Title: Fault-tolerant logical gates in holographic stabilizer codes are severely restricted

Speakers: Samuel Cree

Series: Perimeter Institute Quantum Discussions

Date: May 05, 2021 - 4:00 PM

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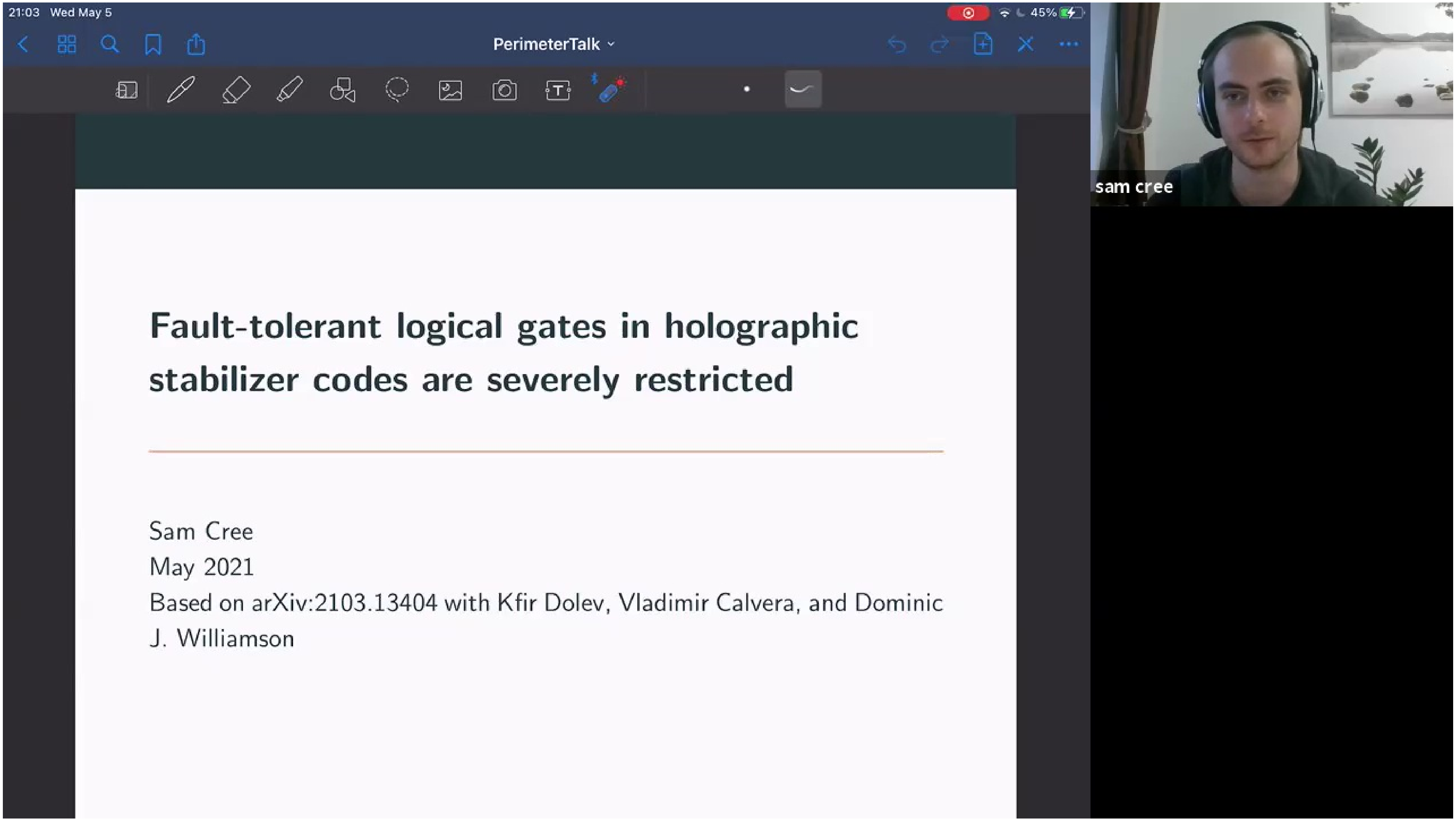
Abstract: We evaluate the usefulness of holographic stabilizer codes for practical purposes by studying their allowed sets of fault-tolerantly implementable gates. We treat them as subsystem codes and show that the set of transversally implementable logical operations is contained in the Clifford group for sufficiently localized logical subsystems. As well as proving this concretely for several specific codes, we argue that this restriction naturally arises in any stabilizer subsystem code that comes close to capturing certain properties of holography. We extend these results to approximate encodings, locality-preserving gates, certain codes whose logical algebras have non-trivial centers, and discuss cases where restrictions can be made to other levels of the Clifford hierarchy. A few auxiliary results may also be of interest, including a general definition of entanglement wedge map for any subsystem code, and a thorough classification of different correctability properties for regions in a subsystem code.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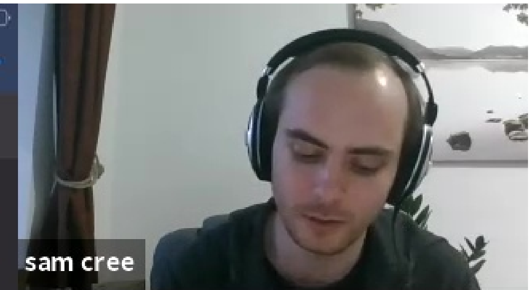
Fault-tolerant logical gates in holographic stabilizer codes are severely restricted
Sam Cree
May 2021
Based on arXiv:2103.13404 with Kfir Dolev, Vladimir Calvera, and Dominic J. Williamson




Motivation

Restrictions on fault-tolerant logical gates

Holographic code example

Complementary recovery and fault-tolerant logical gates

What makes a code holographic?

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# Holographic codes

- First appeared in 2015 [PYHP15]

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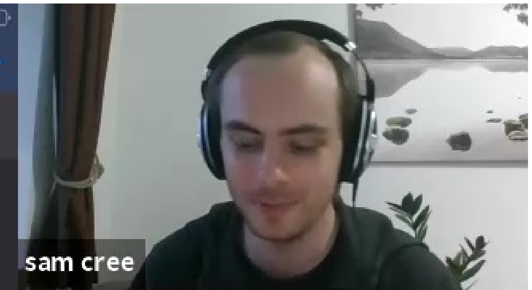

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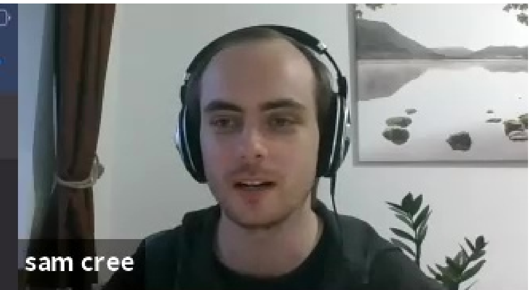
## Holographic codes

- First appeared in 2015 [PYHP15]
- Inspired by quantum gravity
- Good error threshold! [HCM$^+$20]
- Varying levels of protection (long/short-term memory?)
- Useful for practical quantum computing?





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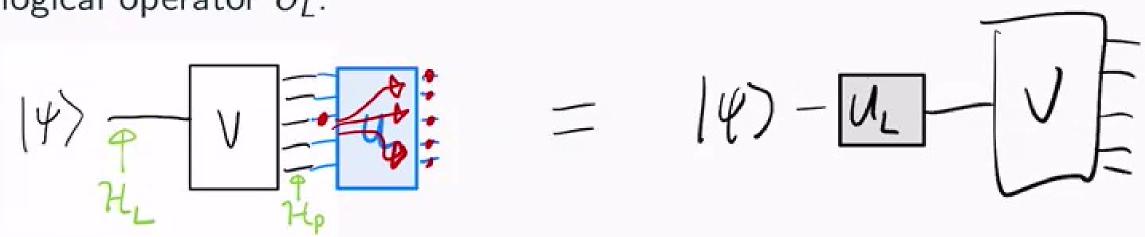
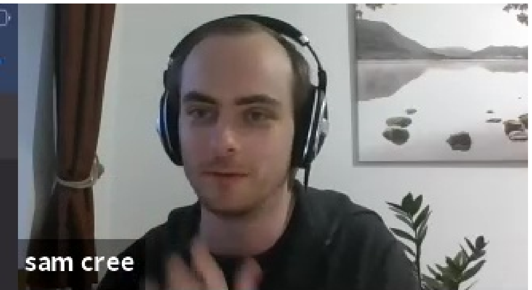
# Fault-tolerant logical gates

Suppose we want to apply $U_L$ to our encoded state, $V|\psi\rangle$.

Use a *codespace-preserving* (CSP) operator that *implements* the logical operator $U_L$:



# Fault-tolerant logical gates

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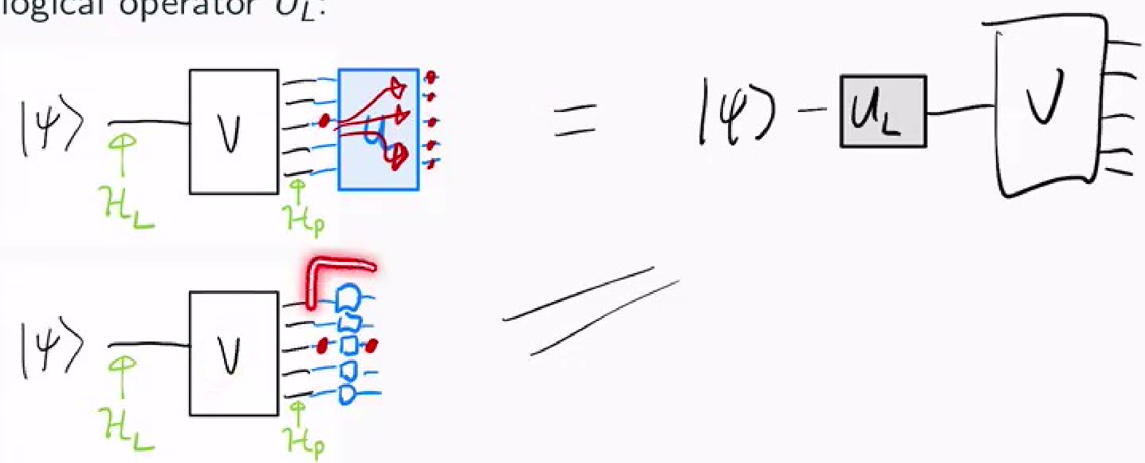


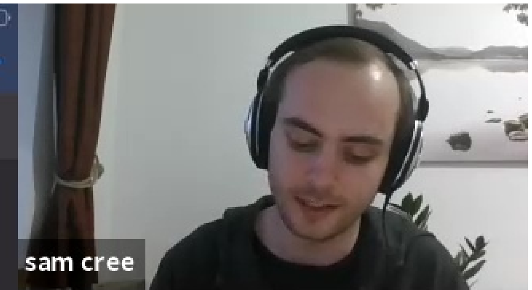

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## Fault-tolerant logical gates

