Title: Summing over geometries in string theory

Speakers: Lorenz Eberhardt

Series: Quantum Fields and Strings

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Abstract: I discuss the question how string theory achieves a sum over bulk geometries with fixed asymptotic boundary conditions. I analyze this problem with the help of the tensionless string on AdS3xS3xT4 (with one unit of NS-NS flux) that was recently understood to be dual to the symmetric orbifold of T4. I argue that large stringy corrections around a fixed background can be interpreted as different semiclassical geometries, thus making a sum over semi-classical geometries superfluous.

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Summing over Geometries in String Theory

Lorenz Eberhardt

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LE : [2008.07533], [2102.12355]

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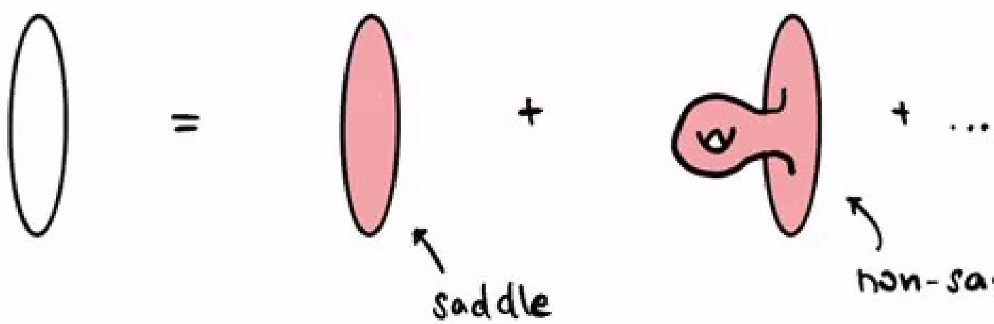
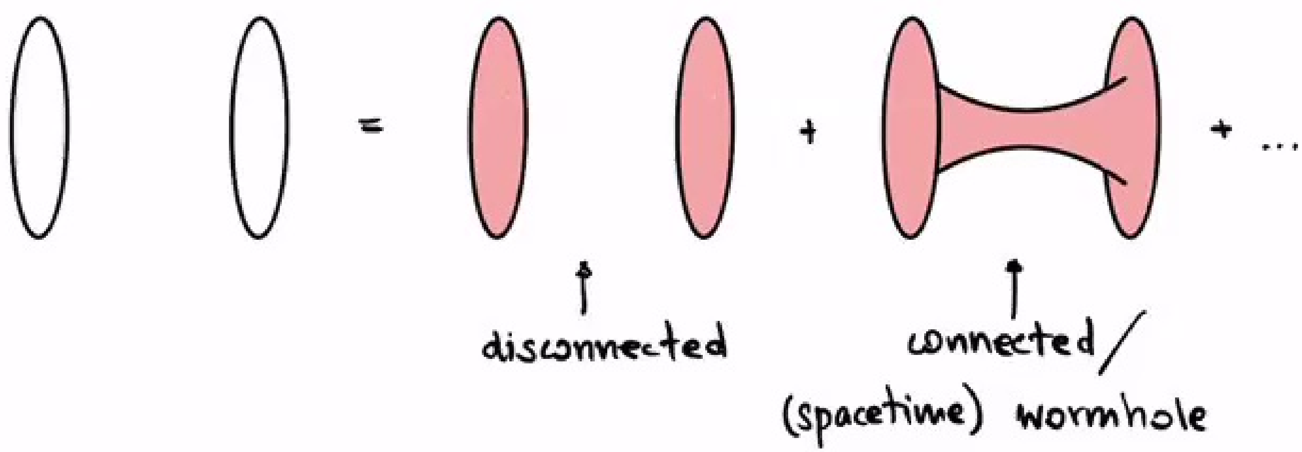
The gravitational path integral

- The Euclidean gravitational path integral includes a priori all possible geometries with given boundary conditions

e.g. JT gravity (2d bulk / 1d boundary)

