

Title: Fault-tolerant qubit from a constant number of components

Speakers: Kianna Wan

Series: Perimeter Institute Quantum Discussions

Date: April 28, 2021 - 4:00 PM

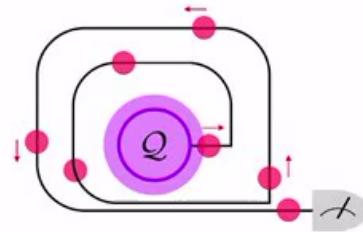
URL: <http://pirsa.org/21040036>

Abstract: With gate error rates in multiple technologies now below the threshold required for fault-tolerant quantum computation, the major remaining obstacle to useful quantum computation is scaling, a challenge greatly amplified by the huge overhead imposed by quantum error correction itself. I'll discuss a new fault-tolerant quantum computing scheme that can nonetheless be assembled from a small number of experimental components, potentially dramatically reducing the engineering challenges associated with building a large-scale fault-tolerant quantum computer. The architecture couples a single controllable qubit to a pair of delay lines which terminate in a detector. Below a threshold value for the error rate associated with the controllable qubit, the logical error rate decays exponentially with the square root of the delay line coherence time. The required gates can be implemented using existing technologies in quantum photonic and phononic systems. With continued incremental improvements in only a few components, we expect these systems to be promising candidates for demonstrating fault-tolerant quantum computation with comparatively modest experimental effort.



Fault-tolerant qubit from a constant number of components

Kianna Wan, Soonwon Choi, Isaac H. Kim, Noah Shutty,
& Patrick Hayden



motivation

previous work:

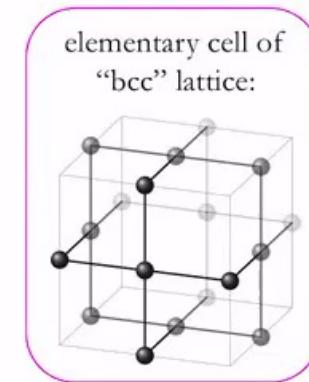
[Lindner & Rudolph '09] – 1D cluster states

[Pichler, Choi, Zoller, Lukin '17] – 2D cluster states
(universal MBQC)

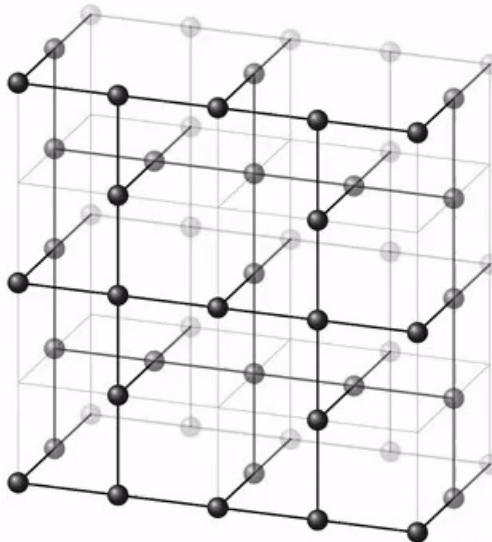
our goal: prepare 3D cluster states on bcc lattice

(*fault-tolerant* universal MBQC [Raussendorf *et al.*])

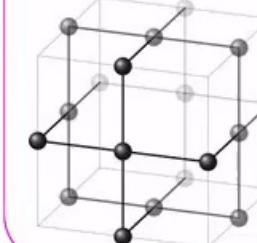
1. using an experimentally feasible setup...
2. ...while preserving fault-tolerance



prologue: cluster states & FTMBQC

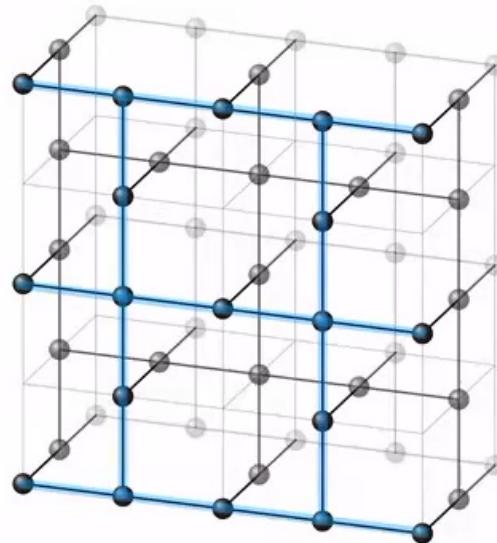


elementary cell of
“bcc” lattice:

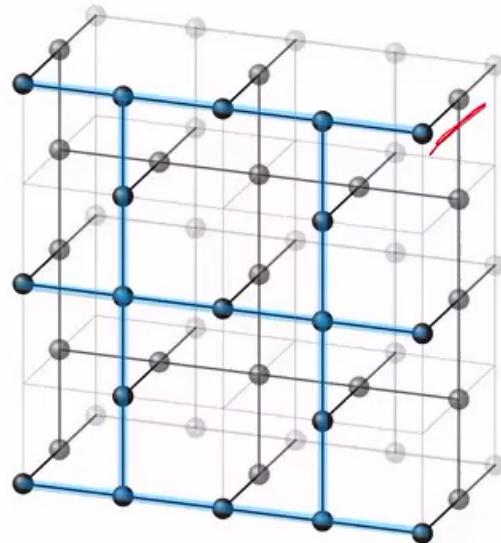


kianna

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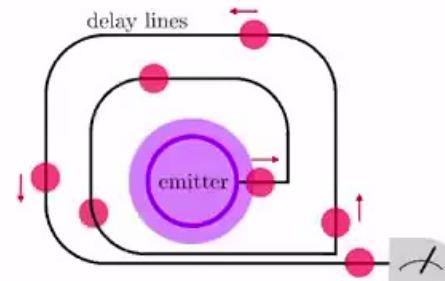
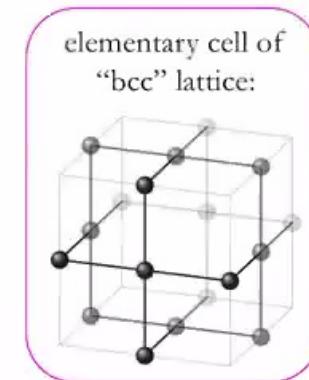
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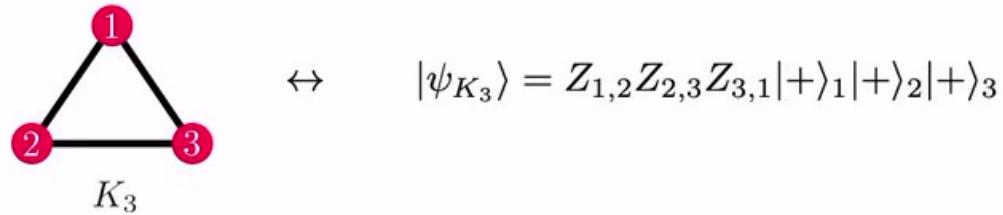


cluster states

any undirected graph $G = (V, E)$ defines a cluster state $|\psi_G\rangle$:

$$|\psi_G\rangle := \left[\prod_{(i,j) \in E} cZ_{i,j} \right] \bigotimes_{k \in V} |+\rangle_k$$

e.g.,





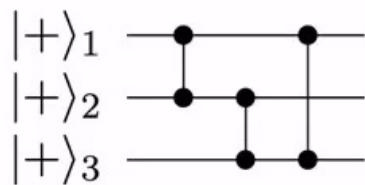
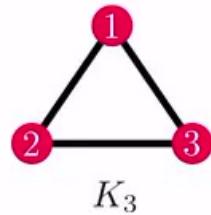
cluster states

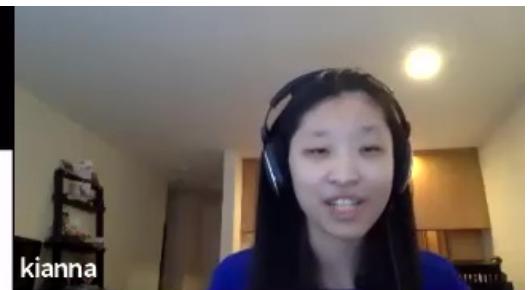
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\Rightarrow very simple circuit!

e.g.,





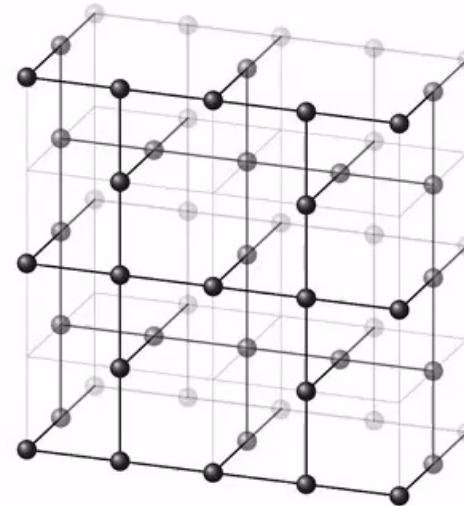
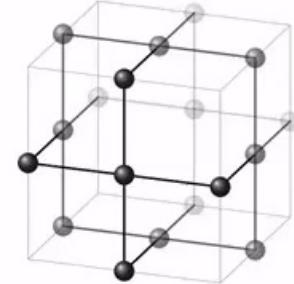
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cluster states

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\Rightarrow very simple circuit!

however, requires interactions between $|E|$ distinct pairs of qubits

instead, introduce a single ancilla, \mathcal{Q} , that interacts with each of the “identical” **data qubits** ($i \in V$) one by one
 \rightarrow calibrate only a constant number of physically distinct interactions

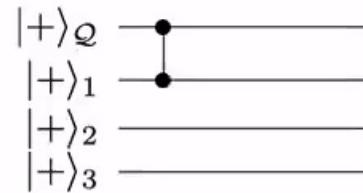
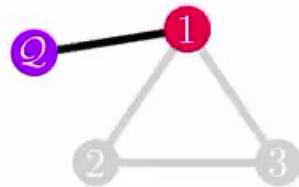


