Title: Emergent cosmology from tensorial group field theories for quantum gravity

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Series: Quantum Gravity

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Abstract: We introduce the basic elements of tensorial group field theories (TGFTs) for quantum gravity, emphasizing how they encode quantum geometry and their relation with canonical loop quantum gravity and spin foam models. Next, we discuss briefly the issue of continuum limit and how it could be understood in this framework.

In the bulk of the talk, we overview the work on the extraction of an effective cosmological dynamics from TGFTs, inspired by the idea of the universe as a quantum gravity condensate. In this context, we emphasize: the need for appropriately coarse-grained states capturing collective dynamics and the role of relational observables and their construction. We discuss what the theory says (so far) about the fate of the big bang singularity at the beginning of our universe and how it suggests a quantum gravity origin for (phantom) dark energy.

Emergent cosmology from tensorial group field theories for quantum gravity

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QG seminar, Perimeter Institute - 03.6.2021

The framework:

Tensorial (Group) Field Theories

Tensorial Group Field Theories: key elements

Quantum field theories for the fundamental QG degrees of freedom, generating superposition of (simplicial) cellular complexes, from which spacetime should emerge in some approximation and "phase" of theory

generalization of matrix models for 2d gravity
(Ambjorn, David, Migdal, Duplantier, Kazakov, ...)

- field theory action with non-local interactions, describing how simplices connect to form higher-cells

details depend on (class of) models

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

"combinatorial non-locality"
in pairing of field arguments

note on nomenclature

tensor models

tensorial field theories

group field theories

tensorial group field theories

different criteria adopted in the literature

here:

TGFTs for general formalism

"quantum-geometric TGFTs" = class of models with stronger relation to LQG, spin foam models, simplicial gravity path integrals, quantum geometry

basic object is tensor

domain of tensor is finite set

domain of tensor is group manifold

domain of tensor is more general manifold

interactions are combinatorially non-local

interactions are bubble (unitary) invariants (tensorial nature of field is prominent))

interactions are simplicial (or other non-bubble)

propagator involves differential operator (proper "dynamical" field theory)

propagator only involves correlations

tensor possesses special group-theoretic symmetries

interaction kernels implement additional group- theoretic conditions on dynamics

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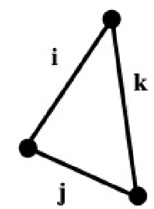
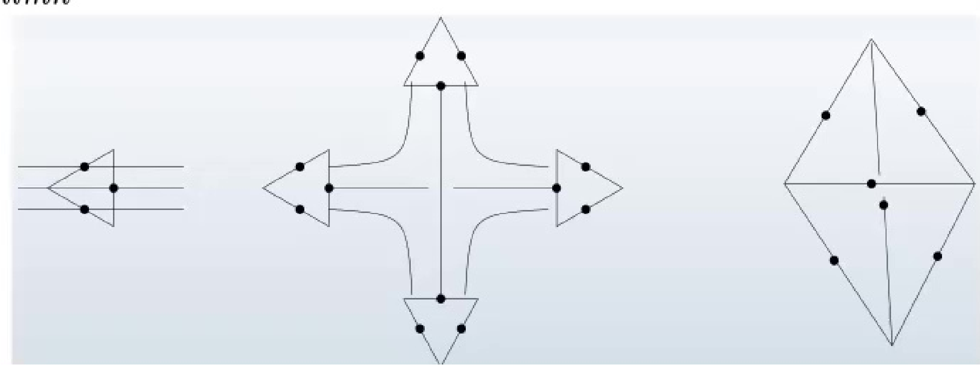
"combinatorial non-locality"
in pairing of field arguments

- example of combinatorial non-locality: rank-3 simplicial tensor model

fundamental building block of (quantum) space: $T_{ijk} \,:\, \mathbb{Z}_N^{\times 3} \rightarrow \mathbb{R}$



$$S(T) = \frac{1}{2}\sum_{i,j,k} T_{ijk}T_{kji} - \frac{\lambda}{4!\sqrt{N^3}}\sum_{ijklmn} T_{ijk}T_{klm}T_{mjn}T_{nli}$$



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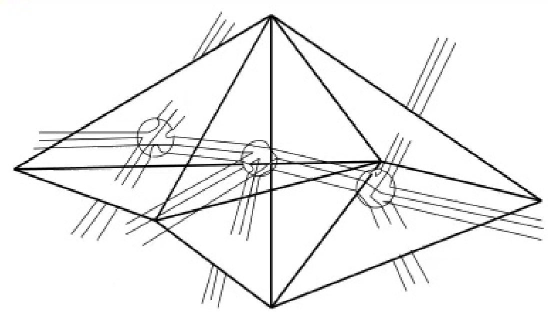
"combinatorial non-locality"
in pairing of field arguments

- Feynman diagrams are dual to cellular complexes
- perturbative expansion of quantum dynamics gives sum over cellular complexes of all topologies

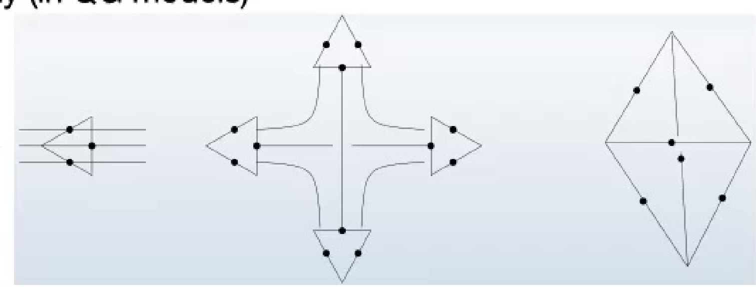


$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma$$

- Feynman amplitudes should be related to discrete gravity (in QG models)

example:

$$S(T) = \frac{1}{2}\sum_{i,j,k} T_{ijk}T_{kji} - \frac{\lambda}{4!\sqrt{N^3}}\sum_{ijklmn} T_{ijk}T_{klm}T_{mjn}T_{nli}$$



amplitudes purely combinatorial -
geometry of equilateral triangulations

TGFT: general aspects

Gurau, Ryan, Tanasa, Bonzom, Rivasseau, Ben Geloun, Benedetti, ...

- link to lattice gravity

  in simplest models, amplitudes depend purely on combinatorics of Feynman diagrams

  link to discrete gravity in terms of dynamical triangulations: dual simplicial complexes as equilateral

  in richer, quantum geometric models, much richer lattice gravity/geometry (see later)

- making mathematical sense of the formalism:

  - controlling the TGFT perturbative expansion

    colors added to label combinatorial structures allow precise control over their topology

    large-N limit can be defined and controlled (melonic dominance)

    multiple-scaling limits can be also defined for simple models

    such tools crucial for TGFT renormalization

  - resumming the perturbative series and critical behaviour

Tensorial Group Field Theories: key elements

Quantum field theories for the fundamental QG degrees of freedom, from which spacetime should emerge in some approximation, possibly only in some "phase" of the theory

- field theory action with non-local interactions, describing how simplices connect to form higher-cells

details depend on (class of) models

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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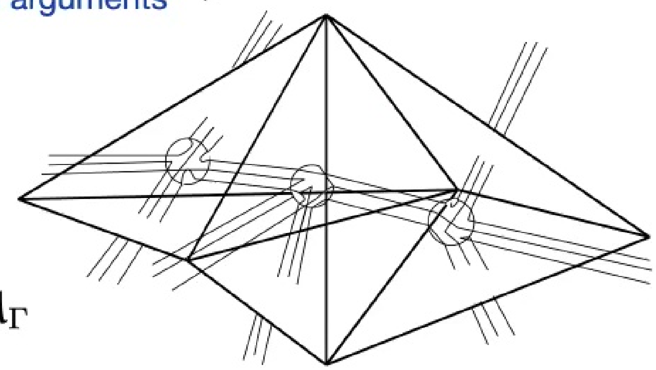

