Title: Nonlocal cosmological models from infrared quantum gravity effects

Speakers: Enis Belgacem

Series: Quantum Gravity

Date: May 13, 2021 - 2:30 PM

URL: http://pirsa.org/21050014

Abstract: The issue of whether quantum effects can affect gravity at cosmological distances still lacks a fundamental understanding, but there are indications of a non-trivial gravitational infrared dynamics. This possibility is appealing for building alternatives to the standard cosmological model and explaining the accelerated expansion of the Universe. In this talk I will discuss some large scale modifications of general relativity due to nonlocal terms, which are assumed to arise at the level of quantum effective action. Nonlocality is a general feature of quantum effective actions for theories with massless degrees of freedom and dynamical mass generation is a typical non-perturbative IR effect. Among several models, cosmological requirements select a single structure of the nonlocal term describing a mass for the conformal mode of the metric. The model fits very well cosmological data and has strong signatures in the tensor sector that could be tested in the future by gravitational-wave detections.

Nonlocal cosmological models from infrared quantum gravity effects

Enis Belgacem

Utrecht University

Main reference:

E. Belgacem, Y. Dirian, A. Finke, S. Foffa and M. Maggiore, ***Gravity in the infrared and effective nonlocal models***
JCAP 04 (2020) 010 [arXiv:2001.07619]

Work done at the University of Geneva

QG seminar, Perimeter Institute

May 13th 2021

OUTLINE

1. Motivations and theoretical framework

- IR quantum effects in gravity and the quantum effective action
- Nonlocality and gauge-invariant/diff.-invariant mass terms
- Nonlocal gravity models, focus on the RT model

2. Phenomenology and cosmological predictions

- FRW background and dynamical dark energy
- Scalar perturbations
- Bayesian parameter estimation and comparison with ΛCDM
- Tensor perturbations and modified GW propagation

Modified gravity in cosmology

- Standard cosmological model ΛCDM in agreement with observational data
- But it is not fully satisfactory from a theoretical point of view: the cosmological constant needs to be fine tuned to explain quantitatively the current accelerated expansion
- Dark energy is the less tested sector
- Modified gravity is interesting: dynamical dark energy and tests of GR at cosmological scales
- There are alternatives to ΛCDM and cosmological data to test them

---

A possible approach:

Can quantum effects in gravity have cosmological consequences?

- No first principle answer to this question is currently available
- **Nonlocal models** studies phenomelogically this possibility

The quantum effective action

When including quantum effects the relevant quantity to consider is not the classical action $S[\varphi]$,

but rather the quantum effective action $\Gamma[\phi]$

Construction (using a scalar field in flat space-time for simplicity)

$$Z[J] = e^{iW[J]} \equiv \int D\varphi \; e^{iS[\varphi]+i\int J\varphi} \qquad \frac{\delta W[J]}{\delta J(x)} = \langle 0|\varphi(x)|0\rangle_J \equiv \phi[J]$$

$$\Gamma[\phi] \equiv W[J] - \int \phi J \qquad \frac{\delta\Gamma[\phi]}{\delta\phi(x)} = -J(x)$$

$$e^{i\Gamma[\phi]} = \int D\varphi \; e^{iS[\phi+\varphi]-i\int \frac{\delta\Gamma[\phi]}{\delta\phi}\varphi}$$

- The quantum effective action gives the equations of motion for the v.e.v.s of the fields and is obtained by integrating over the quantum fluctuations around them.

- The quantum effective action is not a low energy Wilson effective action. It has the same regime of validity as the fundamental action.

For gravity: quantum fluctuations of the matter fields and of the metric.

Integration over quantum fluctuations of matter fields gives (with $\phi = 0\,, \quad J = 0$)

$$e^{i\Gamma[g_{\mu\nu}]} = e^{iS_{\rm EH}[g_{\mu\nu}]} \int D\varphi \; e^{iS_m[g_{\mu\nu};\varphi]} \equiv e^{iS_{\rm EH}[g_{\mu\nu}]} \, e^{i\Gamma_m[g_{\mu\nu}]}$$

$$\Gamma = S_{\rm EH} + \Gamma_m \qquad \langle 0|T^{\mu\nu}|0\rangle = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma_m}{\delta g_{\mu\nu}} \qquad G^{\mu\nu} = 8\pi G \langle 0|T^{\mu\nu}|0\rangle$$

- Then one would have to integrate over quantum fluctuations of the metric, including the Faddeev-Popov determinant and a gauge-fixing term

The physical interpretation of the final results is determined by the boundary conditions in the path integral:

Feynman path integral	Schwinger-Keldysh path integral
in-out matrix elements	in-in matrix elements
The Feynman propagator gives acausal EOM	The **retarded propagator** gives **causal** EOM
in-out v.e.v.s appear as intermediate steps in QFT computations, but by themselves they are not physical	in-in v.e.v.s are physical and $\langle 0_{\rm in}|\hat{g}_{\mu\nu}|0_{\rm in}\rangle$ is a semiclassical metric

For gravity: quantum fluctuations of the matter fields and of the metric.

