

Title: TBA

Speakers: Mikhail Solon

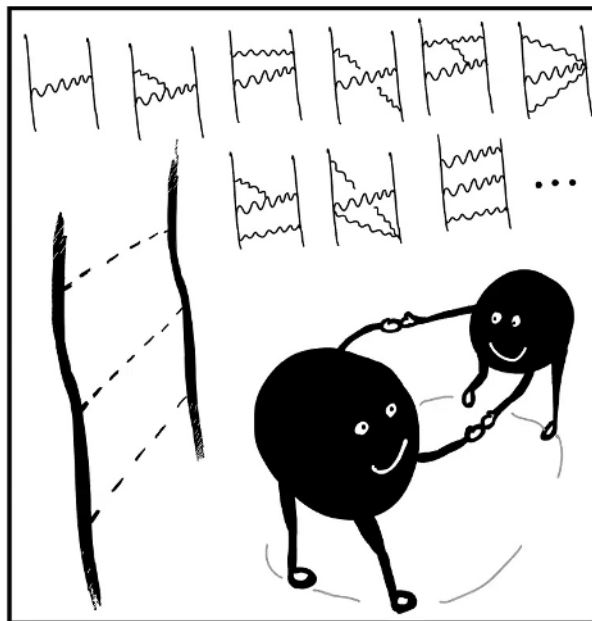
Series: Particle Physics

Date: June 15, 2021 - 1:00 PM

URL: <http://pirsa.org/21060000>

Abstract: Abstract: TBD

Zoom Link: <https://pitp.zoom.us/j/96516977019?pwd=WVlpZG5WTTUwbFJVZ2wvcXdNWUR5Zz09>



Binary Black Holes and Scattering Amplitudes

Mikhail Solon

Bhaumik Institute for Theoretical Physics, UCLA

Quantum Field Theory Tools



Precision Gravitational Wave Science

Quantum Field Theory Tools



Precision Gravitational Wave Science

Cheung, Rothstein, MS (PRL 18)

Bern, Cheung, Roiban, Shen, MS, Zeng (PRL 19, JHEP 19)

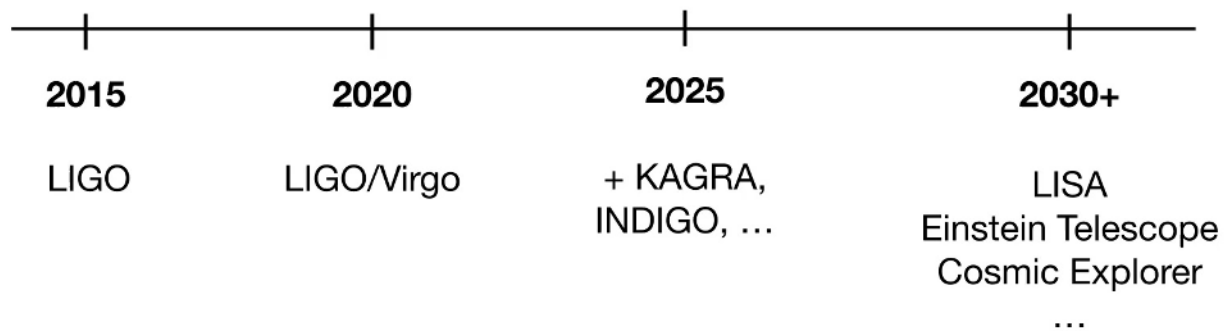
Cheung, MS (JHEP 20, PRL 20)

Cheung, Shah, MS (PRD 20)

Bern, Parra-Martinez, Roiban, Ruf, Shen, MS, Zeng (PRL 21)

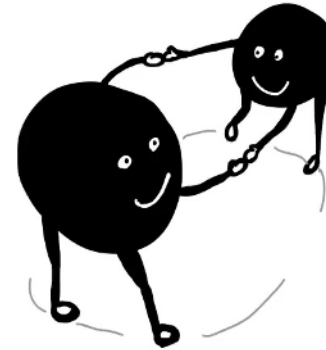
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Gravitational Wave Science



Gravitational Wave Science

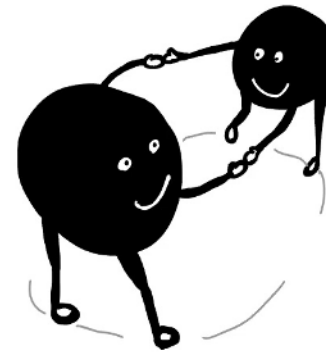
$$G_{ab} = 8\pi T_{ab} \rightarrow$$



Decades of heroic effort.

Gravitational Wave Science

$$G_{ab} = 8\pi T_{ab} \rightarrow$$



Decades of heroic effort.

post-Newtonian

$$G \sim v^2 \ll 1$$

4PN₍₂₀₁₄₎ → 6PN

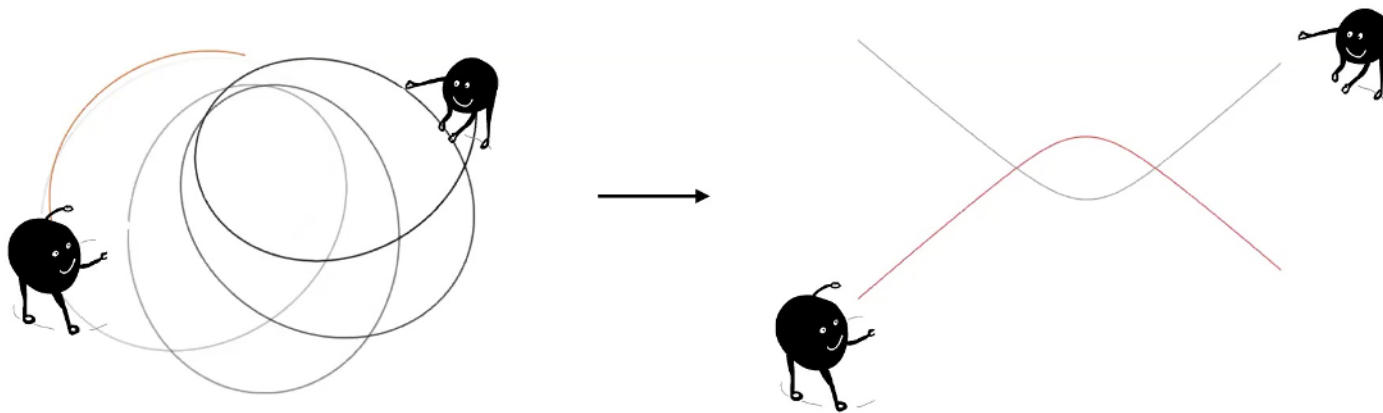
self-force

$$m/M \ll 1$$

1SF₍₁₉₉₀₎ → 2SF

numerical relativity

Turn it into an easier problem



Relativity

Tools of Theoretical High Energy Physics

onshell methods, advanced multiloop integration, effective field theory

Factorize states from underlying dynamics

$$\langle \Psi | \bar{u} \mathcal{O} u | \Psi \rangle$$

Does it work?

