Title: Evolving quantum state for black holes

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Series: Quantum Gravity

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Abstract: A fundamental problem of quantum gravity is to understand the quantum evolution of black holes. While aspects of their evolution are understood asymptotically, a more detailed description of their evolving wavefunction can be provided. This gives a possible foundation for studying effects that unitarize this evolution, which in turn may provide important clues regarding the quantum nature of gravity.

Evolving quantum state for black holes

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Perimeter Institute Seminar

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Based partly on: **SG: 2006.10834 + in preparation**

SG & J. Perkins, work in progress

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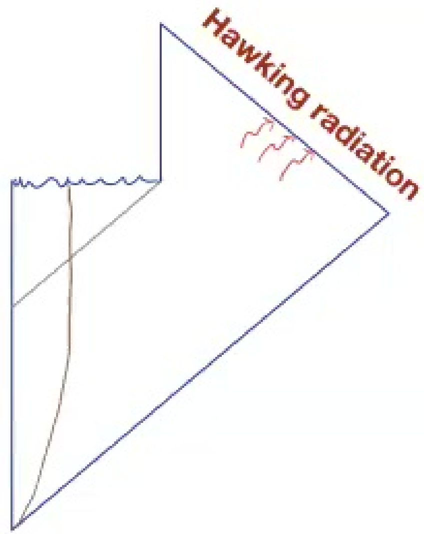
Introduction

Assume there is a quantum mechanical theory of gravity

One of the biggest puzzles is how BHs behave in that theory

(Key problem: guide)

Have ~ asymptotic description — Hawking, etc.:



Of course leads to a contradiction — unitarity violated (not QM)

We seek organizational principles for QG.

Is a black hole a quantum subsystem?

A very basic structural question.

What is a subsystem, in gravity? *Subtle question*

Donnelly and SG, 1507.07921, 1607.01025, 1706.03104, 1805.11095

SG, 1903.06160

SG and S. Weinberg, 1911.09115

Negative answer suggested to possibly resolve problem - soft hair

Hawking; Hawking, Perry, Strominger

… counterarguments: 1805.11095, 1903.06160, 1907.06644

How do they evolve?

Hawking: non unitary

"BH theorem:"

If BHs do behave as subsystems, evolution is unitary, and they disappear, then there *must* be couplings that transfer information out

First goal: understand all of this more sharply in standard LQFT description:

BHs as evolving quantum subsystems

Surprisingly though many general statements made, less precise understanding

Nice to see explicit description.

BH theorem implies must be modifications; can we understand as "small" corrections to this picture?

SG: 1701.08765 + preceding refs; overview 1905.08807

(a few more words at end...)

At the same time, address some long-standing questions:

"Where" is Hawking radiation produced?

Is there a transplanckian problem?

How do we treat interacting theories? Can't trace back free modes

This talk: confined largely to standard LQFT description

Possible modifications discussed elsewhere, and could certainly describe further, on another occasion. Understanding these is likely an important guide towards the deeper principles of QG — just like discovering the correct physics of the atom was for QM

In the LQFT description, how do we describe the evolving quantum state of a BH?

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{D-2} \qquad f(r) = 1 - \mu(r)$$

