatorname{arccosh}(\sigma) \right] \\
& + h_{12} \mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{13} \mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right) \mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{14} \mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right),
\end{aligned}$$

$$h_1 = \frac{1151 - 3336\sigma + 3148\sigma^2 - 912\sigma^3 + 339\sigma^4 - 552\sigma^5 + 210\sigma^6}{12(\sigma^2-1)}$$

$$h_2 = \frac{1}{2}\left(5 - 76\sigma + 150\sigma^2 - 60\sigma^3 - 35\sigma^4\right)$$

$$h_3 = \sigma \frac{(-3 + 2\sigma^2)}{4(\sigma^2-1)}\left(11 - 30\sigma^2 + 35\sigma^4\right)$$

$$\begin{aligned} h_4 = {} & \frac{1}{144(\sigma^2-1)^2\sigma^7}(-45 + 207\sigma^2 - 1471\sigma^4 + 13349\sigma^6 \\ & - 37566\sigma^7 + 104753\sigma^8 - 12312\sigma^9 - 102759\sigma^{10} - 105498\sigma^{11} \\ & + 134745\sigma^{12} + 83844\sigma^{13} - 101979\sigma^{14} + 13644\sigma^{15} + 10800\sigma^{16}) \end{aligned}$$

$$\begin{aligned} h_5 = {} & \frac{1}{4(\sigma^2-1)}(1759 - 4768\sigma + 3407\sigma^2 - 1316\sigma^3 + 957\sigma^4 \\ & - 672\sigma^5 + 341\sigma^6 + 100\sigma^7) \end{aligned}$$

$$\begin{aligned} h_6 = {} & \frac{1}{24(\sigma^2-1)^2}(1237 + 7959\sigma - 25183\sigma^2 + 12915\sigma^3 + 18102\sigma^4 \\ & - 12105\sigma^5 - 9572\sigma^6 + 2973\sigma^7 + 5816\sigma^8 - 2046\sigma^9) \end{aligned}$$

$$h_7 = 2\sigma \frac{(-852 - 283\sigma^2 - 140\sigma^4 + 75\sigma^6)}{3(\sigma^2-1)}$$

$$\begin{aligned} h_8 = {} & \frac{\sigma}{8(\sigma^2-1)^2}(-304 - 99\sigma + 672\sigma^2 + 402\sigma^3 - 192\sigma^4 - 719\sigma^5 \\ & - 416\sigma^6 + 540\sigma^7 + 240\sigma^8 - 140\sigma^9) \end{aligned}$$

$$h_9 = \frac{1}{2}\left(52 - 532\sigma + 351\sigma^2 - 420\sigma^3 + 30\sigma^4 - 25\sigma^6\right)$$

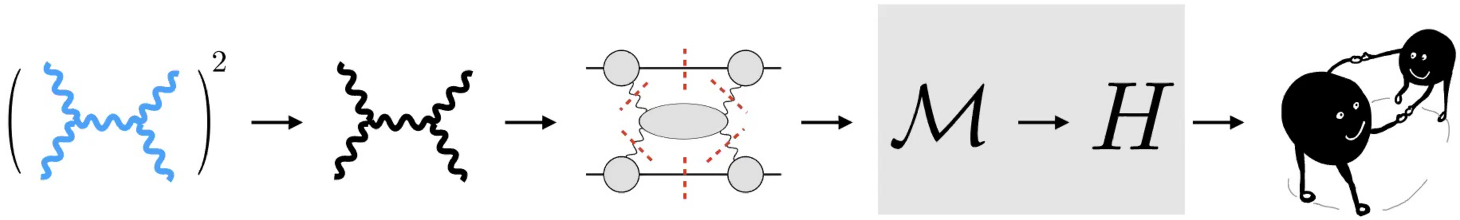
$$h_{10} = 2\left(27 + 90\sigma^2 + 35\sigma^4\right)$$

$$h_{11} = 20 + 111\sigma^2 + 30\sigma^4 - 25\sigma^6$$

$$h_{12} = \frac{834 + 2095\sigma + 1200\sigma^2}{2(\sigma^2-1)}$$

$$h_{13} = -\frac{1183 + 2929\sigma + 2660\sigma^2 + 1200\sigma^3}{2(\sigma^2-1)}$$

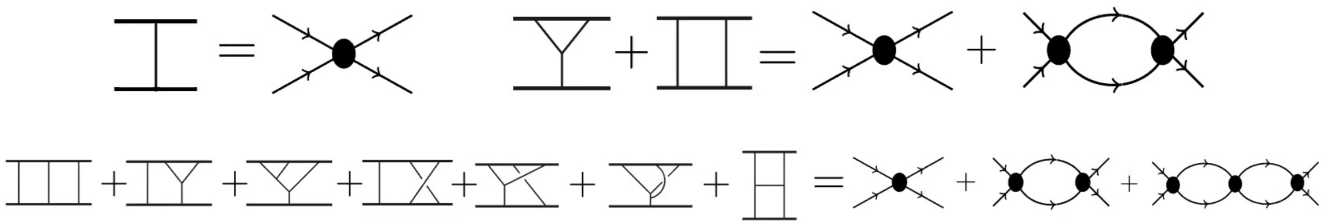
$$h_{14} = \frac{7\left(169 + 380\sigma^2\right)}{4(\sigma-1)}$$