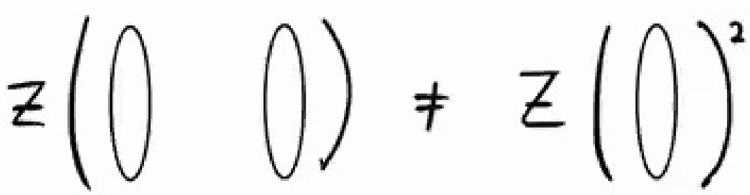




The gravitational path integral

- Including all possible geometries is very important to explain a number of phenomena:

  - Page curve for the entropy of Hawking radiation [Penington, Shenker, Stanford, Yang '19]
  - Ramp in the spectral form factor $|Z(\beta+it)|^2$. [Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka '16]
  - Ensemble AdS/CFT correspondence



due to connected contributions

$\Rightarrow$ JT gravity is not dual to a single quantum mechanical system, but to an ensemble [Saad, Shenker, Stanford '19

String theory

- String theory should work differently.
- We have AdS/CFT correspondences without ensemble average:

$$AdS_5 \times S^5 / \mathcal{N}=4 \text{ SYM}, \qquad AdS_3 \times S^3 \times T^4 / \text{Sym}^N(T^4)$$

[Maldacena '97]

- How does string theory achieve a sum over geometries without ensemble average?
- Usually we only consider string theory on a fixed background but here we should sum also over backgrounds

$$Z_{\text{string}}(\text{bdry conditions}) \overset{?}{=} \sum_{\text{bulks } \mathcal{M}} \exp\left(\sum_{g=0}^{\infty} g_s^{2g-2} \int D[\text{fields}]\; e^{-S_{\mathcal{M}}[\text{fields}]} \right)$$

we know only how to + D instantons

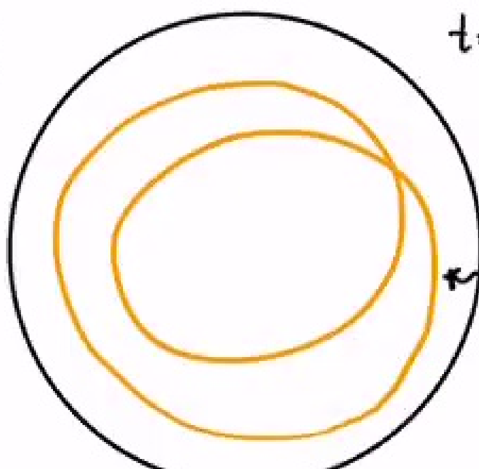
The tensionless string on AdS_3

- We will discuss this with a controllable model [LE, Gaberdiel, Gopakumar '18]

$$\boxed{\begin{array}{l}\text{String theory on } AdS_3 \times S^3 \times T^4 \text{ with one unit of NSNS flux} \\ \quad = \text{Symmetric product } \mathrm{Sym}^N(T^4) \qquad (AdS_3 \to \mathcal{M}_3)\end{array}}$$

N: number of fundamental strings in the background

- The string is tensionless: String excitations are very light, there massless higher spin fields in the spectrum
- Generic string state:



$t=0$ surface of AdS_3

string winds around the asymptotic boundary of AdS_3

The tensionless string on AdS_3

- Even the graviton is such a winding state

  $\Rightarrow$ No local notion of spacetime geometry ?

- We can still talk about background geometry

  background geometry $\Rightarrow$ worldsheet theory

  this remains well-defined

- Sum over geometries becomes sum over worldsheet theories

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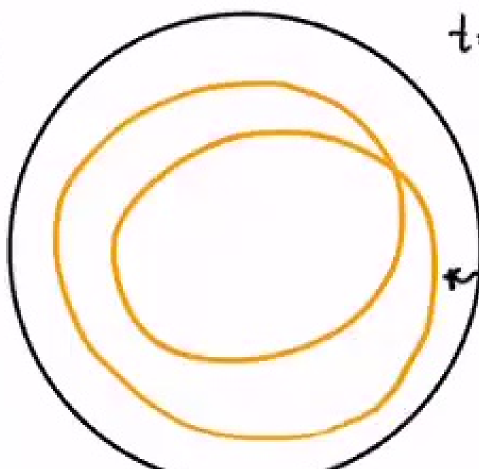


string winds around the asymptotic boundary of AdS_3

Bulk geometries

- We can consider this string theory on any geometry of the form

$$\mathcal{M}_3 \times S^3 \times T^4$$

hyperbolic 3-manifold (locally Euclidean AdS_3)