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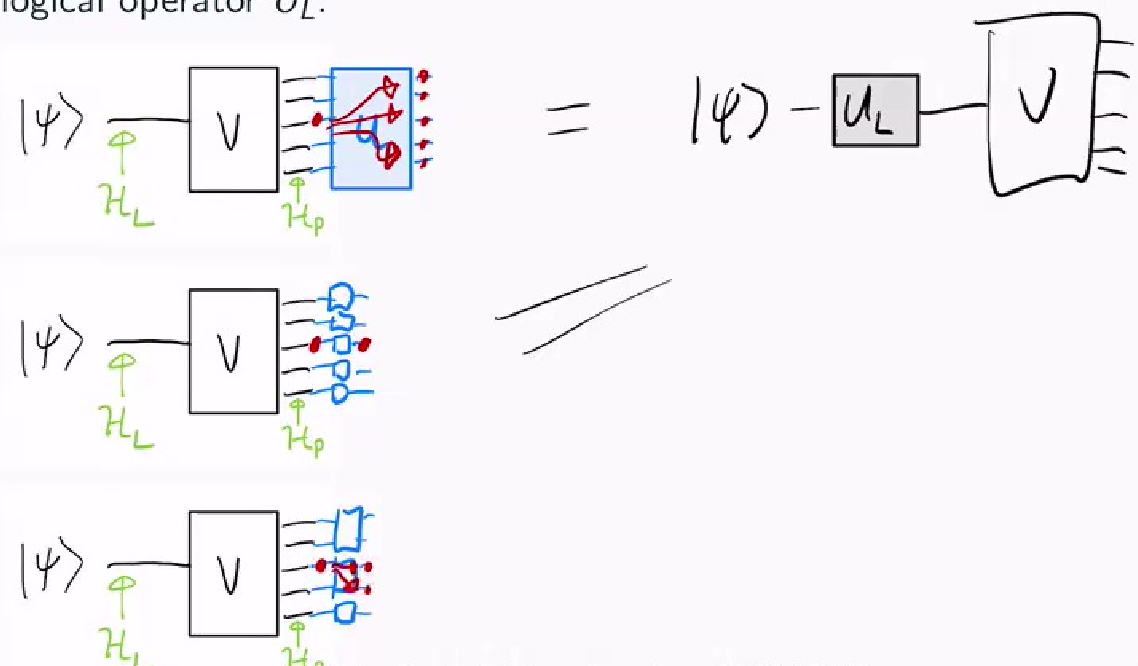
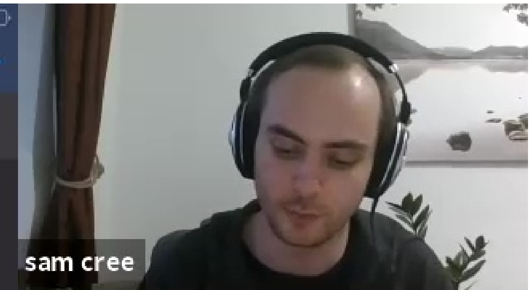


# Transversal gate sets

[EK09]: Transversal gate set cannot be universal for a code that can correct local errors.

**One strategy: code-switching.**

$$\text{Clifford Group} + \text{any non-Clifford gate} = \text{Universal}$$

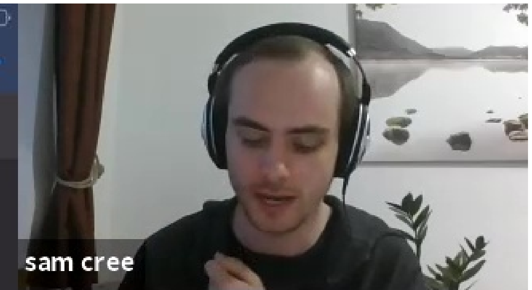
*Which codes can implement transversal non-Clifford gates?*

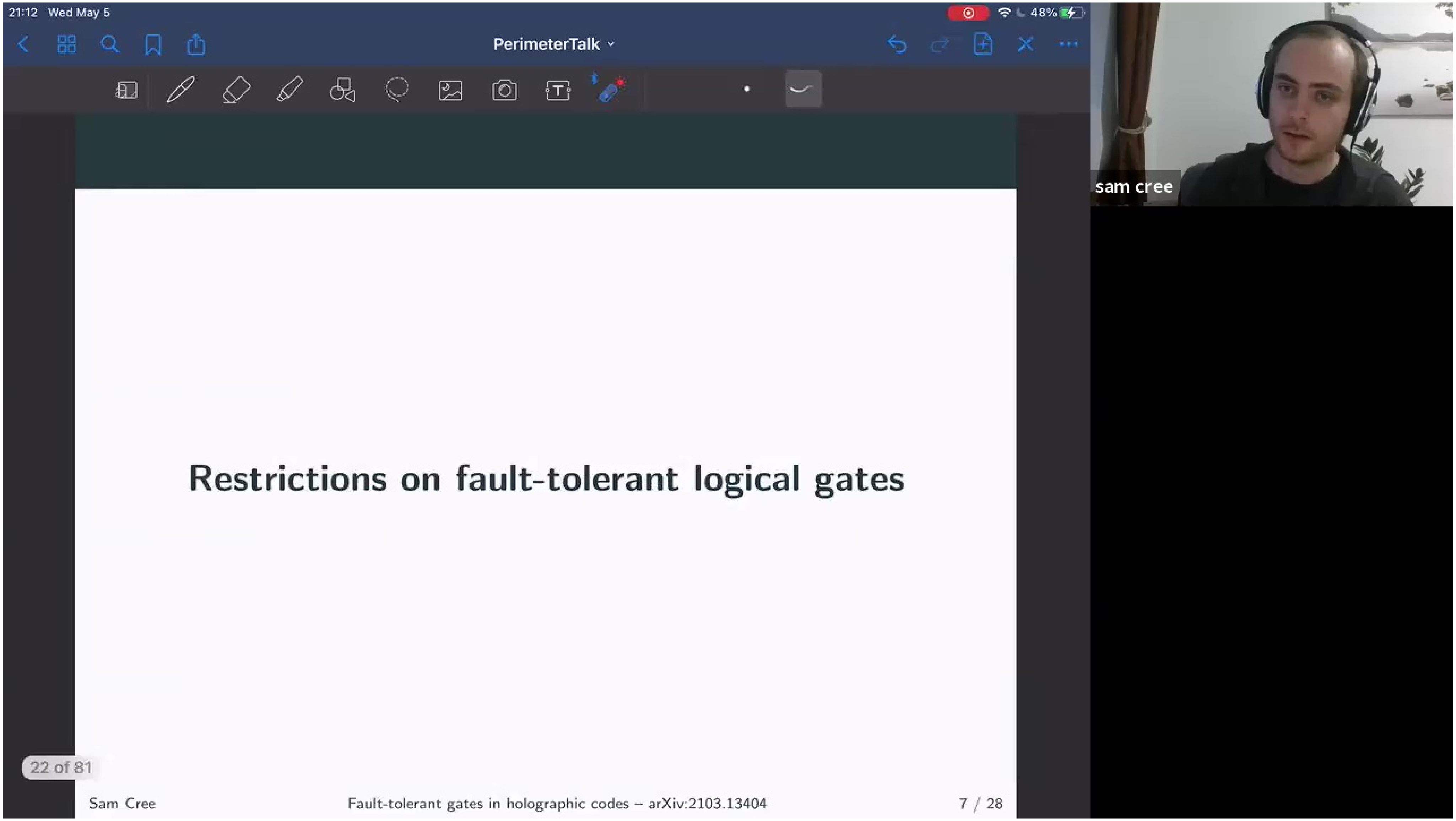
[BK13]: 2D topological stabilizer codes cannot.

[PY15]: 2D topological *subsystem* stabilizer codes cannot.

Our claim: *Holographic* subsystem stabilizer codes cannot.



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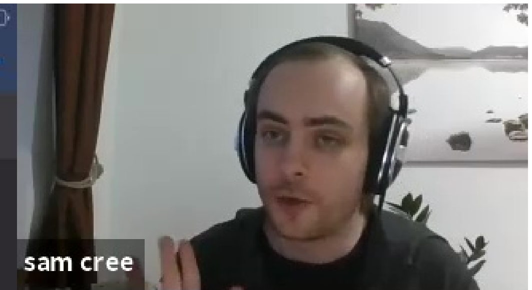


## Stabilizer codes + Clifford group

**Definition (Stabilizer Code)**
For an abelian subgroup of the physical Pauli group $\mathcal{S} \subset \mathcal{P}$ such that $-\mathbb{1} \notin \mathcal{S}$, a stabilizer code is one whose code subspace is the space of simultaneous $+1$ eigenvectors for all of $\mathcal{S}$.

In stabilizer codes, codespace-preserving Pauli operators implement logical Pauli operators [NC10].

**Definition (Clifford group)**
The *Clifford group* is the set of all unitary logical operators $U$ such that $\forall P \in \mathcal{P}_L, UPU^\dagger P^\dagger \in \mathcal{P}_L$.

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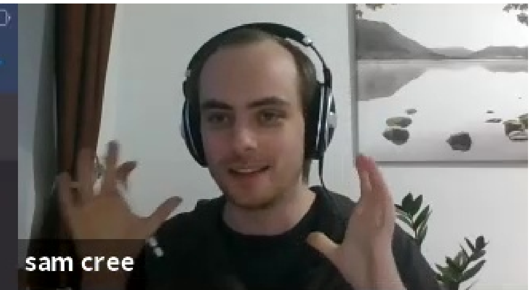
## Core Lemma (from Bravyi-Koenig)