$$\mu(r) = \left(\frac{R}{r}\right)^{D-3} \;, D \geq 4$$

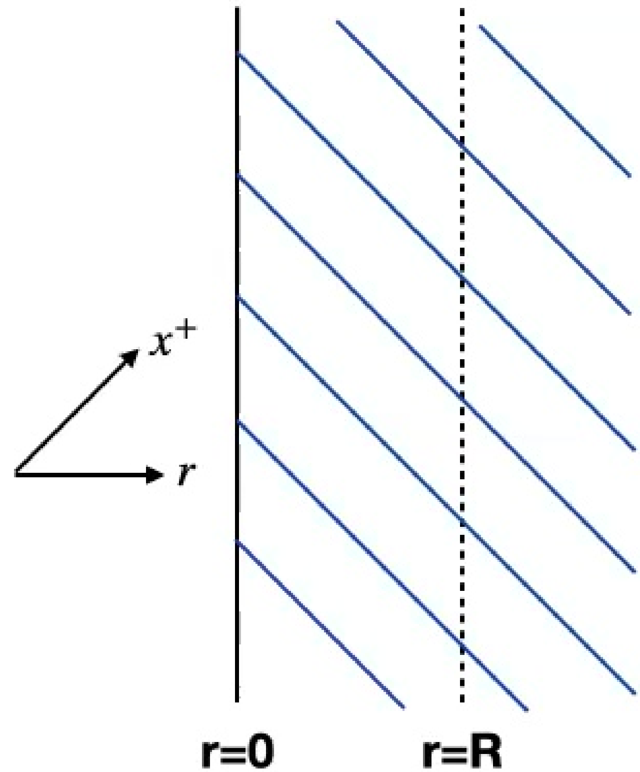
$$\mu(r) = Me^{-2r} = e^{-2(r-R)} \;, D = 2$$

(dilaton grav., e.g. "CGHS")

Eddington-Finkelstein

$$ds^2 = -f(r)dx^{+2} + 2dx^+dr + r^2 d\Omega^2_{D-2}$$

Eddington-Finkelstein diagram



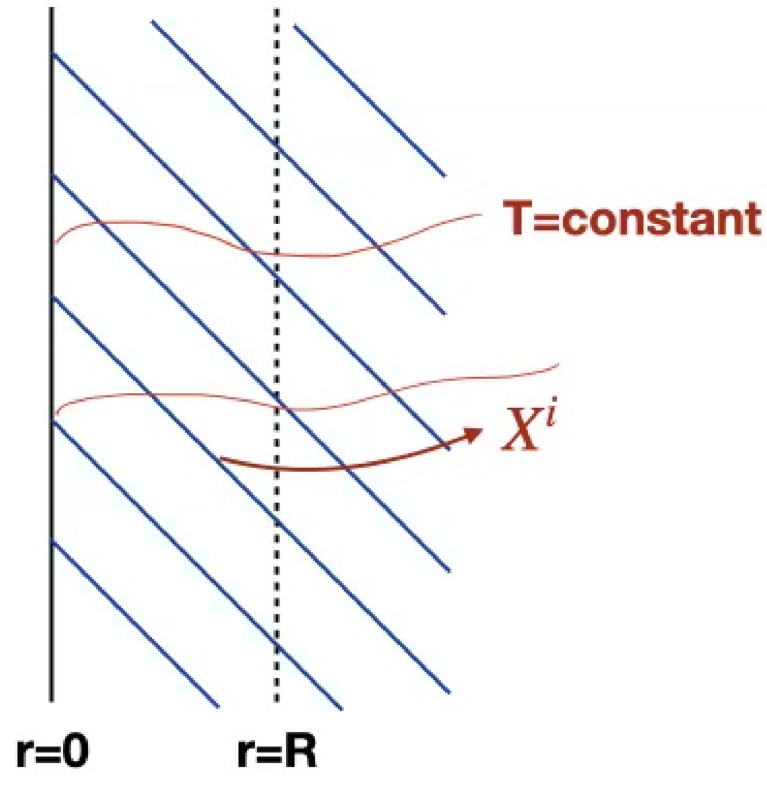
Physical quantum black holes:

$$\partial_t R \sim \frac{1}{MR} \ll 1$$

~ Killing vector

… essentially stationary

Evolving quantum state: need choice of slices



Parameterize slices:

$$T(x^\mu), X^i(x^\mu)$$

ADM metric:

$$ds^2 = -N^2 dT^2 + q_{ij}(dX^i + N^i dT)(dX^j + N^j dT)$$

Exploit symmetries:

Rotational: $T(x^+, r), X(x^+, r)$

θ, ϕ

Time trans.:

$$T = x^+ - s(r)$$

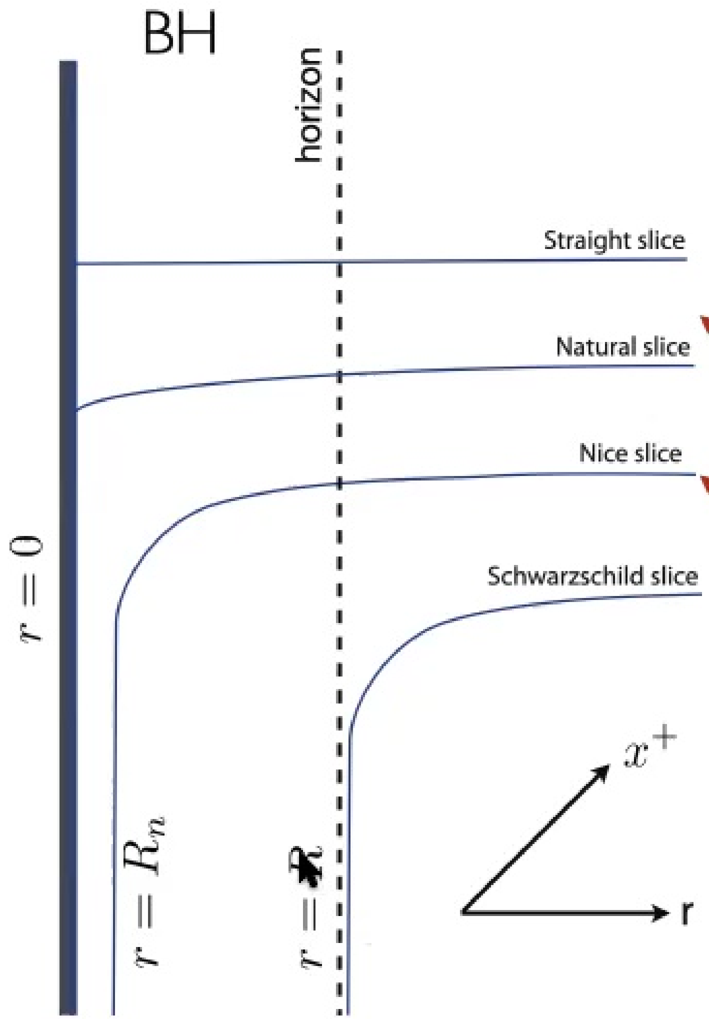
"stationary slicing"

$$X = X(r)$$

Earlier examples: Jacobson; Melnikov, Weinstein

Different classes of slicings:

$$x^+ = T + s(r)$$



Schwarzschild: $s(r) = r_*(r)$

Nice: $s(r) \to -\infty$ **as** $r \to R_n$

Straight: $s(r) = r$

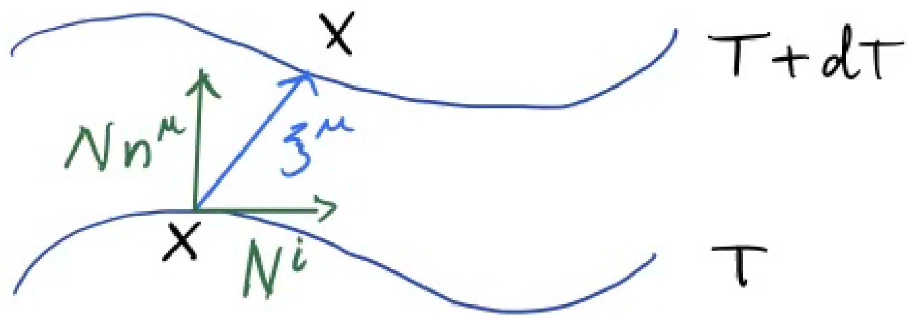
Not Cauchy

Cauchy

$\Big\}\; D \geq 4$

But: Natural, straight slices Cauchy for D=2

Spatial coordinates and different "pictures"



$$\xi^\mu = \left(\frac{\partial}{\partial T}\right)_X^\mu$$

E.g. free scalar ϕ

$$H_\xi = \int d^{D-1}X\sqrt{q}\left[\frac{1}{2}N(\pi^2 + q^{ij}\partial_i\phi\partial_j\phi) + N^i\pi\partial_i\phi\right] = \int d^{D-1}X\sqrt{q}n^\mu\xi^\nu T_{\mu\nu}$$

$$|\Psi_H(T,\xi)\rangle = e^{-i\int_{-\infty}^{T} dT' H_\xi(T')}|\Psi_0\rangle$$

depends on choice of T *and* X

Simple example - 2d flat space, R moving scalar $\phi(x^-)$: $x^\pm = t \pm x$

$(T, X) = (t, x)$ $\quad H_\xi = \frac{1}{2}\int dx[\dot{\phi}^2 + (\partial_x\phi)^2]$