Integration over quantum fluctuations of matter fields gives (with $\phi = 0\,, \quad J = 0$)

$$e^{i\Gamma[g_{\mu\nu}]} = e^{iS_{\rm EH}[g_{\mu\nu}]} \int D\varphi \; e^{iS_m[g_{\mu\nu};\varphi]} \equiv e^{iS_{\rm EH}[g_{\mu\nu}]} \, e^{i\Gamma_m[g_{\mu\nu}]}$$

$$\Gamma = S_{\rm EH} + \Gamma_m \qquad \langle 0|T^{\mu\nu}|0\rangle = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma_m}{\delta g_{\mu\nu}} \qquad G^{\mu\nu} = 8\pi G \langle 0|T^{\mu\nu}|0\rangle$$

- Then one would have to integrate over quantum fluctuations of the metric, including the Faddeev-Popov determinant and a gauge-fixing term

The physical interpretation of the final results is determined by the boundary conditions in the path integral:

Feynman path integral	Schwinger-Keldysh path integral
in-out matrix elements	in-in matrix elements
The Feynman propagator gives acausal EOM	The **retarded propagator** gives **causal** EOM
in-out v.e.v.s appear as intermediate steps in QFT computations, but by themselves they are not physical	in-in v.e.v.s are physical and $\langle 0_{\rm in}|\hat{g}_{\mu\nu}|0_{\rm in}\rangle$ is a semiclassical metric

- The classical action is local, but the quantum effective action is nonlocal when the theory contains **massless** (or light) particles!

Quantum fluctuations of the **graviton** $\Longrightarrow$ Nonlocal terms, relevant in the IR

Example: QED

The quantum fluctuations due to the electron affect the dynamics of the photon

$$\Gamma_{\rm QED}[A_\mu] = -\frac{1}{4}\int d^4x \left[F_{\mu\nu}\frac{1}{e^2(\Box)}F^{\mu\nu} + \mathcal{O}(F^4)\right] + \text{ fermionic terms}$$

$$|\Box/m_e^2| \gg 1 \text{ (light electron)} \Longrightarrow \frac{1}{e^2(\Box)} \simeq \frac{1}{e^2(m_e)} - \frac{1}{12\pi^2}\log\left(\frac{-\Box}{m_e^2}\right)$$

It just translates the running of the coupling constant from momentum space to coordinate space

$$\log\left(\frac{-\Box}{m_e^2}\right) \equiv \int_0^\infty dx \left[\frac{1}{x+m_e^2} - \frac{1}{x-\Box}\right] \text{ is a nonlocal operator}$$

Anomaly-induced effective action in D=2: the Polyakov action

Gravity in D=2 spacetime dimensions and N **conformally-coupled** matter fields

$$S = \int d^2x \sqrt{-g}(\kappa R - \lambda) + S_m$$

Exact trace anomaly $\langle 0|T^a_a|0\rangle = \frac{N}{24\pi} R$

$g_{ab} = e^{2\sigma}\bar{g}_{ab}$ $\sigma(x)$ conformal mode In D=2 we can always write locally $\bar{g}_{ab} = \eta_{ab}$

$$\frac{\delta\Gamma_m}{\delta\sigma} = \sqrt{-g}\,\langle 0|T^a_a|0\rangle = -\frac{N}{12\pi}\,\Box_\eta\sigma \quad \Longrightarrow \quad \Gamma_m[\sigma] - \Gamma_m[0] = -\frac{N}{24\pi}\int d^2x\,\sigma\Box_\eta\sigma$$

$\Gamma_m[0] = 0$ because, in D=2, $\sigma = 0$ corresponds to flat-space time

$$\Box_g = e^{-2\sigma}\Box_\eta\,,\ R = -2\Box_g\sigma\,,\ \sqrt{-g} = e^{2\sigma} \Longrightarrow \Gamma_m[g_{\mu\nu}] = -\frac{N}{96\pi}\int d^2x\sqrt{-g}\,R\Box_g^{-1}R$$

The covariant form requires nonlocal operators

Then we have to integrate over quantum fluctuations of the metric (i.e. of the conformal mode) and, as in every gauge theory, there are contributions from Faddeev-Popov ghosts.

Total quantum effective action $\Gamma = -\frac{N-25}{96\pi}\int d^2x\sqrt{-g}\,R\frac{1}{\Box}R - \lambda\int d^2x\,\sqrt{-g}$

Reintroducing $g_{ab} = e^{2\sigma}\bar{g}_{ab}$, apart from terms depending only on $\bar{g}_{ab}$, we find

$$\Gamma = \int d^2x\sqrt{-\bar{g}}\left(\frac{N-25}{24\pi}\bar{g}^{ab}\partial_a\sigma\partial_b\sigma + \frac{N-25}{24\pi}\overline{R}\sigma - \lambda e^{2\sigma}\right)$$

Despite the appearance there is **no ghost** in the theory, independently from the sign of N-25

The degrees of freedom must be counted from the fundamental action, not from the QEA

At the quantum level, the states associated to the Faddeev-Popov ghosts and to the conformal mode are eliminated by the condition that physical states have zero BRST charge.

Analogously to the Gupta-Bleuler condition for QED, here one has to impose $\langle s'|T^{ab}_{\rm tot}|s\rangle = 0$

J. Polchinski, "A Two-Dimensional Model for Quantum Gravity," Nucl. Phys. B324 (1989) 123–140

IR quantum effects in gravity

1. Strong IR divergences in the graviton propagator in de Sitter, due to quantum fluctuations of the conformal mode. Antoniadis, Mottola 1991,1992

A typical IR effect due to the presence of massless particles is dynamical mass generation

2. A massless scalar field φ with $\lambda\varphi^4$ self-interaction, in a given de Sitter background, develops a mass dynamically $m^2_{dyn} \propto H^2\sqrt{\lambda}$ Starobinski, Yokoyama 1994; Riotto, Sloth 2008; Burgess et al. 2010; Rajaraman 2010

3. More recently non-perturbative techniques started to be used to study the IR regime:

- lattice gravity — Knorr, Saueressig 1804.03846 using numerical simulations of CDT, but continuum limit not studied yet

  ↓ Results there suggest generation of a mass for the conformal mode in the IR

- functional renormalization group equations — Wetterich 1704.08040, Morris 1802.04281 strong IR quantum gravity effects

Also relevant: Reuter, Weyer 0410119, find singularity of some RG trajectories at a finite momentum scale when evolving the RG flow towards the IR

↓ It could signal dynamical mass generation,
but it could also be an artifact of the truncation in the action functionals space

Can the generation of a mass for the conformal mode od the graviton be compatible with diffeomorphism invariance?