$$\begin{aligned} &G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \end{aligned}$$

Does it work?

$$\begin{aligned} & G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ & G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ & G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ & G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ & G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ & G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ & G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \end{aligned}$$

Cheung, Rothstein, MS PRL18

Does it work?

$$\begin{aligned} &G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \\ &G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \dots) \end{aligned}$$

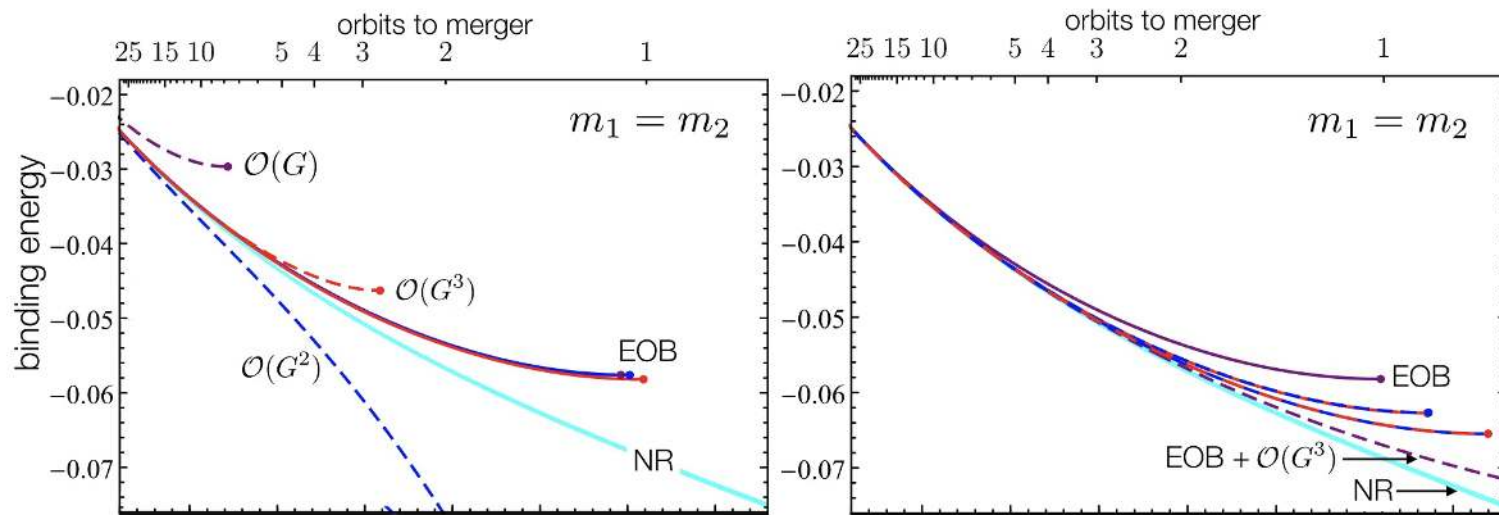
Cheung, Rothstein, MS PRL18

Bern, Cheung, Roiban, Shen, MS, Zeng PRL19

Bern, Parra-Martinez, Roiban, Ruf, Shen, MS, Zeng PRL 21

Theorists at LIGO are interested

Antonelli, Buonanno, Steinhoff, van de Meent, Vines 2019



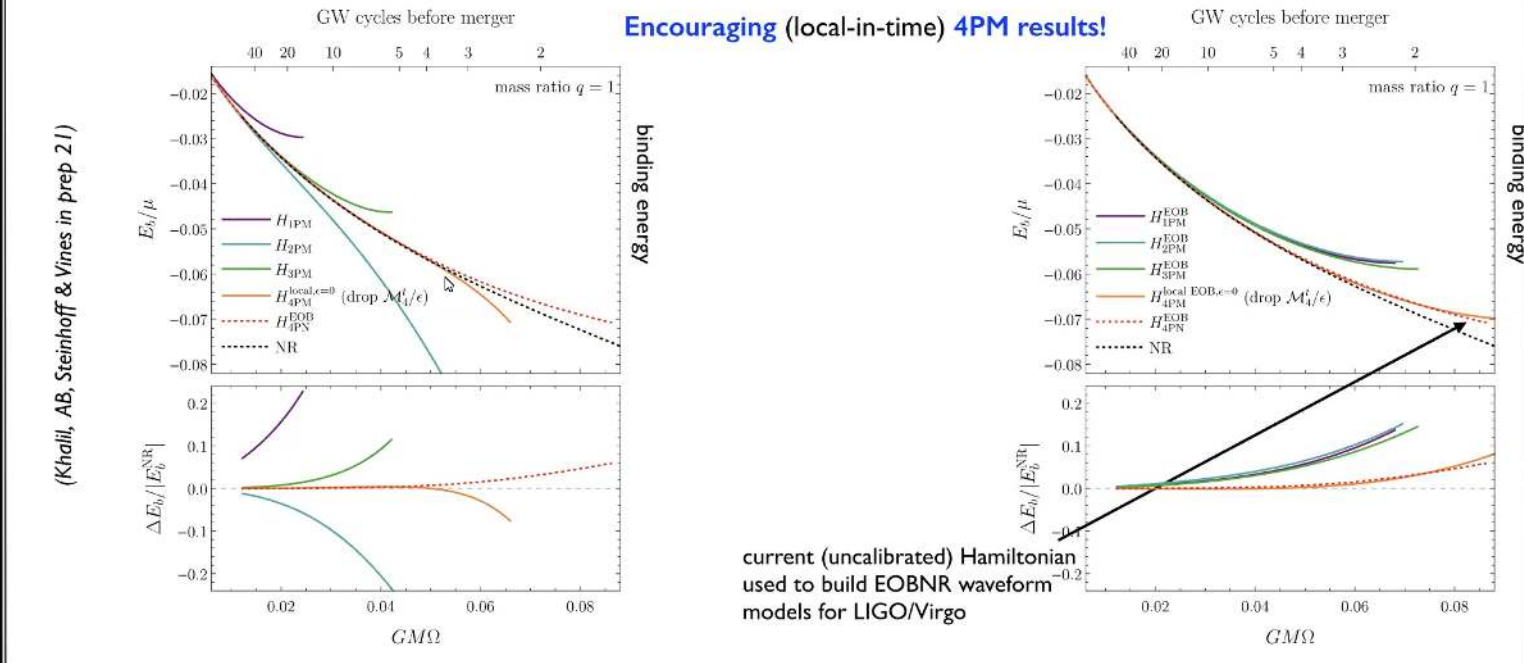
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NEW! from Alessandra Buonanno's recent talk at GGI

Crucial to push PM calculations at higher order, and resum them in EOB formalism.

