Title: D-branes and Orbit Average

Speakers: Shota Komatsu

Series: Quantum Fields and Strings

Date: May 04, 2021 - 2:00 PM

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Abstract: I will explain how to compute correlation functions of two heavy operators and a light BPS single-trace operator at strong coupling using a dual description of D-branes absorbing a supergravity mode. Our approach is inspired by the large charge expansion of CFT and resolves some confusions in the literature on the holographic computation involving heavy operators. In particular, we point out two important effects which are often missed; the first one is an average over classical configurations of the heavy state, which physically amounts to projecting the state to an eigenstate of quantum numbers. The second one is the contribution from wave functions of the heavy state. Time permitting I will also comment on possible applications to states dual to black holes and fuzzballs.

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D-branes and Orbit Average

Shota Komatsu (CERN)

Perimeter Institute, String Seminar

Based on arXiv:2103.16580

with Peihe Yang, Junbao Wu, Yunfeng Jiang

Tianjin University CERN

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Plan

I. Introduction : Large charge , Large Δ in holography

II. Correlators of D-branes : Review & Puzzle

III. New Approach

IV. Discussion : BHs & Fuzzballs

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I. Introduction

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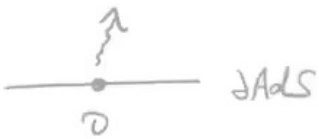
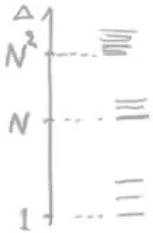
Operators and their duals in AdS/CFT



- AdS/CFT relates

  $\mathcal{O}|_{CFT} \longleftrightarrow$ Excitation in AdS



- $\Delta \sim \mathcal{O}(1) \to$ graviton, light matter

  $\Delta \sim \mathcal{O}(\lambda^{1/4}, \lambda^{1/2}) \to$ Stringy states

  $\Delta \sim \mathcal{O}(N) \to$ D-branes

  $\Delta \sim \mathcal{O}(N^2) \to$ deformed geometry $\begin{cases} \text{LLM, fuzzballs } (1/4 \text{ BPS}) \\ \text{BHs } \sim e^{N^2} \end{cases}$

- Operators with large Δ often come with other large quantum numbers ($U(1)$ (R-)charge J, Spin S)

  $\Delta \gtrsim J, S$

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Large Charge Expansion in CFT

Hellerman, Orlando, Reffert, Watanabe
Monin, Pirtskhalava, Rattazzi, Seibold
...

- (Semi-) universal prediction of CFT data @ large charge

  $\Delta \sim J^{\frac{d}{d-1}}$ for CFT_d

- CFT on $R_t \times S^{d-1}$ with radius R

  $\rightsquigarrow$ Energy $= \Delta/R$, charge $\sim J$

- Assume that, in $R \to \infty$ limit, the large charge state becomes a state on $R^{1,d-1}$ with finite energy & charge densities

  $\rightsquigarrow$ Energy $\overset{R\to\infty}{\sim} \varepsilon \times R^{d-1}$, charge $\overset{R\to\infty}{\sim} j \times R^{d-1}$ $\longrightarrow$ $\Delta \sim R^d \sim J^{d/d-1}$

- $1/J$ corrections can be computed by EFT methods

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Large Charge in $\underline{S}$ CFT

- $\Delta \sim J^{d/d-1}$ is incorrect for superconformal theories.

  BPS states : $\Delta \propto \# J \ll J^{d/d-1}$

- BPS state in SCFT $\xrightarrow{R \to \infty}$ a state on $R^{1,d-1}$ with zero energy

  ← a state on a vacuum manifold

- For $\mathcal{N}=2$ SCFT in 4d, [Hellerman, Maeda]

  (large charge expansion) ← (Coulomb-branch EFT)

  $\mathrm{Im}[\mathcal{F}''] \sim \int d^4x \, \partial_\mu A J^\mu \bar{A}$ + higher der.

- For rank 1 $\mathcal{N}=2$ SCFT ($\sim$ 1-dimensional CB moduli)

  $\langle \mathcal{O}_J \, \bar{\mathcal{O}}_J \rangle \sim J! \, (\#)^{2J} J^{\alpha}$ ⇝ Agreement with localization

  $(\alpha \sim a_{UV} - a_{IR})$

  [Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu]

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Large Charge S CFT with Higher Rank

- Analysis is more complicated for theories w/. higher rank

EFT
1. More possibilities of symmetry breaking patterns.
   $SU(2) \to U(1) \times U(1)$ vs. $SU(3) \to (U(1))^3$, $U(1) \times SU(2)$
2. Classification of SUSic higher derivative terms (F-terms)

3. Degeneracy of BPS ops.
   $SU(2)$: $\mathrm{Tr}[Z^{2J}] = (\mathrm{Tr}[Z^2])^J$, $SU(N)$: $\mathrm{Tr}[Z^{2J}] \neq (\mathrm{Tr}[Z^2])^J$

4. "Gram-Schmidting" of localization results is more complicated for $SU(N)$

- 1 & 3 are related.