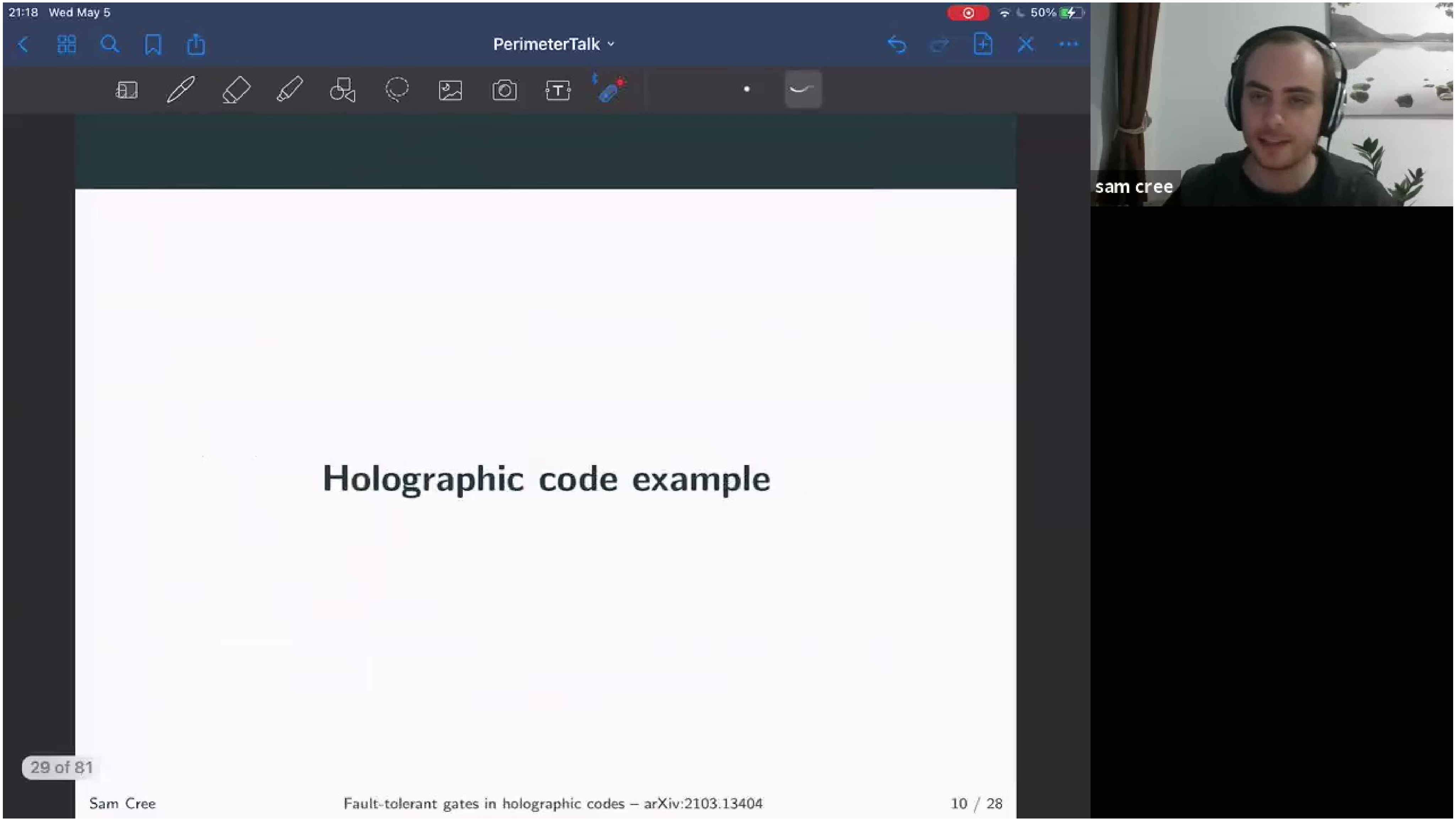
A region $R \subset \mathcal{H}_p$ is *correctable* if the encoded state can still be recovered after $R$ is erased.

**Core Lemma**
*For a stabilizer code, let $U$ be a unitary transversal CSP operator. If the physical space $\mathcal{H}_p$ can be partitioned into three correctable regions $R_1$, $R_2$ and $R_3$, then the logical unitary implemented by $U$ is in the Clifford group, $\mathcal{C}_2$.*

A region $R \subset \mathcal{H}_p$ is correctable iff every logical operator $A_L$ can be implemented by a CSP operator supported on its complement, $R^c$.



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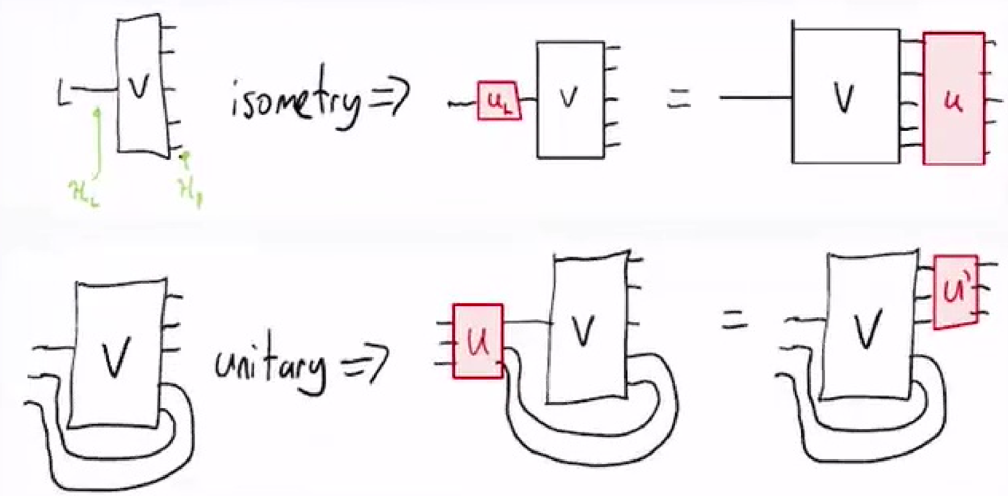
# HaPPY

We can "push" operators through isometries.

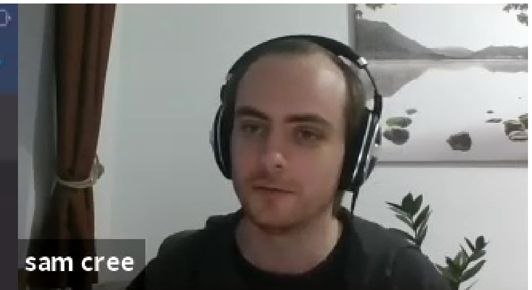



A *perfect tensor* is a unitary for any 3-3 grouping of legs.

Then any operation on $\leq 3$ legs can be "pushed" to the other side.

We can use this to construct codes!





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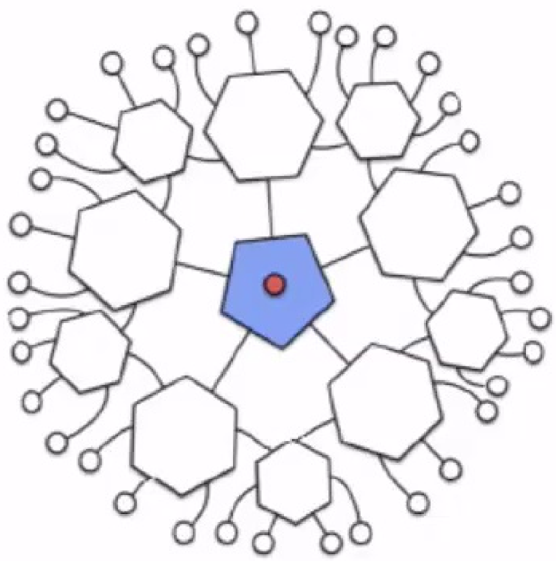

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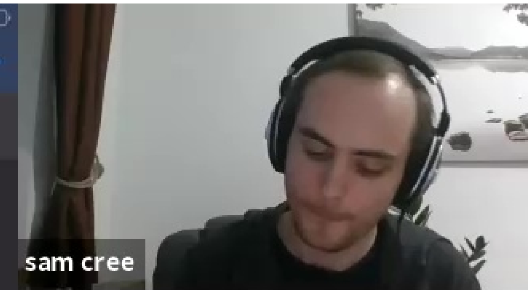

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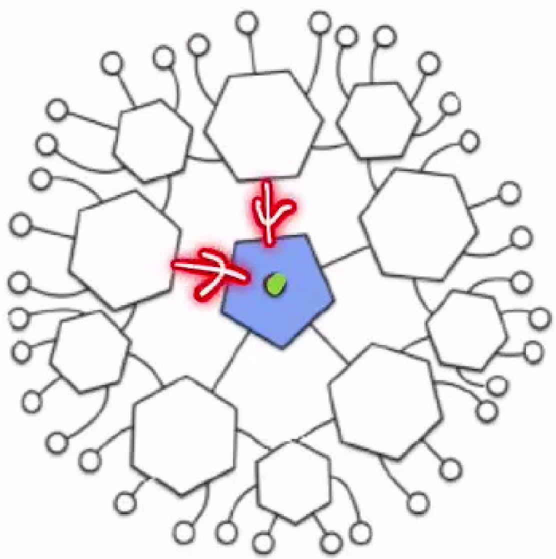