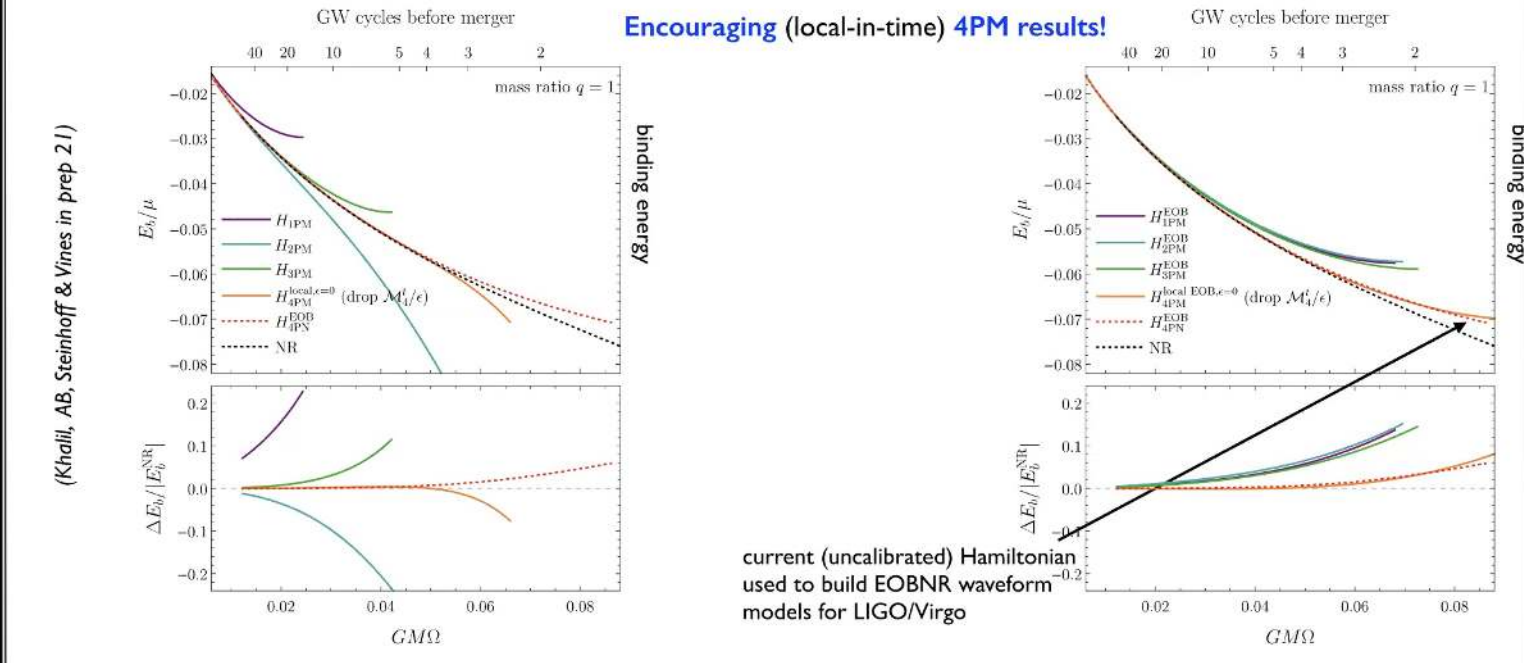
(Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)



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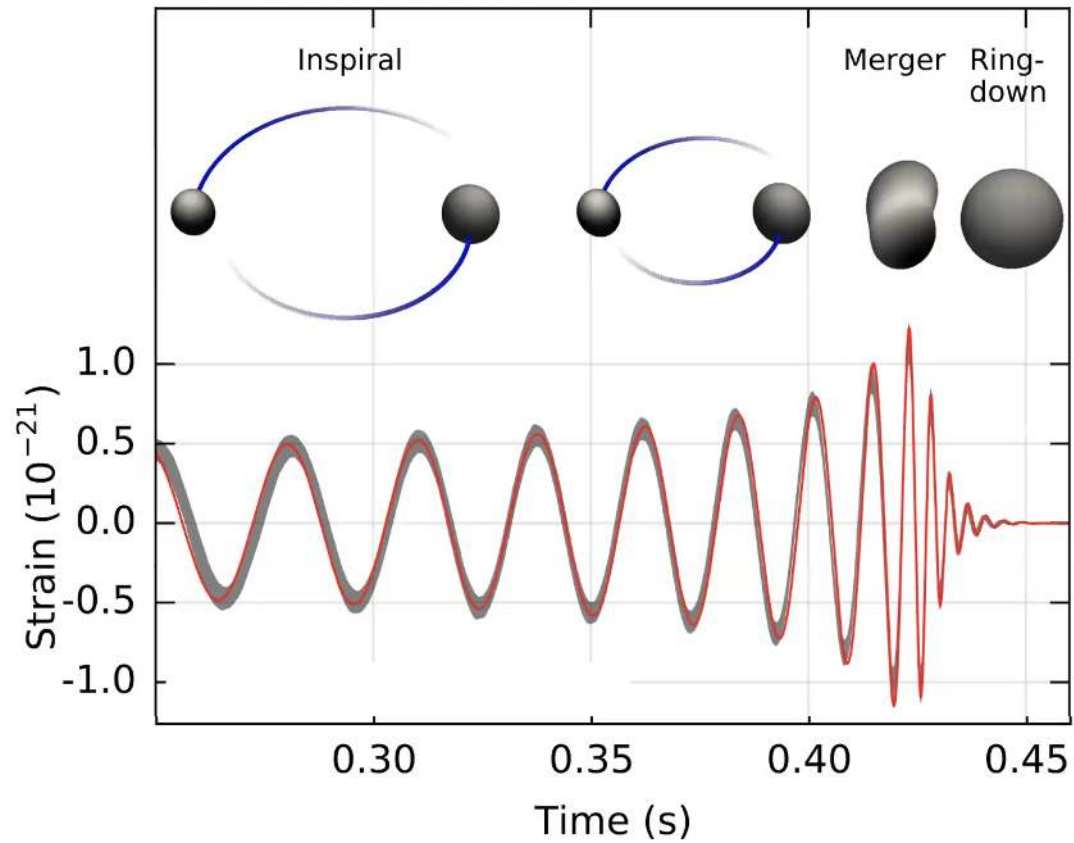