Nonlocality allows for it!

Example: massive electrodynamics in a nonlocal but gauge-invariant form

$$S = \int d^4x \left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}m_\gamma^2\, A_\mu A^\mu - j_\mu A^\mu\right)$$

Action local but not gauge-invariant

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu$$

Apply ∂_ν and find $\partial_\nu A^\nu = 0$ for $m_\gamma \neq 0$

Then $\left(\Box - m_\gamma^2\right) A^\nu = j^\nu$

$$S = \int d^4x \left[-\tfrac{1}{4}F_{\mu\nu}\left(1 - \frac{m_\gamma^2}{\Box}\right)F^{\mu\nu} - j_\mu A^\mu\right]$$

Action nonlocal but gauge-invariant

$$\left(1 - \frac{m_\gamma^2}{\Box}\right)\partial_\mu F^{\mu\nu} = j^\nu$$

Choose the Lorentz gauge $\partial_\nu A^\nu = 0$

and find again $\left(\Box - m_\gamma^2\right) A^\nu = j^\nu$

Same EOM

How to write a diff-invariant mass term for the conformal mode?

Following Foffa, Maggiore, Mitsou 2014; EB, Dirian, Finke, Foffa, Maggiore 2020 — let's start from linearized gravity over Minkowski

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad \kappa = (32\pi G)^{1/2}$$

$$S^{(2)}_{\rm EH} = \frac{1}{2}\int d^4x\, h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma}$$

$$S^{(1)}_{\rm int} = \frac{\kappa}{2}\int d^4x\, h_{\mu\nu}T^{\mu\nu}$$

$$\Longrightarrow \quad \mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma} = -\frac{\kappa}{2}T^{\mu\nu}$$

Decomposition $h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + \frac{1}{2}(\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu) + \frac{1}{3}\eta_{\mu\nu}s$ and similarly for $T_{\mu\nu}$

Under a gauge transf. $h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{2}(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu)$, then

$$\begin{cases} h^{\rm TT}_{\mu\nu} \to h^{\rm TT}_{\mu\nu} \\ \epsilon_\mu \to \epsilon_\mu - \xi_\mu \\ s \to s \end{cases}$$

Observe that the inverse relation of the metric decomposition is nonlocal.

For the gauge-invariant parts

$$s = P^{\mu\nu}h_{\mu\nu}$$

$$h^{TT}_{\mu\nu} = \left(P^\rho_\mu P^\sigma_\nu - \frac{1}{3}P_{\mu\nu}P^{\rho\sigma}\right)h_{\rho\sigma}$$

with $P^\nu_\mu = \delta^\nu_\mu - \frac{\partial_\mu\partial^\nu}{\Box}$

Can the generation of a mass for the conformal mode od the graviton be compatible with diffeomorphism invariance?

Nonlocality allows for it!

Example: massive electrodynamics in a nonlocal but gauge-invariant form

$$S = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu\right)$$

Action local but not gauge-invariant

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu$$

Apply ∂_ν and find $\partial_\nu A^\nu = 0$ for $m_\gamma \neq 0$

Then $\left(\Box - m_\gamma^2\right) A^\nu = j^\nu$

$$S = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m_\gamma^2}{\Box}\right) F^{\mu\nu} - j_\mu A^\mu\right]$$

Action nonlocal but gauge-invariant

$$\left(1 - \frac{m_\gamma^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

Choose the Lorentz gauge $\partial_\nu A^\nu = 0$

and find again $\left(\Box - m_\gamma^2\right) A^\nu = j^\nu$

Same EOM

How to write a diff-invariant mass term for the conformal mode?

Following Foffa, Maggiore, Mitsou 2014; EB, Dirian, Finke, Foffa, Maggiore 2020 — let's start from linearized gravity over Minkowski

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad \kappa = (32\pi G)^{1/2}$$

$$S^{(2)}_{\rm EH} = \tfrac{1}{2}\int d^4x\, h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma}$$

$$S^{(1)}_{\rm int} = \tfrac{\kappa}{2}\int d^4x\, h_{\mu\nu}T^{\mu\nu} \qquad \Longrightarrow \qquad \mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma} = -\tfrac{\kappa}{2}T^{\mu\nu}$$

Decomposition $h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + \frac{1}{2}(\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu) + \frac{1}{3}\eta_{\mu\nu}s$ and similarly for $T_{\mu\nu}$

Under a gauge transf. $h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{2}(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu)$, then

$$\begin{cases} h^{\rm TT}_{\mu\nu} \to h^{\rm TT}_{\mu\nu} \\ \epsilon_\mu \to \epsilon_\mu - \xi_\mu \\ s \to s \end{cases}$$

Observe that the inverse relation of the metric decomposition is nonlocal.

For the gauge-invariant parts

$$s = P^{\mu\nu}h_{\mu\nu}$$

$$h^{TT}_{\mu\nu} = \left(P^\rho_\mu P^\sigma_\nu - \tfrac{1}{3}P_{\mu\nu}P^{\rho\sigma}\right)h_{\rho\sigma}$$

with $P^\nu_\mu = \delta^\nu_\mu - \frac{\partial_\mu\partial^\nu}{\Box}$

The action can be written as

$$S^{(2)}_{\rm EH} = \frac{1}{2}\int d^4x\,\left(h^{TT}_{\mu\nu}\Box h^{\mu\nu,TT} - \frac{2}{3}s\Box s\right)$$

$$S^{(1)}_{\rm int} = \frac{\kappa}{2}\int d^4x\,\left(h^{TT}_{\mu\nu}\,T^{\mu\nu,TT} + \frac{1}{3}sT\right)$$

and the corresponding EOM

$$\Box h^{TT}_{\mu\nu} = -\frac{\kappa}{2}T^{TT}_{\mu\nu} \qquad \Box s = \frac{\kappa}{4}T$$

Add mass term for the conformal mode

$$\Gamma^{(2)} = \frac{1}{2}\int d^4x\,\left[h^{TT}_{\mu\nu}\Box h^{\mu\nu,TT} - \frac{2}{3}s\left(\Box + \mathbf{m^2}\right)s\right]$$

We expect the mass for the conformal mode to be the most relevant. Furthermore, after different trials it has been found that only in this case there is a viable cosmology.