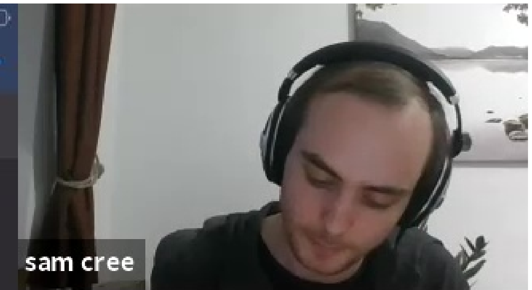

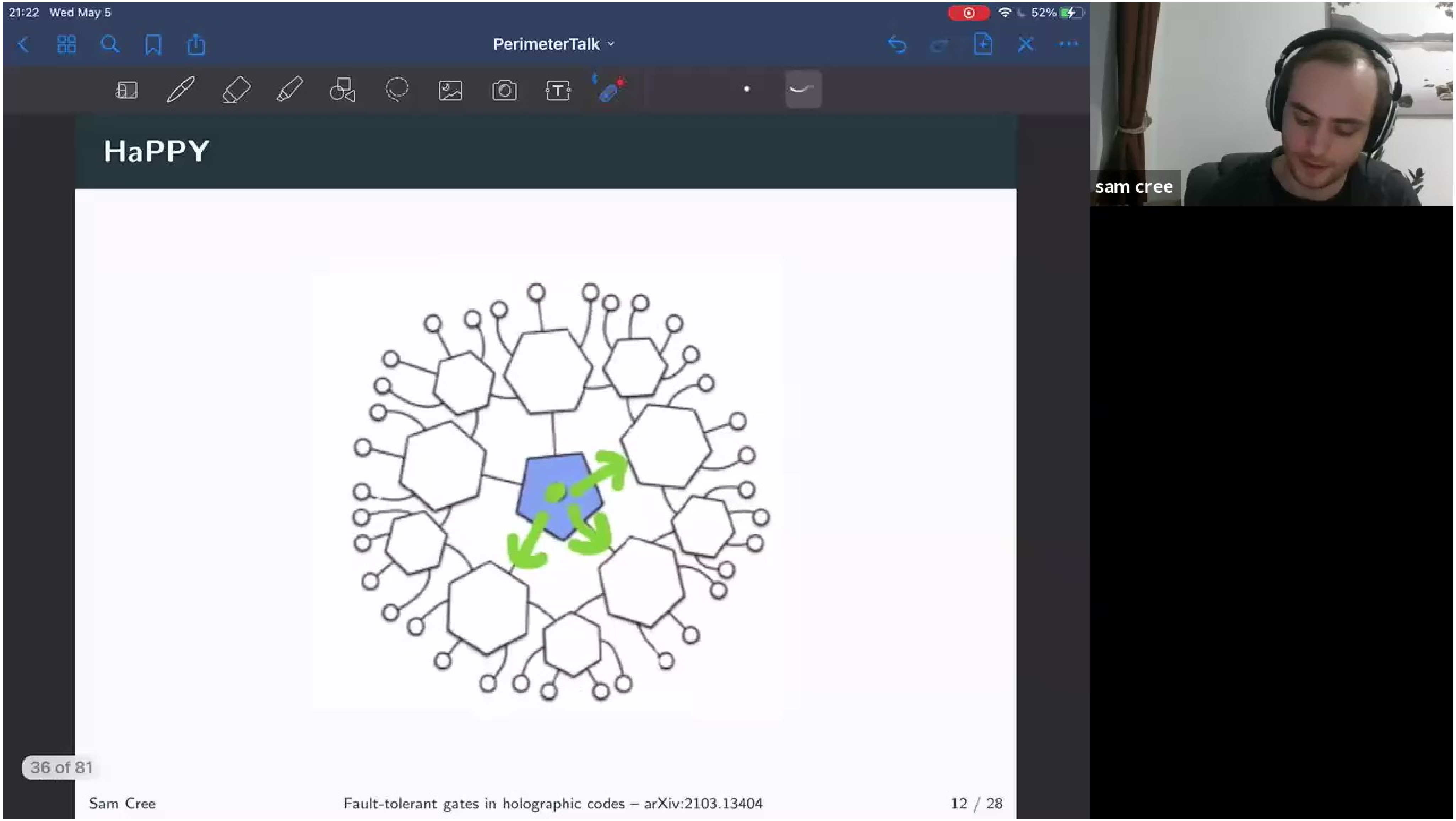
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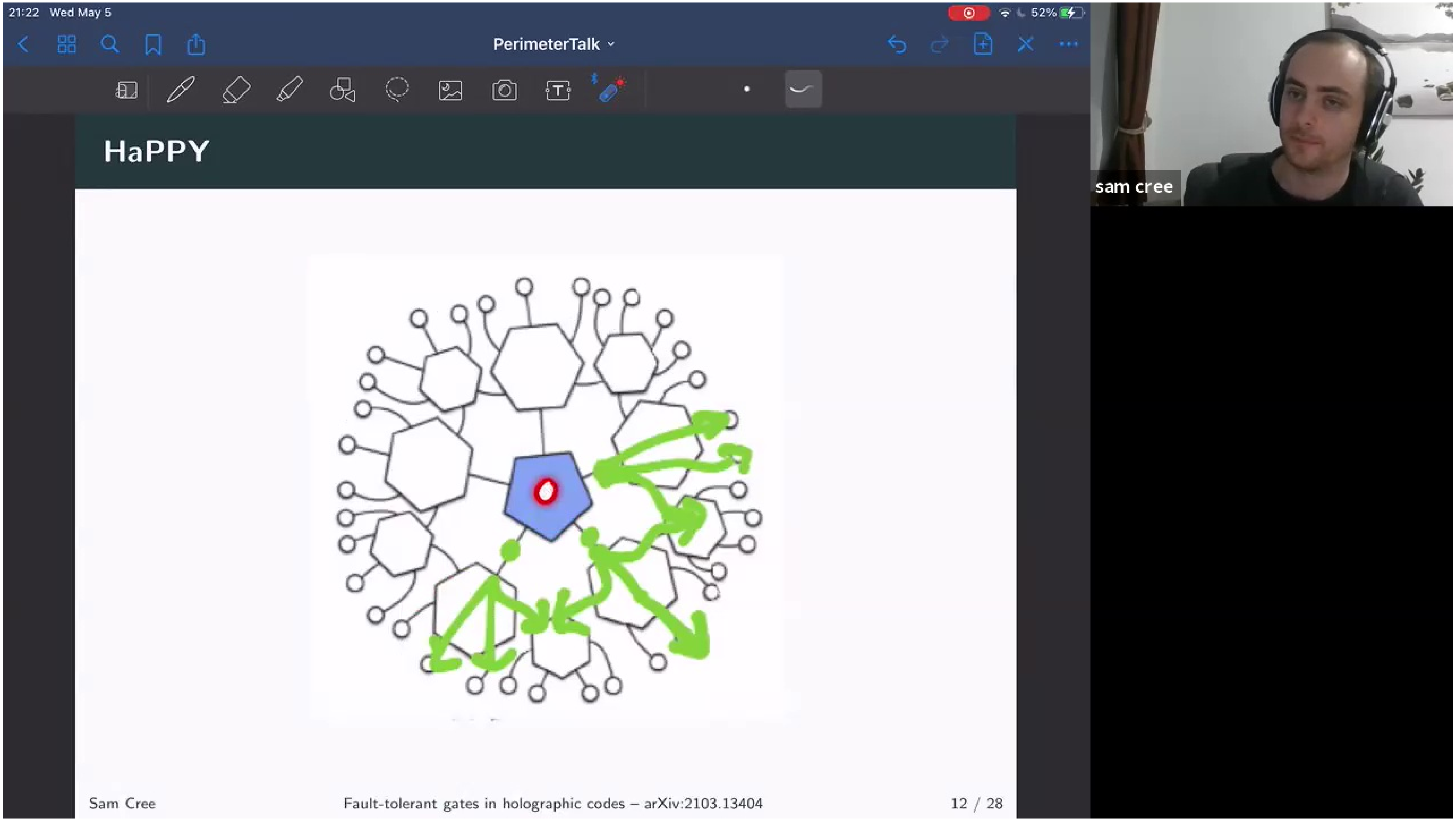
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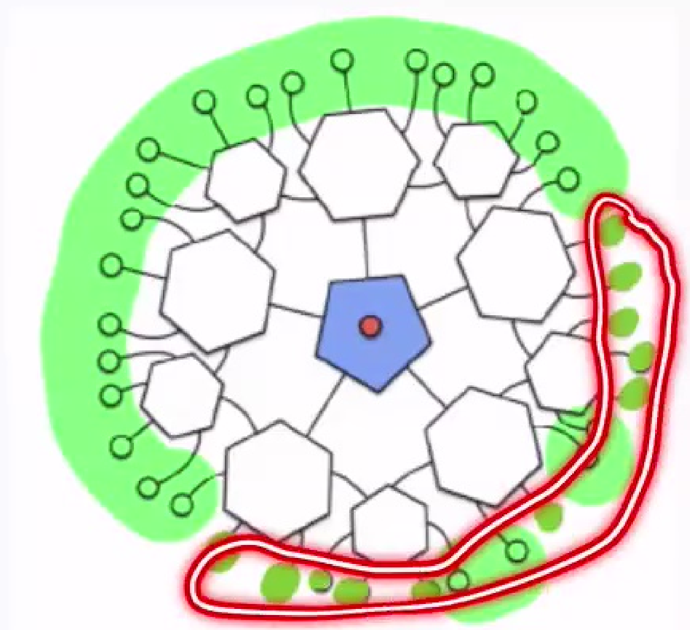
# HaPPY



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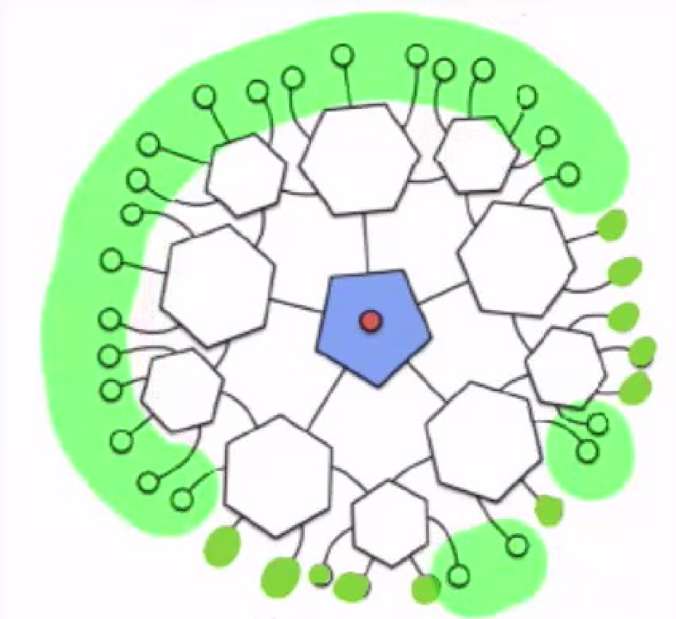

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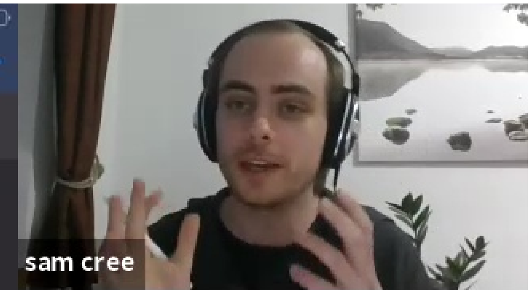

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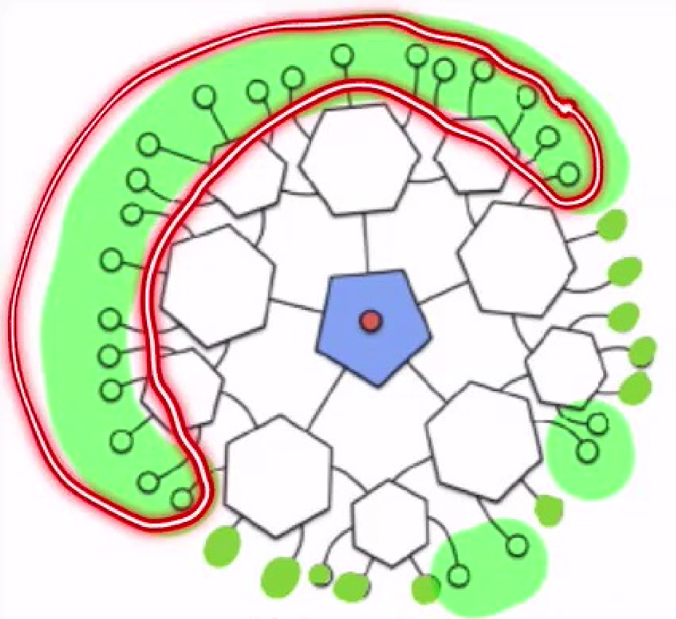
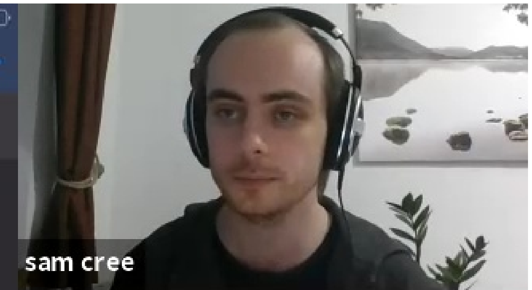
## HaPPY