Interesting exchanges between GR and HEP

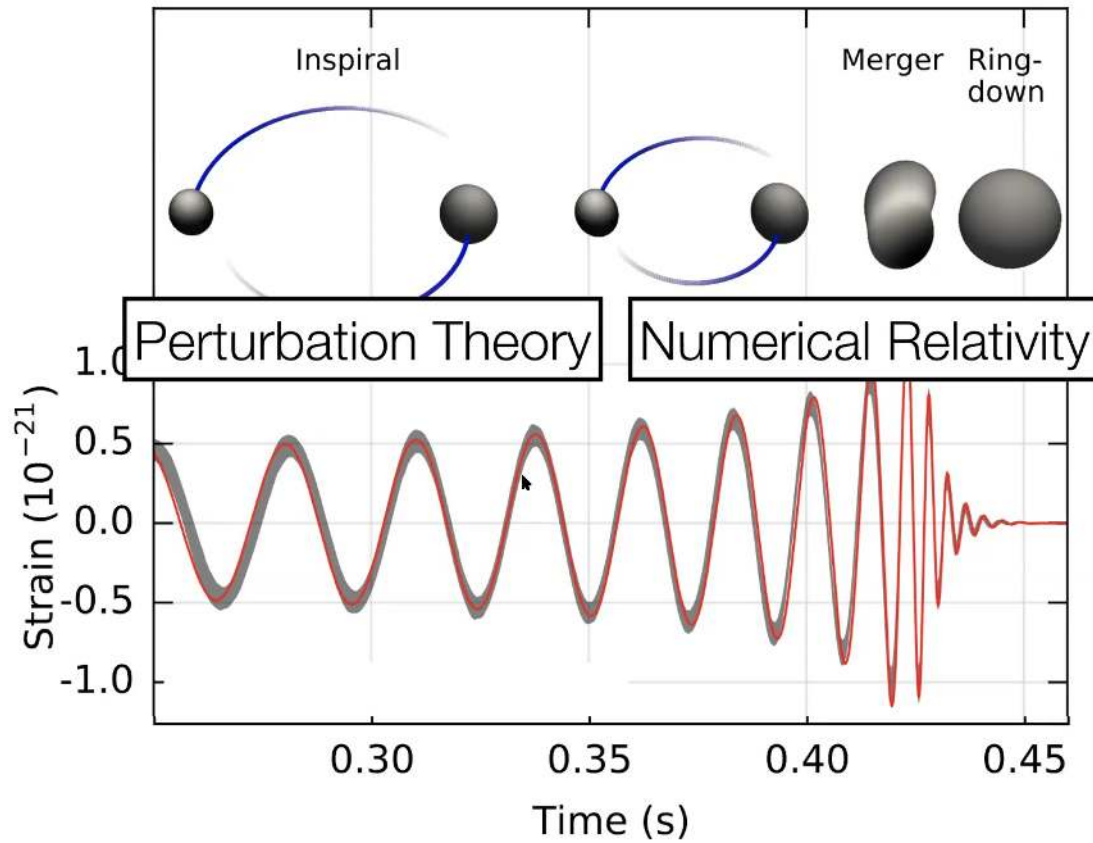
Bini, Damour, Geralico 2019, 2020, ... Blümlein, Maier, Marquard, Schäfer 2021

Damour 2019 Damour 2020

Hybrid Waveforms

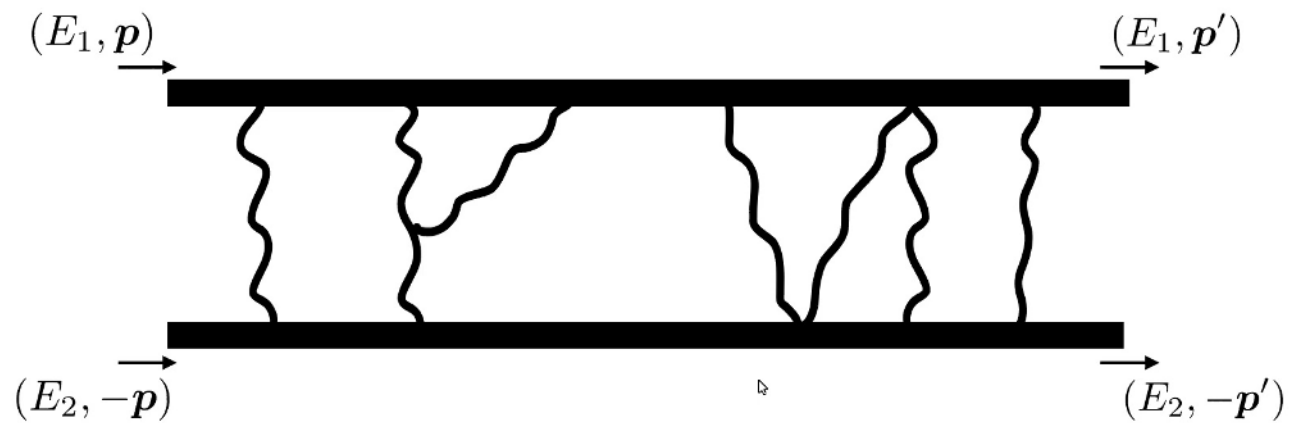


Hybrid Waveforms

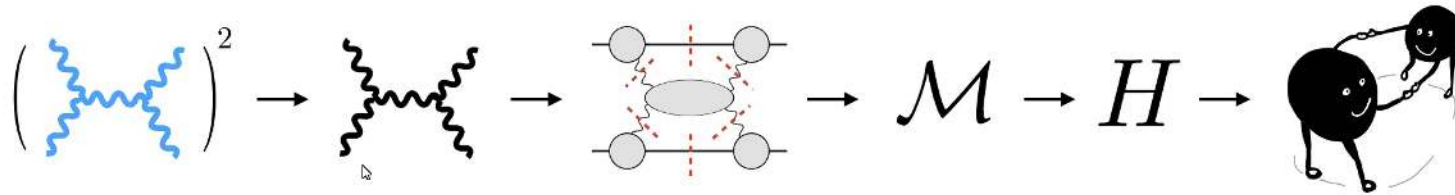


Scalable pipeline using tools from QFT ?