In terms of a not-rescaled $h_{\mu\nu}$

$$\Gamma^{(2)} = \frac{1}{64\pi G}\int d^4x\,\left[h_{\mu\nu}\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma} - \frac{2}{3}\mathbf{m^2}\left(P^{\mu\nu}h_{\mu\nu}\right)^2\right]$$

$$\mathcal{E}^{\mu\nu,\rho\sigma}h_{\rho\sigma} - \frac{2}{3}\mathbf{m^2}P^{\mu\nu}P^{\rho\sigma}h_{\rho\sigma} = -16\pi G T^{\mu\nu}$$

Covariantizations

Key ingredients $R^{(1)} = -\Box\left(P^{\mu\nu} h_{\mu\nu}\right)$ $G^{(1)}_{\mu\nu} = -\frac{1}{2}\mathcal{E}_{\mu\nu,\rho\sigma} h^{\rho\sigma}$

A natural covariantization of $\Gamma^{(2)}$ leads to

$$\Gamma_{\rm RR} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{6} R\frac{1}{\Box^2}R\right]$$

"RR model" Maggiore, Mancarella 2013

while starting directly from the equations of motion

$$G_{\mu\nu} - \frac{m^2}{3}\left(g_{\mu\nu}\Box^{-1}R\right)^{\rm T} = 8\pi G\, T_{\mu\nu}$$

"RT model" Maggiore 2014

Transverse part of a tensor $S_{\mu\nu} = S^{\rm T}_{\mu\nu} + \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$ with $\nabla^\mu S^{\rm T}_{\mu\nu} = 0$

RR and RT equivalent when linearized over Minkowski, but they are different in general

$\Longrightarrow$ they give different cosmological predictions

RR shares most of the nice properties of RT:

- viable cosmological background evolution
- stable scalar and tensor perturbations
- fit CMB, BAO, type Ia SNe and structure formation data, at the same level as ΛCDM
- speed of gravitational waves equals the speed of light

But **RR excluded from Lunar Laser Ranging** EB, Finke, Frassino, Maggiore 2019

The RT model is immune to LLR and, to date, passes all the observational tests

Focus on the RT model

$$G_{\mu\nu} - \frac{m^2}{3}\left(g_{\mu\nu}\Box^{-1}R\right)^{\rm T} = 8\pi G\, T_{\mu\nu}$$

Localization

Define auxiliary fields $U,\ S_\mu$

$$U = -\Box^{-1}R \qquad S_{\mu\nu} = -U g_{\mu\nu}$$

$$S_{\mu\nu} = S^{\rm T}_{\mu\nu} + \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$$

with $\nabla^\mu S^{\rm T}_{\mu\nu} = 0$

"Localized" RT equations

$$G_{\mu\nu} + \frac{m^2}{6}\left(2U g_{\mu\nu} + \nabla_\mu S_\nu + \nabla_\nu S_\mu\right) = 8\pi G T_{\mu\nu}$$

$$\Box U = -R$$

$$(\delta^\mu_\nu \Box + \nabla^\mu \nabla_\nu) S_\mu = -2\partial_\nu U$$

RR shares most of the nice properties of RT:

- viable cosmological background evolution
- stable scalar and tensor perturbations
- fit CMB, BAO, type Ia SNe and structure formation data, at the same level as ΛCDM
- speed of gravitational waves equals the speed of light

But **RR excluded from Lunar Laser Ranging** EB, Finke, Frassino, Maggiore 2019

The RT model is immune to LLR and, to date, passes all the observational tests

Focus on the RT model
$$G_{\mu\nu} - \frac{m^2}{3}\left(g_{\mu\nu}\Box^{-1}R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$$

Localization

Define auxiliary fields $U,\ S_\mu$

$$U = -\Box^{-1}R \qquad S_{\mu\nu} = -U g_{\mu\nu}$$
$$S_{\mu\nu} = S^{\mathrm{T}}_{\mu\nu} + \tfrac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$$
with $\nabla^\mu S^{\mathrm{T}}_{\mu\nu} = 0$

"Localized" RT equations

$$G_{\mu\nu} + \frac{m^2}{6}\left(2U g_{\mu\nu} + \nabla_\mu S_\nu + \nabla_\nu S_\mu\right) = 8\pi G T_{\mu\nu}$$
$$\Box U = -R$$
$$(\delta^\mu_\nu \Box + \nabla^\mu \nabla_\nu) S_\mu = -2\partial_\nu U$$

OBSERVATIONS

1) Auxiliary fields for localization are not new degrees of freedom

This is clear considering the case of 2D Polyakov action, where we have a fundamental derivation:

We have seen that, using $\sigma = -\frac{1}{2}\Box^{-1}R$, we get $\Gamma_m[g_{\mu\nu}] = -\frac{N-25}{96\pi}\int d^2x\,\sqrt{-g}\,R\Box^{-1}R$

We can localize the theory by introducing $U = -\Box^{-1}R$

U is completely determined by the metric as $U = 2\sigma$ and is not a generic solution of $\Box U = -R$

We are not free to choose any homogeneous solution $U_{\rm hom}$ of $\Box U_{\rm hom} = 0$

At the quantum level, there are no creation/annihilation operators associated to U because there are no free coefficients coming from $U_{\rm hom}$ that can be promoted to operators.