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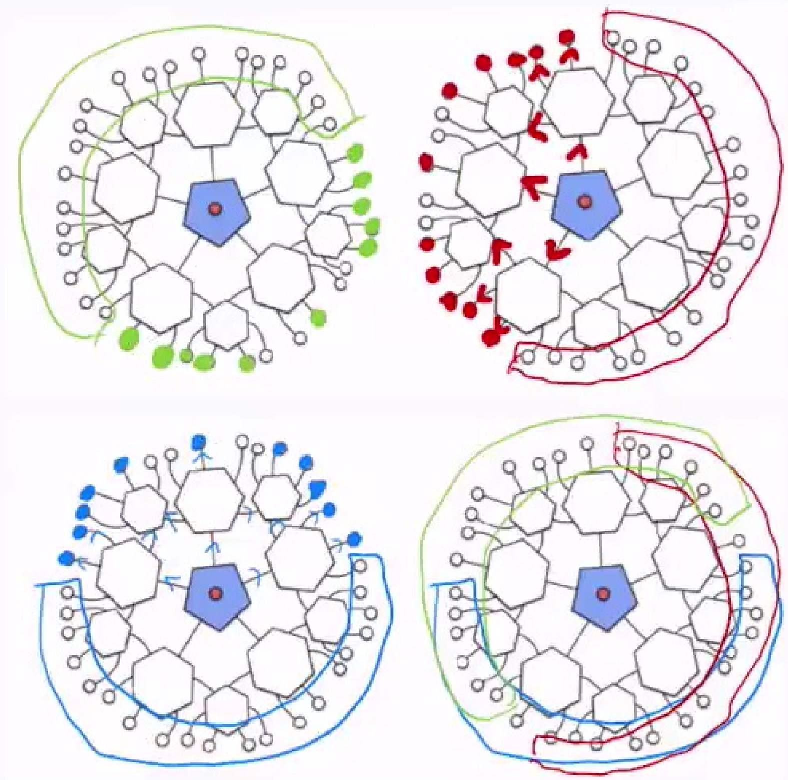
# Non-Cliffords in HaPPY?



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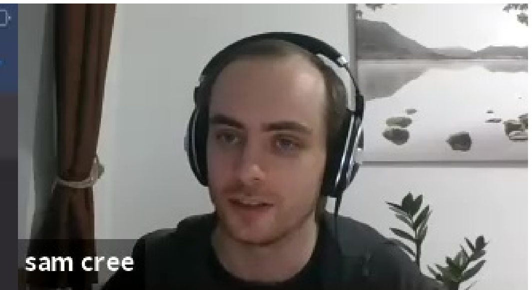

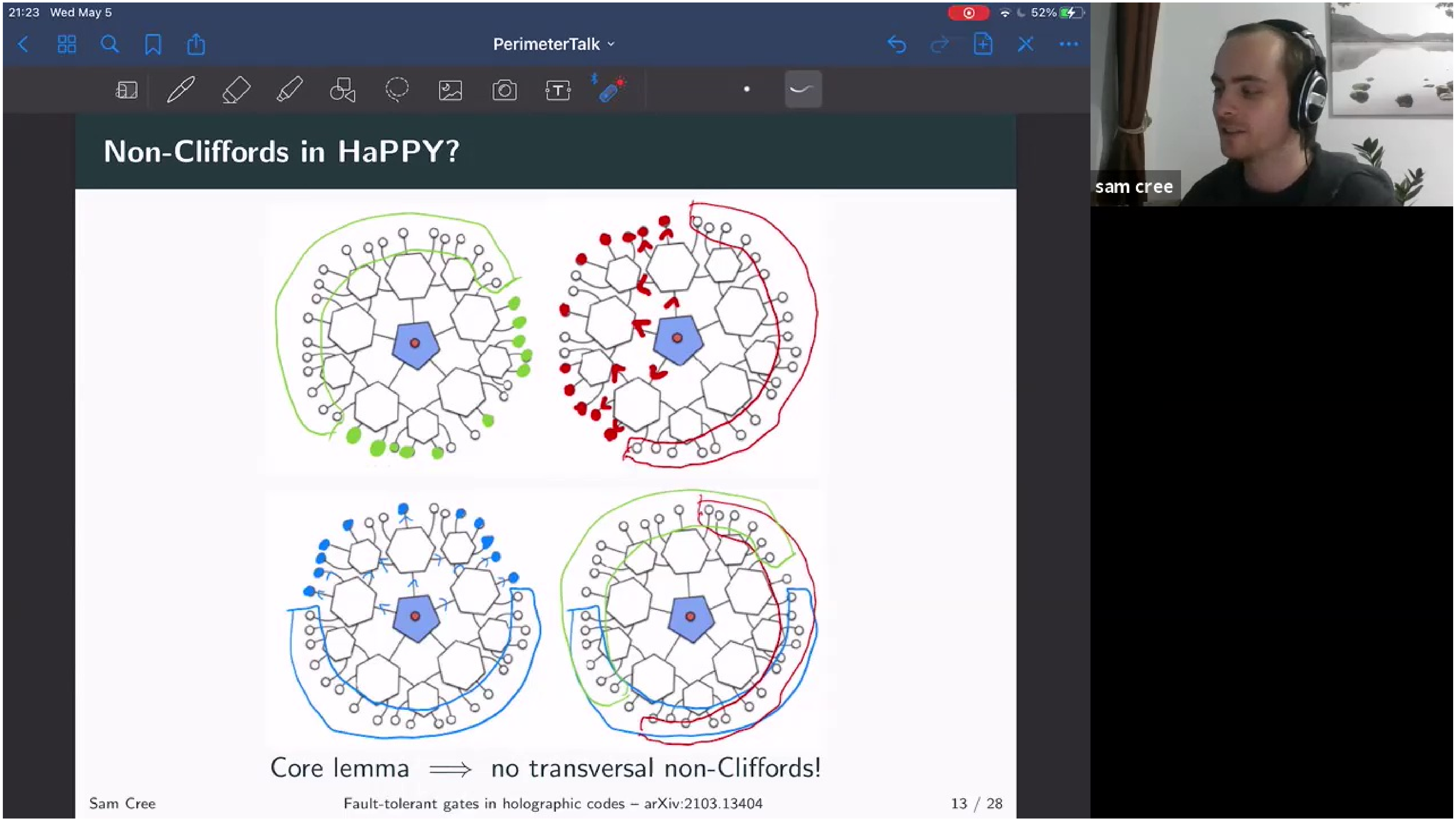
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Non-Cliffords in HaPPY?
Core lemma ⟹ no transversal non-Cliffords!
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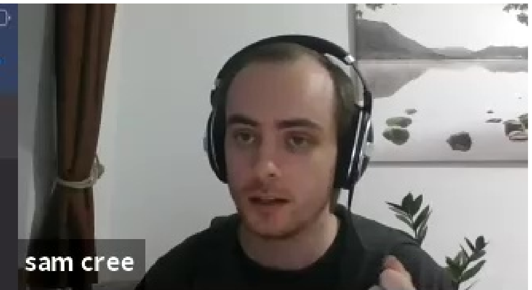
# Non-Cliffords in HaPPY?

Seems to be common among holographic codes. Is this a coincidence, or is there some underlying reason?





# Complementary recovery and fault-tolerant logical gates

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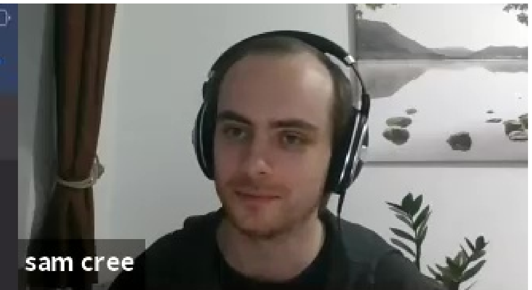

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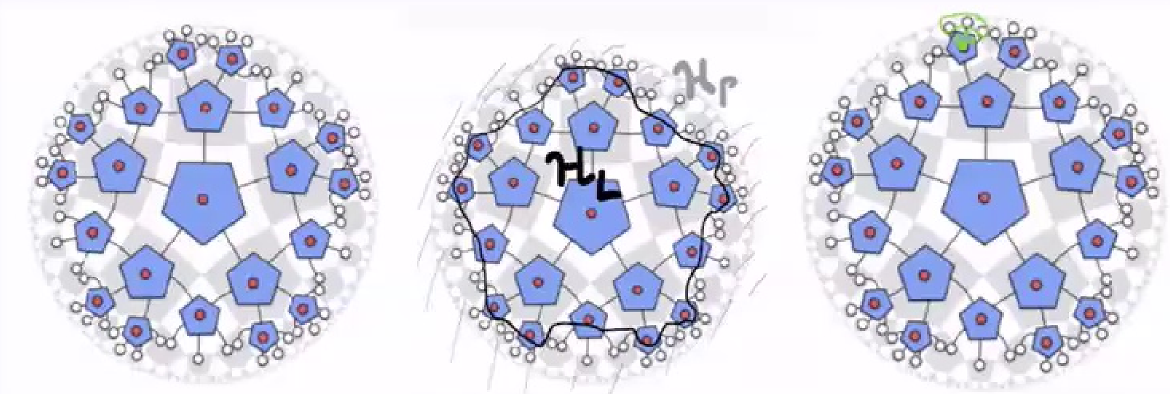
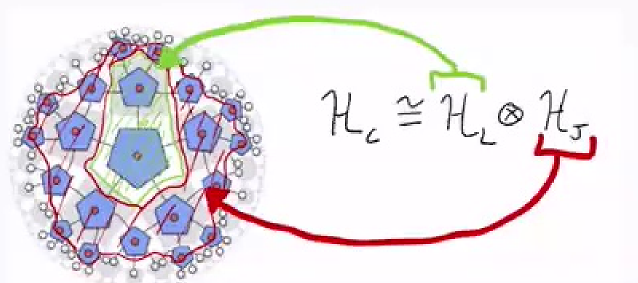
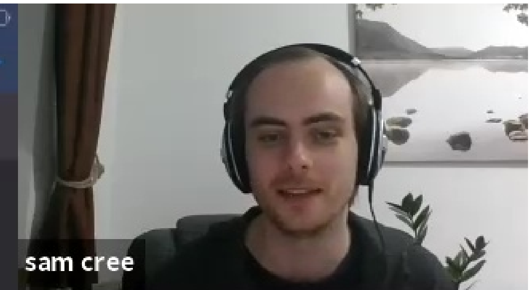
# Subsystem stabilizer codes

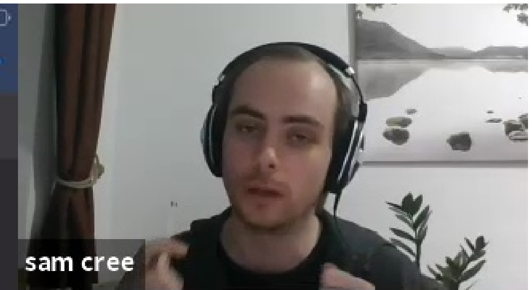


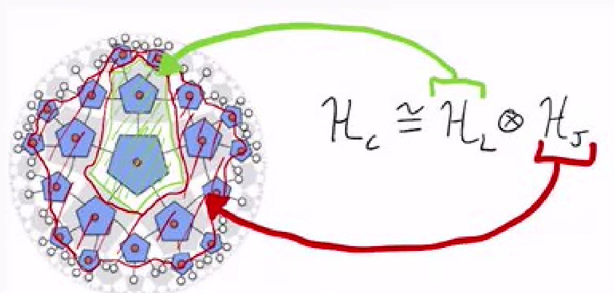

Things get a bit more complicated, but a variant of the Core Lemma still holds [PY15].