$(T, X) = (t, x^-)$ $\quad H_\xi = 0$

… same dynamics

$X(x^+, r)$ **sometimes useful (e.g. nice slice evolution)**

Or, stationary picture: $X(r)$

E.g. $X = r$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{D-2}$$

$$ds^2 = -N^2 dT^2 + q_{ij}(dX^i + N^i dT)(dX^j + N^j dT)$$

ADM variables:

$$N^2 = \frac{1}{s'(2 - fs')} \qquad N_r = 1 - fs'$$

$$q_{rr} = s'(2 - fs')$$

Schrodinger quantization

$$\phi(X^i, T) = \sum_A \left[a_A \phi_A(X^i) + a_A^\dagger \phi_A^*(X^i)\right] \quad , \quad \pi(X^i, T) = \sum_A \left[a_A \pi_A(X^i) + a_A^\dagger \pi_A^*(X^i)\right]$$

$$\gamma_i(X) = (\phi_i(X), \pi_i(X))$$

$$J\gamma_A = i\gamma_A \quad , \quad J\gamma_A^* = -i\gamma_A^*$$

$$(\gamma_A, \gamma_B) = \delta_{AB} \quad , \quad (\gamma_A, \gamma_B^*) = 0 \qquad\qquad (\gamma_1, \gamma_2) = i\int d^{D-1}X\sqrt{q}(\phi_1^*\pi_2 - \pi_1^*\phi_2)$$

$$[a_A, a_B^\dagger] = \delta_{AB} \quad , \quad [a_A, a_B] = [a_A^\dagger, a_B^\dagger] = 0$$

$$a_A|0\rangle = 0$$

- Leads to perfectly regular description of state, including near horizon
- Regular description of production of Hawking radiation, in atmosphere region ~R
- Subsystem decomposition ~ LQFT

Note: sometimes problematic in time-dependent backgrounds?

Torre, Varadarajan; Cortez, Marugan + others; Agullo Ashtekar; Much, Oeckl

But here time independent. Though — different pictures

2d example:

(D>2 technically more complicated — reflection; WIP w/ J. Perkins)

Metric $$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} = -f(r)\,dx^{+2} + 2dx^+dr = \begin{cases} -f(r)\,dx^+dx^- \ , \ r > R \\ f(r)\,dx^+d\hat{x}^- \ , \ r < R \end{cases}$$

$$x^\pm = t \pm r_*(r)$$

E.g. dilaton gravity/CGHS: $f(r) = 1 - Me^{-2r} = 1 - e^{-2(r-R)}$

Focus on right movers (i.e. outgoing): $\partial_+\phi = 0$

Quantization depends on choices:

1) Slices
2) Coordinates on slices
3) Modes

(ultimately expect physics independent of choices)

Evolution for regular modes:

$$\phi(r) = \int_0^\infty \frac{dk}{4\pi k}(a_k e^{ikr} + a_k^\dagger e^{-ikr})$$

(T independent: Schrodinger pic.)

$$H_\xi = \frac{1}{2}\int dr\, \gamma(r)\partial_r\phi\partial_r\phi = \int_0^\infty \frac{dk}{4\pi}\frac{dk'}{4\pi}\left[A(k,k')a_k^\dagger a_{k'} + B(k,k')a_k^\dagger a_{k'}^\dagger + \mathrm{h.c.}\right]$$

$$\gamma(r) = \frac{f}{2-f} = \tanh(r-R)$$

Particle creation

$$A(k,k') = \frac{1}{2}\int dr\gamma(r)e^{-i(k-k')r}\ ,\ B(k,k') = -\frac{1}{2}\int dr\gamma(r)e^{-i(k+k')r}\ .$$