RR shares most of the nice properties of RT:

- viable cosmological background evolution
- stable scalar and tensor perturbations
- fit CMB, BAO, type Ia SNe and structure formation data, at the same level as ΛCDM
- speed of gravitational waves equals the speed of light

But **RR excluded from Lunar Laser Ranging** EB, Finke, Frassino, Maggiore 2019

The RT model is immune to LLR and, to date, passes all the observational tests

Focus on the RT model $$G_{\mu\nu} - \frac{m^2}{3}\left(g_{\mu\nu}\Box^{-1}R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$$

Localization

Define auxiliary fields $U,\ S_\mu$

$$U = -\Box^{-1}R \qquad S_{\mu\nu} = -U g_{\mu\nu}$$

$$S_{\mu\nu} = S^{\mathrm{T}}_{\mu\nu} + \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$$

with $\nabla^\mu S^{\mathrm{T}}_{\mu\nu} = 0$

"Localized" RT equations

$$G_{\mu\nu} + \frac{m^2}{6}\left(2U g_{\mu\nu} + \nabla_\mu S_\nu + \nabla_\nu S_\mu\right) = 8\pi G T_{\mu\nu}$$

$$\Box U = -R$$

$$(\delta^\mu_\nu \Box + \nabla^\mu \nabla_\nu) S_\mu = -2\partial_\nu U$$

2) The RR and RT models have the same propagating degrees of freedom as GR

Ingredients for counting the degrees of freedom

Linearize over Minkowski

3+1 decomposition of the metric

$$h_{00} = 2\psi \qquad h_{0i} = \beta_i + \partial_i \gamma$$

$$h_{ij} = -2\phi\, \delta_{ij} + \left(\partial_i \partial_j - \tfrac{1}{3}\delta_{ij}\nabla^2\right)\lambda + \tfrac{1}{2}(\partial_i v_j + \partial_j v_i) + H_{ij}^{\rm TT}$$

with $\partial_i \beta^i = 0,\ \partial_i v^i = 0,\ \partial^j H_{ij}^{\rm TT} = 0,\ \delta^{ij} H_{ij}^{\rm TT} = 0$

6 gauge-invariant variables:

$$\Phi = -\phi - (1/6)\nabla^2 \lambda \qquad \Psi = \psi - \dot{\gamma} + (1/2)\ddot{\lambda}$$

$$\Xi_i = \beta_i - (1/2)\dot{v}_i \qquad H_{ij}^{\rm TT}$$

3+1 decomposition of the energy-momentum tensor

$$T_{00} = \rho \qquad T_{0i} = K_i + \partial_i K$$

$$T_{ij} = P\, \delta_{ij} + \left(\partial_i \partial_j - \tfrac{1}{3}\delta_{ij}\nabla^2\right)\Sigma + \tfrac{1}{2}(\partial_i \Sigma_j + \partial_j \Sigma_i) + \Sigma_{ij}$$

with $\partial_i K^i = 0,\ \partial_i \Sigma^i = 0,\ \partial^j \Sigma_{ij} = 0,\ \delta^{ij}\Sigma_{ij} = 0$

GR

$$\nabla^2\Phi = -4\pi G\rho\,, \quad \nabla^2\Psi = -4\pi G(\rho - 2\nabla^2\Sigma)$$

$$\nabla^2\Xi_i = -16\pi G K_i\,, \quad \Box H_{ij}^{\rm TT} = -16\pi G\Sigma_{ij}$$

RT (or RR)

They are equivalent when linearized over Minkowski. For this computations it is more convenient to use the RR model, as it only needs scalar auxiliary fields.

$$U = -\Box^{-1}R \qquad S = -\Box^{-1}U$$

$$\nabla^2\left(\Phi - \frac{m^2}{6}S\right) = -4\pi G\rho\,, \quad \Phi - \Psi - \frac{m^2}{3}S = -8\pi G\Sigma$$

$$\nabla^2\Xi_i = -16\pi G K_i\,, \quad \Box H_{ij}^{\rm TT} = -16\pi G\Sigma_{ij}$$

$$(\Box + m^2)U = -8\pi G(\rho - 3P)\,, \quad \Box S = -U$$

- In both cases the 2 GW polarizations in $H_{ij}^{\rm TT}$ are the only propagating degrees of freedom
- In both cases the conformal mode is fully determined by the energy-momentum tensor as $s = 6\Phi - 2\Box^{-1}\nabla^2(\Phi + \Psi) = 6\Phi + 16\pi G\Box^{-1}(\rho - \nabla^2\Sigma)$ and has to vanish if $\rho = 0\,, \Sigma = 0$

s is non-radiative and there is no ghost problem with its kinetic term in

$$S^{(2)}_{\rm EH} = \frac{1}{2}\int d^4x\,\left(h^{TT}_{\mu\nu}\Box h^{\mu\nu,TT} - \frac{2}{3}s\Box s\right) \quad \text{or} \quad \Gamma^{(2)} = \frac{1}{2}\int d^4x\,\left[h^{TT}_{\mu\nu}\Box h^{\mu\nu,TT} - \frac{2}{3}s\left(\Box + m^2\right)s\right]$$

Observation: matter perturbative loop corrections are not relevant for cosmology

One-loop corrections from matter fields can be organized in an expansion in curvature

$$\Gamma_{\rm one-loop} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - R\,k_R(\Box)R - C_{\mu\nu\rho\sigma}k_W(\Box)C^{\mu\nu\rho\sigma} + GB\right]$$