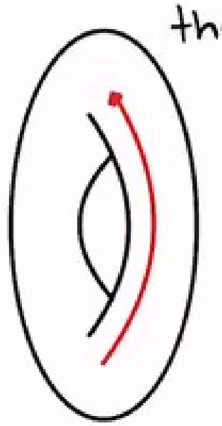
- Every $\mathcal{M}_3$ can be written as

$$\mathcal{M}_3 = \mathbb{H}^3/\Gamma \quad , \quad \Gamma \subset SL(2,\mathbb{C}) \text{ discrete subgroup}$$

Examples

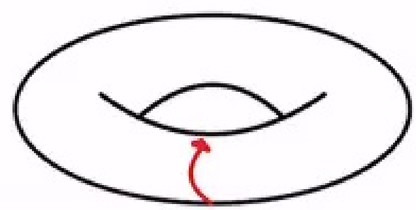
- $\Gamma \cong \mathbb{Z}$: $\mathcal{M}_3$ = solid torus = thermal AdS_3 / BTZ black hole

thermal AdS_3



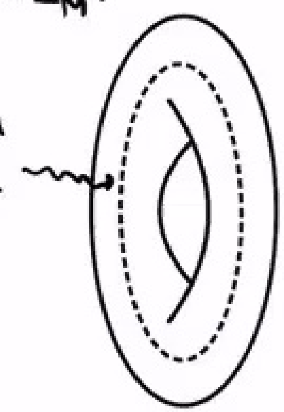
Euclidean boundary time

BTZ

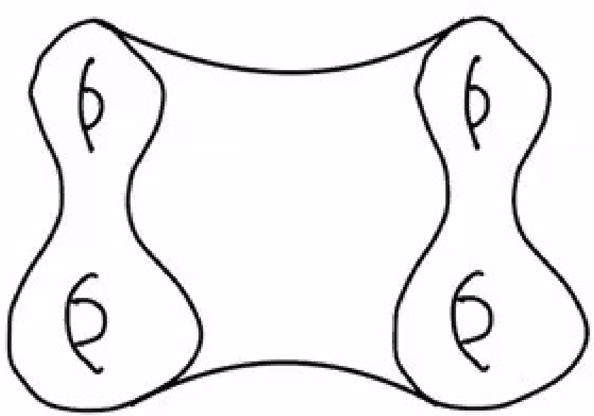


$\Gamma = \mathbb{Z} \times \mathbb{Z}_M$:

conical defect



- $\Gamma \cong$ Fuchsian group $= \left\langle A_1, \dots, A_g, B_1, \dots, B_g \in SL(2,\mathbb{R}) \,\middle|\, \prod_{i=1}^{g} [A_i, B_i] = 1 \right\rangle$



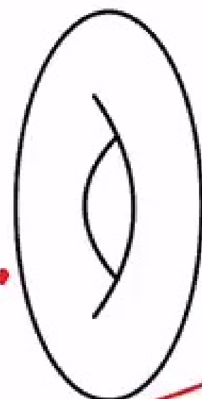
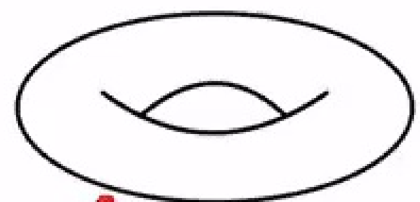
wormhole with two genus g boundaries

We necessarily have $g \geq 2$.

Thermal partition function

- The simplest object to compute is the boundary torus partition function $Z_{Sym(T^4)}$.