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Toy Model : Complex Matrix Model.

$Z, \bar{Z}$: $N \times N$ complex matrices.

$$\boxed{\langle \mathcal{O}\, \bar{\mathcal{O}} \rangle = \int dZ\, d\bar{Z}\; \mathcal{O}\, \bar{\mathcal{O}}\; e^{-N\, \mathrm{Tr}[Z\bar{Z}]}}$$

- Using the result by Ginibre, this reduces to eigenvalues.

$$\int \prod_j d^2 z_j \prod_{j<k} |z_j - z_k|^2 \times \mathcal{O}(z) \times \bar{\mathcal{O}}(\bar{z}) \times \exp\left[-N \sum_k |z_i|^2\right]$$

- Take "single trace op" $\mathcal{O} = \mathrm{Tr}[Z^J]$

$$\rightsquigarrow \quad \cdots \sum_k z_k^J \cdots \quad \underset{\text{permutation inv.}}{=} \quad \cdots N \times z_1^J \cdots$$

$\rightsquigarrow$ source term only for z_1 $\rightsquigarrow$ $U(N) \to U(1) \times U(N-1)$

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Toy Model : Complex Matrix Model.

$$\int \prod_j d^2 z_j \prod_{j<k} |z_j - z_k|^2 \times \mathcal{O}(z) \times \bar{\mathcal{O}}(\bar{z}) \times \exp\left[-N \sum_k |z_i|^2\right]$$

- "Double-trace operator" $\mathcal{O} = \mathrm{Tr}[Z^{J_1}]\,\mathrm{Tr}[Z^{J_2}]$

$$\rightsquigarrow \sum_{n,m} (z_n)^{J_1} (z_m)^{J_2} = \sum_{n \neq m} \underbrace{(z_n)^{J_1} (z_m)^{J_2}}_{U(N) \to U(1) \times U(1) \times U(N-2)} + \sum_n \underbrace{(z_n)^{J_1+J_2}}_{U(N) \to U(1) \times U(N-1)}$$

$\rightsquigarrow$ Mixture of sym breaking patterns

- Correct op. : $\mathrm{Tr}[Z^{J_1}]\,\mathrm{Tr}[Z^{J_2}] - \mathrm{Tr}[Z^{J_1+J_2}]$

- Can be generalized to other sym breaking patterns (sth similar should work for general $\mathcal{N}=2$ SCFT)

But general expressions are rather complicated....

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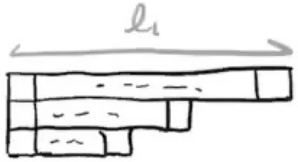
Good Basis @ Large N

- @ large N, there exist a nice basis of $\mathcal{O}$'s.

  Schur polynomial basis : $\chi_R[Z] \equiv \sum_{\sigma \in S_J} \chi_R(\sigma)\, Z^{a_1}{}_{b_{\sigma_1}} \cdots Z^{a_J}{}_{b_{\sigma_J}}$

  Young tableau

- 

  l_k : magnitude of source term for k-th eigenvalue.

  $\rightsquigarrow U(1) \times U(1) \times U(1) \times U(N-3)$

- 

  $\rightsquigarrow \chi_R[Z] = \mathrm{Tr}[Z^J] + (1/N \text{ suppressed terms})$

- They also have nice holographic descriptions.

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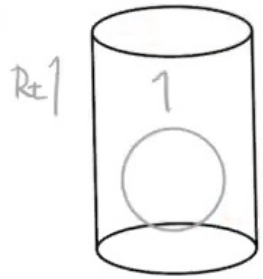
Holography for Schur polynomials

- In $\mathcal{N}=4$ SYM, $\chi_{\square\cdots\square}(Z)$ $(U(1)\times U(N-1))$ is dual to.

dual giant graviton



- spherical D3 brane $R_t \times S^3 \subset AdS_5$
- size ~ charge

- In the large charge limit, dual giant graviton approaches AdS bdy and can be approximated by a flat D3-brane

$\mathcal{N}=4$ SYM in CB

⇝ geometrical realization of large charge expansion.

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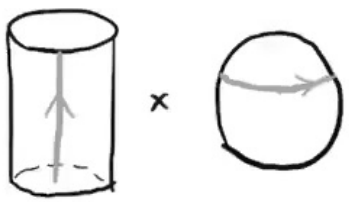
Giant Gravitons.