Barvinsky, Vilkovisky 1987

The contribution from a matter field of mass M, in the limit of light fields $|\Box/M^2| \gg 1$, is

$$k_R\left(\frac{-\Box}{M^2}\right) = \alpha\log\left(\frac{-\Box}{M^2}\right) + \beta\left(\frac{M^2}{-\Box}\right) + \gamma\left(\frac{M^2}{-\Box}\right)\log\left(\frac{-\Box}{M^2}\right) + \delta\left(\frac{M^2}{-\Box}\right)^2 + \dots$$

Gorbar, Shapiro 2003

The terms with coefficients α, β, γ have little effects on cosmology today

Codello, Jain 2017

The term with coefficient δ is of the RR form, but it is not viable for cosmology:

The mass m in the RR action would be $m = \mathcal{O}(M^2/M_P)$

But the perturbative expansion is valid today only if $M \ll H_0$

$\Longrightarrow$ $m \ll H_0\,(H_0/M_P)$

but dynamical DE needs $m \simeq H_0$

In RR actually $m < H_0$, but it is definitely not suppressed by H_0/M_P

Background cosmology of the RT model

FRW metric $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$

Auxiliary fields $U,\ S_0$

$$H^2 - \frac{m^2}{9}(U - \dot{S}_0) = \frac{8\pi G}{3}\rho$$
$$\ddot{U} + 3H\dot{U} = 6\dot{H} + 12H^2$$
$$\ddot{S}_0 + 3H\dot{S}_0 - 3H^2 S_0 = \dot{U}$$

Initial conditions for the auxiliary fields:

Maggiore 2016
EB, Cusin, Foffa, Maggiore, Mancarella 2018

- irrelevant starting from RD and MD
- In inflation, one of the auxiliary fields grows exponentially and this affects the subsequent evolution $\Longrightarrow$ introduce a parameter ΔN = number of e-folds from the moment in inflation when auxiliary fields have initial conditions O(1) to the end of inflation.

Options for ΔN

- initial conditions of O(1) set in RD, i.e. $\Delta N = 0$, that we call "**RT minimal**"
- $\Delta N = 34, 50, 64$ corresponding to $(\Delta N)_{\min}$ for $M_{\rm infl} = \{10^3, 10^{10}, 10^{16}\}$ GeV

$$M_{\rm infl} = (\rho_{\rm infl})^{1/4} \qquad\qquad (\Delta N)_{\min} \simeq 64 - \log \frac{10^{16}\,{\rm GeV}}{M_{\rm infl}}$$

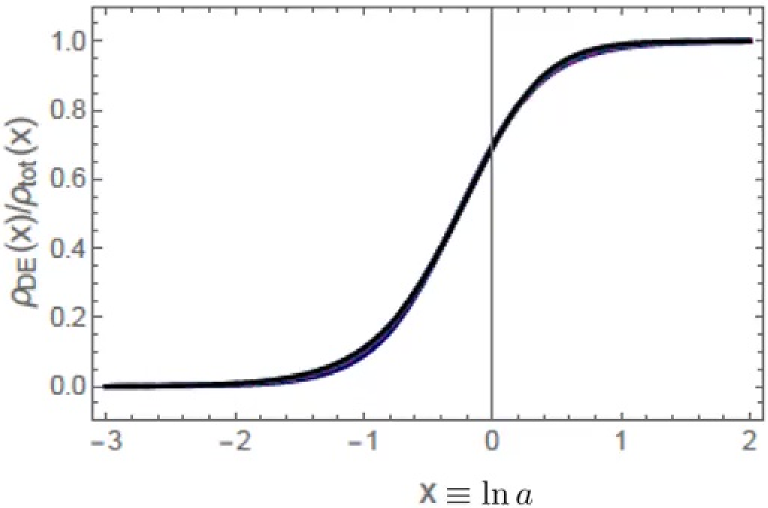
From EB, Cusin, Foffa, Maggiore, Mancarella 2018 **at sufficiently large ΔN the results saturate** to a limiting curve:

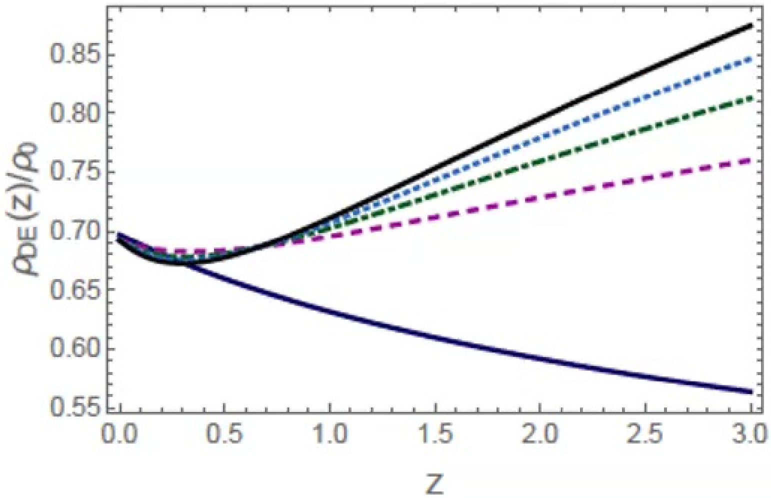
More precisely, for any given value of $M_{\rm infl}$, if ΔN is large enough we reach the same limiting cosmology. For $M_{\rm infl} = 10^{16}{\rm GeV}$ this happens at $\Delta N \simeq 70$