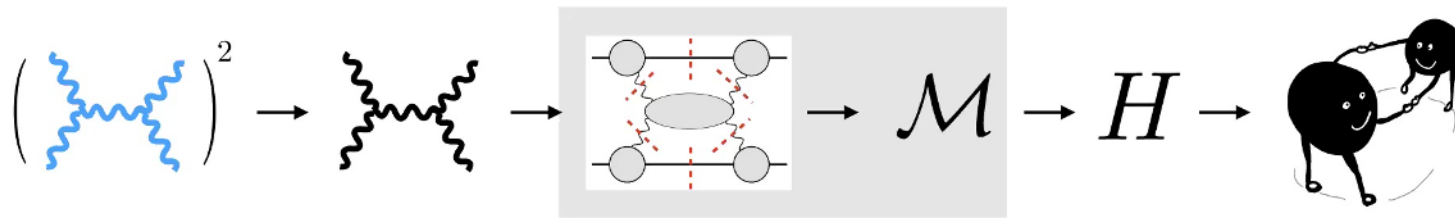
$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} \sum_{i=1,2} (\nabla \phi_i \nabla \phi_i - m_i^2 \phi_i^2) \right]$$



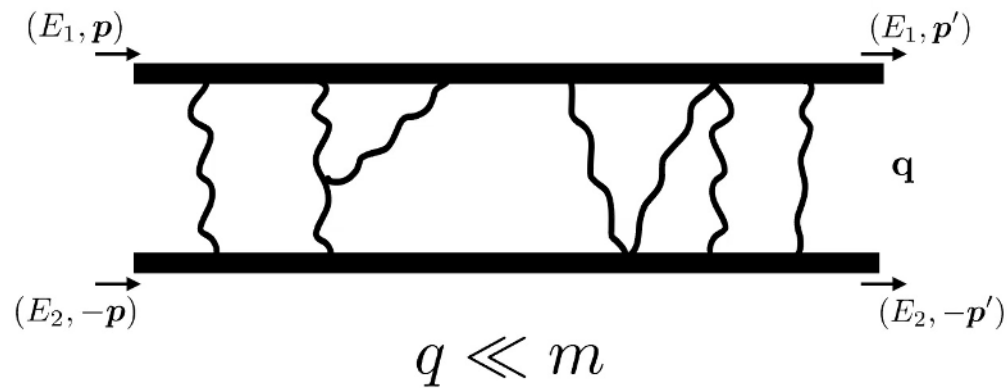
Scalable pipeline using tools from QFT:



Classical limit is taken at the earliest stages.

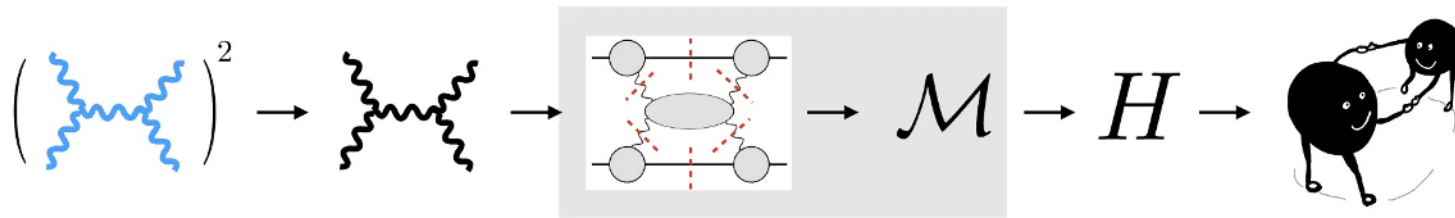


Full integration of quantum loops won't scale

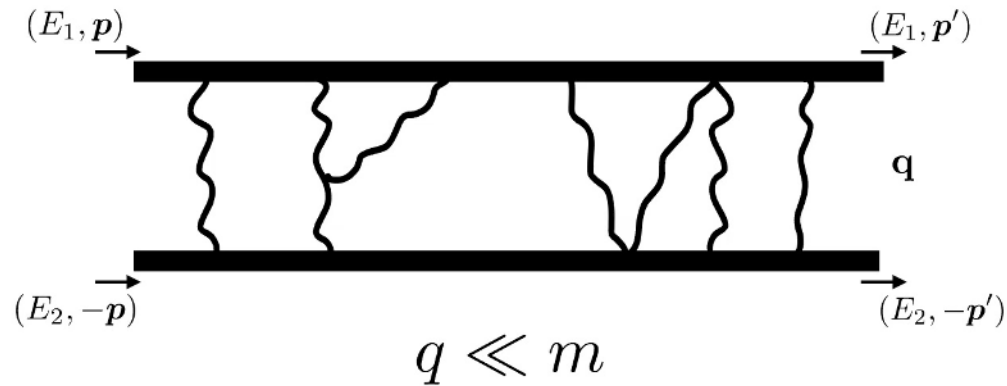


“In reality classical effects are smaller than quantum.” - Aneesh Manohar

$$\mathcal{M}_{\text{QED}} \sim \left(\frac{Q^2}{Q_{\text{pl}}^2} \frac{q}{m} \right)^c \left(\frac{q}{m} \right)^{\mathcal{Q}} \quad e \sim 10^{-1} Q_{\text{pl}}$$



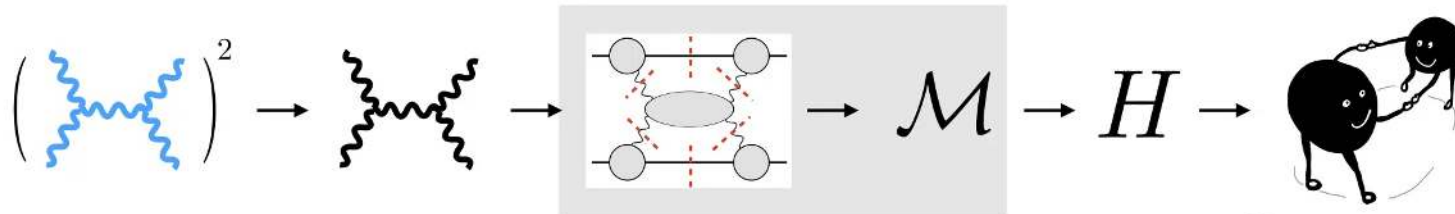
Full integration of quantum loops won't scale



“In reality classical effects are smaller than quantum.” - Aneesh Manohar

$$\mathcal{M}_{\text{QED}} \sim \left(\frac{Q^2 q}{Q_{\text{pl}}^2 m} \right)^c \left(\frac{q}{m} \right)^{\mathcal{Q}} \quad e \sim 10^{-1} Q_{\text{pl}}$$

$$\mathcal{M}_{\text{GR}} \sim \left(\frac{m^2 q}{m_{\text{pl}}^2 m} \right)^c \left(\frac{q}{m} \right)^{\mathcal{Q}} \quad M_{\odot} \sim 10^{38} m_{\text{pl}}$$



Classical limit is simple.