- Interestingly, AdS / CFT provides an EFT description also in the "opposite" limit

$$\chi_{\square}(Z) \sim \epsilon^{a_1 \cdots a_J c_1 \cdots c_{N-J}} \, \epsilon_{b_1 \cdots b_J c_1 \cdots c_{N-J}} \, Z_{a_1}{}^{b_1} \cdots Z_{a_J}{}^{b_J}$$

sub-determinant operator (expansion of $\det[I+Z]$)

- 

D3-brane : pointlike in AdS_5
extended in S^5
(radius $\cos^2\theta_0 = J/N$)

- Would be nice to develop field theory understanding ...

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II. Correlators of D-branes.

Review & Puzzles

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Three-point functions of Giant Gravitons

- Consider $\langle \mathcal{D}_J(x_1, Y_1)\, \mathcal{D}_{J+k}(x_2, Y_2)\, \mathcal{D}_L(x_3, Y_3) \rangle$ in $\mathcal{N}=4$ SYM

  $\mathcal{D}(N)$ $\mathcal{D}(1)$

  $\mathcal{D}_J(x, Y) \equiv \chi_{\square\square\square\,J}(Y\cdot\Phi)$, $\mathcal{D}_L(x, Y) \equiv \mathrm{Tr}[(Y\cdot\Phi)^L]$

- $Y\cdot\Phi \equiv \sum_{I=1}^{6} Y_I \Phi_I$, $Y_I Y^I = 0$ $\rightsquigarrow$ All ops are ½ BPS.

- Thanks to SUSY, $\langle \mathcal{D}_J\, \mathcal{D}_{J+k}\, \mathcal{D}_L \rangle$ is independent of g_{YM}^2

  [Baggio, de Boer, Papadodimas]

- We can directly compare field theory & holography.

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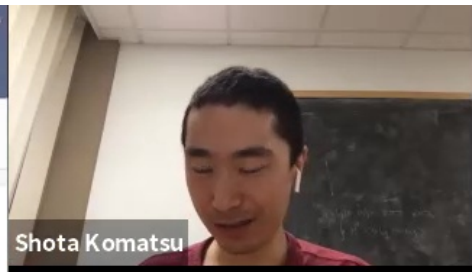

Prediction from field theory [SK, Yang, Wu, Jiang]

$$C_{D_J D_{J+k} \mathcal{O}_L} = -\frac{\sqrt{L}}{2}\left(i^{L-k} + (-i)^{L-k}\right) \times \frac{\Gamma\left(\frac{L+k}{2}\right)\cos^2\theta_0 \sin^2\theta_0}{\Gamma(1+k)\,\Gamma\left(1+\frac{L-k}{2}\right)}\, {}_2F_1\left(\frac{2+k-L}{2}, \frac{2+k+L}{2}, 1+k; \sin^2\theta_0\right)$$

$(\cos^2\theta_0 \equiv J/N)$

Structure const.

- Complicated result owing to the combinatorics of Wick contractions. (Nobody computed it before...)

- Our goal is to reproduce it from holography.

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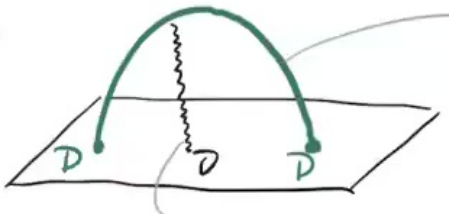
Holographic Description

- D_J : giant graviton D3-brane

  $\mathcal{O}_L$: supergravity mode with KK momentum L

- 

  classical configuration of D-brane (geodesic)

  quantum SUGRA fluctuation sourced by $\mathcal{O}$

- We need to compute how the D-brane gets perturbed by a SUGRA mode.

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Approaches in the literature

[Bissi, Kristjansen, Young, Zarbos 2011]

$$\langle D_J \; D_{J+k} \; \mathcal{O}_L \rangle$$

- D_J : dual to D-brane in AdS $\qquad S_{DBI} \sim N \int d^4\sigma \sqrt{\det\left[g_{\mu\nu} \, \partial_a X^\mu \partial_b X^\nu \right]}$

  $\rightsquigarrow$ world volume is described by DBI + WZ

- $\mathcal{O}_L$ sources quantum fluctuations of metric etc.

$$\delta g_{\mu\nu} \sim \left(\frac{2z}{z^2 + (x - x_3)^2} \right)^L \quad : \text{Bulk-to-bdy propagator}$$

$\mathcal{O}_L(x_3) \rightsquigarrow$

$$\delta g_{AB} \sim Y_L(\Omega) \quad : \text{Spherical harmonics}$$

- Perturb $S_{DBI} + S_{WZ}$ by δg $\rightsquigarrow$ $\boxed{\delta S_{DBI} + \delta S_{WZ} \sim \langle D_J \; D_{J+k} \; \mathcal{O}_L \rangle}$

