Title: The standard model, left/right symmetry, and the "magic square"

Speakers: Latham Boyle

Collection: Octonions and the Standard Model

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Abstract: Recently, an intriguing connection between the exceptional Jordan algebra h_3(O) and the standard model of particle physics was noticed by Dubois-Violette and Todorov (with further interpretation by Baez). How do the standard model fermions fit into this story? I will explain how they may be neatly incorporated by complexifying h_3(O) or, relatedly, by passing from RxO to CxO in the so-called "magic square" of normed division algebras. This, in turn, suggests that the standard model, with gauge group SU(3)xSU(2)xU(1), is embedded in a left/right-symmetric theory, with gauge group SU(3)xSU(2)xSU(2)xU(1). This theory is not only experimentally viable, but offers some explanatory advantages over the standard model (including an elegant solution to the standard model's "strong CP problem"). Ramond's formulation of the magic square, based on triality, provides further insights, and possible hints about where to go next.

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The Standard Model, Left/Right Symmetry, and the Magic Square

Latham Boyle (Perimeter)

Mostly based on arXiv: 2006.16265

Some general references:

- J. Baez "The Octonions"
- I. Yokota "Exceptional Lie Groups" arXiv: 0902.0431

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Questions:

1) Why $G_{SM} = [SU(3)_c \times SU(2)_L \times U(1)_Y]/\mathbb{Z}_6$?

2) Why
$$\rho^{(1)}_{SM} = \underset{q_L}{(3, 2, \tfrac{1}{6})} \oplus \underset{\bar{d}_R}{(\bar{3}, 1, \tfrac{1}{3})} \oplus \underset{\bar{u}_R}{(\bar{3}, 1, -\tfrac{2}{3})}$$
$$\oplus \underset{l_L}{(1, 2, -\tfrac{1}{2})} \oplus \underset{\bar{e}_R}{(1, 1, 1)} \oplus \underset{\bar{\nu}_R}{(1, 1, 0)} \ ?$$

3) Why 3 generations

$$\rho_{SM} = \rho^{(1)}_{SM} \oplus \rho^{(1)}_{SM} \oplus \rho^{(1)}_{SM} \ ?$$

Old Hints:

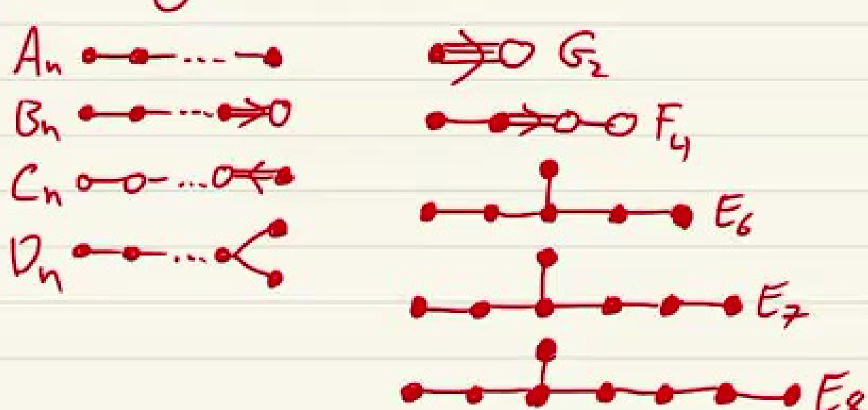
1) Grand Unification

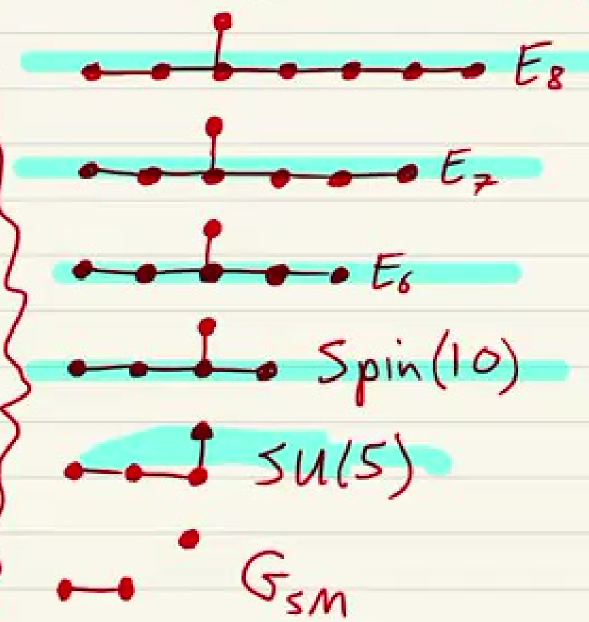
i) $SU(5) \longrightarrow G_{SM}$ ii) $Spin(10) \longrightarrow G_{SM}$

$\Lambda \mathbb{C}^5 \longrightarrow \rho^{(1)}_{SM}$ $16 \longrightarrow \rho^{(1)}_{SM}$

2) Exceptional?