E.g. begin with $|0\rangle_r$ **(can generalize to time-dependent formation/initial state)**

1) Slices

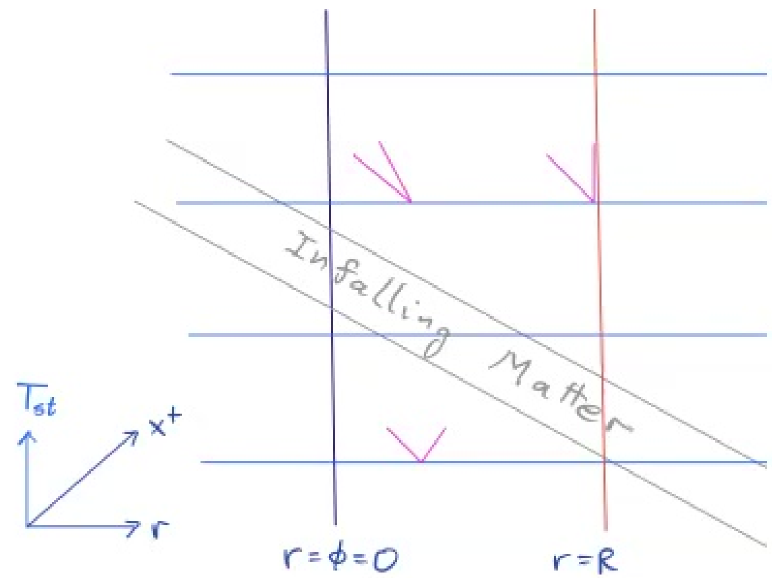
e.g straight: $x^+ = T + r$

CGHS: $2\sinh(r-R)\, e^{-(T-R)} = \begin{cases} e^{-x^-} , & r > R \\ -e^{\hat{x}^-} , & r < R \end{cases}$

2) Spatial coordinate/picture

$x^-, \hat{x}^-$ $(H_\xi = 0)$

r **(focus on this)**



3) Modes

Regular modes: e^{ikr} **at** $T = T_0$

Energy eigenmodes: $e^{-i\omega x^-}$, $e^{-i\omega \hat{x}^-}$ **singular at r=R!**

(used in traditional Hawking derivation)

Evolution for regular modes:

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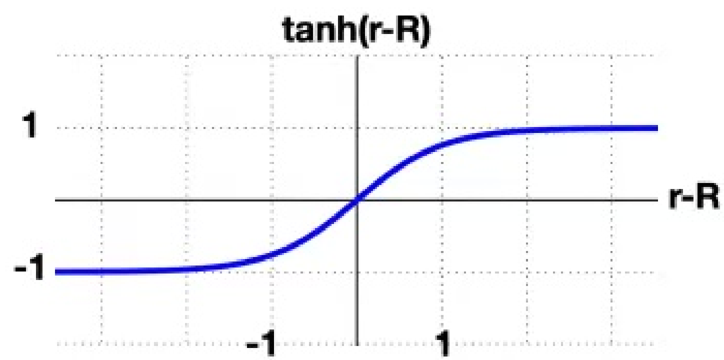


+ Production in atmosphere

+ Thermal spectrum $B(k,k') \sim e^{-\pi(k+k')/2}$

+ No transplanckian physics

- Evolution complicated

$$B(k,k') = \frac{i\pi}{\sinh\left[\pi(k+k')/2\right]}e^{-i(k+k')R}$$

Simplified, though singular, description: energy eigenmodes

Evolution for energy eigenmodes:

$$\phi = \theta(r-R)\int_0^\infty \frac{d\omega}{4\pi\omega}(b_\omega e^{-i\omega x^-} + b_\omega^\dagger e^{i\omega x^-}) + \theta(R-r)\int_0^\infty \frac{d\omega}{4\pi\omega}(\hat{b}_\omega e^{-i\omega \hat{x}^-} + \hat{b}_\omega^\dagger e^{i\omega \hat{x}^-})$$

$$H_\xi = \int_0^\infty \frac{d\omega}{4\pi\omega}\omega(b_\omega^\dagger b_\omega - \hat{b}_\omega^\dagger \hat{b}_\omega)$$

$$|0\rangle_r \sim \exp\left\{-\frac{1}{2}(b^\dagger\ \hat{b}^\dagger)\mathscr{B}^*\mathscr{A}^{-1}\begin{pmatrix} b^\dagger \\ \hat{b}^\dagger \end{pmatrix}\right\}|\hat{0},0\rangle \quad \sim \quad \sum_{\{n_\omega\}} e^{-\pi\int d\omega\omega n_\omega}|\{\widehat{n_\omega}\}\rangle|\{n_\omega\}\rangle$$