- Justification : $\underbrace{\int \mathcal{D}X}_{\text{path integral of D-brane}} \; e^{-S_{DBI} + S_{WZ}} \xrightarrow{\delta} \int \mathcal{D}X \; \delta(S_{DBI} + S_{WZ}) \; e^{-S_{DBI} + S_{WZ}}$

$$\xrightarrow{\text{saddle}} \delta(S_{DBI} + S_{WZ})\Big|_{X^*} \; e^{-S^*_{DBI} + S^*_{WZ}}$$

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"Puzzles"

$$\langle \mathcal{D}_J \, \mathcal{D}_{J+k} \, \mathcal{D}_L \rangle \sim \delta \, S_{DBI} + \delta \, S_{WZ}$$

- Result is insensitive to k since the classical sol. is determined only by J & "δ" is determined by L
- Result for $k=L$ (extremal correlator) does not match the field theory result [Bissi, Kristjansen, Young, Zoubos]
  - Later, a "regularization" prescription that reproduces field-theory answer was proposed. [Kristjansen, Mori, Young] But the prescription seems self-inconsistent.
- Some results for non-extremal correlator ($k \neq L$) were found to agree [Caputa, de Mello Koch, Zoubos]. But in general they disagree. (even for $k=0$)

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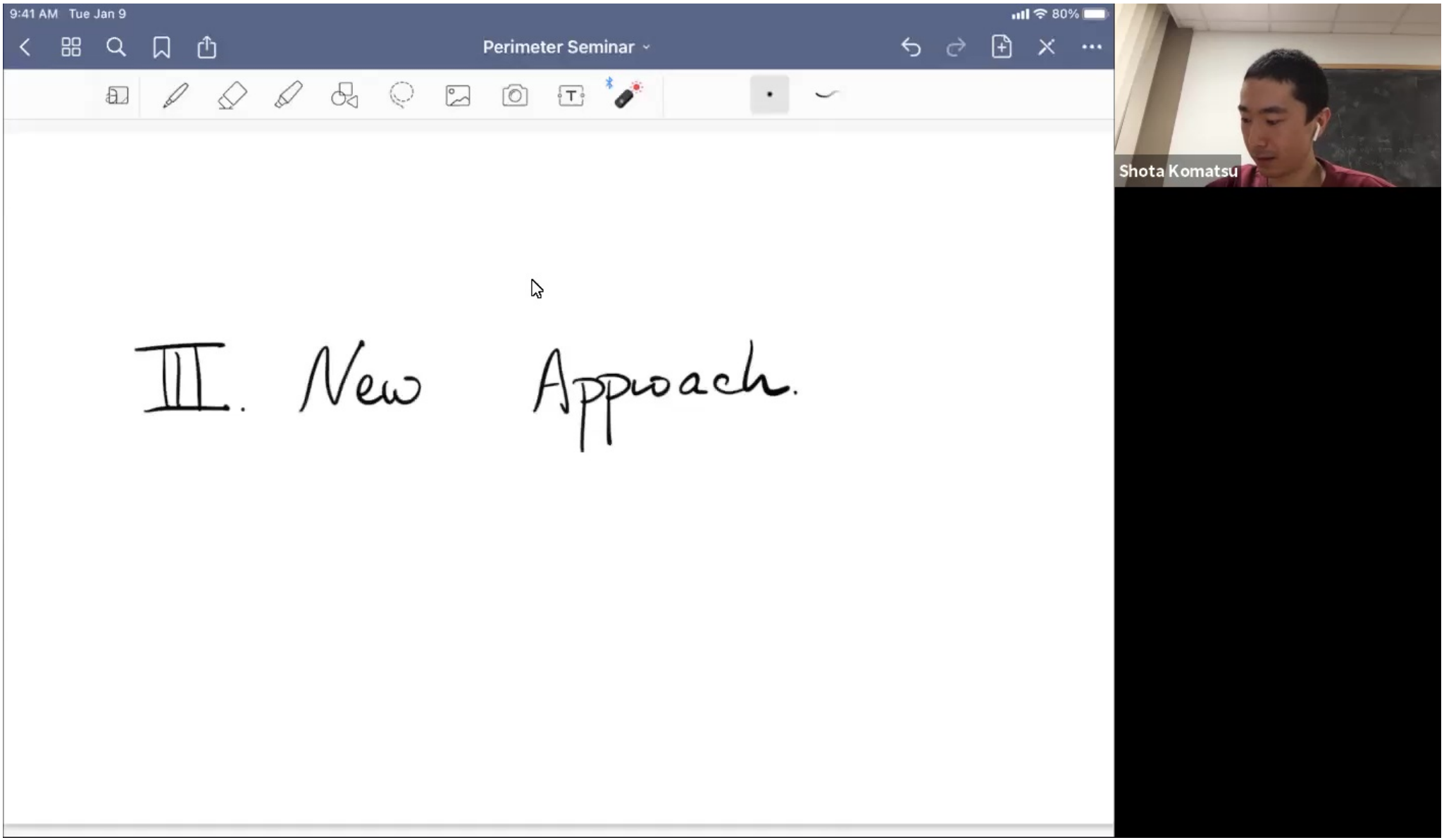
Our Conclusion

- Previous approaches miss 2 important effects.