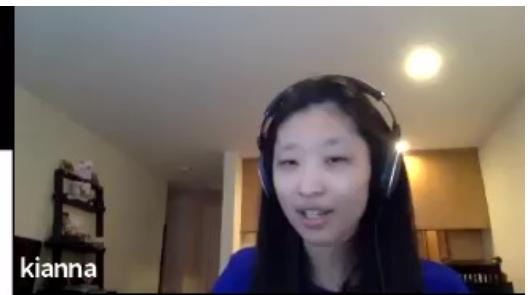
preparing cluster states

abstract problem: prepare cluster states using interactions only between \mathcal{Q} and **data qubits** (no two-qubit gates between data qubits)

simplest solution: use SWAP gates

e.g.,



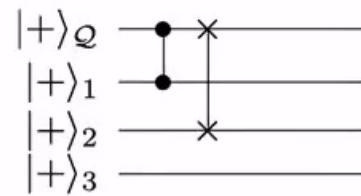
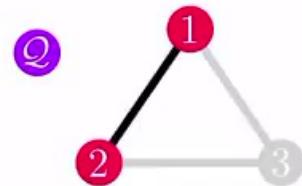


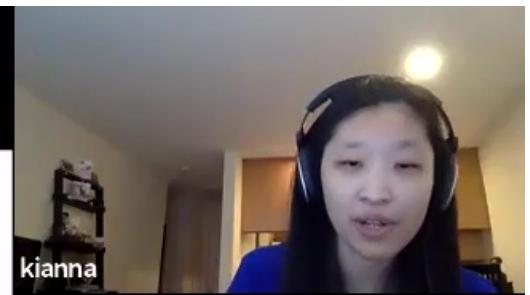
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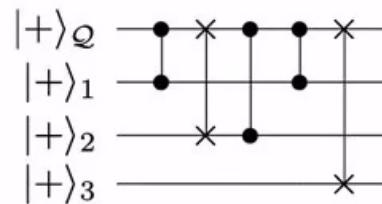
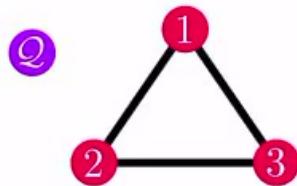


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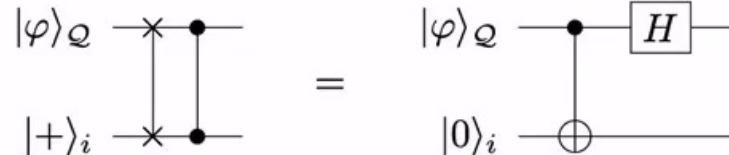
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practical challenge: dual-rail SWAP gate ???

(encoding scheme in which
qubit loss is detectable)

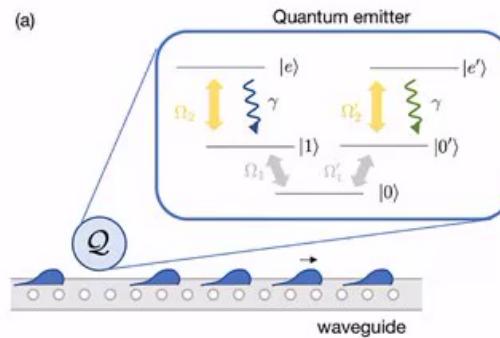
fix:



implementable!



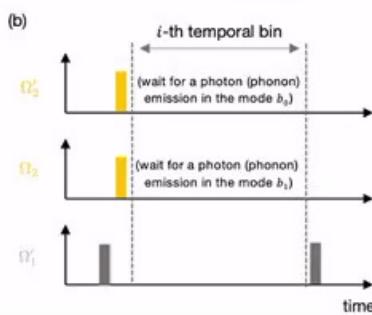
dual-rail implementation of CNOT gate



$$|0\rangle_Q |\emptyset\rangle_i |\phi_0\rangle_{\text{rest}} + |1\rangle_Q |\emptyset\rangle_i |\phi_1\rangle_{\text{rest}}$$

$$\mapsto |0'\rangle_Q |\emptyset\rangle_i |\phi_0\rangle_{\text{rest}} + |1\rangle_Q |\emptyset\rangle_i |\phi_1\rangle_{\text{rest}}$$

$$\mapsto |0'\rangle_Q \left(b_0^\dagger |\emptyset\rangle_i \right) |\phi_0\rangle_{\text{rest}} + |1\rangle_Q \left(b_1^\dagger |\emptyset\rangle_i \right) |\phi_1\rangle_{\text{rest}}$$

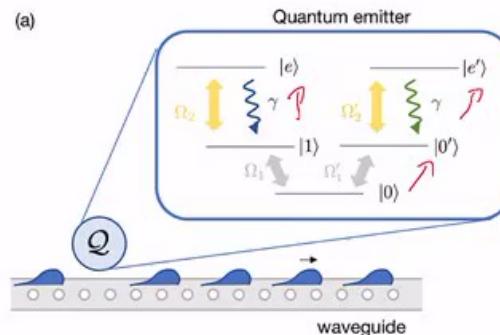


$$\mapsto |0\rangle_Q \left(b_0^\dagger |\emptyset\rangle_i \right) |\phi_0\rangle_{\text{rest}} + |1\rangle_Q \left(b_1^\dagger |\emptyset\rangle_i \right) |\phi_1\rangle_{\text{rest}}$$

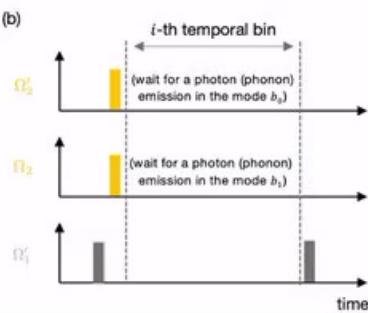
data qubit states: $|0\rangle_i \equiv b_0^\dagger |\emptyset\rangle_i$ and $|1\rangle_i \equiv b_1^\dagger |\emptyset\rangle_i$



dual-rail implementation of CNOT gate

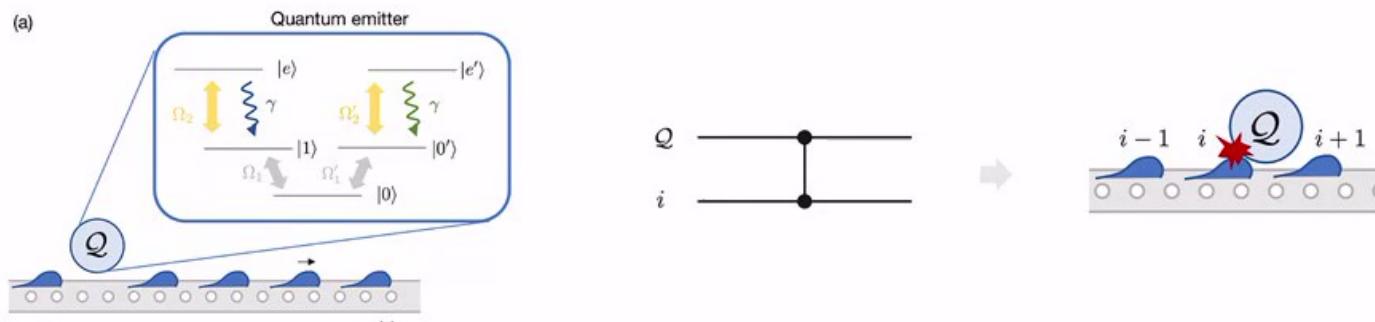


$$\begin{aligned}
 &|0\rangle_Q |\emptyset\rangle_i |\phi_0\rangle_{\text{rest}} + |1\rangle_Q |\emptyset\rangle_i |\phi_1\rangle_{\text{rest}} \\
 &\mapsto |0'\rangle_Q |\emptyset\rangle_i |\phi_0\rangle_{\text{rest}} + |1\rangle_Q |\emptyset\rangle_i |\phi_1\rangle_{\text{rest}} \\
 &\mapsto |0'\rangle_Q (b_0^\dagger |\emptyset\rangle_i) |\phi_0\rangle_{\text{rest}} + |1\rangle_Q (b_1^\dagger |\emptyset\rangle_i) |\phi_1\rangle_{\text{rest}} \\
 &\mapsto |0\rangle_Q (b_0^\dagger |\emptyset\rangle_i) |\phi_0\rangle_{\text{rest}} + |1\rangle_Q (b_1^\dagger |\emptyset\rangle_i) |\phi_1\rangle_{\text{rest}}
 \end{aligned}$$



data qubit states: $|0\rangle_i \equiv b_0^\dagger |\emptyset\rangle_i$ and $|1\rangle_i \equiv b_1^\dagger |\emptyset\rangle_i$

dual-rail implementation of CZ gate



$$\text{CZ :} \quad \begin{aligned} |00\rangle &\mapsto |00\rangle \\ |01\rangle &\mapsto |01\rangle \\ |10\rangle &\mapsto |10\rangle \\ |11\rangle &\mapsto -|11\rangle \end{aligned}$$





preparing cluster states

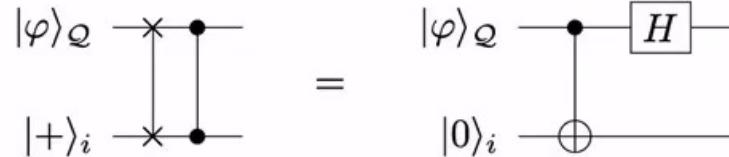
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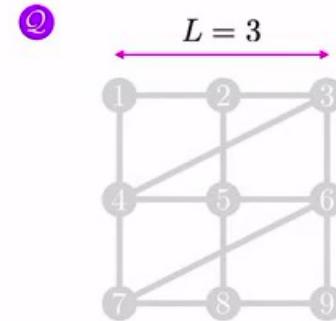
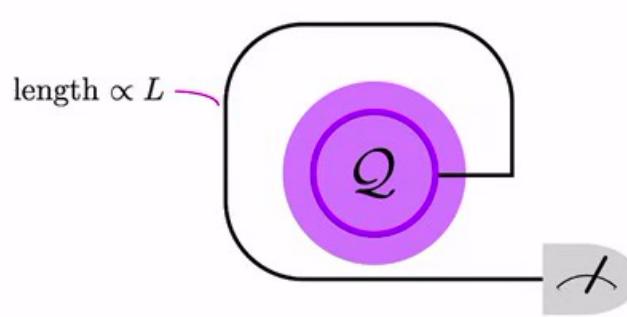
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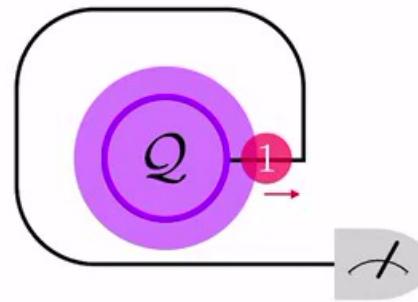
fix:



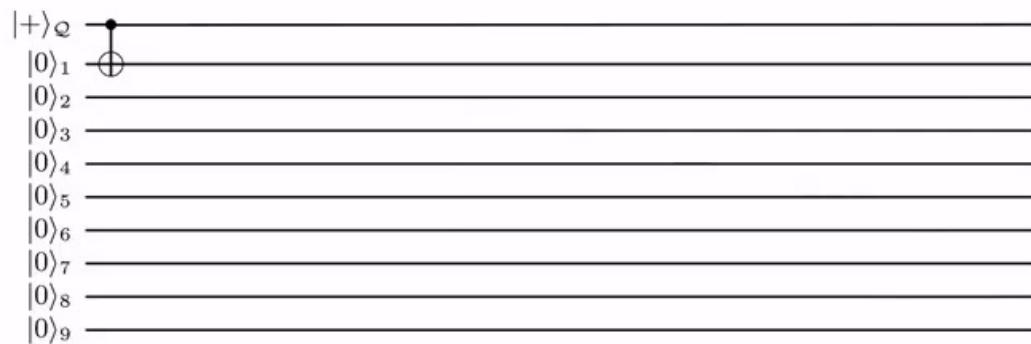
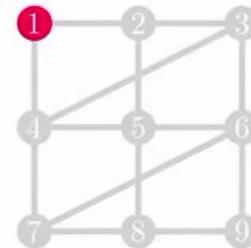


$|+\rangle_{\mathcal{Q}}$ _____
 $|0\rangle_1$ _____
 $|0\rangle_2$ _____
 $|0\rangle_3$ _____
 $|0\rangle_4$ _____
 $|0\rangle_5$ _____
 $|0\rangle_6$ _____
 $|0\rangle_7$ _____
 $|0\rangle_8$ _____
 $|0\rangle_9$ _____



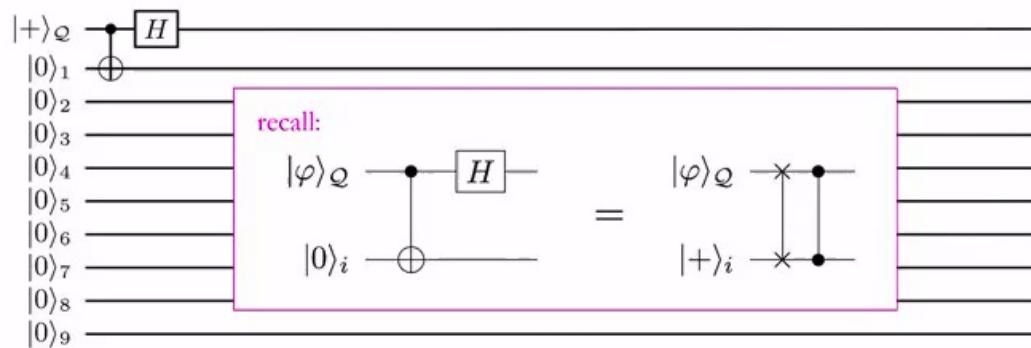
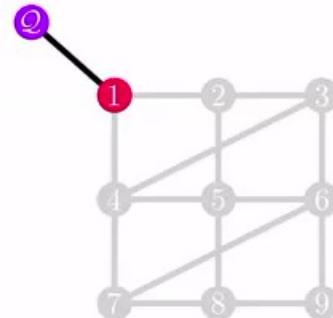
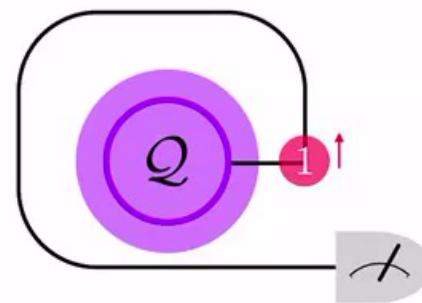
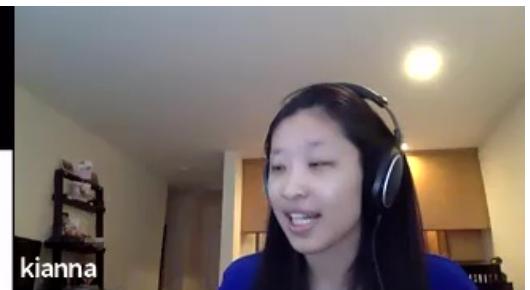


\mathcal{Q}

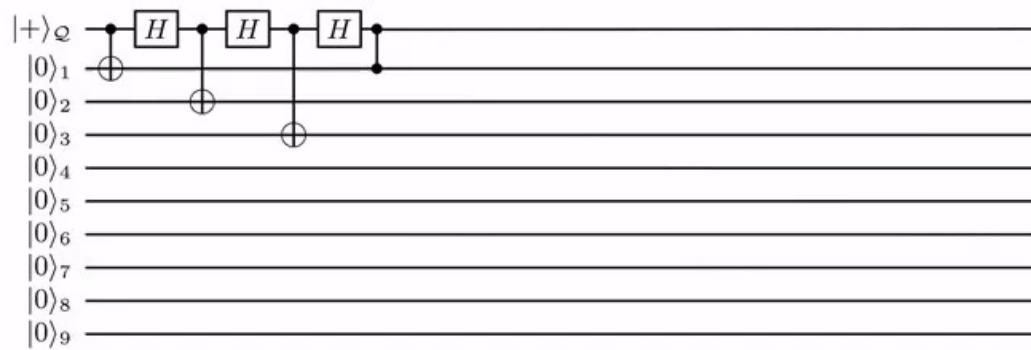
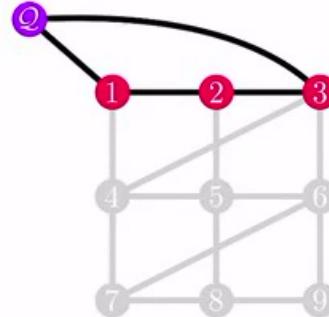
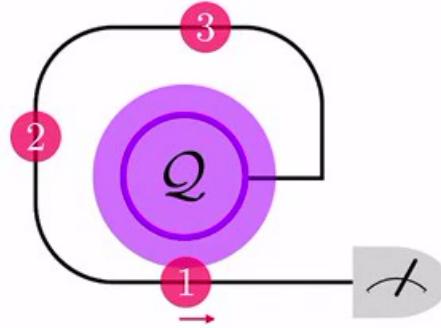


$$\text{"CNOT": } (\alpha|0\rangle + \beta|1\rangle)_Q |\emptyset\rangle_i \mapsto \alpha|0\rangle_Q|0\rangle_i + \beta|1\rangle_Q|1\rangle_i$$



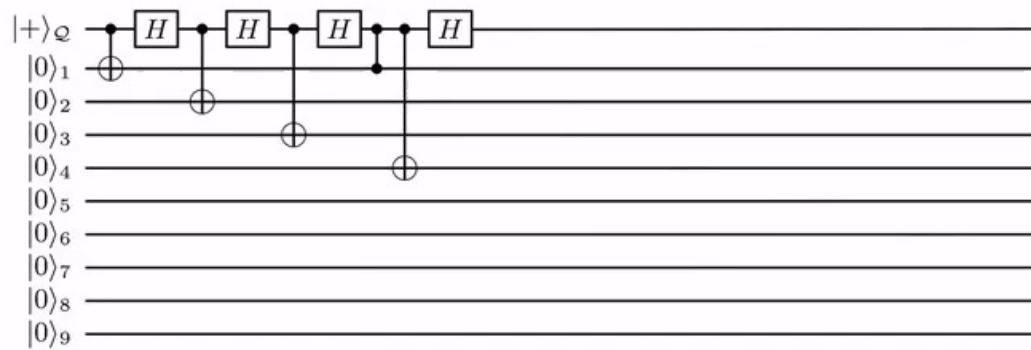
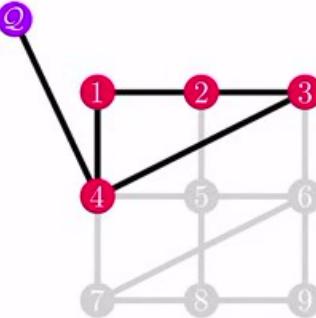
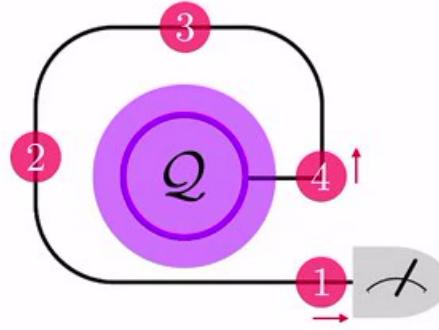


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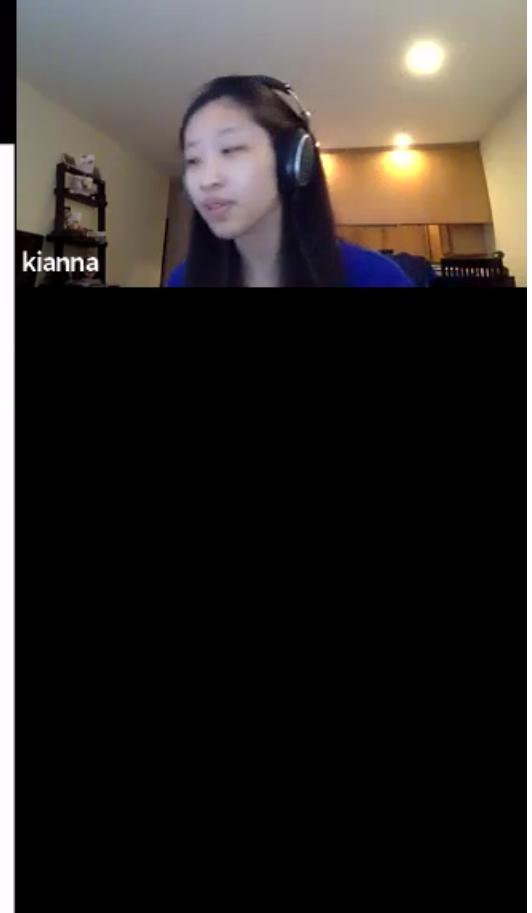