↑ **Bogolubov**

near-horizon
(need localized wavepackets)

↓

Paired excitations
Evolve away from horizon

Nice fact: for $r-R \gg 1,\ b_\omega^\dagger \sim a_\omega^\dagger$: excitations identified

Indeed, r modes nicely interpolate:

$$2\sinh(r-R)\,e^{-(T-R)} = -X^- = \begin{cases} e^{-x^-}, & r > R \\ -e^{\hat{x}^-}, & r < R \end{cases}$$

↑ **Kruskal**

$|r-R| \ll 1$ $\qquad 2(r-R)e^{-T-R} \approx -X^-$ $\qquad |0\rangle_r \sim |0\rangle_{X^-} \to \sum_{\{n_\omega\}} e^{-\pi \int d\omega \omega n_\omega} |\{\widehat{n_\omega}\}\rangle |\{n_\omega\}\rangle$

$r - R \gg 1$ $\qquad T - r \approx x^-$

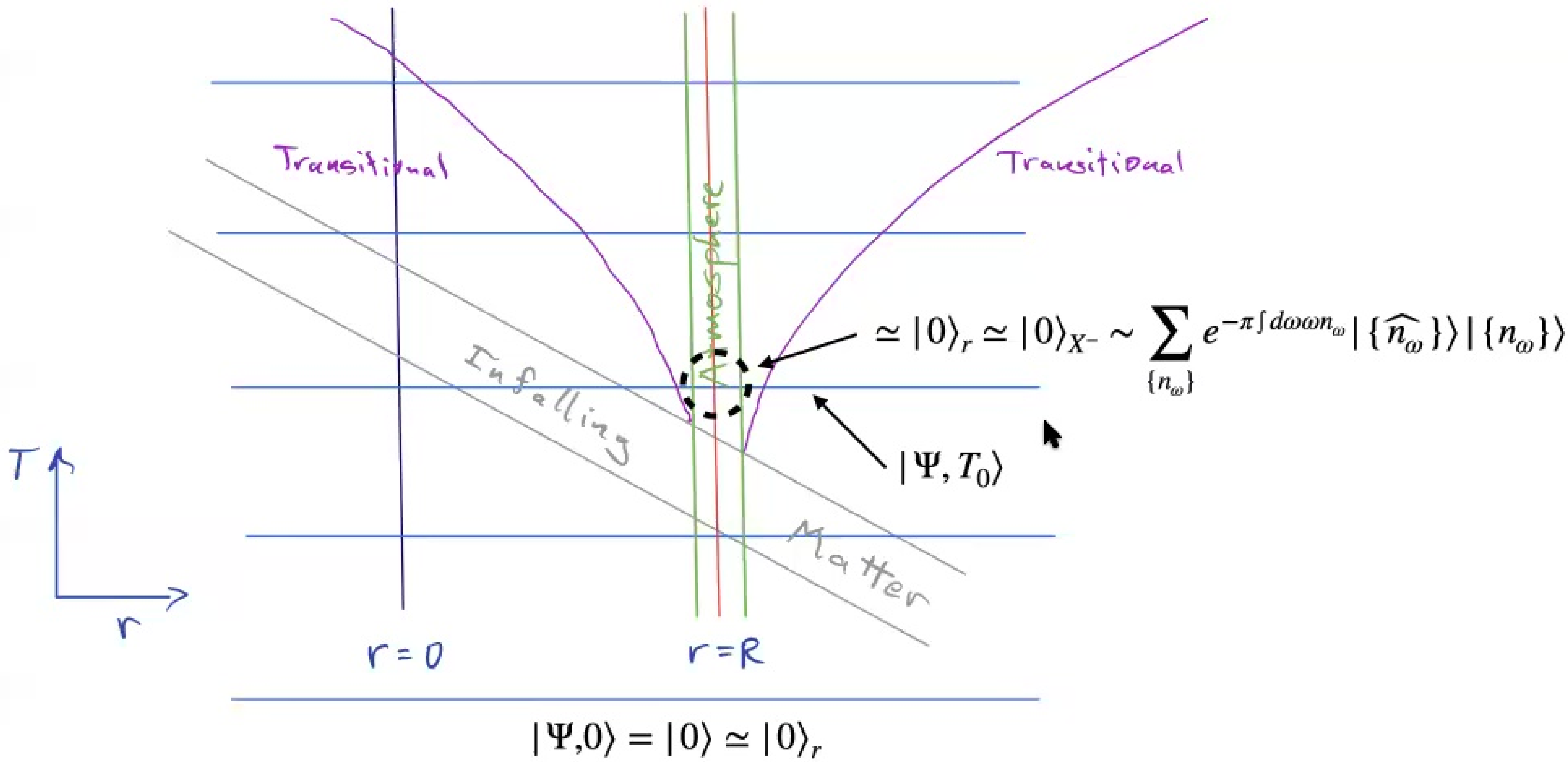
$R - r \gg 1$ $\qquad -T - r + 2R \approx \hat{x}^-$