1. Orbit Average

2. Wavefunctions of heavy states

- Once these are included, the results agree with results from field theory.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III. New Approach.
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Quantum Mechanical Toy Model

[Moriu, Pirtskhalava, Rattazzi, Seibold]

- Consider QM on S^1 ($\theta \in [0, 2\pi]$) w/. $U(1)$ sym

$$\theta \to \theta + c$$

- Compute $\langle J+k | \underbrace{\mathcal{O}(t=0)}_{\text{light}} \underbrace{|J\rangle}_{\text{heavy w/. } U(1) \text{ charge } J}$ in the WKB limit

$$J \to \infty, \quad \hbar \to 0, \quad \hbar J : \text{fixed}.$$

- $$\langle J+k | \mathcal{O}(t=0) | J\rangle = \int \mathcal{D}\theta \; e^{-i(J+k)\theta(t=+\varepsilon)} \, \mathcal{O}[\theta(t=0)] \, e^{iJ\theta(t=-\varepsilon)} \, e^{\frac{i}{\hbar}S[\theta]}$$

$\downarrow$ Saddle pnt approx. $\theta = \theta_0^*$ (Sol. of saddle eq. with source terms)

$$\sim e^{-i(J+k)\theta_0^*} \, \mathcal{O}[\theta_0^*(t=0)] \, e^{iJ\theta_0^*} \, e^{\frac{i}{\hbar}S[\theta_0^*]}$$

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Quantum Mechanical Toy Model

- $\langle J+k | \mathcal{O} | J \rangle \sim e^{-i(J+k)\theta_0^*} \, \mathcal{O}[\theta_0^*(t=0)] \, e^{iJ\theta_0^*} \, e^{\frac{i}{\hbar} S[\theta_0^*]}$

- This is incorrect. Because of $U(1)$ sym. $(\theta \to \theta + c)$ we can construct a family of saddle-pnt sol.

$$\theta_c^* \equiv \theta_0^* + c$$

- The correct formula is "average over c"

$$\langle J+k | \mathcal{O} | J \rangle \sim \int_0^{2\pi} \frac{dc}{2\pi} \, e^{-i(J+k)\theta_c^*} \, \mathcal{O}[\theta_c^*(t=0)] \, e^{iJ\theta_c^*} \, e^{\frac{i}{\hbar} S[\theta_c^*]}$$

$$= \boxed{e^{\frac{i}{\hbar} S[\theta_0^*]} \int_0^{2\pi} \frac{dc}{2\pi} \, e^{-ikc} \, \mathcal{O}[\theta_c^*(t=0)]}$$

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Quantum Mechanical Toy Model

- $\langle J+k | \mathcal{O} | J \rangle \sim e^{\frac{i}{\hbar} S[\theta_0^*]} \underbrace{\int_0^{2\pi} \frac{dc}{2\pi}}_{\text{orbit average}} \underbrace{e^{-ikc}}_{\text{wave function}} \mathcal{O}[\theta_c^*(0)]$

[Bajnok, Janik] for HHL string correlators.

- For $\mathcal{O}_p \sim e^{ip\theta}$ (charge p operator), we get

$$\langle J+k | \mathcal{O}_p | J \rangle \propto \int_0^{2\pi} \frac{dc}{2\pi} e^{-ikc} e^{ipc} = \underbrace{\delta_{k,p}}_{\text{charge conservation}}$$

- Orbit average & wave functions are crucial for reproducing the charge conservation.

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Orbit Average and Symmetry Breaking

- Generalization : If $\exists$ multiple commuting charges
  $\rightarrow$ average over multi-dimensional moduli of sols.

- Extreme case : In integrable theories, we have ∞ many charges.
  $\rightsquigarrow$ Do we need to ∞-dim integral ? $\rightarrow$ No.