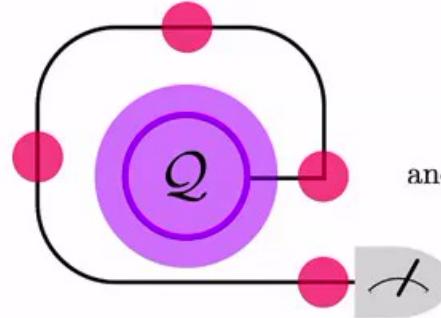
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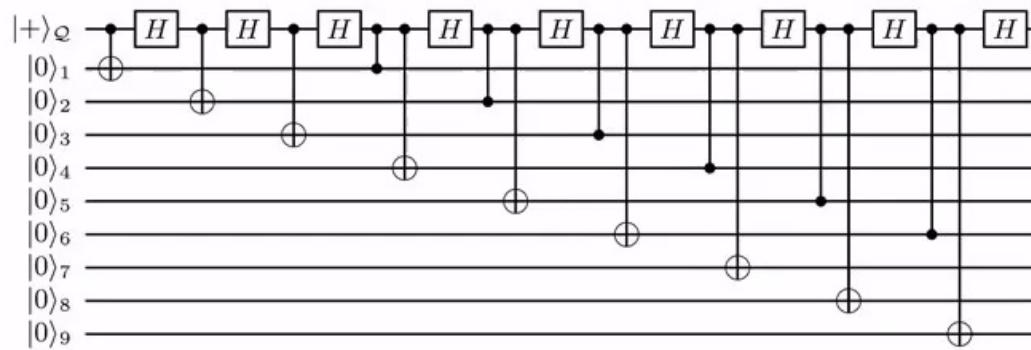
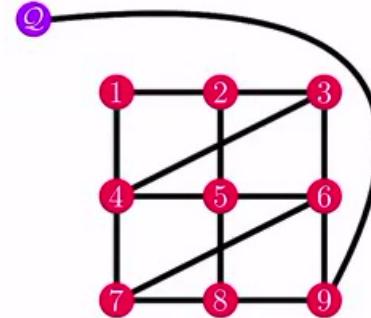


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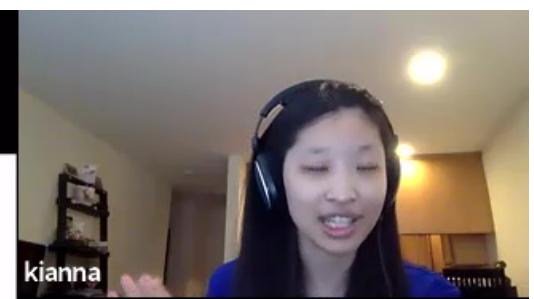
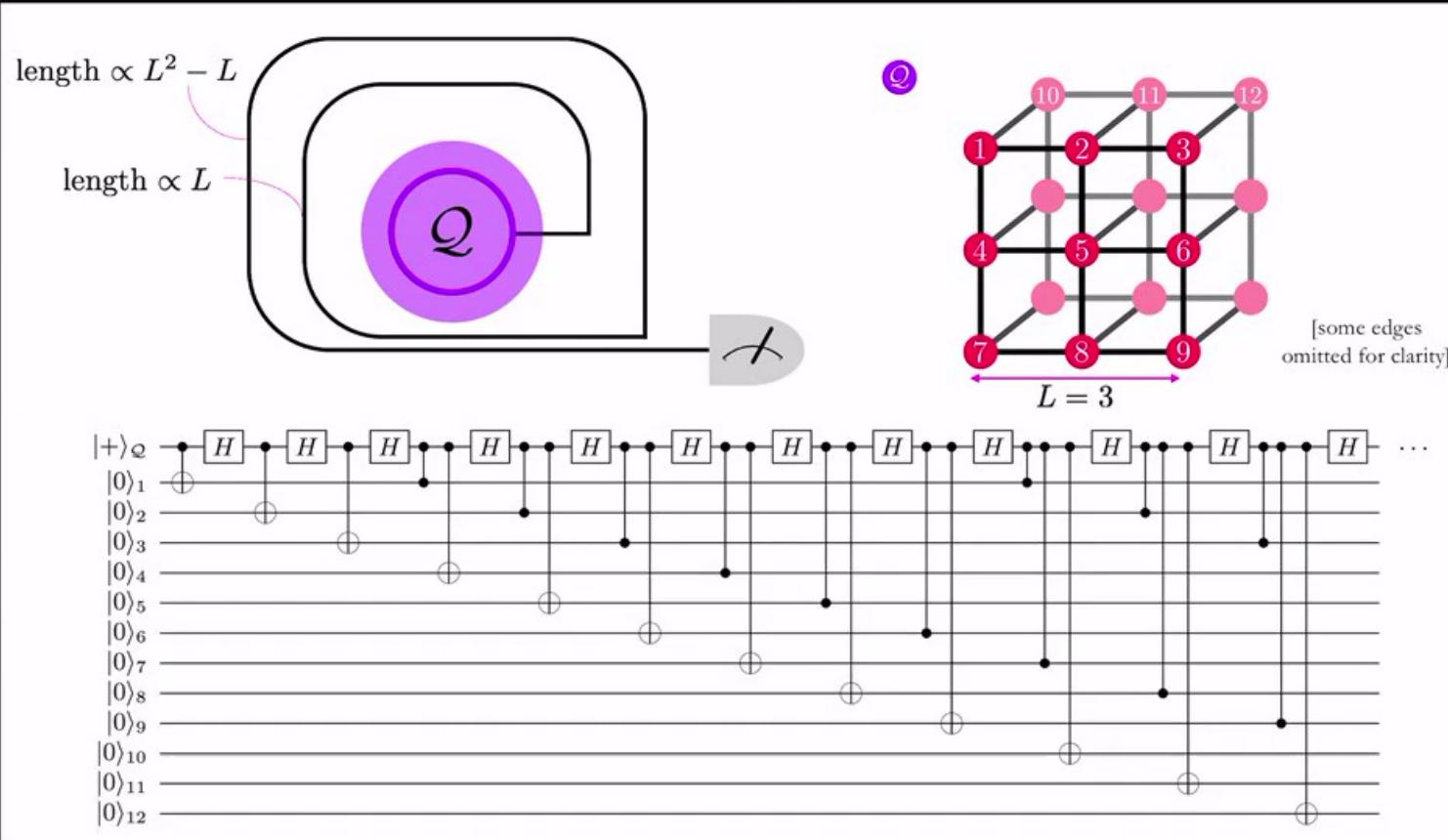


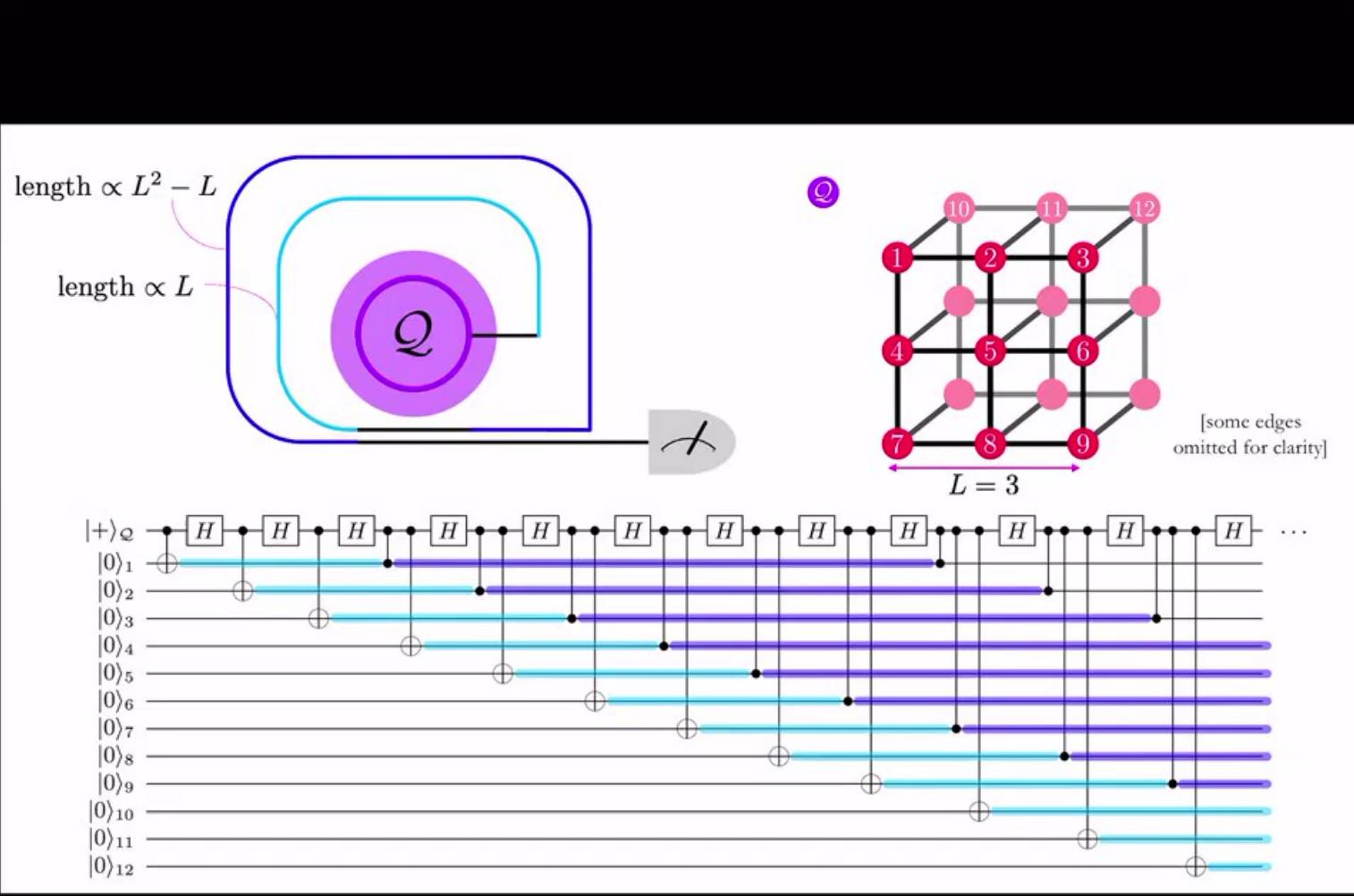
and so on...