$|0\rangle_r \to |\hat{0},0\rangle$

$b^\dagger_\omega \sim a^\dagger_\omega$

$\hat{b}^\dagger_\omega \sim a^\dagger_\omega$

Full picture of evolution:



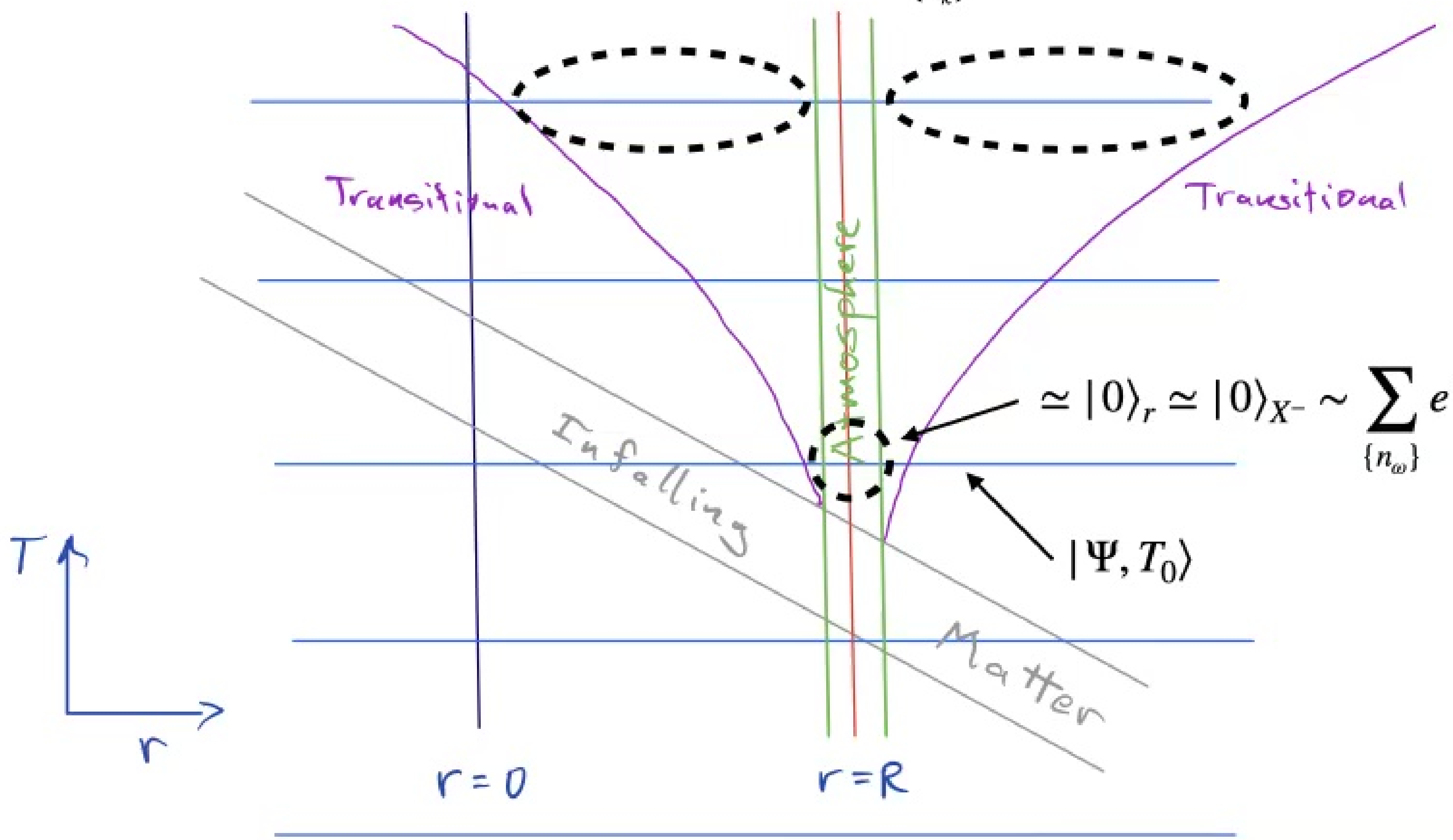
- **Explicit description of evolving state**