- Classical (= saddle) sols. are invariant under most of those symmetries. we only need syms broken by solns.
  ($x^* \xrightarrow[\text{broken gen.}]{}$ new solution)

- Empirical Rule : (dim. of moduli) = (# of nonzero commuting charges)

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Quantum Mechanical Toy Model

- $\langle J+k | \mathcal{O} | J \rangle \sim e^{-i(J+k)\theta_0^*} \, \mathcal{O}[\theta_0^*(t=0)] \, e^{iJ\theta_0^*} \, e^{\frac{i}{\hbar} S[\theta_0^*]}$

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$$= \boxed{e^{\frac{i}{\hbar} S[\theta_0^*]} \int_0^{2\pi} \frac{dc}{2\pi} \, e^{-ikc} \, \mathcal{O}[\theta_c^*(t=0)]}$$

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Orbit Average and Symmetry Breaking

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- Empirical Rule : (dim. of moduli) = $\begin{pmatrix} \text{\# of nonzero} \\ \text{commuting charges} \end{pmatrix}$

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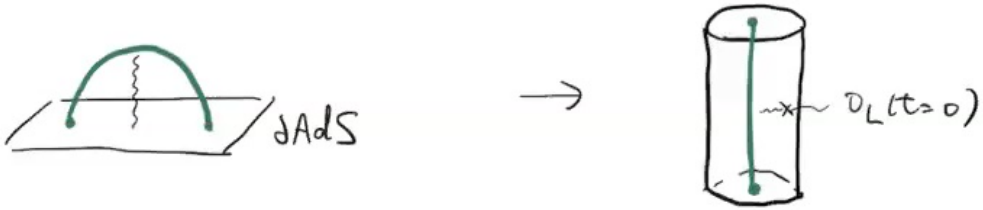
Application to giant gravitons.

- To apply the previous arguments, it's better to consider 3pt as a matrix element $\langle D_{J+k} | \mathcal{O}_L(t=0) | D_J \rangle$



* We gauge-fix $\sigma^0 = t$

- $\mathcal{O}_L(t=0) \rightsquigarrow$ sources quantum fluctuations (δg etc)

  $\rightsquigarrow$ Operator on the worldvolume of D-brane @ $\sigma^0 = 0$

  $\hat{\mathcal{O}}_L = \int d^3\sigma \; \delta\mathcal{L}_{DBI} + \delta\mathcal{L}_{WZ}$ | $\delta S_{DBI} = \int d^4\sigma \; \delta\mathcal{L}_{DBI}$

- $\underbrace{\langle D_{J+k} | \mathcal{O}_L(t=0) | D_J \rangle}_{\text{defined in } \mathcal{N}=4 \text{ SYM}} = \underbrace{\langle \hat{D}_{J+k} | \hat{\mathcal{O}}_L(\sigma^0=0) | \hat{D}_J \rangle}_{\text{defined on w.v. of D-brane}}$

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Application to giant gravitons

- We next evaluate $\langle \hat{D}_{J+k} | \hat{O}_L | \hat{D}_J \rangle$ semiclassically.
- Orbit average & wavefunction are important.
- Orbit average : Giant graviton $\rightsquigarrow$ 2 charges Δ, J

  $\rightsquigarrow$ Average over 2 directions

  $\Delta \longleftrightarrow \tau_0$: Conjugate to dilatation (EAdS time)

  $J \longleftrightarrow \varphi_0$: S^5 angle (Conjugate to R-sym rotation)
- Wavefunction

$$\Psi = e^{-\Delta \tau_0} \times e^{iJ\varphi_0}$$

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"Master Formula"

$\to \int d^3\sigma \left(\delta\mathcal{L}_{DBI} + \delta\mathcal{L}_{WZ}\right)$

$$\langle \hat{D}_{J+k} | \hat{\delta}_L | \hat{D}_J \rangle = \underbrace{\int_0^\infty d\tau_0 \int_0^{2\pi} \frac{d\varphi_0}{2\pi}}_{\text{Orbit average}} \hat{\delta}_L \left[X^*_{\tau_0,\varphi_0}(t=0)\right] \times \underbrace{e^{(\Delta_{J+k}-\Delta_J)\tau_0} \times e^{-i((J+k)-J)\varphi_0}}_{\text{wavefunction}}$$

- In [Bissi et al.], we have $\int d^4\sigma\,(\cdots)$.

  This splits into $d\tau_0$ & $d^3\sigma$.
- Result consistent with charge conservation.
- Result sensitive to k
- Result fully agrees with field theory

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$$C_{P_J P_{J+k} \partial_L} = -\frac{\sqrt{L}}{2}\left(i^{L-k} + (-i)^{L-k}\right) \qquad \left(\cos^2\theta_0 \equiv J/N\right)$$

$$\times \frac{\Gamma\left(\frac{L+k}{2}\right)\cos^2\theta_0 \sin^2\theta_0}{\Gamma(1+k)\,\Gamma\left(1+\frac{L-k}{2}\right)}\, {}_2F_1\left(\frac{2+k-L}{2}, \frac{2+k+L}{2}, 1+k ; \sin^2\theta_0\right)$$

- Computation can be generalized to ABJM (both in IIA limit & M-theory limit)

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Some subtleties about extremal limit

- In the extremal limit ($k=L$), we have

$$\sim \underbrace{(k-L)}_{0} \times \underbrace{\int dt_0 \, (\cdots)}_{\infty} + (\text{finite})$$

- Setting $0 \times \infty$ to zero $\rightarrow$ result doesn't agree with Schur-pol. operators or single-particle basis [Aprile et al]
- Analytic continuation from $k-L \neq 0$ $\rightsquigarrow$ Reproduce result for Schur pol.
- But analytic contin. is NOT unique.