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- quantum geometric models:

  Feynman amplitudes can be represented equivalently as: lattice gravity path integrals (like quantum Regge calculus) or spin foam models (covariant LQG dynamics)

  quantum states are second quantized simplices/spin networks

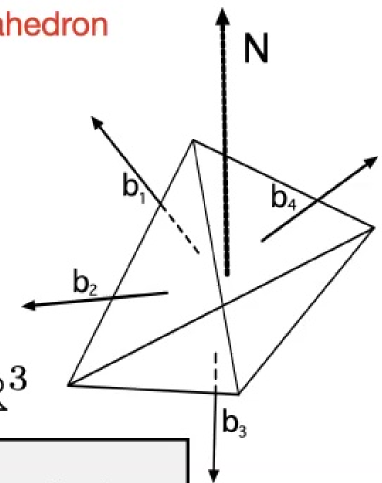
TGFTs for Quantum Gravity:

Quantum geometric models
and relation to canonical LQG and spin
foam models

TGFT "atom of space" in quantum geometric models

Barbieri '97; Baez, Barrett, '99; Rovelli, Speziale, '06; Freidel, Krasnov, '07; Livine, Speziale, '07; Bianchi, Dona, Speziale, '10; Baratin, DO, '11; ……

Elementary building block of 3d space: single polyhedron - simplest example: a tetrahedron



Classical geometry in group-theoretic variables

4 vectors normal to triangles that close (lying in hypersurface with normal N)

$$\boxed{A_i\, n_i^I = b_i^I \in \mathbb{R}^{3,1} \qquad b_i \cdot N = 0 \qquad \sum_i b_i = 0}$$

choosing N=(1,0,0,0): $b_i^I \leftrightarrow (0, b_i^a) \quad b_i^a \in \mathbb{R}^3$

upon assigning a symplectic structure to vector space, isomorphism: $\mathfrak{su}(2) \simeq \mathbb{R}^3$

part of classical phase space $[\mathcal{T}^* SU(2)]^{\times 4}$

Phase space for tetrahedron

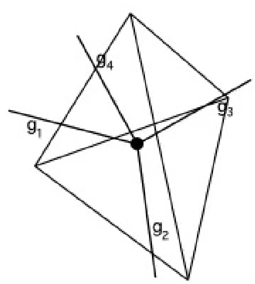
conjugate variables: group elements $\{g_i\} =$ discrete connection (extrinsic geometry) on links dual to triangles

note:

equivalently: constrained area 2-forms ~ so(3,1) Lie algebra elements - leading to SO(3,1)-based phase space $B_i^{IJ} = N^I \wedge b_i^J \; (\sim \star e \wedge e)$

starting point of construction of spin foam models

one can map SO(3,1) description to/from SO(3) one



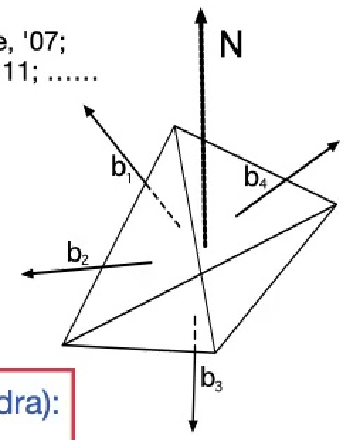
subject to the closure + simplicity constraints --> submanifold of same phase space

extended geometric triangulations: tetrahedra with boundary data identified, and assigned parallel transports along paths connecting respective centers

Quantum geometry

Barbieri '97; Baez, Barrett, '99; Rovelli, Speziale, '06; Livine, Speziale, '07; L. Freidel, K. Krasnov, '7; Bianchi, Dona, Speziale, '10; Baratin, DO, '11; ……

Hilbert space
$$\mathcal{H}_v = L^2\left(G^d; d\mu_{Haar}\right) \xrightarrow[\text{+ constraints}]{} \mathcal{H}_{tet}$$



note: can work with G= SO(3,1) or G = SO(3), and define map between Hilbert spaces

different representations for Hilbert space of quantum tetrahedron (collection of tetrahedra):

- group representation (functions on group manifold)
- Lie algebra representation (non-commutative functions on Lie algebra)
- spin representation (functions labeled by eigenvalues of compatible operators)

$$\psi(g_1, g_2, g_3, g_4) = \sum_{j_i, m_i, n_i} \psi^{j_1 j_2 j_3 j_4}_{m_1 n_1 \ldots m_4 n_4} D^{j_1}_{m_1 n_1}(g_1) \ldots D^{j_4}_{m_4 n_4}(g_4)$$ Hilbert space of spin network vertex

a lot of interesting quantum geometry, from quantizing geometric phase space observables

Hilbert space of quantum geometric TGFTs: Fock space

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym \left\{ \left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)} \right) \right\}$$ (bosonic statistics assumed)

$$\left[\hat{\varphi}(\vec{g}), \hat{\varphi}^\dagger(\vec{g}')\right] = \mathbb{I}_G(\vec{g}, \vec{g}') \qquad \left[\hat{\varphi}(\vec{g}), \hat{\varphi}(\vec{g}')\right] = \left[\hat{\varphi}^\dagger(\vec{g}), \hat{\varphi}^\dagger(\vec{g}')\right] = 0$$

all geometric operators have 2nd quantized counterpart

e.g. total space volume (extensive quantity): $$\hat{V}_{tot} = \int [dg_i][dg'_j] \hat{\varphi}^\dagger(g_i)\, V(g_i, g'_j)\, \hat{\varphi}(g'_j) = \sum_{J_i} \hat{\varphi}^\dagger(J_i)\, V(J_i)\, \hat{\varphi}(J_j)$$

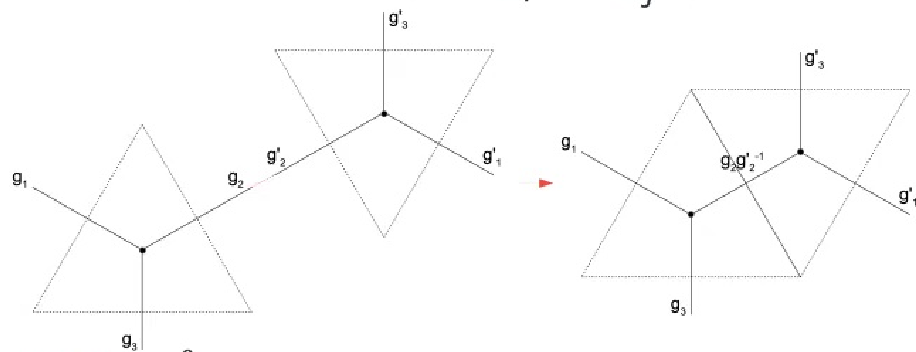
volume of single tetrahedron (from simplicial geometry)

Entanglement graphs

E. Colafranceschi, DO, '20

- graphs structures are associated to special entangled TGFT states within Fock space
- start with many-body wavefunction for N TGFT quanta (= N vertices) $\psi(\ldots, g_i^x, \ldots, g_j^y, \ldots)$
- associated to vertices x,y connected by link i if invariant under group action:

$$\int dh \psi(\ldots, g_i^x h, \ldots, g_i^y h, \ldots) = \psi(\ldots, g_i^x g_i^{y-1}, \ldots)$$



- can be enforced by "link map":

$$P_i^{x\otimes y} : \mathcal{H}_i^x \otimes \mathcal{H}_i^y \rightarrow \mathrm{Inv}(\mathcal{H}_i^x \otimes \mathcal{H}_i^y) \qquad P_i^{x\otimes y} := \int dh dg_i^x dg_i^y \; |g_i^x\rangle\langle g_i^x h| \otimes |g_i^y\rangle\langle g_i^y h|$$

which entangles x and y along link i

- can generalize to arbitrary graph:

$$|\psi_\gamma\rangle = \prod_{A_{xy}^i = 1} P_i^{x\otimes y} |\psi\rangle$$

state associated to graph

"disconnected" state

adjacency matrix of graph

- applies to states in pre-Fock space $\text{pre-}\mathcal{F}(\mathcal{H}) = \oplus_{V=1}^{\infty} \mathcal{H}_V$ then projected by symmetrization
- graphs = patterns of entanglement among TGFT quanta - "primitive entanglement/geometry correspondence"
- TGFT states are generalized and 2nd quantized random tensor network states