Killing-Cartan:





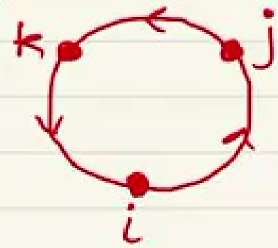
New Hint:

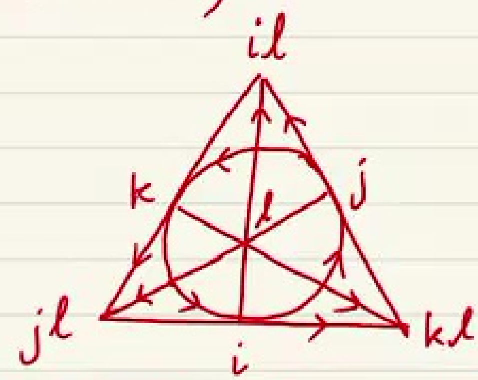
Dubois-Violette + Todorov (+ Baez) (2018)

$$h_3(\mathbb{O}) \longleftrightarrow G_{SM}$$

What about $\mathfrak{g}_{SM}$?

4 Normed Division Algebras: $\mathbb{K} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ (Hurwitz)





$\mathbb{C}$: $z = a_0 + a_1 i$

$\mathbb{H}$: $q = a_0 + a_1 i + a_2 j + a_3 k$

$\mathbb{O}$: $x = a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 il + a_6 jl + a_7 kl$

Euclidean Jordan Algebras (Jordan, Wigner, von Neumann)

1) $J\mathrm{spin}(n)$
2) $h_n(\mathbb{R})$
3) $h_n(\mathbb{C})$
4) $h_n(\mathbb{H})$

$h_3(\mathbb{O})$ ("Exceptional Jordan Algebra")

$$Y = \begin{pmatrix} \alpha_1 & x_3 & x_2^* \\ x_3^* & \alpha_2 & x_1 \\ x_2 & x_1^* & \alpha_3 \end{pmatrix} \in h_3(\mathbb{O})$$

$$Y_1 \circ Y_2 = \frac{1}{2}\{Y_1, Y_2\} = \frac{1}{2}(Y_1 Y_2 + Y_2 Y_1)$$

G_{SM} from $h_3(\mathbb{O})$

$$G = F_4 = \{\gamma \mid \det(y) = \det(\gamma y),\ \langle y_1 | y_2 \rangle = \mathrm{Tr}[y_1 \circ y_2] = \langle \gamma y_1 | \gamma y_2 \rangle \}$$

$H_1 = Spin(9)$: preserves $\Pi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longleftrightarrow 2+1$ split $\left(\begin{array}{cc|c} \alpha_1 & x_3 & x_2^* \\ x_3^* & \alpha_2 & x_1 \\ \hline x_2 & x_1^* & \alpha_3 \end{array}\right)$

(a pt in $\mathbb{O}P^2$)

$H_2 = [SU(3) \times SU(3)]/\mathbb{Z}_3$: preserves $\mathbb{O} = \mathbb{C} \oplus \mathbb{C}^3$ split

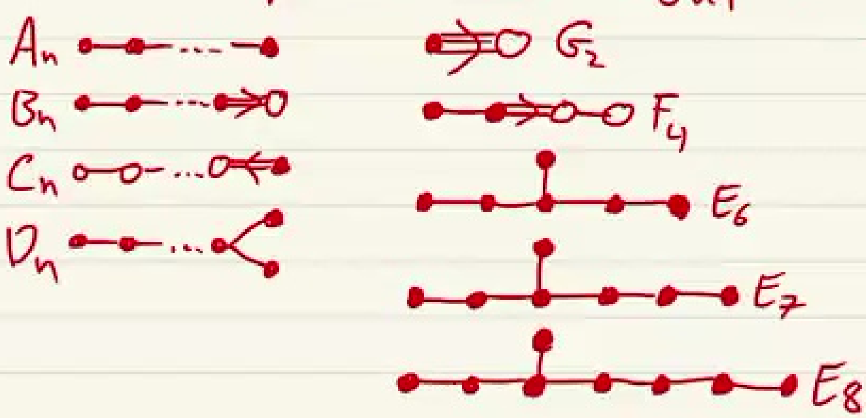
$H_1 \cap H_2 = G_{SM} = [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$ (Dubois-Violette + Todorov, 2018)

Baez's Interpretations: i) fix M_{9+1} in $h_3(\mathbb{O})$ and M_{3+1} in M_{9+1}.

ii) fix $\mathbb{O}$-bit in $\mathbb{O}$-trit and $\mathbb{C}$-bit in $\mathbb{O}$-bit.