29 of 38 tremal limit is sensitive to non-planar corrections

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"Master Formula"

$\to \int d^3\sigma \left(\delta\mathcal{L}_{DBI} + \delta\mathcal{L}_{WZ}\right)$

$$\langle \hat{D}_{J+k} | \hat{\delta}_L | \hat{D}_J \rangle = \underbrace{\int_0^\infty d\tau_0 \int_0^{2\pi} \frac{d\varphi_0}{2\pi}}_{\text{Orbit average}} \hat{\delta}_L \left[X^*_{\tau_0,\varphi_0}(t=0)\right] \times \underbrace{e^{(\Delta_{J+k}-\Delta_J)\tau_0} \times e^{-i((J+k)-J)\varphi_0}}_{\text{wavefunction}}$$

- In [Bissi et al.], we have $\int d^4\sigma\,(\cdots)$.

  This splits into $d\tau_0$ & $d^3\sigma$.
- Result consistent with charge conservation.
- Result sensitive to k
- Result fully agrees with field theory

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$$C_{D_J D_{J+k} \partial_L} = -\frac{\sqrt{L}}{2}\left(i^{L-k} + (-i)^{L-k}\right) \qquad \left(\cos^2\theta_0 \equiv J/N\right)$$

$$\times \frac{\Gamma\left(\frac{L+k}{2}\right)\cos^2\theta_0 \sin^2\theta_0}{\Gamma(1+k)\,\Gamma\left(1+\frac{L-k}{2}\right)}\, {}_2F_1\left(\frac{2+k-L}{2}, \frac{2+k+L}{2}, 1+k ; \sin^2\theta_0\right)$$

- Computation can be generalized to ABJM (both in IIA limit & M-theory limit)

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IV. Conclusion, Discussion

Speculation, Random thoughts ---

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So, what are lessons ... ?

- Another (highly nontrivial) precision test of AdS / CFT. But we believe AdS/CFT anyway, don't we ... ?

- But it's worth emphasizing that we reproduced "off-diagonal" 3pt $\langle D_{J+k} | D_L | D_J \rangle$ by semiclassical D-branes.

  One might say semiclassical computation is insensitive to microscopic $(+k)$ difference. But that is false!

⇝ Can we do this for BHs ... ?

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What we expect for BHs ... ?

- States dual to BHs come with exponential degeneracy ($\sim e^{N^2}$)

- Conservative viewpoint : Computation of semiclassical BH captures average or universal part of such states

- $\langle E_m | O | E_n \rangle = \mathcal{O}_{\text{thermal}}(E_m)\,\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E}, \Delta E)\, r_{mn}$

$\bar{E} = \frac{E_m + E_n}{2}$, $\Delta E = \frac{E_m - E_n}{2}$ Eigenstate thermalization

$\langle r_{mn} \rangle = 0$, $\langle r_{mn}^2 \rangle = 1$

- Average of off-diagonal part $\propto \langle r_{mn} \rangle = 0$ ....

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Some remarks on ETH

$$\langle E_m | O | E_n \rangle = O_{thermal}(E_m)\,\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E}, \Delta E)\, r_{mn}$$

- ETH predicts off diagonal part is suppressed by $e^{-S/2} \sim e^{-N^2/2}$
- But we need to be careful abt for which states this holds
- In 2d CFT, if $|E_m\rangle$ & $|E_n\rangle$ are in the same Verma module $\rightsquigarrow$ $1/(S(E))^{\#}$ [Besken, Datta, Kraus]
- Average over heavy primaries $\rightsquigarrow$ $e^{-S(E)/2}$ Brehm, Das, Datta, ----

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Simpler setups

- $\exists$ states heavy enough to deform geometry but in the same "universality class" as giant gravitons

$\rightsquigarrow$ · 1/2 BPS LLM geometry in $\mathcal{N}=4$ SYM

· 1/4 BPS fuzz ball geometry in D1-D5 CFT

(elliptic genus counts 1/8 BPS)

- LLM geometry : $AdS_2 \times S^3_1 \times S^3_2$ factor.

○ S^3_1 shrinks

○ S^3_2 shrinks

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Simpler Setups

- Non-rotationally sym pattern

⇝ time dependent = breaks dilatation

⇝ we need to orbit average!