G. Chirco, DO, M. Zhang, '17;
E. Colafranceschi, DO, '20

- reformulation in language of tensor networks useful for defining and analyzing holographic QG maps

E. Livine, '21; G. Chirco, E. Colafranceschi, DO, '21

TGFT and Loop Quantum Gravity: quantum states

DO, '13

E. Colafranceschi, DO, '20

- TGFTs can be seen as a "2nd quantized Loop Quantum Gravity" (but even less "spatiotemporal")
- key point: LQG states (spin networks) for connected graphs = entangled quantum many-body TGFT states
  - LQG Hilbert space of any (4-valent) graph is faithfully embedded in TGFT pre-Fock space
  - graph-entangled states form subspace of Fock space = LQG Hilbert space for same graph (scalar product induced by Fock space scalar product)
- full TGFT Fock space obtained by summing over vertices and imposing permutation symmetry

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\}$$

- full Hilbert space of canonical Loop Quantum Gravity is quite different

Ashtekar, Lewandowski, Okolow, Sahlmann, Rovelli, Smolin, Thiemann, Fleischack, ...

(more continuum spacetime ingredients)

- TGFT: abstract graphs - LQG: from embedding into (canonical) manifold
- different way of organizing and intending spin networks: distinctively "discrete-first" perspective in TGFT

- TGFT: states associated to different graphs with same number of vertices ARE NOT orthogonal
- TGFT: states associated to graphs with different number of vertices ARE orthogonal
- no cylindrical equivalence, no projective limit in TGFT - required in LQG to interpret data as corresponding to continuum connection or metric fields
- permutation symmetry imposes label-independence (counterpart in LQG? recall: no embedding)

Tensorial group field theory: dynamics of quantum space

Dynamics governs gluing processes and formation of extended discrete structures

Interactions processes correspond to (simplicial) complexes in one dimension higher

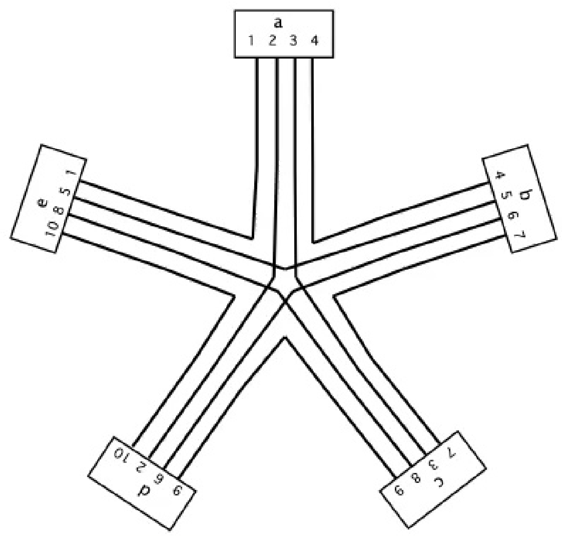
L. Freidel, '06;
DO, '09; DO, '14

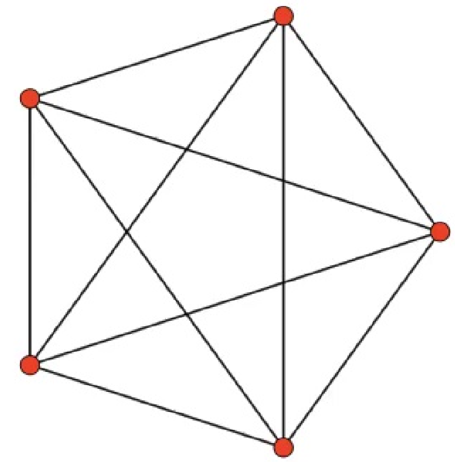
details depend on (class of) models

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

"combinatorial non-locality"
in pairing of field arguments

Example: simplicial interactions





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Feynman diagrams = stranded
diagrams dual to cellular
complexes of arbitrary topology

sum over triangulations/complexes

amplitude for each
triangulation/complex

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sum over triangulations/complexes

amplitude for each triangulation/complex

Feynman amplitudes (model-dependent):

equivalently:

- spin foam models (sum-over-histories of spin networks ~ covariant LQG)
  Reisenberger,Rovelli, '00
- lattice path integrals (group+Lie algebra variables)
  A. Baratin, DO, '11

TGFT and spin foam models

complete definition of SF model: quantum amplitudes for all spin foam complexes + organization principle

sum over spin foam complexes

refinement of spin foam complexes

Bahr, Dittrich, Steinhaus, ...

spin foam model with sum over complexes as perturbative expansion of TGFT (valid for any SF model)

R. De Pietri, L. Freidel, K. Krasnov, C. Rovelli, '99;
M. Reisenberger, C. Rovelli, '00

$$Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I)$$

any combinatorics

DO, J. Ryan, J. Thürigen, '14

$$Z(\Gamma) \leftrightarrow \begin{cases} A_f(J) \\ A_e(J,I) \\ A_v(J,I) \end{cases} \longleftrightarrow \begin{cases} \mathcal{K}(J,I) \sim \mathcal{K}(g) \\ \mathcal{V}(J,I) \sim \mathcal{V}(g) \end{cases} \leftrightarrow S(\varphi,\bar{\varphi})$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})\ldots\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

using local expression of SF amplitudes, needed for composition along (portions of) boundaries

M. Finocchiaro, DO, '18

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\, e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma \qquad Z(\Gamma) \equiv \mathcal{A}_\Gamma$$

advantages:

- prescription for combinatorial weights + parametrize SF ambiguities
- ways to go beyond spin foams (non-perturbative & collective physics)
- QFT tools for extracting physics

Example: EPRL-like TGFT model

TGFT action

$$S = \sum_{\substack{j_{v_{a_i}} \\ m_{v_{a_i}}, \iota_a}} \bar{\varphi}^{j_{v_1}\iota_1}_{m_{v_1}} \varphi^{j_{v_2}\iota_2}_{m_2} (\mathscr{K}_2)^{j_{v_1} j_{v_2} \iota_1 \iota_2}_{m_{v_1}, m_{v_2}} + \frac{1}{5} \sum_{\substack{j_{v_{a_i}} \\ m_{v_{a_i}}, \iota_a}} \left[\left(\prod_{a=1}^{5} \bar{\varphi}^{j_{v_a}\iota_a}_{m_{v_a}} \right) \bar{\mathscr{V}}_5 + \left(\prod_{a=1}^{5} \varphi^{j_{v_a}\iota_a}_{m_{v_a}} \right) \mathscr{V}_5 \right]$$

$$\mathscr{V}_5 := \mathscr{V}_5(j_{v_1}, \ldots, j_{v_5}, m_{v_1}, \ldots, m_{v_5}, \iota_1, \ldots, \iota_5)$$