- In 3d gravity:

$Z_{Sym(T^4)} =$  $+$  $+$ $SL(2,\mathbb{Z})$ family of Euclidean black holes [Maloney, Witten '07]

$+$ conical defect geometries? [Benjamin, Collier, Maloney '20]

$+$ non saddle geometries? [Maxfield, Turiaci '20; Cotler, Jensen '20]

$\boxed{\text{We can compute the perturbative string partition function on these and compare with the boundary CFT}}$

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The grand canonical ensemble

- Recall that

$N = \#$ of fundamental strings in AdS_3

- String perturbation theory does not fix N. [Kim, Porrati '15]

- Instead we fix a chemical potential and let N vary

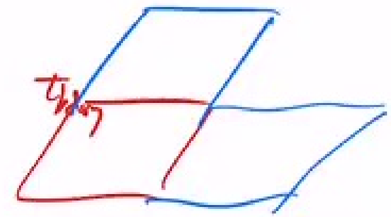
- In the boundary CFT we compute the grand canonical partition function

$$\mathfrak{Z}_{Sym} = \sum_{N=0}^{\infty} p^N Z_{Sym^N(T^4)} .$$

$$= \exp\left(\sum_{L=1}^{\infty} \frac{p^L}{L} \sum_{a|L} \sum_{b \in \mathbb{Z}/\mathbb{Z}a} Z^{T^4}\begin{bmatrix} b/2 \\ a/2 \end{bmatrix}\left(\frac{L\tau_{bdry}+ab}{a^2}\right)\right).$$

$\uparrow$ sum over degree L coverings of the torus

The grand canonical ensemble



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sum over degree L coverings of the torus

Bulk computation

- We can compute the string partition function on thermal AdS_3 (& other geometries) to all orders in g_s.
- It is one-loop exact. [LE '21]
- The worldsheet theory is a $\mathbb{Z}$-orbifold of the global AdS_3 worldsheet theory:

global AdS_3 worldsheet theory = $PSU(1,1|2)_1$ WZW model

× top. twisted T^4

× ghosts

this has a free field realization

$\Longrightarrow$ The worldsheet theory becomes solvable [Berkovits, Vafa, Witten '99]

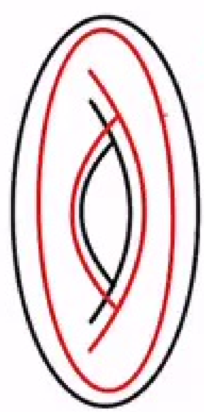
Worldsheet partition function

- The worldsheet torus partition function can be computed: [LE '20]

$$Z^{\text{thermal AdS}_3}_{\text{worldsheet}} = \frac{1}{2} \operatorname{Im} \tau_{\text{bdry}} \sum_{a,b,c,d\in\mathbb{Z}} \delta^{(2)}\left(\tau_{\text{bdry}}(c\tau+d) - a\tau - b\right)$$
$$\times\; p^{\det\begin{pmatrix} a & b \\ c & d\end{pmatrix}} \left| \exp\left(\frac{\pi i \tau_{\text{bdry}}}{2} \det\begin{pmatrix} a & b \\ c & d\end{pmatrix}\right)\right|^2 Z^{T^4}\begin{bmatrix} b/2 \\ a/2 \end{bmatrix}(\tau).$$

- $\sum_{a,b,c,d\in\mathbb{Z}}$ sums over topologically distinct embeddings of the worldsheet

- $p^{\det\begin{pmatrix} a & b \\ c & d\end{pmatrix}}$ is the effect of including the chemical potential on the worldsheet



worldsheet torus inside target space solid torus

Localization

- The worldsheet partition function localizes in $\mathcal{M}_1$ on all holomorphic covering surfaces of the boundary torus

$$\tau_{\text{bdry}} = \frac{a\tau + b}{c\tau + d}$$

- $\gamma : T^2_{\text{ws}} \longrightarrow T^2_{\text{bdry}}$ is a degree $-\det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ covering map.
- This property can be proven for arbitrary genus & bulks $\mathcal{M}_3$.
- Every covering map of a torus is a torus