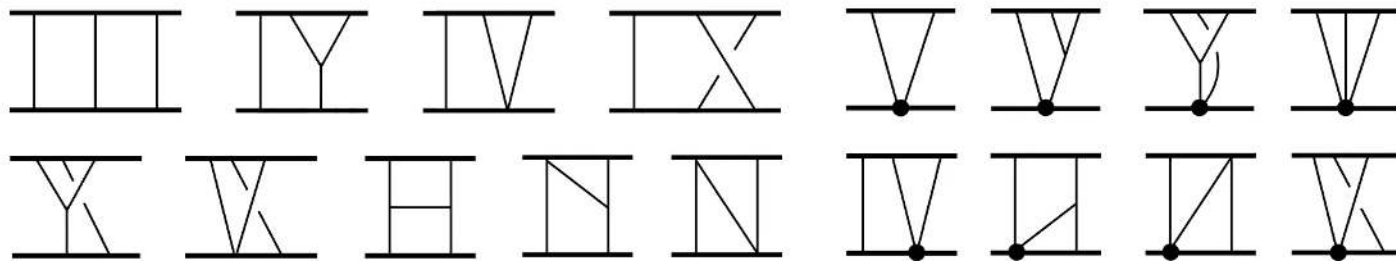
$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} \sum_{i=1,2} (\nabla\phi_i \nabla\phi_i - m_i^2 \phi_i^2) \right] S_{\text{GF}} = -\frac{1}{32\pi G} \int d^D x F_\mu F^\mu$$

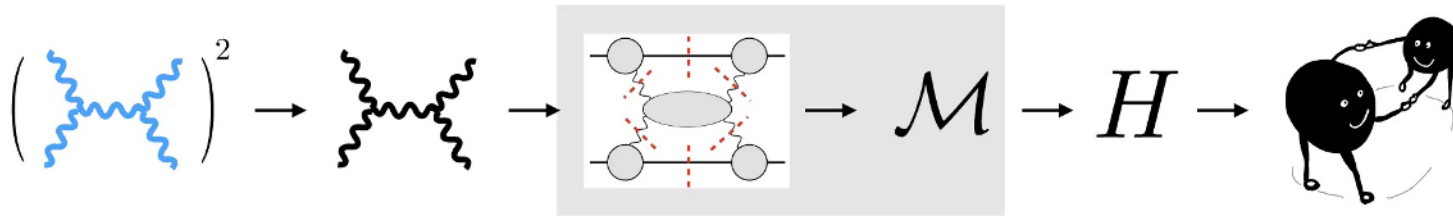
$$F_\mu = \partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \partial^\nu h^{\rho\sigma} (\zeta_1 h_{\mu\nu} \eta_{\rho\sigma} + \zeta_2 h_{\mu\rho} \eta_{\nu\sigma} + \zeta_3 h_{\rho\sigma} \eta_{\mu\nu} + \zeta_4 h_{\nu\sigma} \eta_{\mu\rho} + \zeta_5 \eta_{\mu\nu} \eta_{\rho\sigma} h + \zeta_6 \eta_{\mu\rho} \eta_{\nu\sigma} h)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \xi_1 h_{\mu\rho} h_\nu{}^\rho + \xi_2 h_{\mu\nu} h + \xi_3 \eta_{\mu\nu} h_{\rho\sigma} h^{\rho\sigma} + \xi_4 \eta_{\mu\nu} h^2$$

$$\Delta S = \int d^4 x \sqrt{-g} \frac{1}{4} C_{\mu\alpha\nu\beta} C^{\rho\alpha\sigma\beta} \sum_{i=1,2} \left(\lambda_i \phi_i^2 \delta_\rho^\mu \delta_\sigma^\nu + \frac{\eta_i}{m_i^4} \nabla^\mu \nabla^\nu \phi_i \nabla_\rho \nabla_\sigma \phi_i \right)$$

Cheung, MS





soft : $(\omega, \ell) \sim (|\mathbf{q}|, |\mathbf{q}|) \longrightarrow$ potential : $(\omega, \ell) \sim (|\mathbf{q}||\mathbf{v}|, |\mathbf{q}|)$
radiation : $(\omega, \ell) \sim (|\mathbf{q}||\mathbf{v}|, |\mathbf{q}||\mathbf{v}|)$

NRGR Goldberger, Rothstein

Loop energies

$$\int \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} \mathcal{I}(\omega, \omega') = \sum_{(i,j)} S_{ij} \underbrace{\text{Res}_{\omega_{ij}, \omega'_{ij}}}_{\text{matter poles}} \mathcal{I}(\omega, \omega')$$

$$\text{IV} = \frac{1}{6} \text{IV} - \frac{1}{3} \text{IV}$$

Cheung, Rothstein, MS

$$1 + v^2 + v^4 + v^6 + v^8 + \dots$$

Static limit boundary condition
in the potential region

Obtain from differential equations.

Parra-Martinez, Ruf, Zeng

Amplitudes

$$\mathcal{M}_1 = 16\pi G\nu^2 M^4 |\mathbf{q}|^{-2} (2\sigma^2 - 1)$$

$$\mathcal{M}_2 = 6\pi^2 G^2 \nu^2 M^5 |\mathbf{q}|^{-1} (5\sigma^2 - 1)$$

$$\mathcal{M}_3 = 2\pi G^3 \nu^2 M^6 \log \mathbf{q}^2 \left[\frac{(1 + 2\nu(\sigma - 1))(5 - 60\sigma^2 + 120\sigma^4 - 64\sigma^2)}{3(\sigma^2 - 1)^2} + 8\nu \left(\frac{\sigma(14\sigma^2 + 25)}{3} + (4\sigma^4 - 12\sigma^2 - 3) \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \right) \right]$$

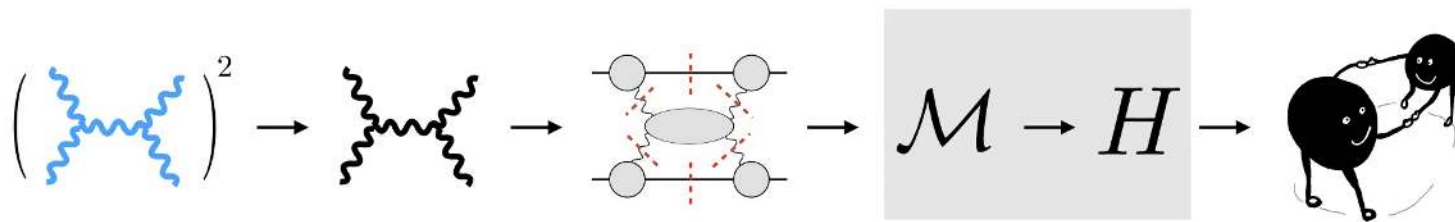
$$\mathcal{M}_4(\mathbf{q}) = G^4 M^7 \nu^2 |\mathbf{q}| \left(\frac{\mathbf{q}^2}{4^{\frac{1}{3}} \tilde{\mu}^2} \right)^{-3\epsilon} \left[\mathcal{M}_4^{\text{P}} + \nu \left(\frac{\mathcal{M}_4^{\text{t}}}{\epsilon} + \mathcal{M}_4^{\text{f}} \right) \right]$$

$$\mathcal{M}_4^{\text{P}} = -\frac{35(1 - 18\sigma^2 + 33\sigma^4)}{8(\sigma^2 - 1)} \quad \mathcal{M}_4^{\text{t}} = h_1 + h_2 \log\left(\frac{\sigma+1}{2}\right) + h_3 \frac{\text{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}$$