4-simplex interaction

both kinetic and interaction kernel contribute simple Kronecker deltas for j, m labels:

$$V = \sum_{j_i, m_i, \iota_i} \Big[\varphi^{j_1 j_2 j_3 j_4 \iota_1}_{m_1 m_2 m_3 m_4} \varphi^{j_4 j_5 j_6 j_7 \iota_2}_{m_4 m_5 m_6 m_7} \varphi^{j_7 j_3 j_8 j_9 \iota_3}_{m_7 m_3 m_8 m_9} \varphi^{j_9 j_6 j_2 j_{10} \iota_4}_{m_9 m_6 m_2 m_{10}} \varphi^{j_{10} j_8 j_5 j_1 \iota_5}_{m_{10} m_8 m_5 m_1}$$

$$\times \tilde{\mathscr{V}}_5(j_1, \ldots, j_{10}; \iota_1, \ldots, \iota_5) \Big] + \text{c.c.}$$

reproduces EPRL-like spin foam amplitudes

with: $$\tilde{\mathscr{V}}_5(j_{ab}, i_a) = \sum_{n_a} \int d\rho_a (n_a^2 + \rho_a^2) \left(\bigotimes_a f^{i_a}_{n_a \rho_a}(j_{ab}) \right)$$

from simplicity constraints:

$$\rho = \gamma n \qquad n = 2j$$

a - tetrahedra, (ab) triangles $\quad 15j_{SL(2,\mathbb{C})}\left((2j_{ab}, 2j_{ab}\gamma); (n_a, \rho_a)\right)$

$$f^i_{n\rho} := i^{m_1 \ldots m_4} \, \bar{C}^{n\rho}_{(j_1, m_1)\ldots(j_4, m_4)}$$

in terms of constrained SL(2,C) irreps and contraction of SL(2,C) invariant tensors

as in Engle, Livine, Pereira, Rovelli, '07 other formulations: Engle, Pereira, '08; Dona', Fanizza, Sarno, Speziale, '19; ....

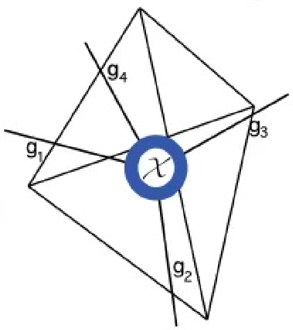
TGFT for 4d quantum geometry coupled to matter

guideline for TGFT model-building from pure quantum geometry extends to matter coupling:

extend domain of TGFT field, the define TGFT action so that Feynman amplitudes are lattice path integrals for gravity coupled to matter

- to later apply relational strategy for reconstruction of "quantum spacetime", need matter degrees of freedom

$\longrightarrow$ work with TGFT models for simplicial geometry coupled to single (free, massless) scalar field



Y. Li, DO, M. Zhang, '17

- TGFT fields now include new real variable (value of scalar field) $\hat{\varphi}(g_I) \longrightarrow \hat{\varphi}(g_I, \chi)$

with consequent extension of field operators, quantum states and operators on Fock space

- corresponding TGFT action is: $S = K + U + \bar{U}$

$$K = \int \mathrm{d}g_I \, \mathrm{d}h_I \int \mathrm{d}\chi \, \mathrm{d}\chi' \;\; \bar{\varphi}(g_I, \chi) K(g_I, h_I; (\chi - \chi')^2) \varphi(h_I, \chi')$$

U = higher order in TGFT field, non-local in quantum geometric data, local in scalar field data

$$U = \int [dg] d\chi \, \varphi(g, \chi) ... \varphi(g', \chi) \, \mathcal{V}(\{g\}; \chi)$$

- only one independent value of scalar field at each 4-simplex --> values of scalar field variables for all TGFT fields in the TGFT interaction should be identified, i.e. interaction is local in scalar field variables

- TGFT propagator takes form of exponential function of differences of values of scalar field variables --> TGFT kinetic term contains (infinite) 2nd derivatives of TGFT field wrt scalar field variable

TGFTs for Quantum Gravity:

continuum limit

How to extract continuum gravitational physics?

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

$$= \sum_\Delta w(\Delta) \int \mathcal{D}g_\Delta\, e^{i\, S_\Delta(g_\Delta)} \equiv \int \mathcal{D}g\, e^{i\, S(g)}$$

TGFT as (non-perturbative) completion of simplicial path integral/spin foam models for quantum gravity

defining full continuum path integral for quantum gravity = defining full TGFT path integral for suitable model

what is continuum physics in TGFT (from perspective of in-built lattice gravity)?

(and taking into account results about continuum limit of classical lattice gravity)

coarse-grained description of QG data

- coarse grained quantum states

no detailed info on lattice data - result of summing over lattice data

collective QG physics

- need distinctively field-theoretic approximations

result of collective quantum dynamics of fundamental discrete degrees of freedom

Extracting continuum spacetime & gravitational physics

* ideally, TGFT free energy itself (and its derivatives) or full TGFT quantum effective action should be used to compute continuum geometric observables and their quantum dynamics

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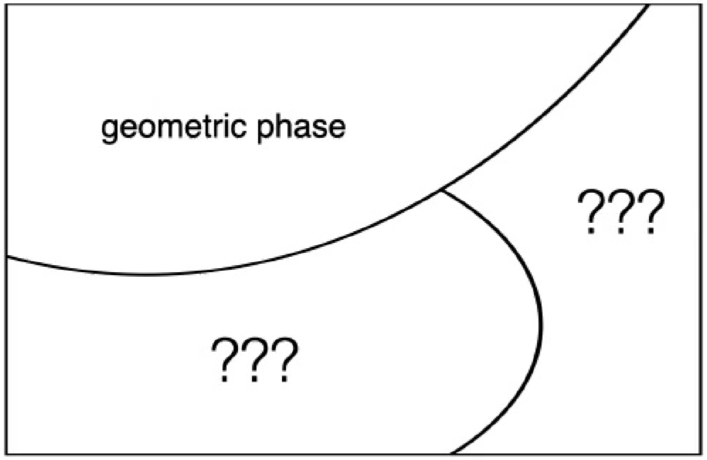
$$F_\lambda(J) = \ln Z_\lambda[J] \qquad \Gamma[\phi] = sup_J \left(J\cdot\phi - F(J)\right) \qquad \langle\varphi\rangle = \phi$$ "mean field"

i.e. evaluate (analytically? numerically?) full quantum dynamics!
(full sum over triangulations weighted by simplicial gravity path integral)

expect different phases
and phase transitions
as result of quantum dynamics
(what are the phases of LQG?)

which ones are "geometric"
in which one does spacetime emerge?
Koslowski, '07; DO, '07



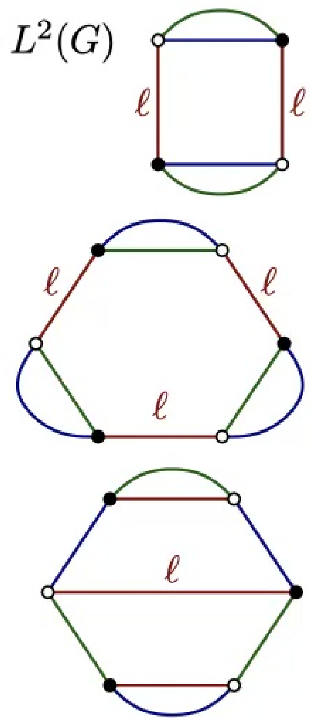
A. Ashtekar, J. Lewandowski, '94 T. Koslowski, H. Sahlmann, '10 B. Dittrich, M. Geiller, '14; B. Bahr, B. Dittrich, M. Geiller, '16; S. Gielen, DO, L. Sindoni, '13 A. Kegeles, DO, C. Tomlin, '16

TGFT (non-perturbative) renormalization: results and insights

mostly for TGFTs with "tensorial" interactions: invariant under unitary transformations

$$U^c : L^2(G) \to L^2(G)$$

theory space identified by (bipartite) r+1 edge coloured graphs b



TGFTs (with dynamical tensorial dofs) abelian, non-abelian, different ranks, with gauge symmetry, different interactions, ...

Benedetti, Ben Geloun, Bonzom, Carrozza, Gurau, Harribey, DO, Rivasseau, Tanasa, Vignes-Tourneret, ...