- To study the limiting curve for background quantities we include the case

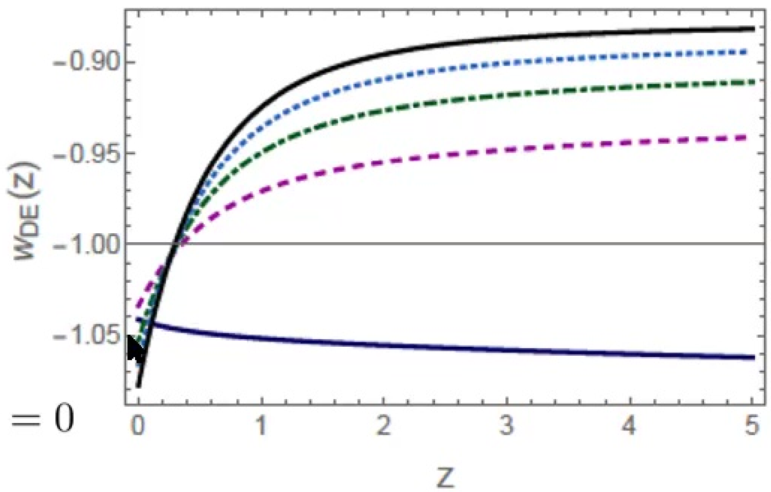
$$(M_{\rm infl} = 10^{16}\,{\rm GeV}, \Delta N = 100)$$

Dynamical dark energy of the RT model



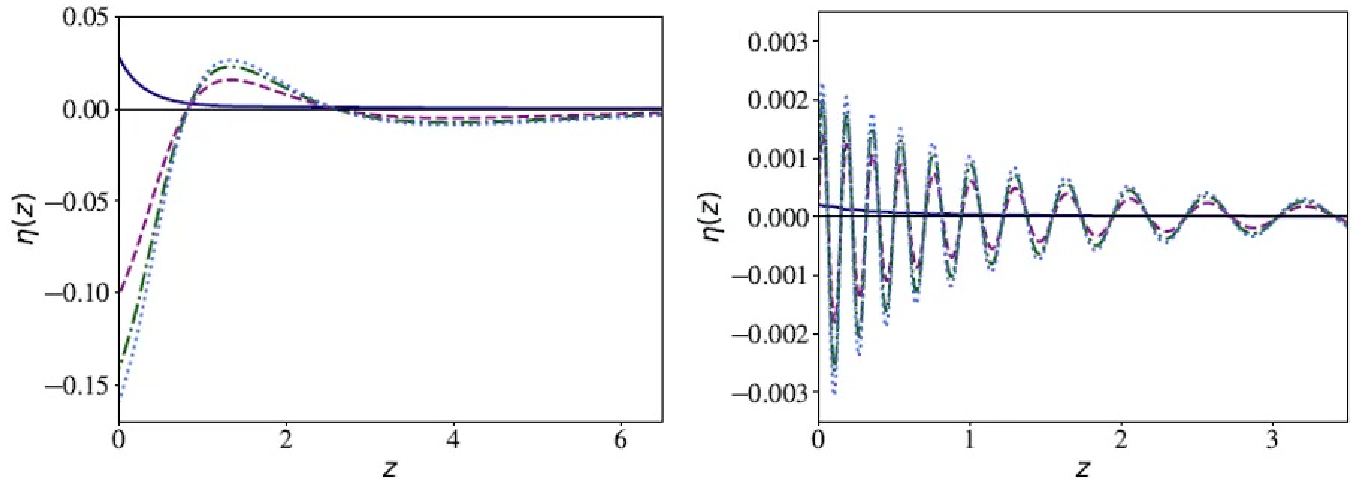


Figures from
EB, Dirian, Finke, Foffa, Maggiore
JCAP 04 (2020) 010



Blue solid: minimal RT
Magenta, dashed: RT with ΔN=34
Green, dot-dashed: RT with ΔN=50
Cyan, dotted: RT with ΔN=64
Black solid: RT in the limit of large ΔN

$$\dot{\rho}_{\rm DE} + 3H(1+w_{\rm DE})\rho_{\rm DE} = 0$$



$$\eta(x;k) = \frac{\Phi_{\mathbf{k}}(x) + \Psi_{\mathbf{k}}(x)}{\Phi_{\mathbf{k}}(x)}$$

Figure from
EB, Dirian, Finke, Foffa, Maggiore
JCAP 04 (2020) 010

η as a function of z, for the minimal RT model (blue solid line) and for RT with $\Delta N = 34$ (magenta, dashed), $\Delta N = 50$ (green, dot-dashed) and $\Delta N = 64$ (cyan, dotted), for $\kappa = 0.1$ (left panel) and $\kappa = 1$ (right panel).

Summary:

In background and scalar perturbations, RT differs from ΛCDM only at percent or sub-percent level

- RT fits current cosmological data at the same level of ΛCDM

Differences for background and scalar perturbations only at the percent/subpercent level

- Potentially within reach of future missions (Euclid, LSST, SKA, DESI)

Biggest surprises for testing nonlocal gravity are in tensor perturbations!

GW propagation in modified gravity

- Tensor perturbations around FRW background, with Fourier modes $h_A(\eta, \mathbf{k})$