$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\, A^\dagger(\boldsymbol{k}')A(\boldsymbol{k})B^\dagger(-\boldsymbol{k}')B(-\boldsymbol{k})$$

Cheung, Rothstein, MS

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$



“amplitude-action” relation

$$i\mathcal{M}(\boldsymbol{q}) = \int_J \left(e^{iI_r(J)} - 1\right)$$

Hamiltonian

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right) \qquad \xi = E_1 E_2 / E^2$$

$$c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3\xi^2}\right]$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\Bigg[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}}$$
$$- \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2}$$
$$+ \frac{2\nu^3(3 - 4\xi)\sigma\left(1 - 2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1 - 2\xi)\left(1 - 2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg],$$

$$c_4 = \frac{M^7\nu^2}{4\xi E^2}\left[\mathcal{M}_4^{\mathrm{p}} + \nu\left(\frac{\mathcal{M}_4^{\mathrm{t}}}{\epsilon} + \mathcal{M}_4^{\mathrm{f}} - 10\mathcal{M}_4^{\mathrm{t}}\right)\right] + \mathcal{D}^3\left[\frac{E^3\xi^3}{3}c_1^4\right] + \mathcal{D}^2\left[\left(\frac{E^3\xi^3}{\boldsymbol{p}^2} + \frac{E\xi(3\xi - 1)}{2}\right)c_1^4 - 2E^2\xi^2 c_1^2 c_2\right]$$
$$+ \left(\mathcal{D} + \frac{1}{\boldsymbol{p}^2}\right)\left[E\xi(2c_1c_3 + c_2^2) + \left(\frac{4\xi - 1}{4E} + \frac{2E^3\xi^3}{\boldsymbol{p}^4} + \frac{E\xi(3\xi - 1)}{\boldsymbol{p}^2}\right)c_1^4 + \left((1 - 3\xi) - \frac{4E^2\xi^2}{\boldsymbol{p}^2}\right)c_1^2 c_2\right],$$

$$\mathcal{D} = \tfrac{d}{d\boldsymbol{p}^2}$$

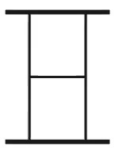
Theoretical Structures

Lorentz invariant, on shell

High-energy behavior

$$\mathcal{M}_3 = \frac{-\pi G^3 \nu^2 m^4 \ln \boldsymbol{q}^2}{6\gamma^2 \xi} \left[\frac{48\nu(3 + 12\sigma^2 - 4\sigma^4)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right] + \cdots$$

$$= -8\pi G^3 s^2 \ln(-t)\, \ln\left(\frac{m_1 m_2}{s}\right) + \cdots$$



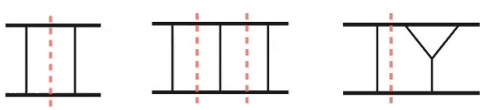
$$\sim \left[\log\left(\frac{s}{m^2}\right) - \log\left(\frac{t}{m^2}\right)\right]^2$$

Regge limit

radiation

Di Vecchia, Heissenberg, Russo, Veneziano 20; Damour 20

Exponentiation



$$i\mathcal{M}(\boldsymbol{q}) = \int_J \left(e^{iI_r(J)} - 1\right)$$

Mass dependence $\mathcal{M}_N \sim G^N \left[m_1^{N+1} m_2^2 + m_1^N m_2^3 + \cdots + \{1 \leftrightarrow 2\}\right]$

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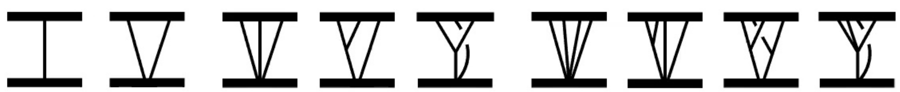
$$\vdots$$

0SF

Test-particle in Schwarzschild (and beyond)

$$\mathcal{M}_0(p,r) = \frac{1}{2E}\left[m^2(1 - g_{rr}^{\text{iso}}(r)) - E^2\left(1 + \frac{g_{rr}^{\text{iso}}(r)}{g_{tt}^{\text{iso}}(r)}\right)\right]$$



Cheung, Shah, MS

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0SF 1SF

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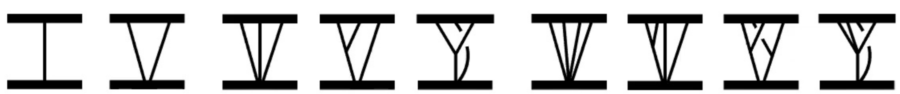

Cheung, Shah, MS

Bootstrap from 1SF (~m/M) results

Bini, Damour, Geralico

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0SF 1SF 2SF

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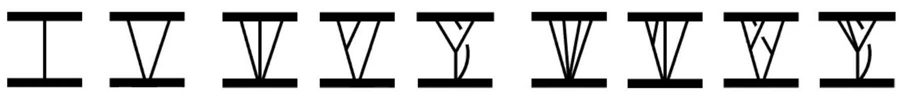

Cheung, Shah, MS

Bootstrap from 1SF (~m/M) results

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Gravitational wave science has opened up a new direction in theoretical high energy physics.

Classical binary dynamics has the hallmarks of a great problem in theoretical high energy physics.

This program is in a nascent stage.

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