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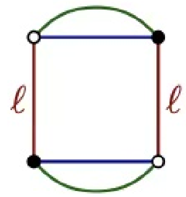

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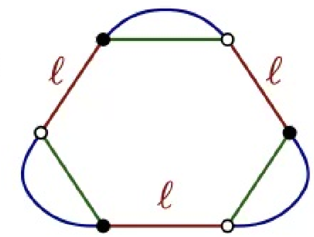

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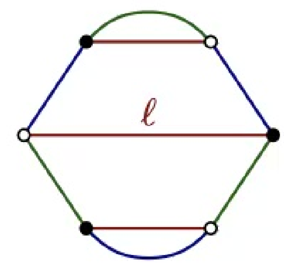
- perturbative renormalizability + asymptotic freedom/safety for several models

What about non-perturbative RG flow? phase transitions?

several results for broad class of models



- [Pithis/Thürigen '20-'21] infinite cyclic-melonic potential on G, LPA
  - small-N (IR): equivalence with O(N) model in d=0
  - symmetry restoration and no phase transition for fixed group size a
  - phase transition (Wilson-Fisher FP, above critical dimension) in limit a-->oo
  - phase transition can be described by mean field theory
  - close to FP, TGFT behaves similarly to local theory in d = dim(G)(r-1)
  - room for asymptotic safety

- need to extend analysis to:
  - improved truncations + better analysis of NGFP
  - non-compact groups (e.g. Lorentz)
  - matter coupling, i.e. both non-local and local directions

Renormalization of quantum geometric TGFTs

quantum amplitudes (and TGFT kernels) much more involved

models based on simplicial interactions, no coloring;

generalization to non-simplicial interactions possible, but then no geometric justification

what we know: C. Perini et al., '08; T. Krajewski et al., '10; J. Ben Geloun et al, '10; A. Riello, '13; V. Bonzom, B. Dittrich, '15; Y. Chen, '16; P. Dona', '18; P. Dona et al, '19; M. Finocchiaro, DO, '20; P. Dona et al., '20

- several results on basic radiative corrections and scaling of amplitudes
- dominant configurations (probably) related to solutions of classical Regge-type discrete gravity dynamics
- hints that needed counterterms are of melonic type
- heuristic arguments connecting (colored) simplicial and tensorial theory spaces
- .... and that's it ....

missing:

- more general power counting theorems
- control over theory space (and symmetries)
- any proper RG analysis
- ......

Extracting continuum spacetime & gravitational physics

✳ ideally, TGFT free energy itself (and its derivatives) or full TGFT quantum effective action should be used to compute continuum geometric observables and their quantum dynamics

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma = \sum_\Delta w(\Delta) \int \mathcal{D}g_\Delta\, e^{i\,S_\Delta(g_\Delta)} \equiv \int \mathcal{D}g\, e^{i\,S(g)}$$

$$F_\lambda(J) = \ln Z_\lambda[J] \qquad \Gamma[\phi] = sup_J\,(J\cdot\phi - F(J)) \qquad \langle\varphi\rangle = \phi$$ "mean field"

i.e. evaluate (analytically? numerically?) full quantum dynamics!
(full sum over triangulations weighted by simplicial gravity path integral)

need approximations that:

- capture collective effects
- correspond to some coarse-graining of fundamental discrete data
- maintain (as possible) quantum nature of fundamental entities

✳ simplest approximation: mean field hydrodynamics

saddle point evaluation of path integral - quantum effective action ~ classical action $\quad \Gamma[\phi] \approx S_\lambda(\phi)$

TGFT condensate hydrodynamics

mean field ~ condensate wavefunction

Extracting continuum spacetime & gravitational physics

Gielen, DO, Sindoni, '13; DO, Sindoni, Wilson-Ewing, '16

✳ simplest approximation: mean field hydrodynamics	saddle point evaluation of path integral - quantum effective action ~ classical action	$\Gamma[\phi] \approx S_\lambda(\phi)$

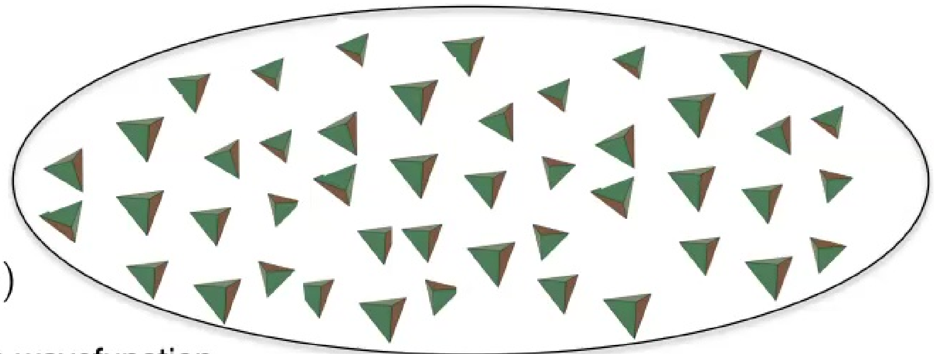
TGFT condensate hydrodynamics   mean field ~ condensate wavefunction

- mean field hydrodynamics corresponds to working with quantum states of the type:

(simplest): TGFT field coherent state

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle$$

$$\hat{\sigma} := \int d^4g\, \sigma(g_I)\hat{\varphi}^\dagger(g_I) \quad \sigma(g_I k) = \sigma(g_I)$$



fully determined by single function: condensate wavefunction

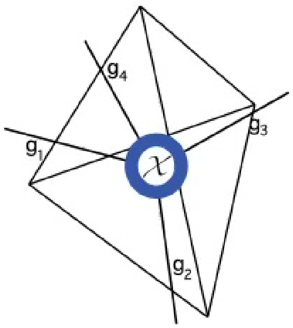
superposition of infinitely many spin networks dofs, “gas”of tetrahedra, all associated with same state ("wavefunction homogeneity") - neglecting connectivity information

Extracting continuum spacetime & gravitational physics

one more step before giving spatiotemporal and physical interpretation to TGFT hydrodynamics

- to apply relational strategy for the reconstruction of "quantum spacetime", need matter degrees of freedom

$\longrightarrow$ work with TGFT models for simplicial geometry coupled to single (free, massless) scalar field



Y. Li, DO, M. Zhang, '17

- TGFT fields including new real variable (value of scalar field) $\quad \hat{\varphi}(g_I) \longrightarrow \hat{\varphi}(g_I, \chi)$

with consequent extension of field operators, quantum states and operators on Fock space

- corresponding TGFT action is: $\quad S = K + U + \bar{U}$

$$K = \int \mathrm{d}g_I \, \mathrm{d}h_I \int \mathrm{d}\chi \, \mathrm{d}\chi' \;\; \bar{\varphi}(g_I, \chi) K(g_I, h_I; (\chi - \chi')^2) \varphi(h_I, \chi')$$

$$U = \int [dg] d\chi \, \varphi(g, \chi) ... \varphi(g', \chi) \, \mathcal{V}(\{g\}; \chi)$$

- TGFT interaction is local in scalar field variables
- TGFT kinetic term contains (infinite) 2nd derivatives of TGFT field wrt scalar field variable

- condensate coherent state: $\quad |\sigma\rangle = N_\sigma \exp\left[\int \mathrm{d}\chi \int \mathrm{d}g_I \, \sigma(g_I, \chi) \hat{\varphi}^\dagger(g_I, \chi)\right] |0\rangle$

$$N_\sigma \equiv e^{-\|\sigma\|^2/2} \qquad \|\sigma\|^2 = \int \mathrm{d}g_I \, \mathrm{d}\chi |\sigma(g_I, \chi)|^2 \equiv \langle \sigma \mid \hat{N} \mid \sigma\rangle$$

Extracting continuum spacetime & gravitational physics

- hydrodynamic level: Gielen, DO, Sindoni, '13; DO, Sindoni, Wilson-Ewing, '16

  only global ("spatial") geometric quantities, a single scalar field value, to be used as "relational clock"

  not enough data for "local physics" $\longrightarrow$ spatiotemporal interpretation: homogeneous cosmology