[LE, Gaberdiel, Gopakumar '19; LE '20; Dei, Gaberdiel, Gopakumar, Knighton '20; LE '20; Knighton '20; LE '21]

$\Longrightarrow$ the string partition function is 1-loop exact

- Localization implies $\int_{\mathcal{M}_g} \longrightarrow \sum_{\text{covering}}$

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String partition function

- We can compute the string partition function on thermal AdS_3 by

$$\int_{\mathcal{M}_1} Z_{\text{worldsheet}}$$

- Easy to compute!

- We also have to account for the effect of the sphere partition function. This depends on the boundary conformal anomaly. We don't know how to compute it from first principles and take the gravity answer for it

$$Z_{\text{sphere}} = \frac{\pi c_{\text{bdry}}}{6} \operatorname{Im} \tau_{\text{bdry}} = \pi N \operatorname{Im} \tau_{\text{bdry}} .$$

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String partition function

- This gives

$$Z_{\text{thermal AdS}_3} = \exp\left(\sum_{a,d=1}^{\infty} \frac{p^{ad}}{ad} \sum_{b \in \mathbb{Z}/\mathbb{Z}a} Z^{T^4}\begin{bmatrix} b/2 \\ a/2 \end{bmatrix}\left(\frac{d\tau_{\text{bdry}} + b}{a} \right)\right)$$

$$= Z_{\text{Sym}(T^4)} \; !$$

- Repeating the computation on other backgrounds,

$$\boxed{\begin{aligned} Z_{\text{Sym}} &= Z_{\text{thermal AdS}} = Z_{\text{BTZ}} \\ &= Z_{SL(2,\mathbb{Z})\text{ family of BH}} = Z_{\text{conical defect}} \end{aligned}}$$

[LE '20]

No sum over geometries

- Instead of

$$\mathcal{Z}_{\text{Sym}} = \sum_{\text{bulks } \mathcal{M}} Z_{\text{string}}(\mathcal{M}),$$