Problem:

- SM rep is "complex" – i.e. ρ_{SM} and $\bar{\rho}_{SM}$ are inequivalent
- SM is chiral,
- So usually consider G_{GUT} with complex reps:



$SU(n)$ $(n \geq 3)$

$SO(4n+2)$ or $Spin(4n+2)$

E_6

- But in $h_3(\mathbb{O})$, G_{SM} arises as subgroup of F_4 and $Spin(9)$ which have no complex reps.

G_{LR} from $h_3^{\mathbb{C}}(\mathbb{O})$

$$\tilde{G} = E_6 = \left\{ \gamma \mid \det(y) = \det(\gamma y),\ \langle y_1 | y_2 \rangle = \mathrm{Tr}[\bar{y}_1 \circ y_2] = \langle \gamma y_1 | \gamma y_2 \rangle \right\}$$

$$\tilde{H}_1 = Spin(10) : \text{preserves } \Pi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to 2+1 \text{ split } \left(\begin{array}{cc|c} \alpha_1 & x_3 & x_2^* \\ x_3^* & \alpha_2 & x_1 \\ \hline x_2 & x_1^* & \alpha_3 \end{array}\right) \leftarrow \text{preserved by } Spin(10)\times U(1)$$

$$\tilde{H}_2 = [SU(3)\times SU(3)\times SU(3)]/\mathbb{Z}_3 \ : \ \text{preserves } \mathbb{O} = \mathbb{C}\oplus\mathbb{C}^3 \text{ split}$$

$$\tilde{H}_1 \cap \tilde{H}_2 = G_{LR} = [SU(3)\times SU(2)_L \times SU(2)_R \times U(1)]/\mathbb{Z}_6$$

$$\begin{pmatrix} x_2^* \\ x_1 \end{pmatrix} \xrightarrow{G_{LR}} \rho_{LR} = \underbrace{(3,2,1,\tfrac{1}{6})}_{q_L} \oplus \underbrace{(\bar{3},1,2,-\tfrac{1}{6})}_{\bar{q}_R} \oplus \underbrace{(1,2,1,-\tfrac{1}{2})}_{l_L} \oplus \underbrace{(1,1,2,\tfrac{1}{2})}_{\bar{l}_R}$$

Left/Right-Symmetric Theory

Standard Model

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	
q_L	3	2	1/6	quarks
u_R	3	1	2/3	$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$
d_R	3	1	$-1/3$	
l_L	1	2	$-1/2$	leptons
ν_R	1	1	0	$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$
e_R	1	1	-1	
h_L	1	2	$-1/2$	Higgs

LR-symmetric version

	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	
q_L	3	2	1	1/6	quarks
q_R	3	1	2	1/6	
h_L, l_L	1	2	1	$-1/2$	leptons +
h_R, l_R	1	1	2	$-1/2$	Higgs

P_{LR}: parity $\vec{x} \rightarrow -\vec{x}$

and swap
$$q_L \leftrightarrow q_R$$
$$l_L \leftrightarrow l_R$$
$$h_L \leftrightarrow h_R$$
$$W_\mu^L \leftrightarrow W_\mu^R$$

LR-symmetric theory features (Hall + Harigaya, 2018)

1) No Yukawa terms. Instead, dimension 5 terms:

$$\frac{1}{\Lambda}\left[\bar{q}_L h_L Y_u h_R^\dagger q_R + \bar{q}_L \tilde{h}_L Y_d \tilde{h}_R^\dagger q_R + \bar{\ell}_L h_L Y_\nu h_R^\dagger \ell_R + \bar{\ell}_L \tilde{h}_L Y_e \tilde{h}_R^\dagger \ell_R + \bar{\ell}_L^c \tilde{h}_L Y_m h_L^\dagger \ell_L + \bar{\ell}_R^c \tilde{h}_R Y_m h_R^\dagger \ell_R\right]$$

2) If $\langle h_R \rangle \gg$ TeV, reduces to SM at low energies.