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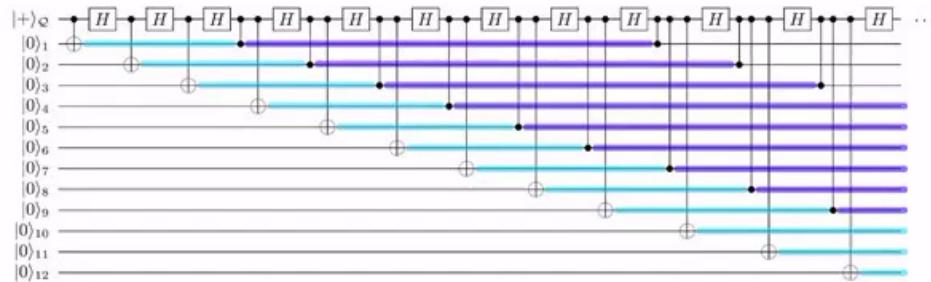


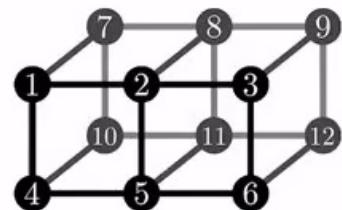
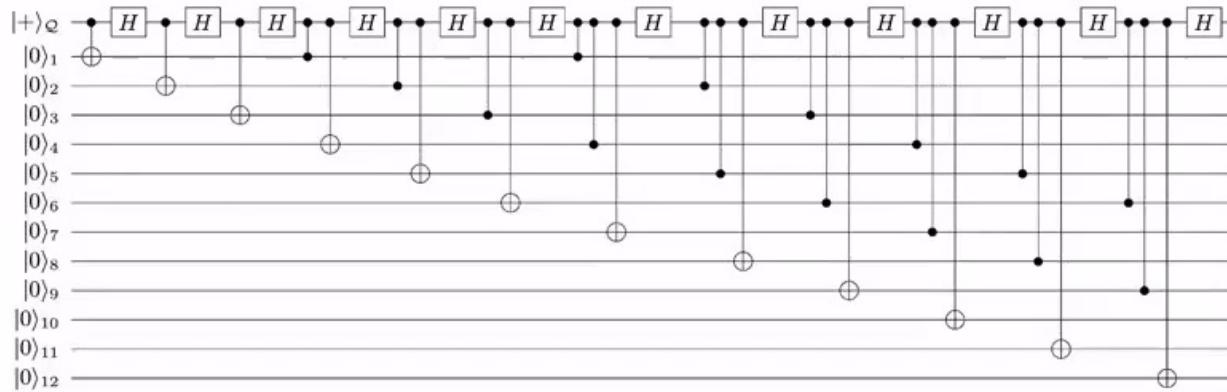


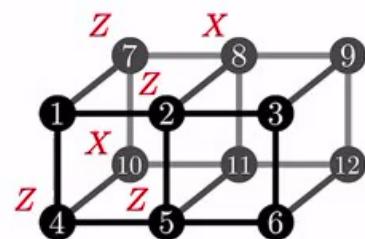
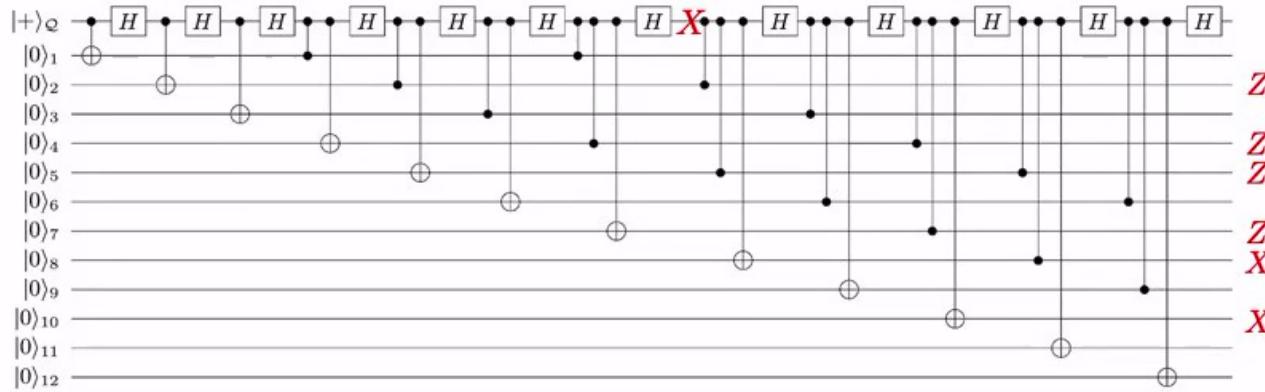
potential concerns

kianna

1. propagation of circuit-level errors

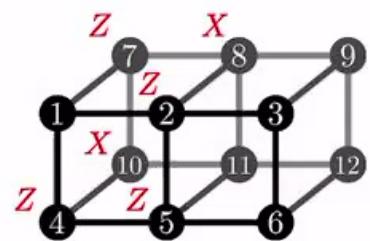
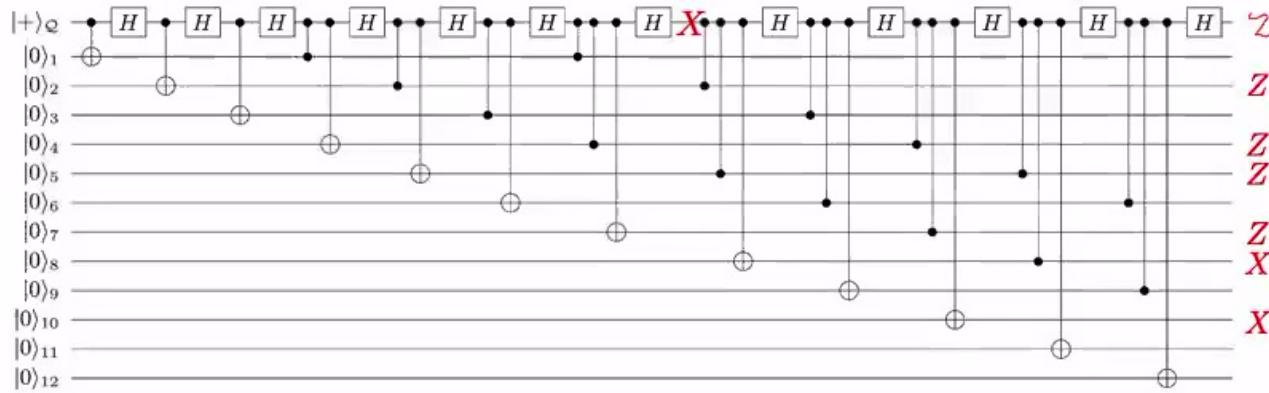


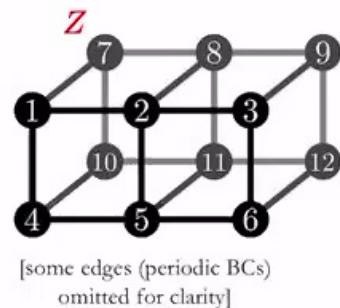
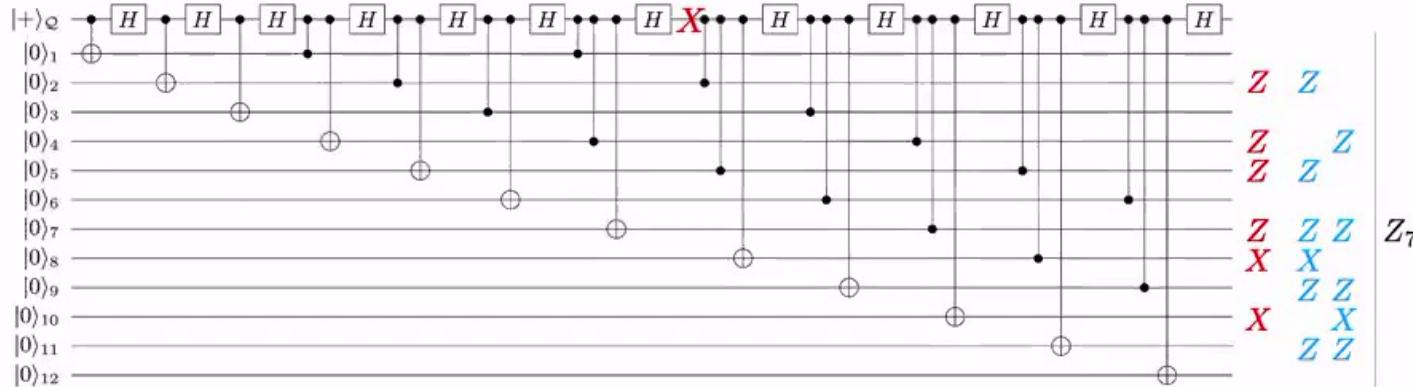




[some edges (periodic BCs)
omitted for clarity]







cluster state stabiliser generators:

$$S_i = X_i \bigotimes_{j \in N(i)} Z_j, \quad N(i) := \{j : (i, j) \in E\}$$

for this lattice,

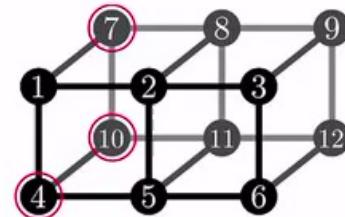
e.g., $S_8 = X_8 Z_2 Z_5 Z_7 Z_9 Z_{11}$

$S_{10} = X_{10} Z_4 Z_7 Z_9 Z_{11}$



all effective errors are local!