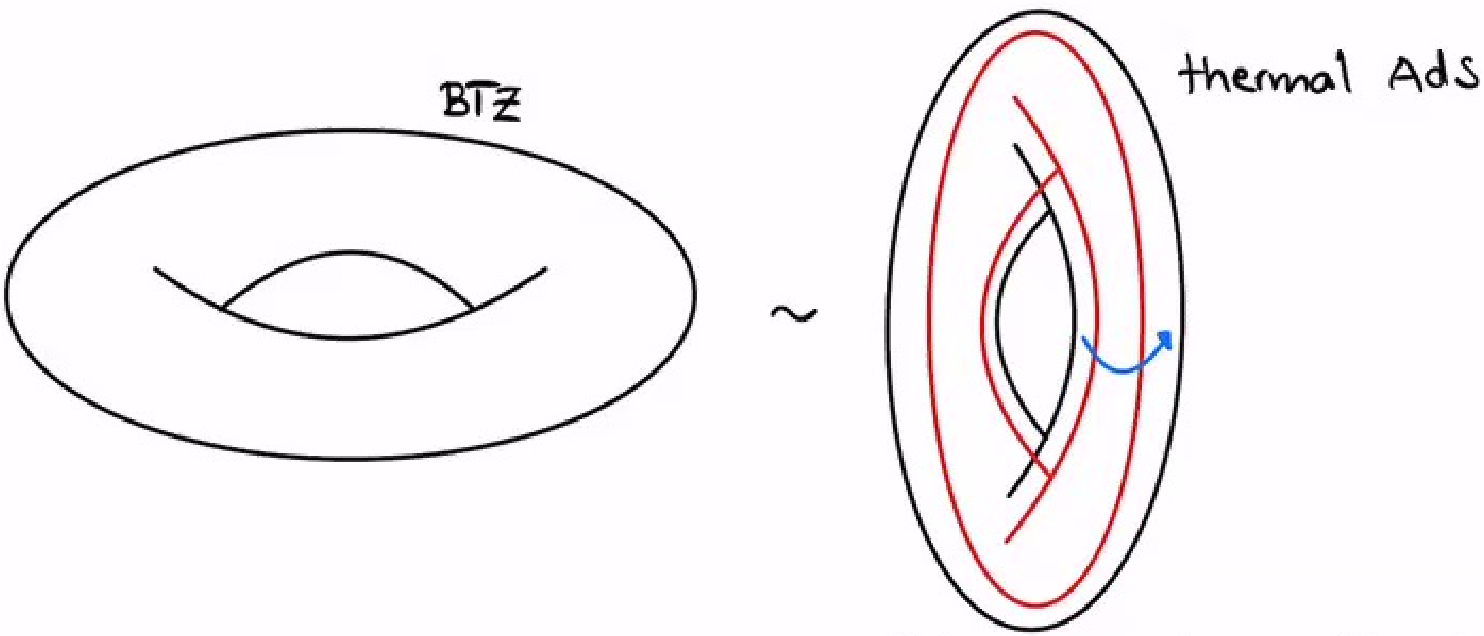
we have

$$\mathcal{Z}_{\text{Sym}} = Z_{\text{string}}(\text{any bulk geometry } \mathcal{M})$$

- Summing over geometries would overcount states.
- Every state in one bulk geometry can be identified with a state in another bulk geometry

Identification of states

- Let us identify the ground state of the BTZ black hole with an excited string state on thermal AdS_3.

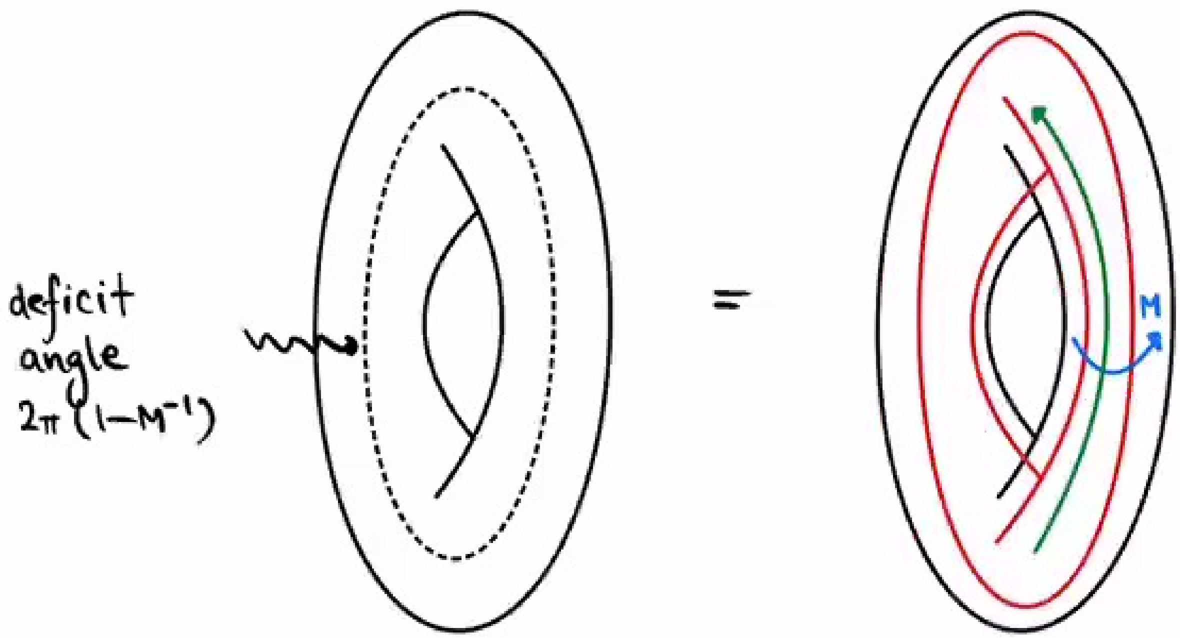


worldsheet winds arbitrarily many times around the boundary

$$\tau = -\frac{N}{\tau_{bdry}}$$

Identification of states

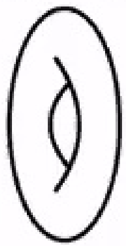
- The conical defect as an excited state in thermal AdS_3:



worldsheet winds M times around spatial cycle & arbitrarily many times around temporal cycle