3) P_{LR} kills $\theta G_{\mu\nu}\tilde{G}^{\mu\nu}$, and $\Rightarrow Y = Y^\dagger$, so no strong CP problem!

4) Explains/predicts that $\lambda_{Higgs} \longrightarrow 0$ at $\Lambda \sim \langle h_R \rangle$

5) If $\langle h_R \rangle \sim 10^{11}$ GeV, also fixes gauge coupling unification

Triality

- $\mathrm{der}(A) = \{\delta \mid \delta(ab) = \delta(a)b + a\delta(b) \quad \forall\, a,b \in A\}$

- $\mathrm{tri}(A) = \{(T_1, T_2, T_3) \mid T_1(ab) = T_2(a)b + aT_3(b) \quad \forall\, a,b \in A\}$

$\mathbb{K}$	$\mathrm{tri}(\mathbb{K})$
$\mathbb{R}$	$-$
$\mathbb{C}$	$u(1) \oplus u(1)$
$\mathbb{H}$	$su(2) \oplus su(2) \oplus su(2)$
$\mathbb{O}$	$so(8)$

$$(T_1, T_2, T_3) \in \mathrm{tri}(\mathbb{K})$$

$$(\overline{T}_3, \overline{T}_1, T_2) \in \mathrm{tri}(\mathbb{K})$$

Triality

- $\mathrm{der}(A) = \{ \delta \mid \delta(ab) = \delta(a)b + a\delta(b) \quad \forall\, a,b \in A \}$

- $\mathrm{tri}(A) = \{ (T_1, T_2, T_3) \mid T_1(ab) = T_2(a)b + aT_3(b) \quad \forall\, a,b \in A \}$

$\mathbb{K}$	$\mathrm{tri}(\mathbb{K})$
$\mathbb{R}$	—
$\mathbb{C}$	$\mathfrak{u}(1) \oplus \mathfrak{u}(1)$
$\mathbb{H}$	$\mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$
$\mathbb{O}$	$\mathfrak{so}(8)$

$(T_1, T_2, T_3) \in \mathrm{tri}(\mathbb{K})$

$(\overline{T}_3, \overline{T}_1, T_2) \in \mathrm{tri}(\mathbb{K})$

Magic Square

$$m(\mathbb{K},\tilde{\mathbb{K}}) = \mathrm{tri}(\mathbb{K}) \oplus \mathrm{tri}(\tilde{\mathbb{K}}) + (\mathbb{K}\otimes\tilde{\mathbb{K}})_1 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_2 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_3$$

Ramond

Barton + Sudbery

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$so(3)$	$su(3)$	$sp(3)$	f_4
$\mathbb{C}$	$su(3)$	$su(3)+su(3)$	$su(6)$	e_6
$\mathbb{H}$	$sp(3)$	$su(6)$	$so(12)$	e_7
$\mathbb{O}$	f_4	e_6	e_7	e_8

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$so(2)$	$so(3)$	$so(5)$	$so(9)$
$\mathbb{C}$	$so(3)$	$so(4)$	$so(6)$	$so(10)$
$\mathbb{H}$	$so(5)$	$so(6)$	$so(8)$	$so(12)$
$\mathbb{O}$	$so(9)$	$so(10)$	$so(12)$	$so(16)$

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$$m(\mathbb{K},\tilde{\mathbb{K}}) = \underbrace{\mathrm{tri}(\mathbb{K}) \oplus \mathrm{tri}(\tilde{\mathbb{K}})}^{\text{bosonic}} + \overbrace{(\mathbb{K}\otimes\tilde{\mathbb{K}})_1 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_2 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_3}^{\text{fermionic}}$$

Lie brackets:

- $\mathrm{tri}(\mathbb{K}) \oplus \mathrm{tri}(\tilde{\mathbb{K}})$ forms a Lie subalgebra
- $[\mathrm{tri}(\mathbb{K}) \oplus \mathrm{tri}(\tilde{\mathbb{K}}), (\mathbb{K}\otimes\tilde{\mathbb{K}})_i] \in (\mathbb{K}\otimes\tilde{\mathbb{K}})_i$
- $[(\mathbb{K}\otimes\tilde{\mathbb{K}})_1, (\mathbb{K}\otimes\tilde{\mathbb{K}})_2] \in (\mathbb{K}\otimes\tilde{\mathbb{K}})_3$ etc.
- $[(\mathbb{K}\otimes\tilde{\mathbb{K}})_i, (\mathbb{K}\otimes\tilde{\mathbb{K}})_i] \in \mathrm{tri}(\mathbb{K}) \oplus \mathrm{tri}(\tilde{\mathbb{K}})$

$$m(\mathbb{C},\mathbb{O}) = e_6 = \underbrace{\mathrm{spin}(10) \oplus u(1)}_{\text{bosonic}} + \underbrace{(\mathbb{C}\otimes\mathbb{O}) + (\mathbb{C}\otimes\mathbb{O})}_{\text{fermionic}}$$

fermionic (16-dim Weyl spinor of Spin(10))