## Subsystem stabilizer codes




Things get a bit more complicated, but a variant of the Core Lemma still holds [PY15]:

**Lemma**
*For a subsystem stabilizer code, let $U$ be a unitary transversal CSP operator that implements a tensor product, $U_L \otimes U_J$. If the physical space $\mathcal{H}_p$ can be partitioned into three regions $R_1$, $R_2$ and $R_3$ that are correctable with respect to $\mathcal{H}_L$, then $U_L \in \mathcal{C}_2$.*

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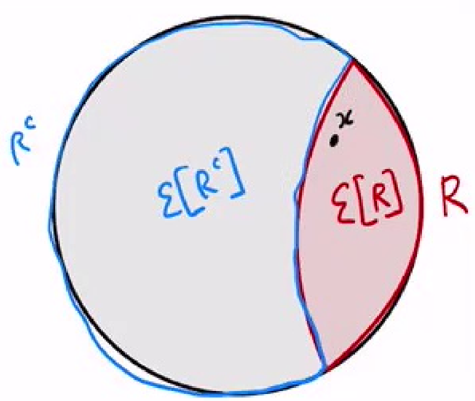
# Entanglement Wedge map

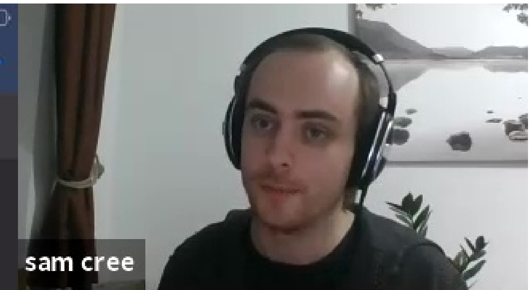
**Definition (Entanglement Wedge)**
The *entanglement wedge* $\mathcal{E}[R]$ of a region $R$ is defined as the union of all individual subsystems with respect to which $R^c$ is correctable.

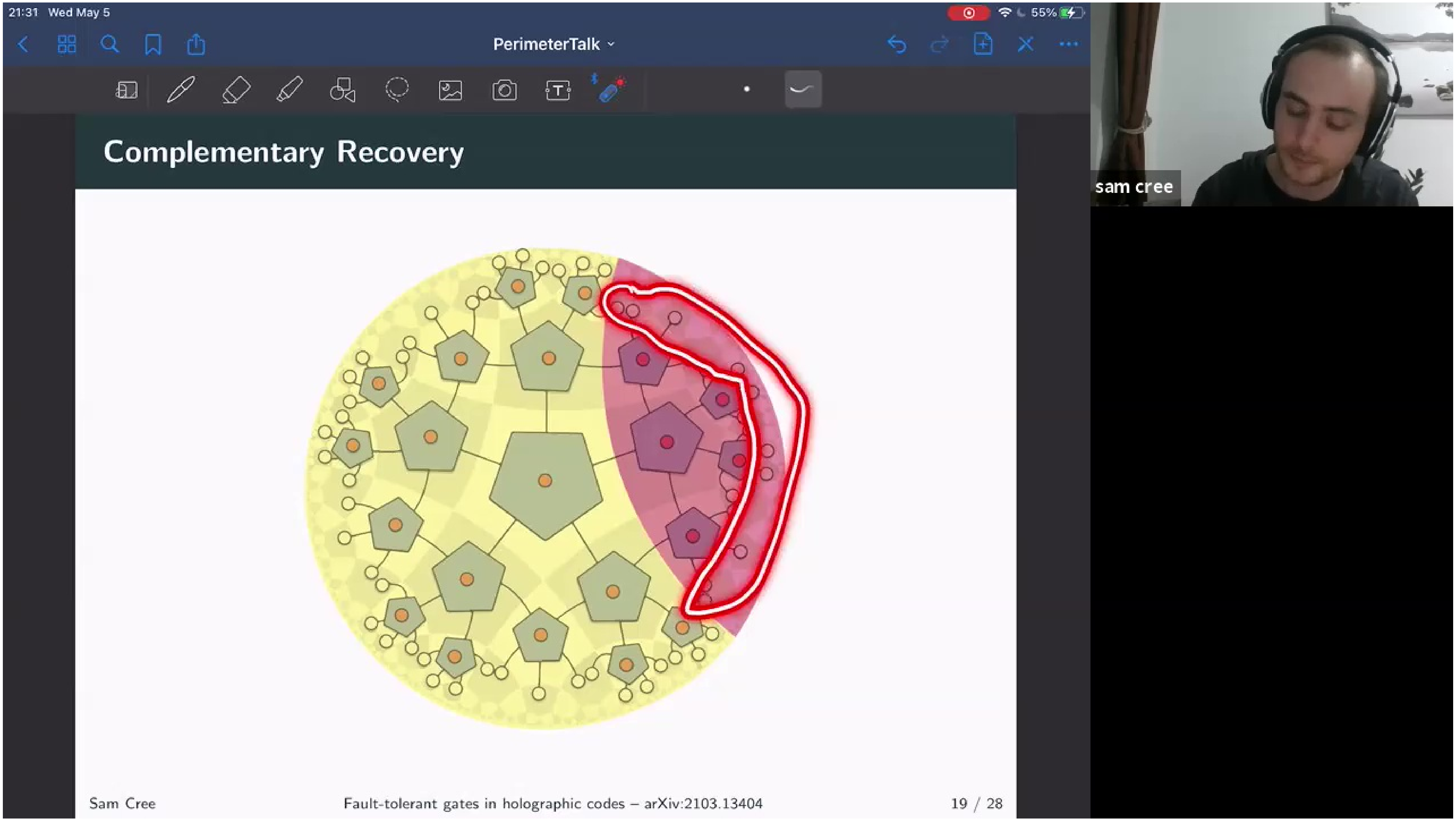



By definition, $x \in \mathcal{E}[R] \iff R^c$ correctable with respect to $x$.

$R$ obeys *complementary recovery* if $\mathcal{E}[R^c] = \mathcal{E}[R]^c$.



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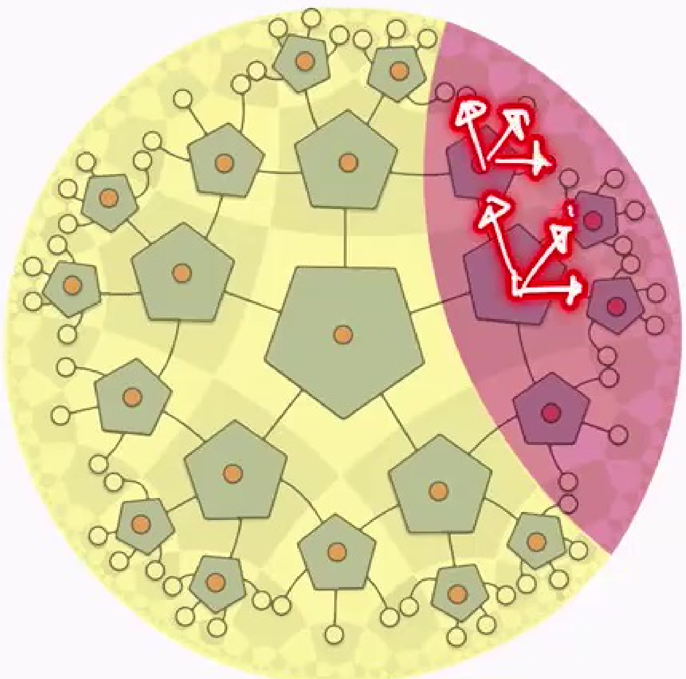
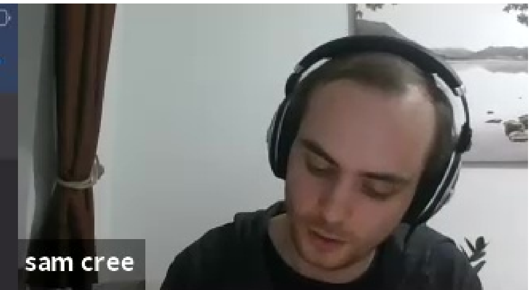
# Complementary Recovery





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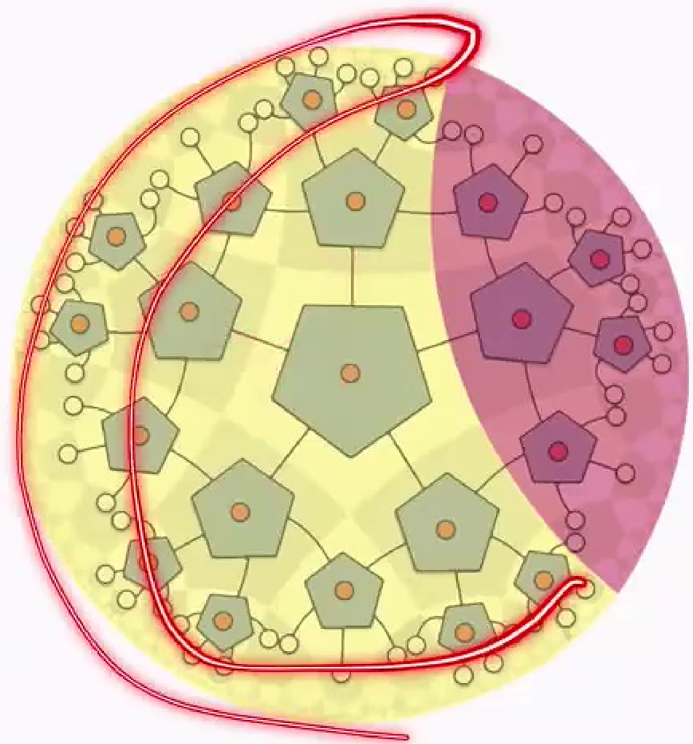
## Complementary Recovery