Hawking Page transition

- Usually we learn in gravity

$Z =$  $+$  $+ \dots$

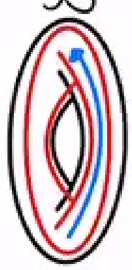
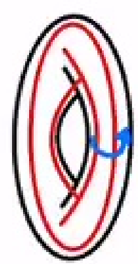
dominates at low temperatures

dominates at high temperatures

$\Rightarrow$ Phase transition!

[Hawking, Page '82]

- Here the HP transition comes from different dominating worldsheet configurations in the same background e.g. thermal AdS_3:

$Z =$  $+$  $+ \dots$

low temperature

high temperature

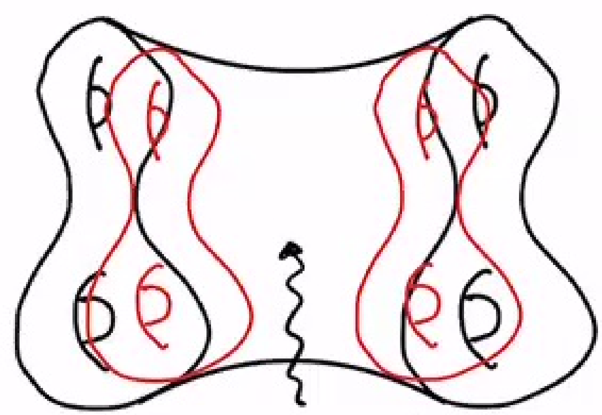
Factorization problem

- The theory can be defined on the genus g wormhole.

- How is it consistent to have a single boundary theory?

- We can compute

$$Z^{\text{string}}_{\text{wormhole}}$$



- Receives contributions from covering maps mapping to only one boundary (due to localization)

- With this one can argue

$$Z_{\text{wormhole}} = Z_L \times Z_R \quad \Rightarrow \text{Factorization restored}$$

Hawking Page transition

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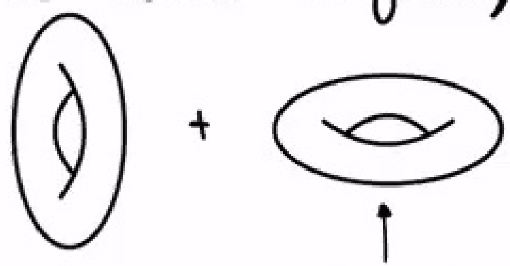
$Z =$  $+ \dots$

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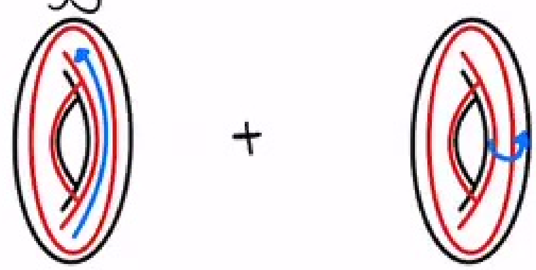
$Z =$  $+ \dots$