- indeed, one has: Gielen, '15

isomorphism between domain of TGFT condensate wavefunction and minisuperpsace

$$\sigma(\mathcal{D}) \qquad \mathcal{D} \simeq \{\text{geometries of tetrahedron}\} \simeq$$
$$\simeq \{\text{continuum spatial geometries at a point}\} \simeq$$
$$\simeq \text{minisuperspace of homogeneous geometries}$$

- general form of resulting (Gross-Pitaevskii) equations of motion for condensate wavefunction (mean field):

$$\int [dg'] d\chi' \mathcal{K}(g,\chi;g',\chi')\sigma(g',\chi') + \lambda \frac{\delta}{\delta\varphi}\mathcal{V}(\varphi)|_{\varphi\equiv\sigma} = 0$$

polynomial functional of condensate wavefunction

cosmology as QG hydrodynamics

remarks:

- formally similar to quantum cosmology, but non-linear; no Hilbert space for cosmological wave functions
- expect approximation corresponding to simplest condensate valid in "mesoscopic TGFT regime"
  - neglected correlations among TGFT quanta, thus need weak interactions
  - interactions grow with density (modulus of condensate wavefunction), thus need smallish densities
  - too small densities correspond to small numbers of TGFT quanta, i.e. no hydrodynamics

TGFT condensate cosmology - bouncing cosmology from EPRL-like model

DO, L. Sindoni, E. Wilson-Ewing, '16; L. Marchetti, DO, '20

Extracting continuum spacetime & gravitational physics

- specialize to EPRL-like TGFT models - quantum geometric data, specific form of kinetic/interaction kernels
  - two more approximations:
    - reduction to isotropic condensate configurations (depending on single variable j): $\sigma_j(\chi)$

      (~ condensate of equilateral tetrahedra)
    - subdominant TGFT interactions
      - consistent with use of field coherent states;
      - required by focus on lattice gravity and spin foams: relevant geometric regime = perturbative one
- in order to recast it in spatiotemporal language, we need to introduce a physical relational frame

  comparison to (quantum) GR can only be done in diffeo-invariant language
- homogeneous cosmological setting: one coarse-grained, dynamical dof should play role of relational clock

- special quantum states in which scalar field behaves "nicely enough" to be good clock

  peaked condensate wavefunction: $\sigma_\epsilon(g_I,\chi) \equiv \eta_\epsilon(g_I;\chi-\chi_0,\pi_0)\tilde{\sigma}(g_I,\chi)$

  peaking function around χ_0 with a typical width given by $\epsilon \ll 1$

  fluctuations in (conjugate) scalar field momentum also small if: $\epsilon\pi_0^2 \gg 1$

TGFT condensate cosmology - bouncing cosmology from EPRL-like model

DO, L. Sindoni, E. Wilson-Ewing, '16; L. Marchetti, DO, '20

- for the reduced condensate wavefunction, one gets the effective dynamics:

$$\boxed{\tilde{\sigma}_j''(\chi_0) - 2i\tilde{\pi}_0\tilde{\sigma}_j'(\chi_0) - E_j^2\tilde{\sigma}(\chi_0) = 0} \qquad \tilde{\pi}_0 = \frac{\pi_0}{\epsilon\pi_0^2 - 1} \qquad E_j^2 = \epsilon^{-1}\frac{2}{\epsilon\pi_0^2 - 1} + \frac{B_j}{A_j}$$

where derivatives are "time derivatives" wrt to χ_0

where A and B are j-dependent coefficients of series expansion of TGFT kinetic term in derivatives wrt χ (0th and 2nd order, respectively, dominant for peaked condensate states)

note: dependence on parameters defining fundamental dynamics and chosen condensate states

- eqn can be recast in hydrodynamic form, using: $\tilde{\sigma}_j \equiv \rho_j \exp[i\theta_j]$ Q = conserved quantity

$$\rho_j''(\chi_0) - \frac{Q_j^2}{\rho_j^3(\chi_0)} - \mu_j^2\rho_j(\chi_0) = 0 \qquad \mu_j^2 = E_j^2 - \tilde{\pi}_0^2 = \frac{\pi_0^2}{\epsilon\pi_0^2 - 1}\left(\frac{2}{\epsilon\pi_0^2} - \frac{1}{\epsilon\pi_0^2 - 1}\right) + \frac{B_j}{A_j}$$

note: there is also second conserved quantity: "energy" $\mathcal{E}_j = (\rho_j')^2 + \frac{Q_j^2}{\rho_j^2} - \mu_j^2\rho_j^2$

next task for "reconstructing space time" is:

recast this effective hydrodynamics in terms of relational spacetime observables

important point: observables defined in fundamental Hilbert space, but used in terms of expectation value in condensate state, thus also spatiotemporal only in approximate, coarse-grained, collective sense

TGFT condensate cosmology - bouncing cosmology from EPRL-like model

DO, L. Sindoni, E. Wilson-Ewing, '16; L. Marchetti, DO, '20

- key observables for emergent cosmology (defined on fundamental TGFT Fock space):

number operator $\hat{N} = \int \mathrm{d}g_I \, \mathrm{d}\chi \;\; \hat{\varphi}^\dagger(g_I, \chi \;)\hat{\varphi}(g_I, \chi \;)$

total universe volume $\hat{V} = \int \mathrm{d}\chi \int \mathrm{d}g_I \, \mathrm{d}g_I' \hat{\varphi}^\dagger(g_I, \chi) V(g_I, g_I') \hat{\varphi}(g_I', \chi)$

scalar field $\hat{X} \equiv \int \mathrm{d}g_I \int \mathrm{d}\chi \, \chi \hat{\varphi}^\dagger(g_I, \chi)\hat{\varphi}(g_I, \chi)$ should be averaged to give physical quantity

total momentum of scalar field $\hat{\Pi} = \frac{1}{i} \int \mathrm{d}g_I \int \mathrm{d}\chi \left[\hat{\varphi}^\dagger(g_I, \chi) \left(\frac{\partial}{\partial \chi} \hat{\varphi}(g_I, \chi) \right) \right]$

- can obtain corresponding relational observable "at given clock time", as functions of scalar field value, by evaluating in expectation value on peaked condensate states:

relational number operator $N(\chi_0) \equiv \langle \hat{N} \rangle_{\sigma;\chi_0,\pi_0} = \sum_j \int \mathrm{d}\chi |\sigma_\epsilon(g_I, \chi; \chi_0, \pi_0)|^2 \simeq \sum_j \rho_j^2(\chi_0)$

relational universe volume $V(\chi_0) \equiv \langle \hat{V} \rangle_{\sigma;\chi_0,\pi_0} = \sum_j V_j |\sigma_j(\chi; \chi_0, \pi_0)|^2 \simeq \sum_j V_j \rho_j^2(\chi_0)$

clock (scalar field) value $\langle \hat{\chi} \rangle_{\sigma_\epsilon;\chi_0,\pi_0} \equiv \frac{\langle \hat{X} \rangle_{\sigma;\chi_0,\pi_0}}{N(\chi_0)} \simeq \chi_0$

relational momentum of scalar field

$$\langle \hat{\Pi} \rangle_{\sigma;\chi_0,\pi_0} = \frac{1}{i} \int \mathrm{d}\chi \sum_j \overline{\sigma}_{\epsilon,j}(\chi; \chi_0, \pi_0) \partial_\chi \sigma_{\epsilon,j}(\chi; \chi_0, \pi_0) \simeq \pi_0 \left(\frac{1}{\epsilon \pi_0^2 - 1} + 1 \right) N(\chi_0) + \sum_j Q_j$$