- Rotationally sym pattern

⇝ time independent = does not break dilatation

⇝ Important difference from D-brane.

- Many to 1 correspondence ... ?

Gravity seems overcomplete ... ?

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What would be nice to achieve ...

- For BHs, we first identify the "orbit" over which we integrate ...
- One candidate is horizon soft hair [Hawking, Perry, Strominger]
- Ideally, we want to have

$$\langle BH' | \mathcal{O}_{light} | BH \rangle \sim \int d[\underbrace{HSG}_{\text{horizon sym group}}]\ \Psi_1^* \mathcal{O} \Psi_2$$

wave functions for soft hair

more dynamical question than reproducing entropy

- Perhaps progress can be made in AdS_3/CFT_2 ... ?

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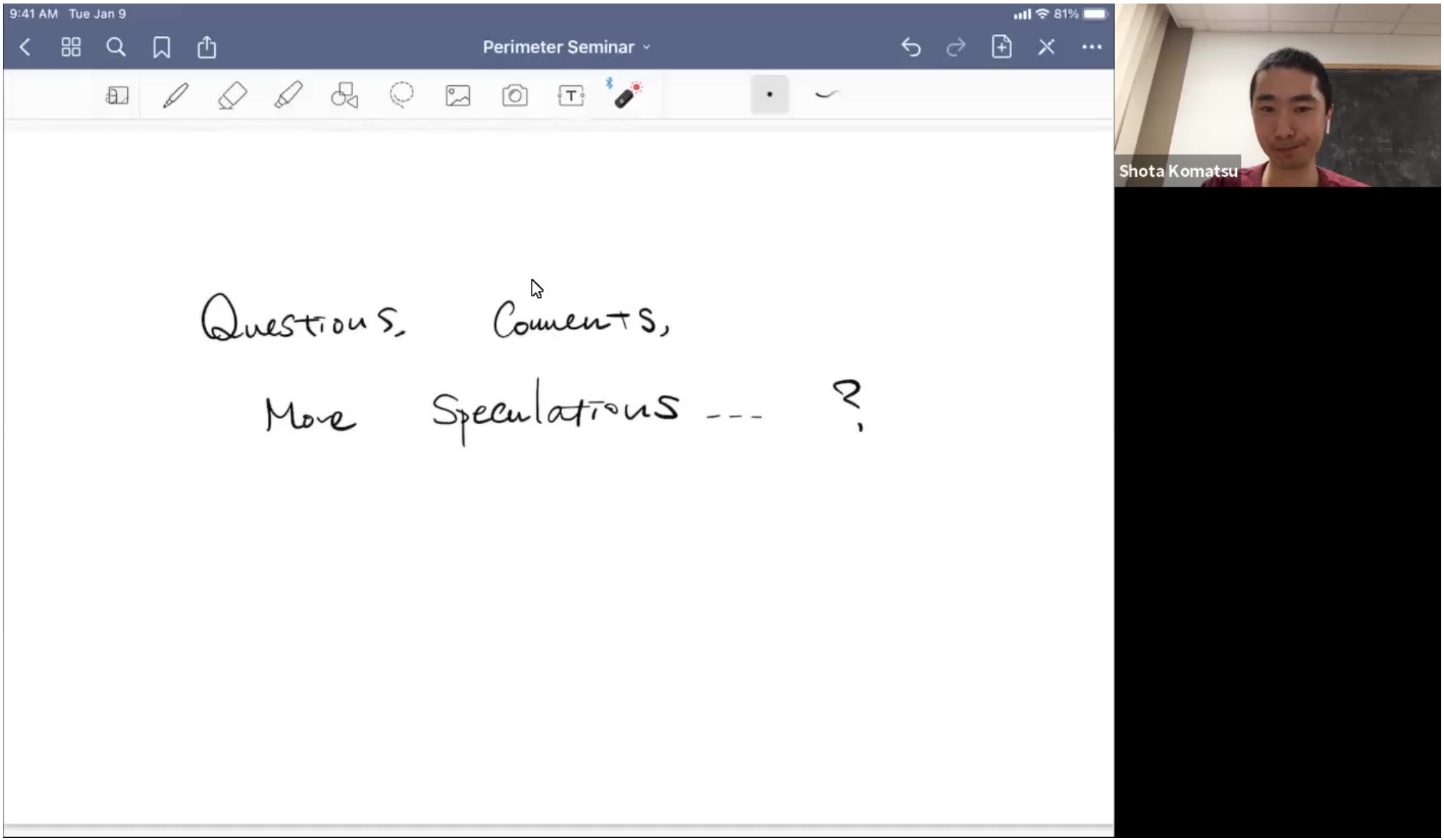
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Questions, Comments,
More Speculations --- ?
Shota Komatsu