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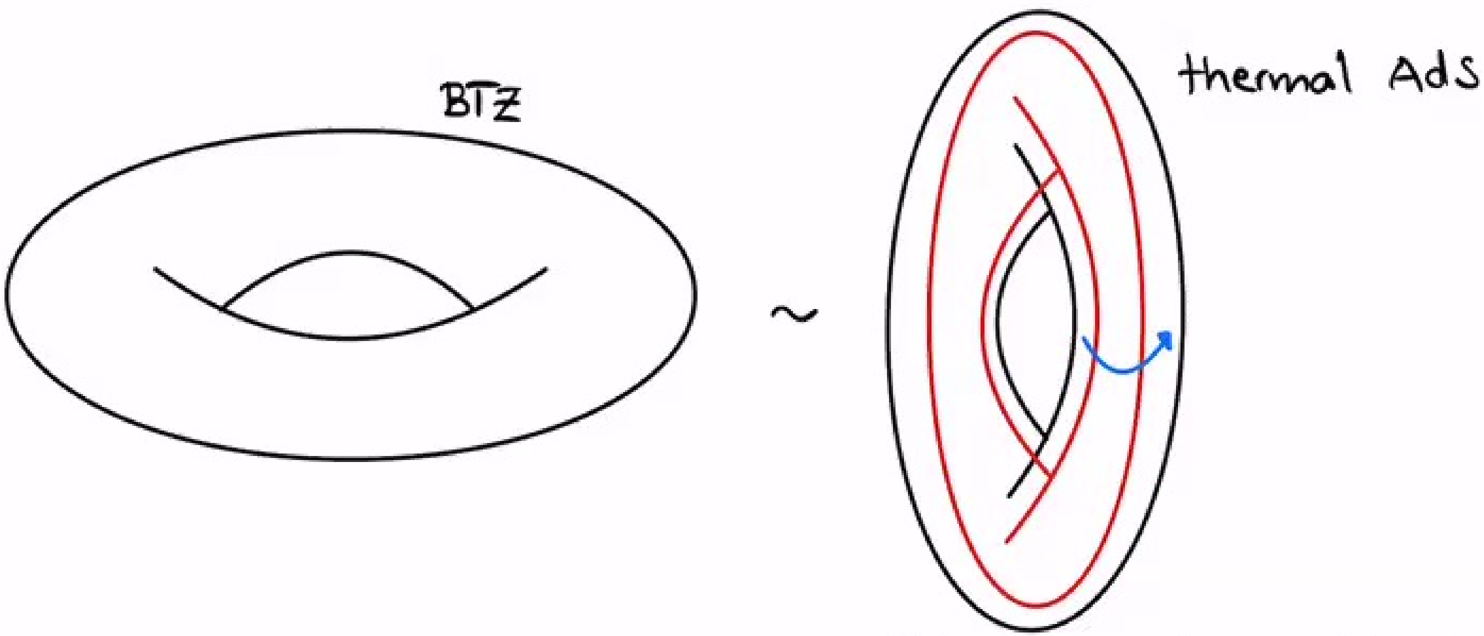

worldsheet winds arbitrarily many times around the boundary

$$\tau = -\frac{N}{\tau_{bdry}}$$

Some lessons

- Worldsheet theory for the tensionless string localizes in $\mathcal{M}_3$ to covering maps of the boundary

- The partition function of the tensionless string on $\mathcal{M}_3 \times S^3 \times T^4$ depends only on $\partial\mathcal{M}_3$.
  - $\Rightarrow$ No sum over geometries
  - $\Rightarrow$ Resolution of the factorization problem

- The natural ensemble is the grand canonical ensemble

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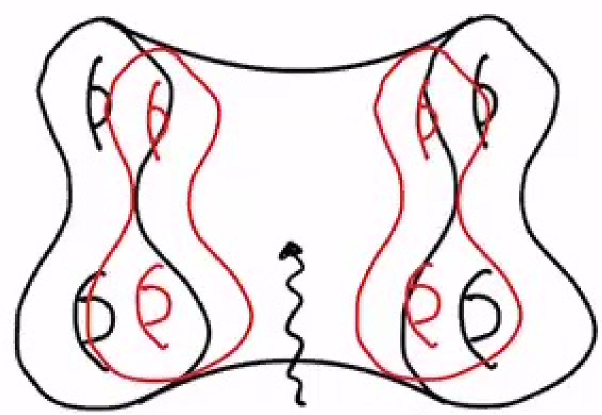

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