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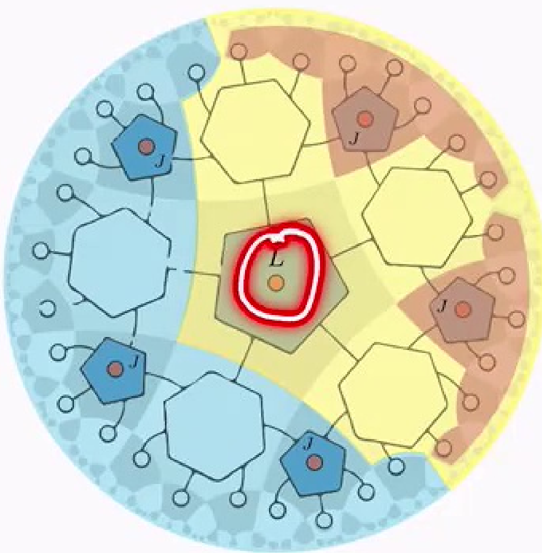
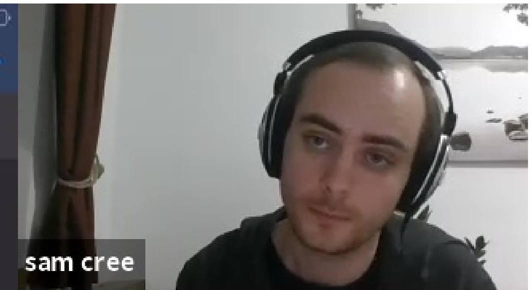
## Basic Argument - single logical qubit



1. Pick a qubit
2. Find smallest non-correctable region, $R$
3. Break into two smaller (correctable!) regions, $R_1$ and $R_2$
4. Complementary recovery $\Longrightarrow$ complement $R^c$ is correctable too





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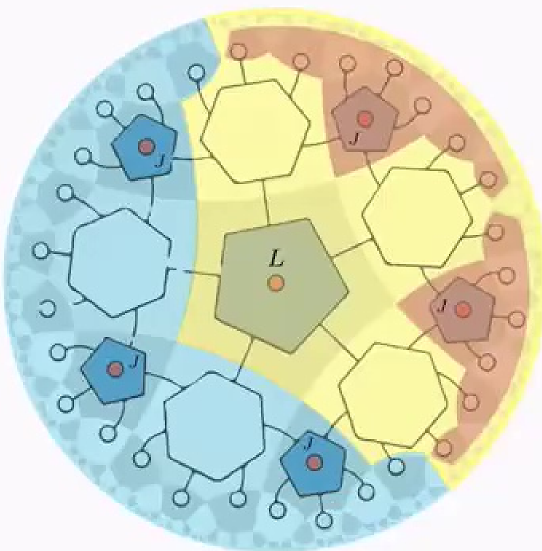
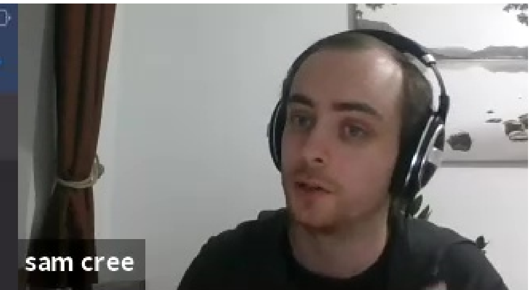
# Basic Argument - single logical qubit




1. Pick a qubit
2. Find smallest non-correctable region, $R$
3. Break into two smaller (correctable!) regions, $R_1$ and $R_2$
4. Complementary recovery $\implies$ complement $R^c$ is correctable too
5. Core lemma on $R_1$, $R_2$ and $R^c$ $\implies$ no transversal non-Cliffords!





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# What makes a code holographic?

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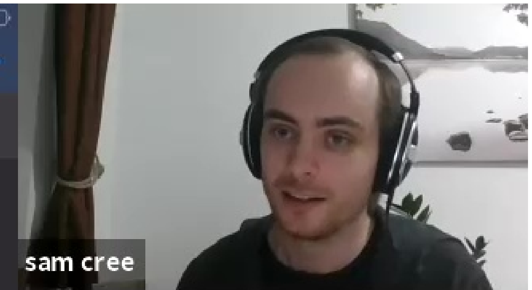

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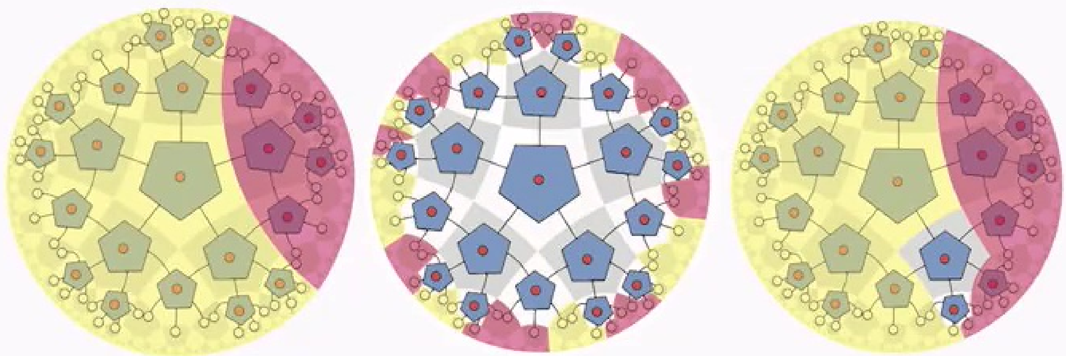
# Complementary Recovery



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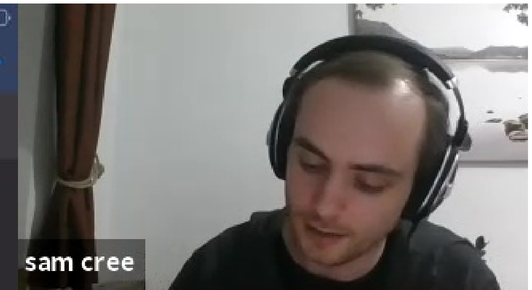

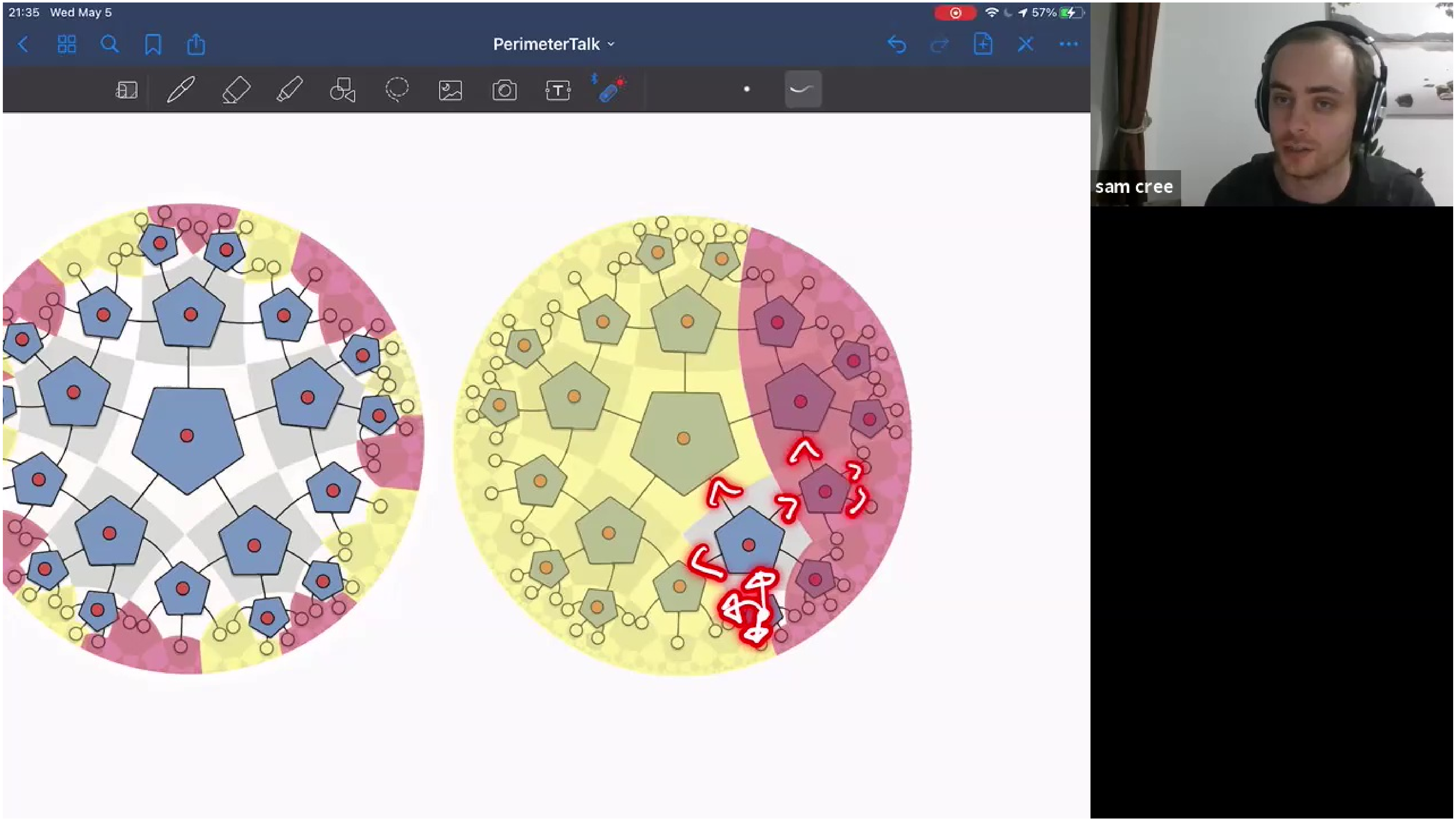
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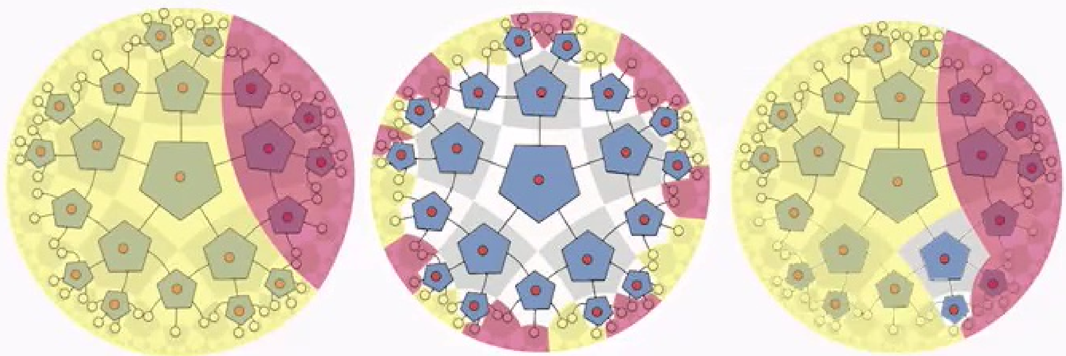
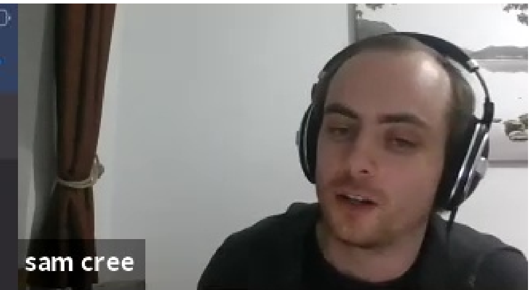
## Complementary Recovery



What if $R$ doesn't satisfy complementary recovery?