$$h''_A + 2\mathcal{H}\left[1 - \delta(\eta)\right] h'_A + k^2 h_A = 0$$

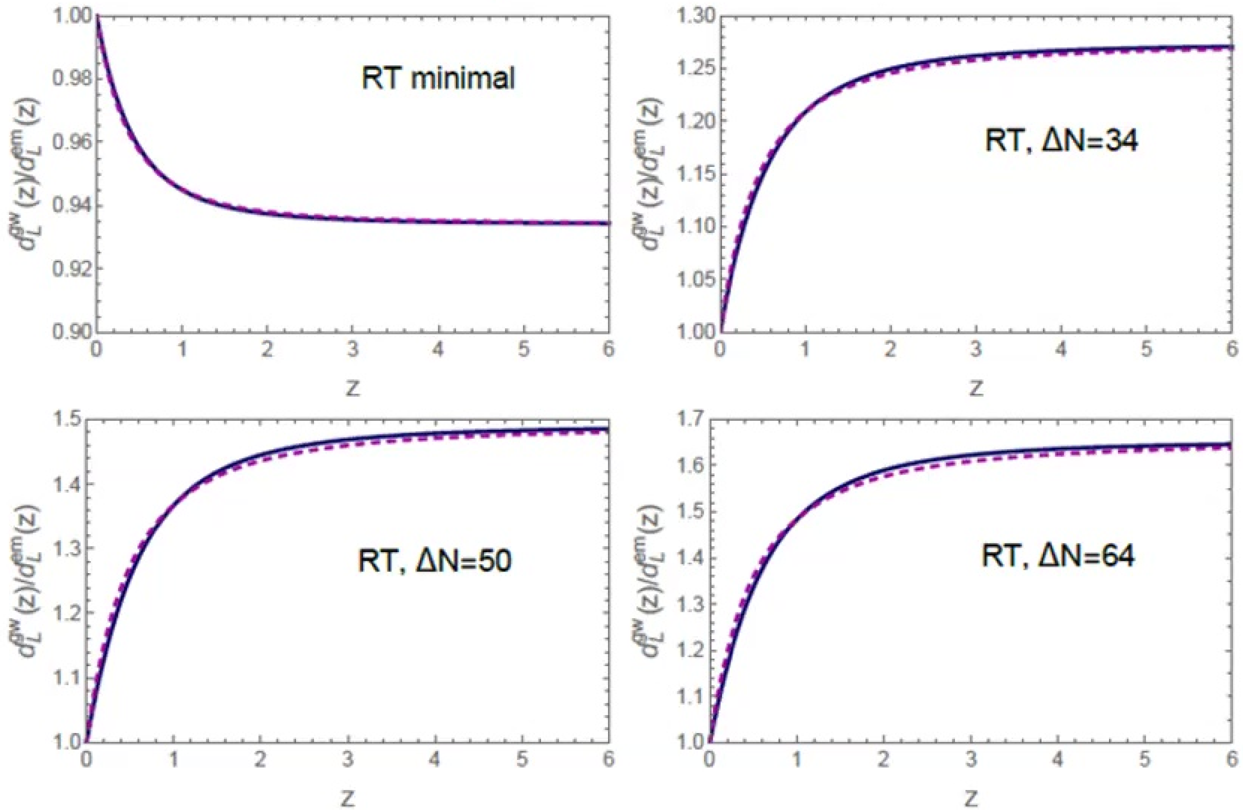
EB, Dirian, Foffa, Maggiore
PRD 2018, 1712.08108
PRD 2018, 1805.08731

- It is a very general feature of modified gravity models, e.g.

  - Scalar-tensor theories: Horndeski (f(R), galileons, Brans-Dicke), DHOST
  - Nonlocal gravity
  - Higher dimensions: DGP
  - Bigravity

Deffayet and Menou 2007
Saltas et al. 2014,
Lombriser and Taylor 2016,
Nishizawa 2017,
EB, Dirian, Foffa, Maggiore 2017, 2018
EB et al. (LISA Cosmology WG), 2019

Tensor sector: predictions of the RT model



Blue solid: numerical integration.
Magenta dashed: fit with a simple parametrization

EB, Dirian, Finke, Foffa, Maggiore
JCAP 1911, 022 (2019)

$$h_A'' + 2\mathcal{H}\left[1 - \delta(\eta)\right] h_A' + k^2 h_A = 0$$

$$\delta(\eta) = \frac{m^2 \bar{S}_0(\eta)}{6H(\eta)}$$

The effect reaches **80%** for large ΔN

Testable at LISA with 1 single event!

EB et al. (LISA Cosmology WG), JCAP 1907 (2019) 024

Testable at ET with 1 single event!

Detectable in a few years also at 2G network

EB, Dirian, Foffa, Howell, Maggiore, Regimbau, JCAP 1908 (2019) 015

EB, Dirian, Finke, Foffa, Maggiore, JCAP 11 (2019) 022

CONCLUSIONS

- Nonlocal gravity is a well motivated approach to the problem of the nature of dark energy
- It is very difficult to build a viable model. Currently we only have one model passing all the tests.
- Having a successful model stimulates research on a more complete understanding of IR quantum gravity
- Background and scalar perturbations: RT close to ΛCDM and fits data at the same level
- Large differences in the propagation of GWs across cosmological distances: testable model

PhD_thesis_Enis_Belgacem.pdf - Adobe Acrobat Reader DC (32-bit)

File Modifica Vista Firma Finestra ?

Home Strumenti PhD_thesis_Enis_Be... 2001.07619_Gravit... 1812.11181_Testin... Accedi

12 (24 di 171) 108%

Segnalibri

set $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$. The maximal information that we can get is the dependence of the QEA on the conformal mode σ.

The covariantization of eq. (1.2.26) is not uniquely determined and a possiblity is given by the Riegert action [64]

$$\Gamma_{\rm anom}[g_{\mu\nu}] = \Gamma_c[g_{\mu\nu}] - \frac{b_3}{12}\int d^4x\,\sqrt{-g}\,R^2 + \frac{1}{8}\int d^4x\sqrt{-g}\left(E - \frac{2}{3}\Box R\right)\Delta_4^{-1}\left[b_2\left(E-\frac{2}{3}\Box R\right) + 2b_1 C^2\right]. \tag{1.2.28}$$

It is interesting that, again, a covariant form requires the use of nonlocal operators. In this case the inverse Paneitz operator is needed, while for $D = 2$ the inverse d'Alembertian appeared.

1.3 Nonlocality and mass terms for gauge theories

Now that the notion of quantum effective action has been discussed, we can explain how it may contain nonlocal mass terms for gauge fields (or for the gravitational field) despite preserving gauge (or diffeomorphism) invariance. We follow the pre-