TGFT condensate cosmology - bouncing cosmology from EPRL-like model

DO, L. Sindoni, E. Wilson-Ewing, '16; L. Marchetti, DO, '20

- effective dynamics for volume - generalised Friedmann equations:

$$\boxed{\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j V_j \rho_j \mathrm{sgn}(\rho_j')\sqrt{\mathcal{E}_j - Q_j^2/\rho_j^2 + \mu_j^2\rho_j^2}}{3\sum_j V_j \rho_j^2}\right)^2}
\qquad
\boxed{\frac{V''}{V} \simeq \frac{2\sum_j V_j\left[\mathcal{E}_j + 2\mu_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}}$$

- classical approx. (Hubble rate small compared to the inverse Planck time - small curvature - large volume) $\rho_j^2 \gg |\mathcal{E}_j|/\,\mu_j^2$ and $\rho_j^4 \gg Q_j^2/\,\mu_j^2$

$$\Rightarrow \boxed{\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j V_j \mu_j \rho_j^2 \mathrm{sgn}(\rho_j')}{3\sum_j V_j\rho_j^2}\right)^2}
\qquad
\boxed{\frac{V''}{V} \simeq \frac{4\sum_j V_j\left[\mu_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}}$$

i.e. Friedmann eqns (in relational time)

under sufficient condition: $\mu_j = 3\pi\tilde{G}$ at least for some dominant j

$$\boxed{\left(\frac{V'}{V}\right)^2 = \frac{V''}{V} = 12\pi\tilde{G}}$$

general lesson: "gravitational" coupling constants are emergent notions, function of fundamental parameters

- one can compute quantum fluctuations of observables:
  - "good clock" requirement for relational dynamics puts constraints on parameters
  - when those are satisfied, volume fluctuations under control for large volumes

L. Marchetti, DO, '20

thus classical (late time) dynamics is robust

TGFT condensate cosmology - bouncing cosmology from EPRL-like model

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- behaviour at small (relational) times, assuming conditions of "good relational clock" are satisfied:
  - there are solutions with singular behaviour (cosmological singularity not always resolved)
  - if at least one coefficient Q or at least one "energy" coefficient is non-zero:

$$\longrightarrow \exists j \;/\; \rho_j(\chi) \neq 0 \;\forall \chi \longrightarrow \boxed{V = \sum_j V_j \rho_j^2}$$

remains positive at all times (with single turning point)

quantum bounce (solving classical singularity)!

- quantum fluctuations remain "small" also at bounce for specific range of parameters (specific class of solutions)

L. Marchetti, DO, '20

TGFT condensate cosmology - bouncing cosmology from EPRL-like model

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$V = \sum_j V_j \rho_j^2$ remains positive at all times (with single turning point)

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L. Marchetti, DO, '20

- simple condensate:

$$\sigma_j(\chi) = 0, \text{ for all } j \neq j_o \longrightarrow \left[\frac{V'}{3V}\right]^2 = \frac{4\pi\tilde{G}}{3} + \frac{4V_{j_o}\mathcal{E}_{j_o}}{9V} \qquad V_0 \approx V_{Pl}\, j_0^{3/2}$$

S. Gielen, '17

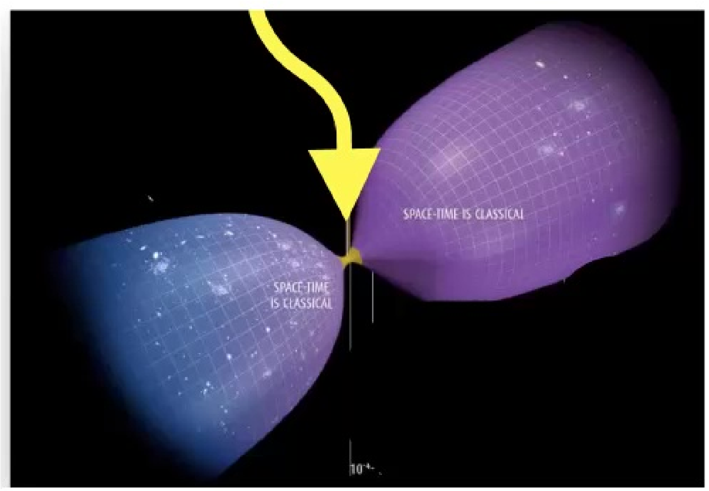
if $\mathcal{E}_{j_o} < 0$ quantum bounce at $V_{min} = V_0 N_{min} = \frac{V_0|\mathcal{E}_0|}{6\pi\tilde{G}}$

quantum fluctuations $\frac{\Delta V}{V}(\chi) \approx \frac{1}{N(\chi)}$ number density cannot be too small (to be expected in hydrodynamic approx)

The fate of cosmological singularities

in Quantum Gravity

within an "emergent spacetime scenario"



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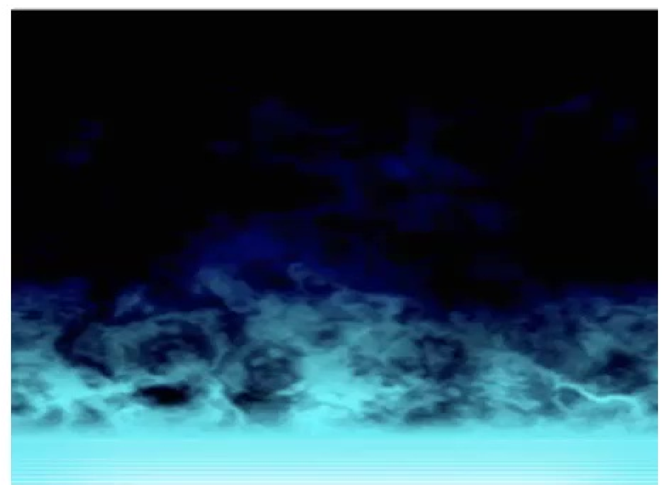

So, what happens to the cosmological singularity in QG?

according to (current description in) TGFT condensate cosmology:

Classical cosmological singularity is replaced by "big bounce" scenario

due to a sort of "quantum gravitational/spacetime pressure", i.e. QG modification of classical GR dynamics, introducing an effective maximal energy density

as in LQC, but from the full QG theory!

more precisely: Classical cosmological singularity is replaced by "big bounce" scenario,
in mean field restriction
of hydrodynamic approximation
within condensate phase

- mean field approximation obviously to be improved, leading to different condensate hydrodynamic eqns

but maybe bouncing scenario is stable

then, yes, cosmic quantum bounce! .... if hydrodynamic approximation holds
.... if "quantum spacetime system" stays within condensate phase

- If hydrodynamic approximation breaks down:

e.g because too few "atoms of space" are involved and/or because fluctuations become too strong

→ disappearance of continuum spacetime

Physical signatures of geometrogenesis

- hypothesis: geometrogenesis will manifest itself in growth of fluctuations close to bouncing region, leading to breakdown of hydrodynamic approximation

$$\left[\frac{V'}{3V}\right]^2 = \frac{4\pi\tilde{G}}{3} + \frac{4V_{j_o}\mathcal{E}_{j_o}}{9V}$$ if $\mathcal{E}_{j_o} < 0$ quantum bounce at $V_{min} = V_0 N_{min} = \frac{V_0|\mathcal{E}_0|}{6\pi\tilde{G}}$

$$V_0 \approx V_{Pl}\, j_0^{3/2}$$

quantum fluctuations $\frac{\Delta V}{V}(\chi) \approx \frac{1}{N(\chi)}$ grow as approaching the bounce

at bounce, maximum of fluctuations: $$\left[\frac{\Delta V}{V}(\chi_c)\right]_{max} \approx \frac{1}{N_{min}(\chi_c)} \simeq \frac{\tilde{G}}{\mathcal{E}_0(\tilde{G})}$$

- expect hydrodynamic approximation to beak down when $\boxed{\frac{\tilde{G}}{\mathcal{E}_0(\tilde{G})} \geq 1}$

- recall: $\tilde{G} = \tilde{G}(\lambda_i^{TGFT})$

$\longrightarrow$ critical value for fundamental coupling constants,

signal of (cosmological) phase transition, i.e. geometrogenesis?