$\mathbb{O} \to \mathbb{C} \oplus \mathbb{C}^3$

bosonic $\downarrow$ G_{LR}

fermionic $\downarrow$ $\mathfrak{g}_{LR}$

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3 generations: (some comments/speculations)

$$m(\mathbb{K},\tilde{\mathbb{K}}) = tri(\mathbb{K}) \oplus tri(\tilde{\mathbb{K}}) + (\mathbb{K}\otimes\tilde{\mathbb{K}})_1 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_2 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_3$$

1) $m(\mathbb{C},\mathbb{O}) = e_6$: 3 overlapping generations?

$$\left(\begin{array}{cc|c} \alpha_1 & X_3 & X_2^* \\ X_3^* & \alpha_2 & X_1 \\ \hline X_2 & X_1^* & \alpha_3 \end{array}\right) \quad \begin{pmatrix} X_2^* \\ X_1 \end{pmatrix} \quad \begin{pmatrix} X_3^* \\ X_2 \end{pmatrix} \quad \begin{pmatrix} X_1^* \\ X_3 \end{pmatrix} \Bigg\} \text{ related by large Bogoliubov transformations?}$$

2) $m(\mathbb{H},\mathbb{O}) = e_7$: right counting to describe 3 gen (internal d.o.f.)

3) $m(\mathbb{O},\mathbb{O}) = e_8$: right counting to describe 3 gen (internal+spacetime d.o.f.)

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iv) A_μ $\longrightarrow$ Lie-algebra valued 1-form

$\longrightarrow$ derivation-valued 1-form (Dubois-Violette)

$\longrightarrow$ triality-valued 1-form (naturally acts on "3 generations")

$\longrightarrow$ triality-valued superconnection (following Quillen, modelled on Ramond's magic square)

Is the standard model (or its LR-symmetric extension) a Yang-Mills theory of such a generalized connection?

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Summary

i) Complexifying $h_3(\mathbb{O}) \to h_3^{\mathbb{C}}(\mathbb{O})$ incorporates SM fermions.

ii) Consequence: SM $\to$ LR-symmetric extension

↳ some advantages (including soln to strong CP problem)

iii) Rephrase above construction: from $m(\mathbb{R},\mathbb{O}) \to m(\mathbb{C},\mathbb{O})$

iv) superalgebra decomposition of $m(\mathbb{K},\tilde{\mathbb{K}})$ agrees w/ Spin(10) GUT

v) Thoughts about 3 generations

$$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} \rightarrow \begin{pmatrix}U_1\psi_1\\ U_2\psi_2\\ U_3\psi_3\end{pmatrix}$$

$$D\left(\quad\right) \rightarrow \begin{matrix}U_1\\ U_2\\ U_3\end{matrix}\left(D\left(\quad\right)\right)$$

$$\mathrm{Tr}\left[F^2\right] \rightarrow$$

Magic Square

$$m(\mathbb{K},\tilde{\mathbb{K}}) = tri(\mathbb{K}) \oplus tri(\tilde{\mathbb{K}}) + (\mathbb{K}\otimes\tilde{\mathbb{K}})_1 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_2 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_3$$

Ramond

Barton+Sudbery

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$\mathbb{O}$	f_4	e_6	e_7	e_8

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$so(2)$	$so(3)$	$so(5)$	$so(9)$
$\mathbb{C}$	$so(3)$	$so(4)$	$so(6)$	$so(10)$
$\mathbb{H}$	$so(5)$	$so(6)$	$so(8)$	$so(12)$
$\mathbb{O}$	$so(9)$	$so(10)$	$so(12)$	$so(16)$

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Magic Square

$su(2)\times$

$so(8)$ $so(8)$

$$m(\mathbb{K},\tilde{\mathbb{K}}) = tri(\mathbb{K}) \oplus tri(\tilde{\mathbb{K}}) + (\mathbb{K}\otimes\tilde{\mathbb{K}})_1 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_2 + (\mathbb{K}\otimes\tilde{\mathbb{K}})_3$$

Ramond

Barton+Sudbery

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
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$\mathbb{O}$	$so(9)$	$so(10)$	$so(12)$	$so(16)$