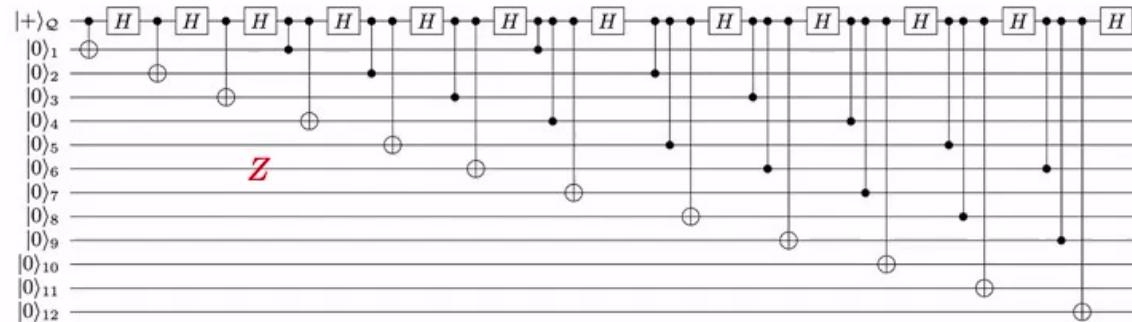
claim: *any* single-qubit circuit-level error \Rightarrow error supported within $\{i\} \cup N(i)$ on the prepared cluster state, for some data qubit i



e.g., $S \subset \{i\} \cup N(i)$ for $i = 10$

claim: single-qubit circuit-level error \Rightarrow error within $\{i\} \cup N(i)$

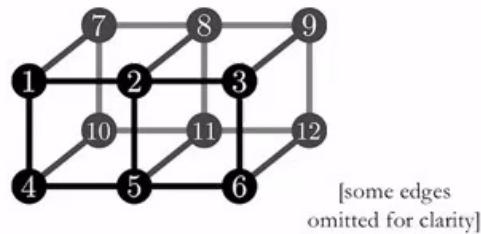
proof idea:



key observations:

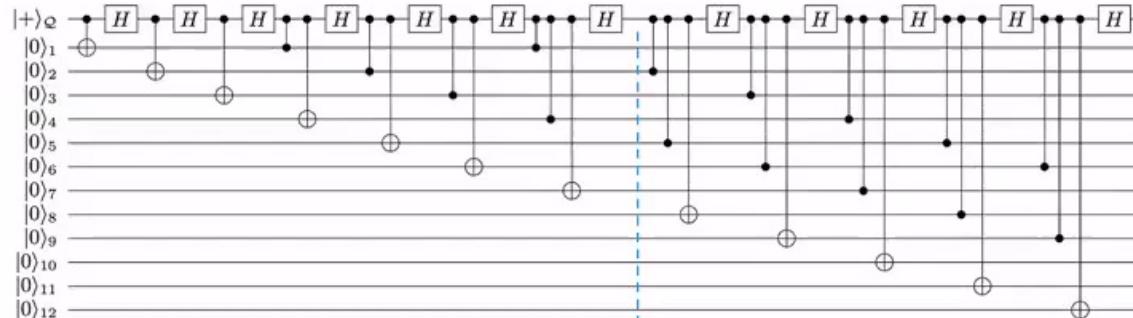
1. Z_i error \Rightarrow no effect, or Z_i on final state
($Z|0\rangle = |0\rangle$; Z commutes with controlled- Z)

Q





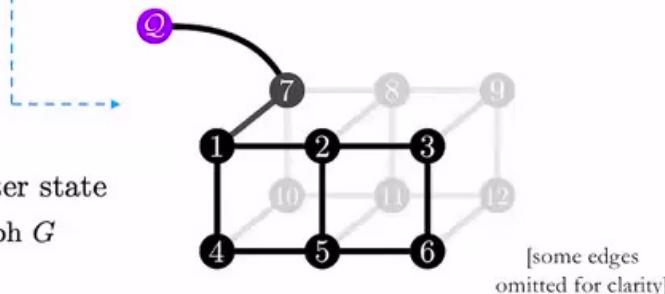
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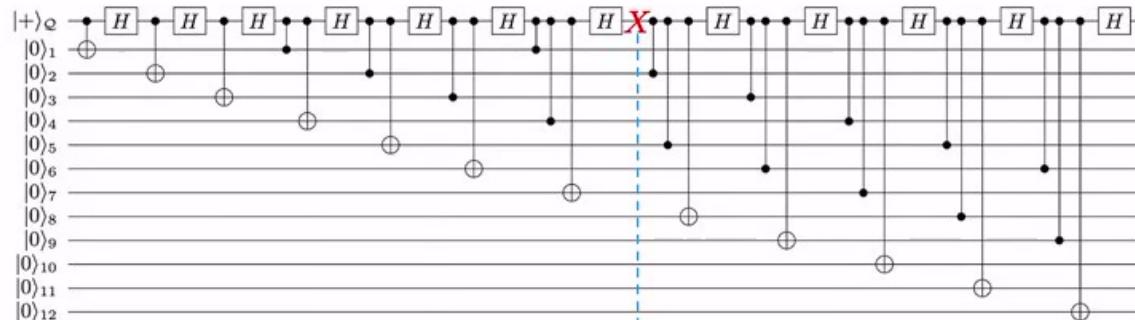
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recall stabilisers: $X_i \bigotimes_{j \in N(i)} Z_j$





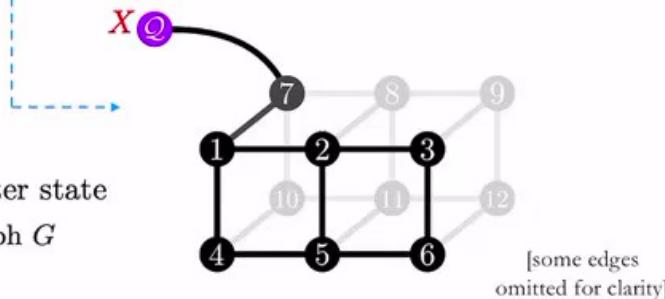
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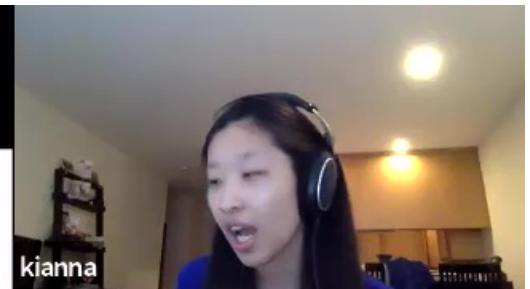