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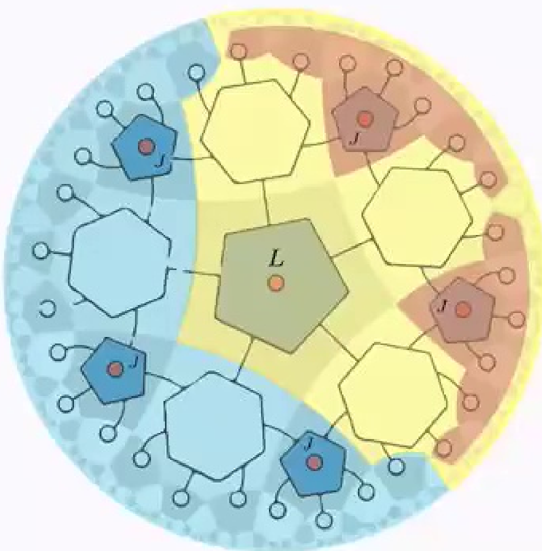
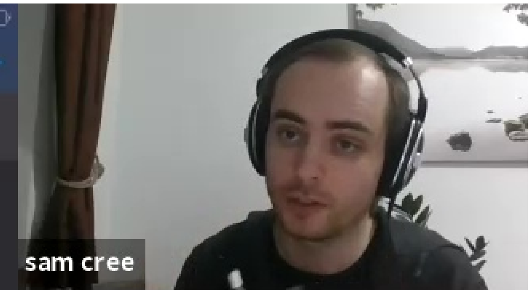
## Basic Argument - single logical qubit




1. Pick a qubit
2. Find smallest non-correctable region, $R$
3. Break into two smaller (correctable!) regions, $R_1$ and $R_2$
4. Complementary recovery $\implies$ complement $R^c$ is correctable too
5. Core lemma on $R_1$, $R_2$ and $R^c$ $\implies$ no transversal non-Cliffords!





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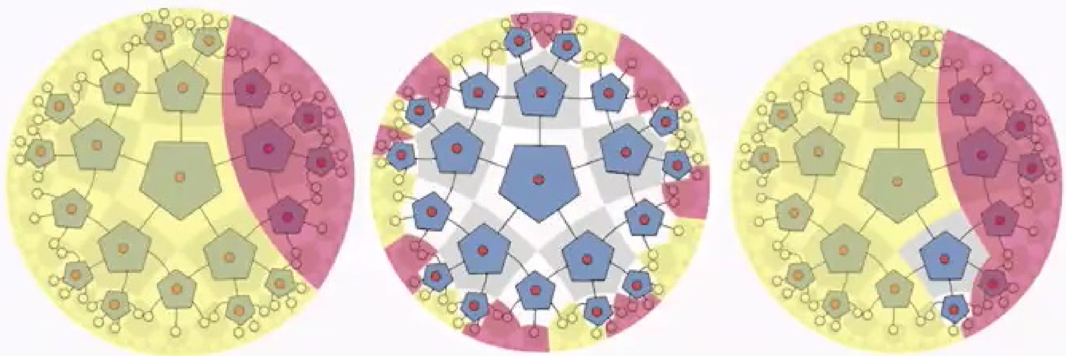
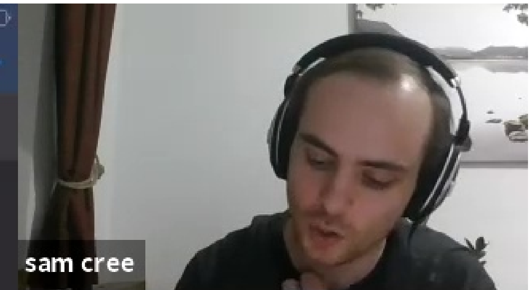


What if $R$ doesn't satisfy complementary recovery?

Just need one region that does (or almost does)

Sam Cree Fault-tolerant gates in holographic codes – arXiv:2103.13404 22 / 28



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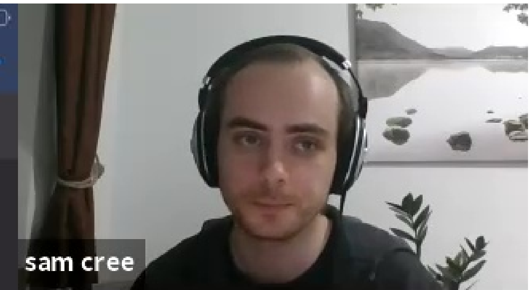
# Extensions

- Almost complementary recovery
- Small bulk regions
- Approximate encodings
- Locality-preserving gates
- Some entanglement wedge surface algebras with non-trivial centers



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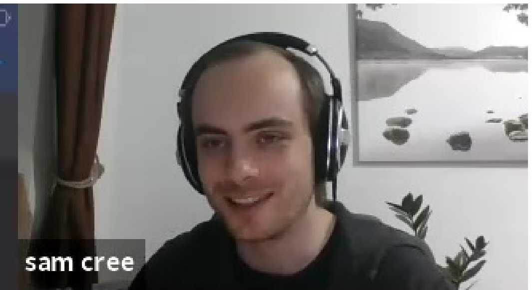
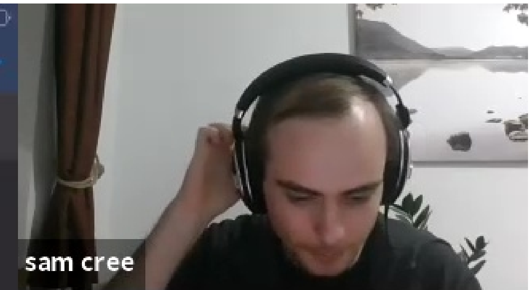
# What makes a code holographic?

Still unclear. But anything which even sometimes almost has complementary recovery should have a severely restricted transversal gate set (i.e. Cliffords only).








# Conclusion

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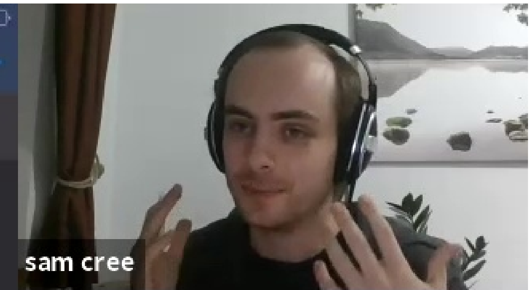
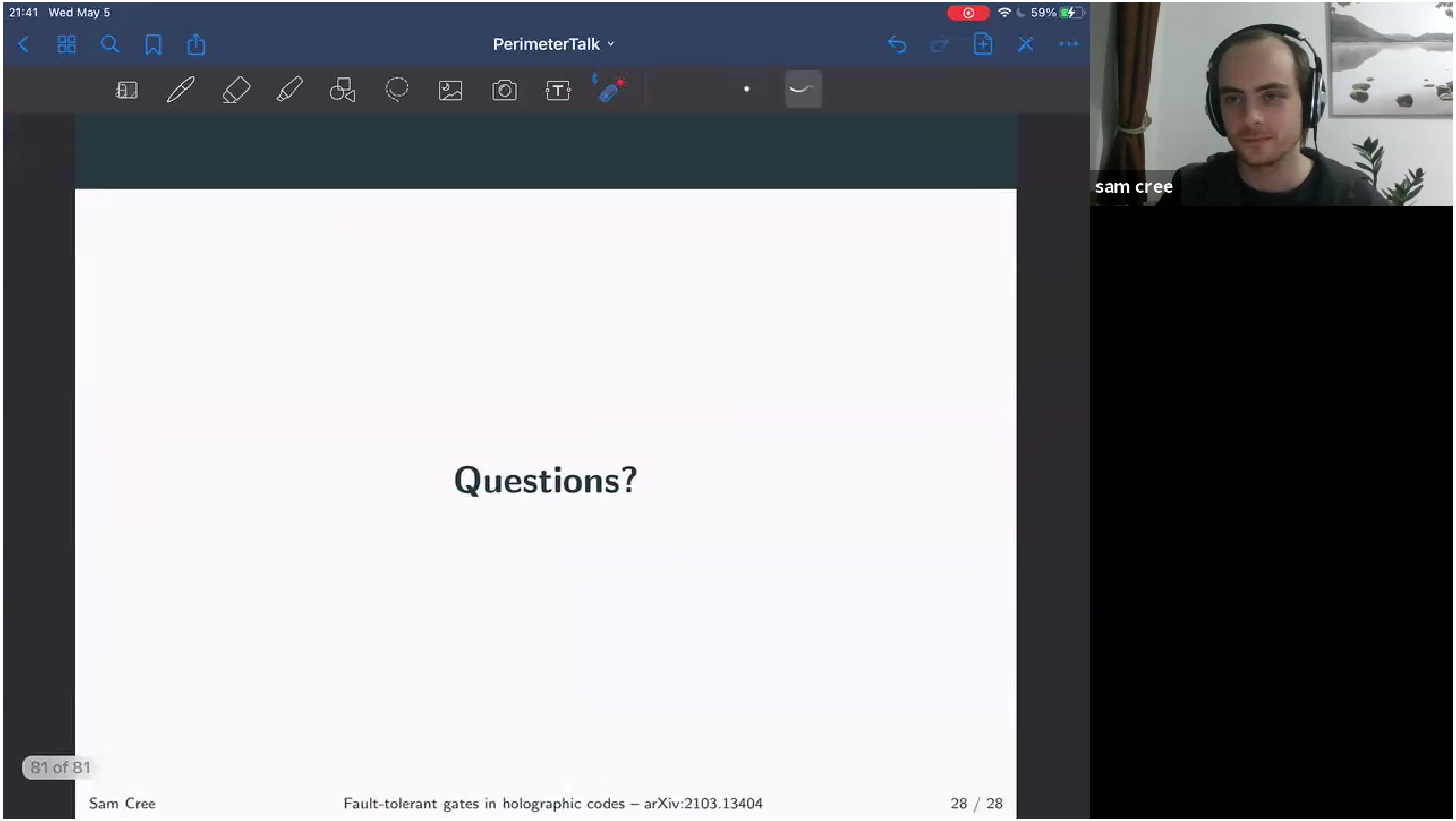
- If a fault-tolerant quantum computing strategy exists involving holographic codes, it either:
  - Implements non-Cliffords without using transversal/locality-preserving gates, or
  - Somehow circumvents restrictions of this work
- Holographic codes still need to be precisely defined
- Entanglement wedge map is defined for any subsystem code
- Some auxiliary results about correctability properties of regions in subsystem codes



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Questions?
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