TGFT condensate cosmology: many other results

- deparametrized reformulation and Hamiltonian evolution

E. Wilson-Ewing, '18, S. Gielen, A. Polaczek, E. Wilson-Ewing, '19; S. Gielen, A. Polaczek, '19

matter scalar field enters GFT action just like time variable ⟶ deparametrization and canonical quantization of fundamental GFT ⟶ relational quantum Hamiltonian evolution + generalised effective cosmological dynamics

- cosmological perturbations

S. Gielen, DO, '17 S. Gielen, '18 F. Gerhardt, DO, E. Wilson-Ewing, '18 L. Marchetti, DO, to appear

formalism can be extended to inhomogeneities; scale invariant power spectrum seems to follow in some generality (preliminary), and corrections can be computed

- cosmological effects of fundamental TGFT interactions

M. De Cesare, A. Pithis, M. Sakellariadou, '16 X. Pang, DO, '21

new cosmological terms modifying background dynamics:

QG-inflation: some choices of parameters lead to QG-generated inflationary phase in early universe (phenomenologically ok?)

QG-dark energy: other (less restrictive) choices produce QG-generated late-time acceleration, matching viable dark energy models

general lesson: in emergent spacetime scenarios, large-scale properties of universe could be of direct QG origin, limits of validity of effective field theory intuition and results

- ....

Effects of TGFT interactions on cosmological dynamics

M. De Cesare, A. Pithis, M. Sakellariadou, '16 X. Pang, DO, '21

- mean field relational dynamics for (isotropic) condensate wavefunction can be extracted from effective action:

$$\left\langle \sigma \left| \frac{\delta S(\hat{\varphi}, \hat{\varphi}^\dagger)}{\delta \hat{\varphi}^\dagger} \right| \sigma \right\rangle = \frac{\delta S(\sigma, \bar{\sigma})}{\delta \bar{\sigma}} = 0 \qquad S(\sigma, \bar{\sigma}) = \int \mathrm{d}\phi \left(\sum_j \bar{\sigma}_j \partial_\phi^2 \sigma_j + \mathcal{V}(\sigma, \bar{\sigma}) \right)$$

- "phenomenological" approach - simple potential: $\boxed{\mathcal{V}(\sigma, \bar{\sigma}) = -\sum_j \left(m_j^2 |\sigma_j|^2 + \frac{2\lambda_j}{n_j} |\sigma_j|^{n_j} + \frac{2\mu_j}{n'_j} |\sigma_j|^{n'_j} \right)}$

assuming: $2 < n_j < n'_j \qquad |\mu_j| \ll |\lambda_j| \ll m^2$

- resulting equation for condensate density

$$\boxed{\rho'_j(\phi) = \frac{1}{\rho_j} \sqrt{2E_j \rho_j^2 - Q_j^2 + m_j^2 \rho_j^4 - \frac{2}{n_j} \lambda_j \rho_j^{n_j+2} - \frac{2}{n'_j} \mu_j \rho^{n'_j+2}}}$$

conserved quantities

- turn this into equation for volume (function of relational time)

in terms equation of state for "emergent matter" component of universe (of QG origin): $\boxed{w = 3 - \frac{2VV''}{(V')^2}}$

$w = p/\rho$

Effects of TGFT interactions on cosmological dynamics

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- now we can analyse what happens in different regimes, due to the presence of interactions

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- free theory: short-lived acceleration after bounce, then Friedman dynamics
- general:
  - for small volumes, interactions are subdominant and dynamics reproduces bouncing scenario
  - as the universe expands after bounce, interactions become more relevant, until they drive evolution
  - beginning of interaction-driven dynamics depends on coupling constants and which modes contributes

- if only single mode contributes: M. De Cesare, A. Pithis, M. Sakellariadou, '16

$$w = \frac{-3Q^2 + 4E\rho^2 + m^2\rho^4 + \left(1 - \frac{4}{n}\right)\lambda\rho^{n+2} + \left(1 - \frac{4}{n'}\right)\mu\rho^{n'+2}}{-Q^2 + 2E\rho^2 + m^2\rho^4 - \frac{2}{n}\lambda\rho^{n+2} - \frac{2}{n'}\mu\rho^{n'+2}} \qquad \begin{array}{c} 2 < n_j < n'_j \\ |\mu_j| \ll |\lambda_j| \ll m^2 \end{array}$$

accelerated phase can last long, depending on parameters QG-inflation

however, either expansion never ends, in which case it is not phenomenologically viable and leads to big rip singularity, or it is followed by recollapse after end of inflation (cyclic universe) - no Friedmann phase - semiclassical physics?

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- if more than one mode contributes: richer and observationally viable phenomenology

  interacting theory: nice Friedmann-like expansion after cosmic bounce

  depending on parameters: +

  phantom-like accelerated phase at later times - QG-dark energy

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  general lesson: in emergent spacetime scenarios, large-scale properties of universe could be of direct QG origin (limited validity of effective field theory intuition and results)

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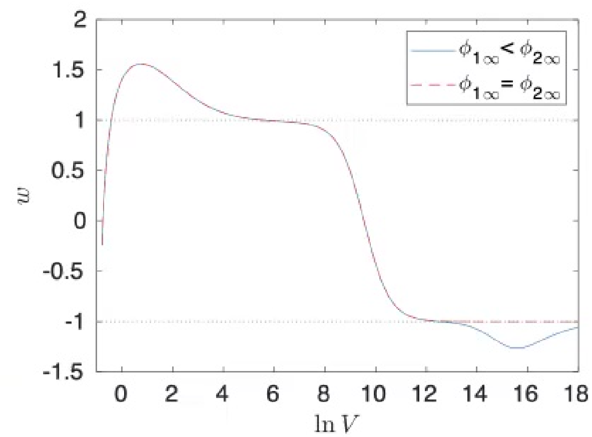
- effective cosmological dynamics $\boxed{w = 3 - \frac{2VV''}{(V')^2}}$ for "emergent matter" component (of QG origin):
- if more than one mode contributes: richer and observationally viable phenomenology
- assume both couplings are very small: interactions only relevant once universe is very large

$\longrightarrow$ very short acceleration after bounce, followed by nice Friedmann expansion

- then interactions become relevant - for two modes j1 and j2
- only one interaction for each mode ($n_1 = n_2 = n$):
- one sees that: $w \leq 2 - \frac{n}{2}$ maximum is reached from below for large volumes (one mode dominates)
- in particular $n = 6$

phantom divide w = -1 can be crossed, then approached from below at large volume

$\longrightarrow$ effective phantom field - QG-dark energy



- however, no Big Rip singularity occurs, because the energy density of the effective phantom field remains finite at large volumes, approaching a constant value

$$w = -1 - \frac{b}{V} \qquad b = 4V_2\rho_2(\phi_{1\infty}) \qquad \rho_\psi = \rho_{\psi 0} e^{-\frac{b}{V}} \approx \rho_{\psi 0} - \frac{\rho_{\psi 0} b}{V}$$

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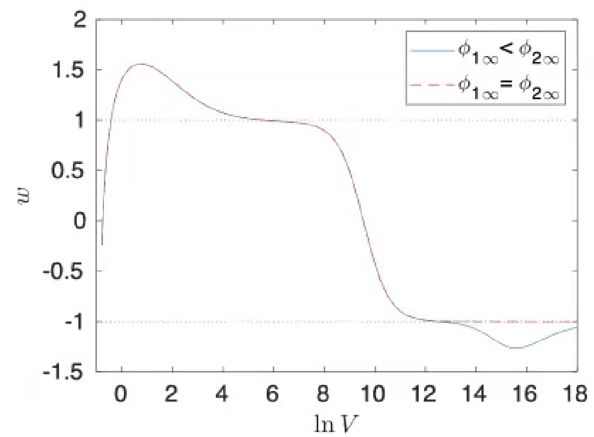

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- QG-produced version of semi-classical model in B. McInnes, '01

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