$$\begin{aligned} \mathcal{M}_4^{\text{f}} = & h_4 + h_5 \log\left(\frac{\sigma+1}{2}\right) + h_6 \frac{\text{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} + h_7 \log(\sigma) - h_2 \frac{2\pi^2}{3} + h_8 \frac{\text{arccosh}^2(\sigma)}{\sigma^2 - 1} + h_9 \left[\text{Li}_2\left(\frac{1-\sigma}{2}\right) + \frac{1}{2} \log^2\left(\frac{\sigma+1}{2}\right) \right] \\ & + h_{10} \left[\text{Li}_2\left(\frac{1-\sigma}{2}\right) - \frac{\pi^2}{6} \right] + h_{11} \left[\text{Li}_2\left(\frac{1-\sigma}{1+\sigma}\right) - \text{Li}_2\left(\frac{\sigma-1}{\sigma+1}\right) + \frac{\pi^2}{3} \right] + h_2 \frac{2\sigma(2\sigma^2 - 3)}{(\sigma^2 - 1)^{3/2}} \left[\text{Li}_2\left(\sqrt{\frac{\sigma-1}{\sigma+1}}\right) - \text{Li}_2\left(-\sqrt{\frac{\sigma-1}{\sigma+1}}\right) \right] \\ & + \frac{2h_3}{\sqrt{\sigma^2 - 1}} \left[\text{Li}_2(1 - \sigma - \sqrt{\sigma^2 - 1}) - \text{Li}_2(1 - \sigma + \sqrt{\sigma^2 - 1}) \right] + 5\text{Li}_2\left(\sqrt{\frac{\sigma-1}{\sigma+1}}\right) - 5\text{Li}_2\left(-\sqrt{\frac{\sigma-1}{\sigma+1}}\right) + 2 \log\left(\frac{\sigma+1}{2}\right) \text{arccosh}(\sigma) \\ & + h_{12} \text{K}^2\left(\frac{\sigma-1}{\sigma+1}\right) + h_{13} \text{K}\left(\frac{\sigma-1}{\sigma+1}\right) \text{E}\left(\frac{\sigma-1}{\sigma+1}\right) + h_{14} \text{E}^2\left(\frac{\sigma-1}{\sigma+1}\right), \end{aligned}$$

$$\begin{aligned} h_1 &= \frac{1151 - 3336\sigma + 3148\sigma^2 - 912\sigma^3 + 339\sigma^4 - 552\sigma^5 + 210\sigma^6}{12(\sigma^2 - 1)} \\ h_2 &= \frac{1}{2} (5 - 76\sigma + 150\sigma^2 - 60\sigma^3 - 35\sigma^4) \\ h_3 &= \sigma \frac{(-3 + 2\sigma^2)}{4(\sigma^2 - 1)} (11 - 30\sigma^2 + 35\sigma^4) \\ h_4 &= \frac{1}{144(\sigma^2 - 1)^2 \sigma^7} (-45 + 207\sigma^2 - 1471\sigma^4 + 13349\sigma^6 \\ &\quad - 37566\sigma^7 + 104753\sigma^8 - 12312\sigma^9 - 102759\sigma^{10} - 105498\sigma^{11} \\ &\quad + 134745\sigma^{12} + 83844\sigma^{13} - 101979\sigma^{14} + 13644\sigma^{15} + 10800\sigma^{16}) \\ h_5 &= \frac{1}{4(\sigma^2 - 1)} (1759 - 4768\sigma + 3407\sigma^2 - 1316\sigma^3 + 957\sigma^4 \\ &\quad - 672\sigma^5 + 341\sigma^6 + 100\sigma^7) \end{aligned}$$

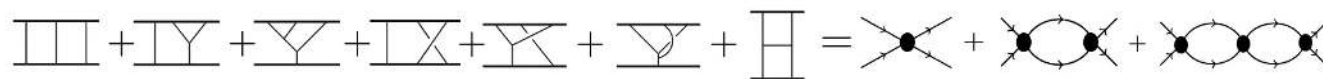
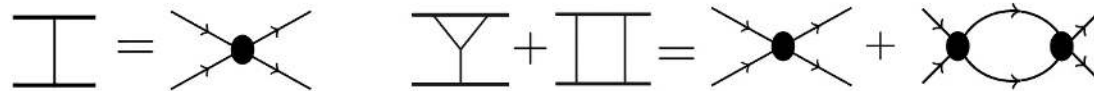
$$\begin{aligned} h_6 &= \frac{1}{24(\sigma^2 - 1)^2} (1237 + 7959\sigma - 25183\sigma^2 + 12915\sigma^3 + 18102\sigma^4 \\ &\quad - 12105\sigma^5 - 9572\sigma^6 + 2973\sigma^7 + 5816\sigma^8 - 2046\sigma^9) \\ h_7 &= 2\sigma \frac{(-852 - 283\sigma^2 - 140\sigma^4 + 75\sigma^6)}{3(\sigma^2 - 1)} \\ h_8 &= \frac{\sigma}{8(\sigma^2 - 1)^2} (-304 - 99\sigma + 672\sigma^2 + 402\sigma^3 - 192\sigma^4 - 719\sigma^5 \\ &\quad - 416\sigma^6 + 540\sigma^7 + 240\sigma^8 - 140\sigma^9) \\ h_9 &= \frac{1}{2} (52 - 532\sigma + 351\sigma^2 - 420\sigma^3 + 30\sigma^4 - 25\sigma^6) \\ h_{10} &= 2(27 + 90\sigma^2 + 35\sigma^4) \\ h_{11} &= 20 + 111\sigma^2 + 30\sigma^4 - 25\sigma^6 \\ h_{12} &= \frac{834 + 2095\sigma + 1200\sigma^2}{2(\sigma^2 - 1)} \\ h_{13} &= -\frac{1183 + 2929\sigma + 2660\sigma^2 + 1200\sigma^3}{2(\sigma^2 - 1)} \\ h_{14} &= \frac{7(169 + 380\sigma^2)}{4(\sigma - 1)} \end{aligned}$$



$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \int_{\mathbf{k}, \mathbf{k}'} V(\mathbf{k}, \mathbf{k}') A^\dagger(\mathbf{k}') A(\mathbf{k}) B^\dagger(-\mathbf{k}') B(-\mathbf{k})$$

Cheung, Rothstein, MS

$$V(\mathbf{p}, \mathbf{r}) = \sum_{i=1}^{\infty} c_i(\mathbf{p}^2) \left(\frac{G}{|\mathbf{r}|} \right)^i$$