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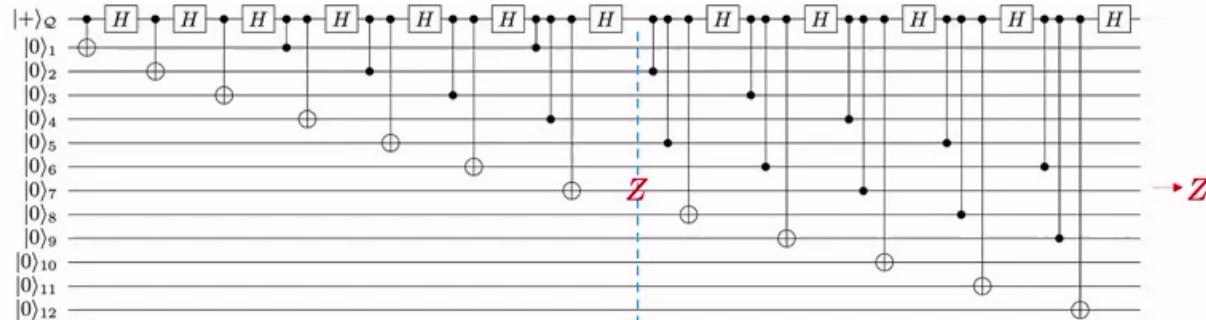
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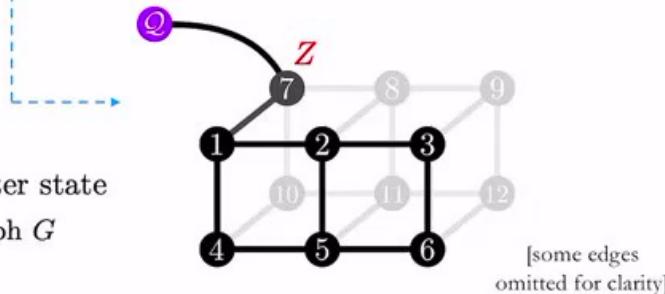
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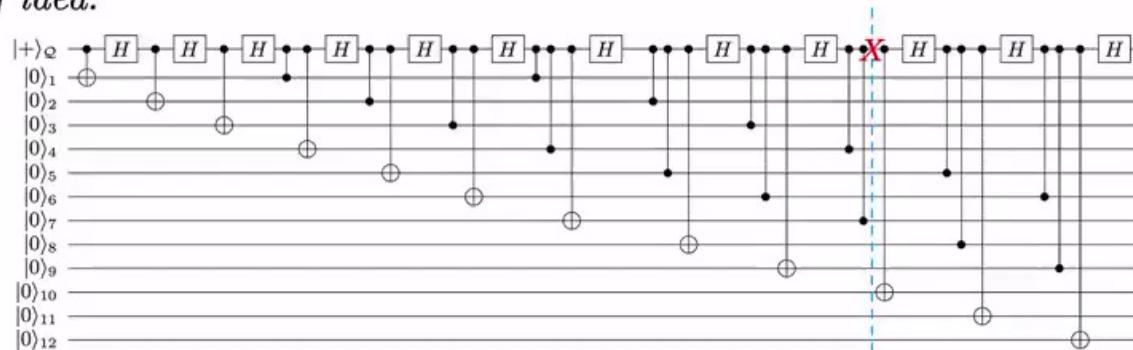
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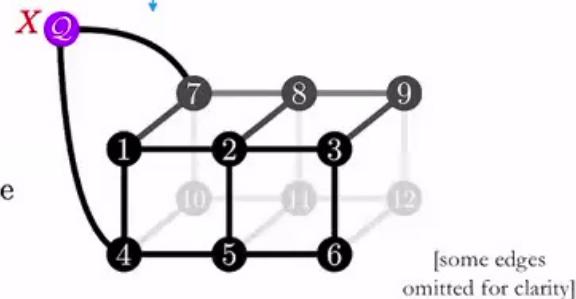
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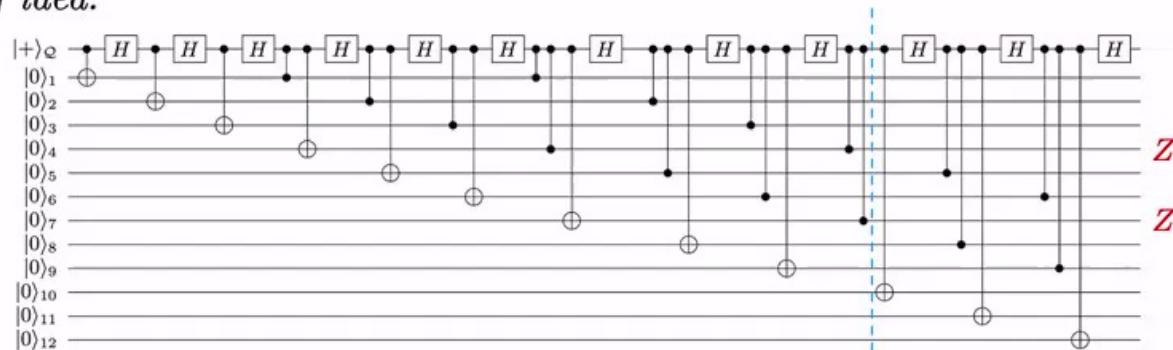
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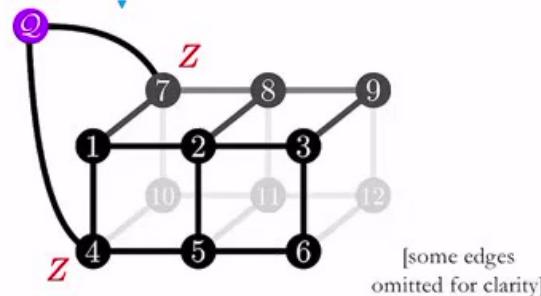
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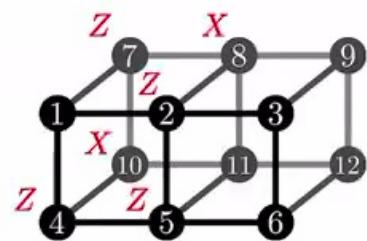
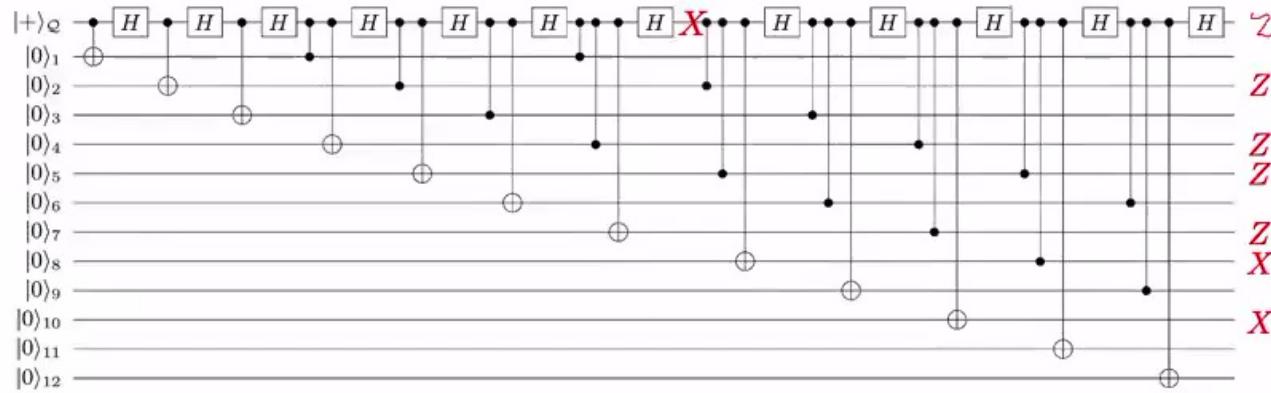


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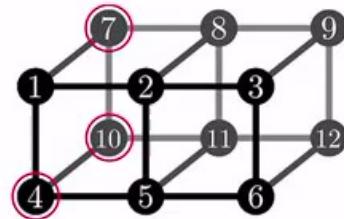


[some edges (periodic BCs)
omitted for clarity]

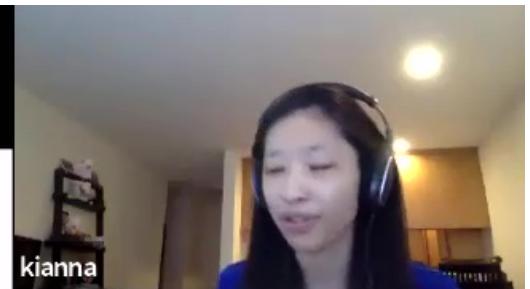


all effective errors are local!

claim: *any* single-qubit circuit-level error \Rightarrow error supported within $\{i\} \cup N(i)$ on the prepared cluster state, for some data qubit i

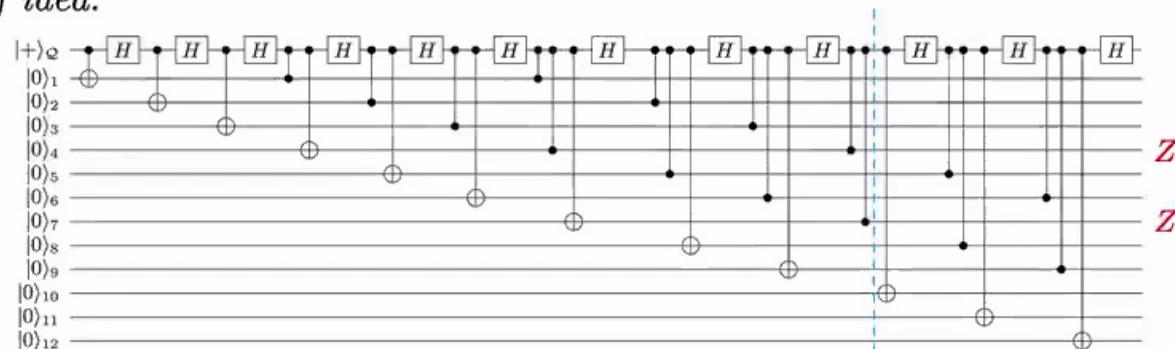


more generally, m single-qubit circuit-level errors $\Rightarrow m$ local errors on the prepared cluster state



claim: single-qubit circuit-level error \Rightarrow error within $\{i\} \cup N(i)$

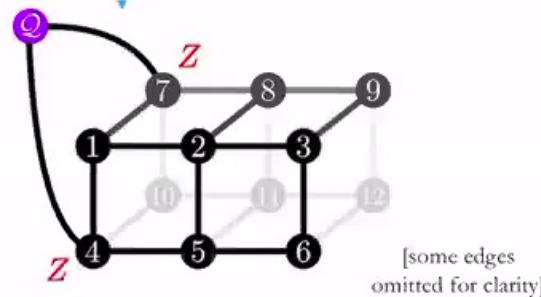
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potential concerns



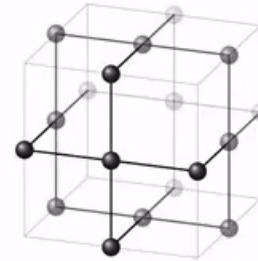
1. propagation of circuit-level errors

effective errors are actually local (\Rightarrow weight $O(1)$ for bcc lattice)

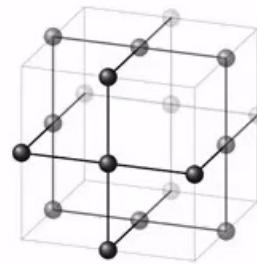
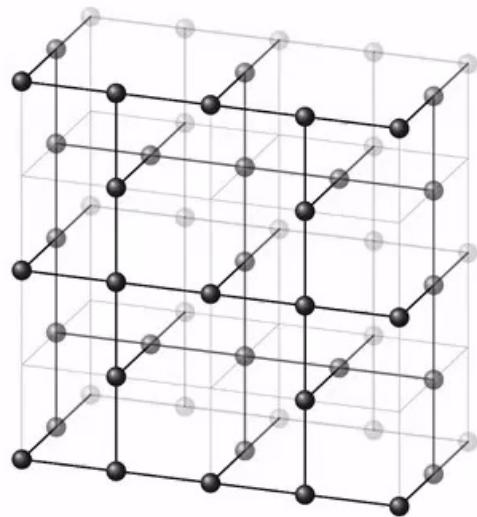
potential concerns

0. cubic lattice → bcc lattice
1. propagation of circuit-level errors

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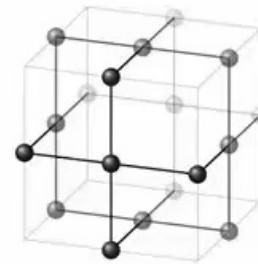
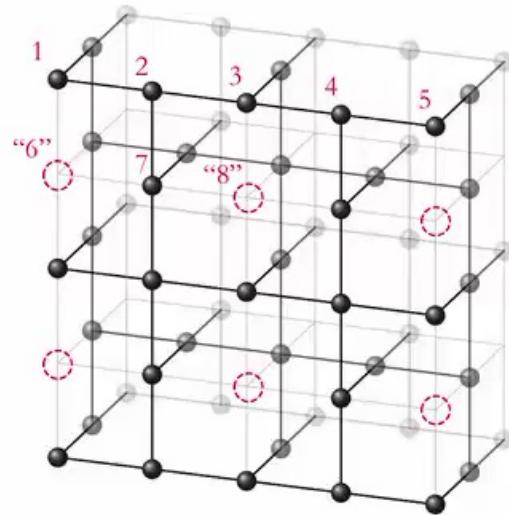