- **Generalizes to $D > 4$ (though more complicated) - WIP w/ J. Perkins**

Full picture of evolution:

$$\simeq |\Psi, T\rangle_{Unruh} \simeq \sum_{\{n_k\}} e^{-\pi \int dk k n_k} |\{\widehat{n_k}\}, \{n_k\}, T\rangle \quad \Rightarrow \rho_{env} = Tr_{BH}|\Psi\rangle\langle\Psi|$$

$$S(T) \propto T$$



$$\simeq |0\rangle_r \simeq |0\rangle_{X^-} \sim \sum_{\{n_\omega\}} e^{-\pi \int d\omega \omega n_\omega} |\{\widehat{n_\omega}\}\rangle |\{n_\omega\}\rangle$$

$$|\Psi, T_0\rangle$$

$$|\Psi, 0\rangle = |0\rangle \simeq |0\rangle_r$$

- Explicit description of evolving state

- Generalizes to $D > 4$ (though more complicated) - WIP w/ J. Perkins

Confirms expectations (in a nice way): no big surprises; greater confidence

Subsystems ~ LQFT

$$H = \int d^{D-1}x\sqrt{q}\left[\frac{1}{2}N(\pi^2 + q^{ij}\partial_i\phi\partial_j\phi) + N^i\pi\partial_i\phi\right] = H_{BH} + H_{environment} + H_I$$

Local evolution

Now can naturally include interactions, e.g. of Standard Model

But: how is the more complete quantum evolution unitary?

Breakdown of subsystem structure?

Within Einstein gravity (e.g. soft hair: info outside?) Hawking, Perry, Strominger

Gravity: add dressing

Subsystems more subtle (see references in intro)

Evidence have subsystems: 1805.11095, 1903.06160, 1907.06644

New quantum gravity effects?

Or: new QG effects modify evolution

Parameterize:

$$\Delta H_I = \sum_{Ab} \int d^{D-1}X \sqrt{q}\, \lambda^A\, \mathcal{O}^b(X)\, G_{Ab}(X)$$

Coefficients

Operators acting on env

Operators acting on BH

Program: further constrain structure, if "small"

$$\Delta H_I = \int d^{D-1}X \sqrt{q}\, H^{\mu\nu}(X) T_{\mu\nu}(X)$$

acts on BH state

acts on env

More discussion: 1905.08807

Or: connection to RWHs? $\Delta H_I = \int \mathcal{O}_{wormhole}$

Must extend to radii $r \sim R$: raises possibility of observational effects - LIGO, EHT

1905.08807, + 1904.05287, 1703.03387, …