“amplitude-action” relation

$$i\mathcal{M}(\mathbf{q}) = \int_J \left(e^{iI_r(J)} - 1 \right)$$

Hamiltonian

$$H(\mathbf{p}, \mathbf{r}) = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} + \sum_{i=1}^3 c_i(\mathbf{p}^2) \left(\frac{G}{|\mathbf{r}|} \right)^i$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi} (1 - 2\sigma^2) \quad \xi = E_1 E_2 / E^2$$

$$c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi} \left[\frac{3}{4} (1 - 5\sigma^2) - \frac{4\nu\sigma (1 - 2\sigma^2)}{\gamma\xi} - \frac{\nu^2 (1 - \xi) (1 - 2\sigma^2)^2}{2\gamma^3 \xi^2} \right]$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi} \left[\frac{1}{12} (3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3) - \frac{4\nu (3 + 12\sigma^2 - 4\sigma^4) \operatorname{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right. \\ \left. - \frac{3\nu\gamma (1 - 2\sigma^2) (1 - 5\sigma^2)}{2(1 + \gamma)(1 + \sigma)} - \frac{3\nu\sigma (7 - 20\sigma^2)}{2\gamma\xi} - \frac{\nu^2 (3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2) (1 - 2\sigma^2)}{4\gamma^3 \xi^2} \right. \\ \left. + \frac{2\nu^3 (3 - 4\xi)\sigma (1 - 2\sigma^2)^2}{\gamma^4 \xi^3} + \frac{\nu^4 (1 - 2\xi) (1 - 2\sigma^2)^3}{2\gamma^6 \xi^4} \right],$$

$$c_4 = \frac{M^7 \nu^2}{4\xi E^2} \left[\mathcal{M}_4^p + \nu \left(\frac{\mathcal{M}_4^t}{\epsilon} + \mathcal{M}_4^f - 10\mathcal{M}_4^i \right) \right] + \mathcal{D}^3 \left[\frac{E^3 \xi^3}{3} c_1^4 \right] + \mathcal{D}^2 \left[\left(\frac{E^3 \xi^3}{\mathbf{p}^2} + \frac{E\xi(3\xi - 1)}{2} \right) c_1^4 - 2E^2 \xi^2 c_1^2 c_2 \right] \\ + \left(\mathcal{D} + \frac{1}{\mathbf{p}^2} \right) \left[E\xi(2c_1 c_3 + c_2^2) + \left(\frac{4\xi - 1}{4E} + \frac{2E^3 \xi^3}{\mathbf{p}^4} + \frac{E\xi(3\xi - 1)}{\mathbf{p}^2} \right) c_1^4 + \left((1 - 3\xi) - \frac{4E^2 \xi^2}{\mathbf{p}^2} \right) c_1^2 c_2 \right],$$

$$\mathcal{D} = \frac{d}{d\mathbf{p}^2}$$

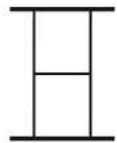
Theoretical Structures

Lorentz invariant, on shell

High-energy behavior

$$\mathcal{M}_3 = \frac{-\pi G^3 \nu^2 m^4 \ln \mathbf{q}^2}{6\gamma^2 \xi} \left[\frac{48\nu(3 + 12\sigma^2 - 4\sigma^4) \operatorname{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right] + \dots$$

$$= -8\pi G^3 s^2 \ln(-t) \ln\left(\frac{m_1 m_2}{s}\right) + \dots$$



$$\sim \left[\log\left(\frac{s}{m^2}\right) - \log\left(\frac{t}{m^2}\right) \right]^2$$

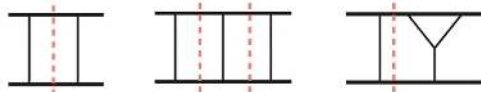
Regge limit



radiation

Di Vecchia, Heissenberg, Russo,
Veneziano 20; Damour 20

Exponentiation



$$i\mathcal{M}(\mathbf{q}) = \int_J \left(e^{iI_r(J)} - 1 \right)$$

Mass dependence $\mathcal{M}_N \sim G^N [m_1^{N+1}m_2^2 + m_1^N m_2^3 + \dots + \{1 \leftrightarrow 2\}]$

$$\mathcal{M}_1 \sim G [m_1^2 m_2^2]$$

$$\mathcal{M}_2 \sim G^2 [m_1^3 m_2^2 + \{1 \leftrightarrow 2\}]$$

$$\mathcal{M}_3 \sim G^3 [m_1^4 m_2^2 + m_1^3 m_2^3 + \{1 \leftrightarrow 2\}]$$

$$\mathcal{M}_4 \sim G^4 [m_1^5 m_2^2 + m_1^4 m_2^3 + \{1 \leftrightarrow 2\}]$$

$$\mathcal{M}_5 \sim G^5 [m_1^6 m_2^2 + m_1^5 m_2^3 + m_1^4 m_2^4 + \{1 \leftrightarrow 2\}]$$

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\vdots OSF

Test-particle in Schwarzschild (and beyond)

$$\mathcal{M}_0^\star(p, r) = \frac{1}{2E} \left[m^2 (1 - g_{rr}^{\text{iso}}(r)) - E^2 \left(1 + \frac{g_{rr}^{\text{iso}}(r)}{g_{tt}^{\text{iso}}(r)} \right) \right]$$



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Mass dependence $\mathcal{M}_N \sim G^N [m_1^{N+1}m_2^2 + m_1^N m_2^3 + \dots + \{1 \leftrightarrow 2\}]$

$$\mathcal{M}_1 \sim G [m_1^2 m_2^2]$$

$$\mathcal{M}_2 \sim G^2 [m_1^3 m_2^2 + \{1 \leftrightarrow 2\}]$$

$$\mathcal{M}_3 \sim G^3 [m_1^4 m_2^2 + m_1^3 m_2^3 + \{1 \leftrightarrow 2\}]$$

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\vdots OSF 1SF

Test-particle in Schwarzschild (and beyond)

Bootstrap from 1SF ($\sim m/M$) results

$$\mathcal{M}_0(p, r) = \frac{1}{2E} \left[m^2 (1 - g_{rr}^{\text{iso}}(r)) - E^2 \left(1 + \frac{g_{rr}^{\text{iso}}(r)}{g_{tt}^{\text{iso}}(r)} \right) \right]$$

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\vdots OSF 1SF 2SF

↘

Test-particle in Schwarzschild (and beyond)

Bootstrap from 1SF ($\sim m/M$) results

Bini, Damour, Gericco

$$\mathcal{M}_0(p, r) = \frac{1}{2E} \left[m^2 (1 - g_{rr}^{\text{iso}}(r)) - E^2 \left(1 + \frac{g_{rr}^{\text{iso}}(r)}{g_{tt}^{\text{iso}}(r)} \right) \right]$$



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Gravitational wave science has opened up a new direction in theoretical high energy physics.

Classical binary dynamics has the hallmarks of a great problem in theoretical high energy physics.

This program is in a nascent stage.

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