0. cubic lattice → bcc lattice





0. cubic lattice \rightarrow bcc lattice



thresholds

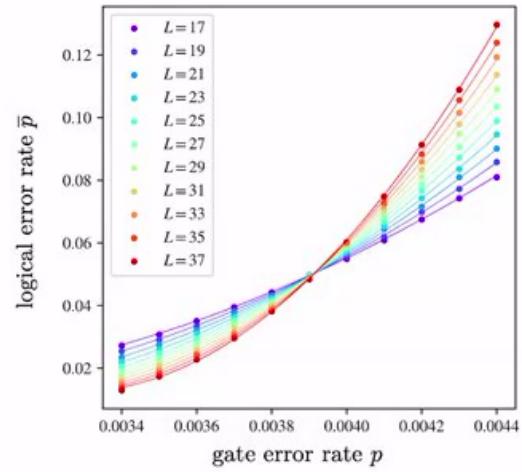
- standard depolarising noise model for “gate errors” (error rate p):
 - single-qubit depolarising noise after each single-qubit operation (gate, measurement, state preparation)
 - two-qubit depolarising noise after each two-qubit gate





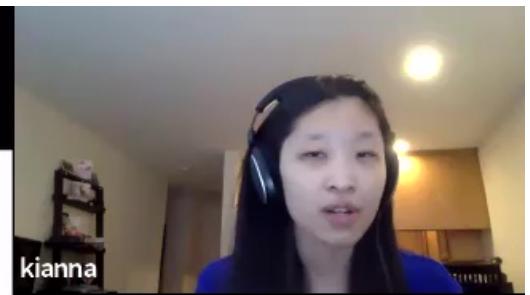
thresholds

- standard depolarising noise model for “gate errors” (error rate p)
- standard MWPM decoder



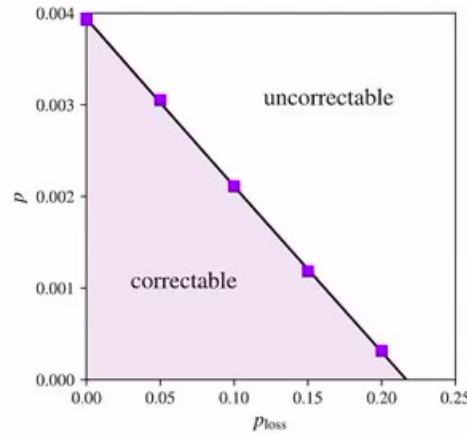
$$\rightarrow p_{\text{th}} \approx 0.39\%$$

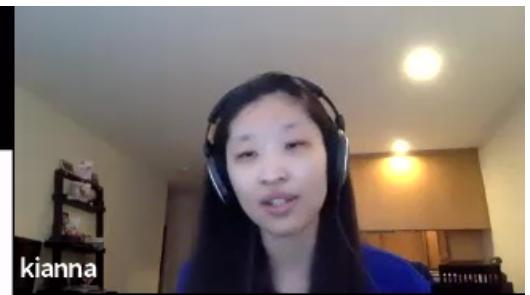
[Raussendorf *et al.*]: $\approx 0.58\%$
(cZ circuit)



thresholds

- standard depolarising noise model for “gate errors” (error rate p)
+ each qubit lost with probability p_{loss}
- generalised MWPM decoder of [Barrett & Stace '10]





potential concerns

0. cubic lattice \rightarrow bcc lattice

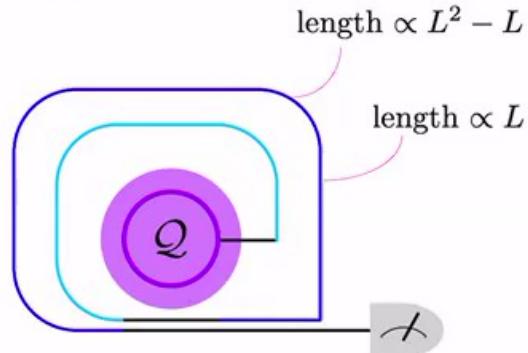
$G_{\text{bcc}} \subset G_{\text{cubic}}$, so omit qubits in $G_{\text{cubic}} \setminus G_{\text{bcc}}$

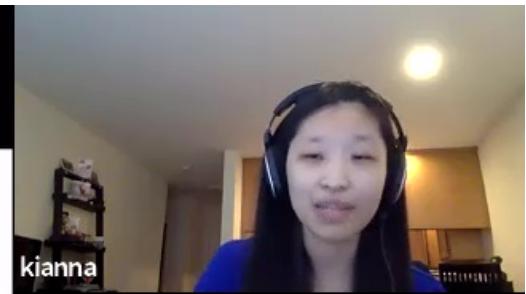
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effective errors are actually local (\Rightarrow weight $O(1)$ for bcc lattice)

2. noisy delay lines \rightarrow errors on idle qubits

total delay line length $\propto L^2$





potential concerns

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total delay line length $\propto L^2$

logical error probability $\sim \exp(-\sqrt{\text{delay line error rate}})$

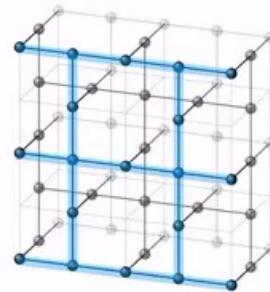


delay line errors

η = delay line error rate

total delay line error probability $\approx \eta L^2$

code distance $\propto L$



$$d = \frac{1}{2}(L + 1)$$

↳ for each η and gate error rate p , \exists an optimal logical error rate \bar{p}_*

for fixed gate error rate p , expect \bar{p}_* to scale with η as

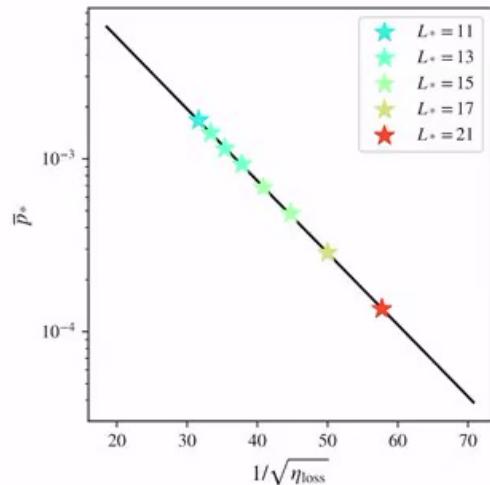
$$\bar{p}_* \propto \exp(-c\sqrt{\eta})$$



delay line errors

e.g.,

- fix gate error $p = 10^{-3}$
- suppose $\eta = \eta_{\text{loss}}$



→ “break-even point” (at which $\bar{p}_* = p = 10^{-3}$)
occurs at $\eta_{\text{loss}} = 7.4 \times 10^{-4}$

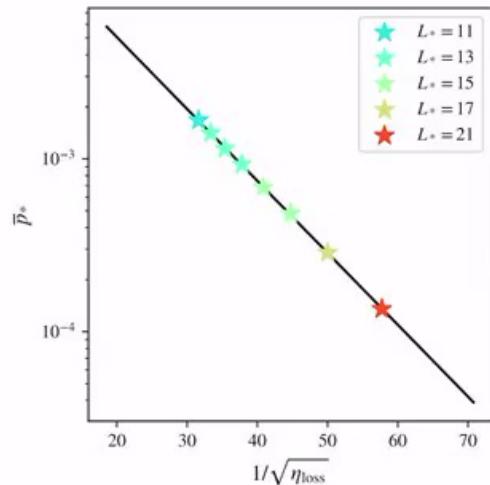
optical fibres: $\eta_{\text{loss}} \approx 9.6 \times 10^{-4}$
(assuming 17 ns between photons)



delay line errors

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optical fibres: $\eta_{\text{loss}} \approx 9.6 \times 10^{-4}$
(assuming 17 ns between photons)

dephasing

“break-even point” occurs at
 $\eta_{\text{dephasing}} = 6.5 \times 10^{-5}$

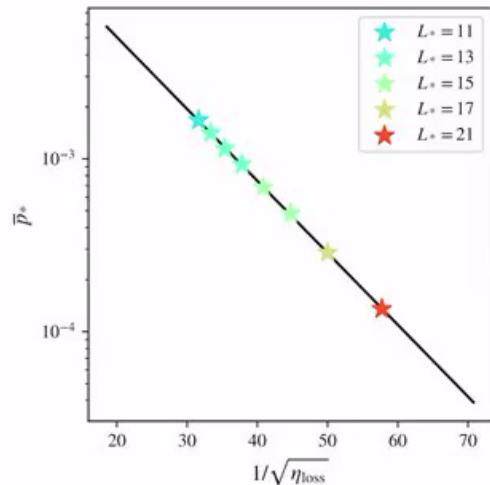
phononic waveguide: $\eta_{\text{dephasing}} \approx 4.8 \times 10^{-4}$
(160 ns between phonons)



delay line errors

e.g.,

- fix gate error $p = 10^{-3}$
- suppose $\eta = \eta_{\text{loss}}$



→ “break-even point” (at which $\bar{p}_* = p = 10^{-3}$)
occurs at $\eta_{\text{loss}} = 7.4 \times 10^{-4}$

optical fibres: $\eta_{\text{loss}} \approx 9.6 \times 10^{-4}$
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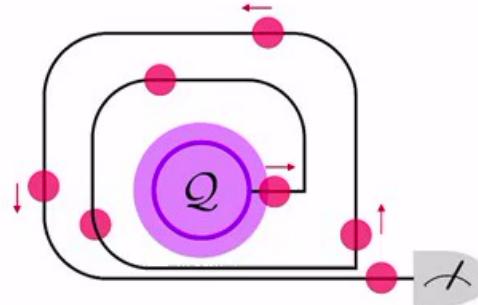
$$\eta_{\text{loss}} = 1.4 \times 10^{-4} \rightarrow \bar{p}_* = 10^{-5} \quad (L_* \approx 30)$$

$$\eta_{\text{loss}} = 2.4 \times 10^{-5} \rightarrow \bar{p}_* = 10^{-10} \quad (L_* \approx 75)$$

$$\eta_{\text{loss}} = 9.5 \times 10^{-6} \rightarrow \bar{p}_* = 10^{-15} \quad (L_* \approx 115)$$

summary

1. constant component overhead
 2. local error propagation
- [open questions: can similar ideas be exploited in other contexts?
better characterisation of these circuits?]
3. exploit relative strengths of emitter and propagating qubits --- very different control and noise characteristics
 4. FTQC could potentially be achieved through incremental improvements to a small number of key components





thanks for listening!
😊

arXiv:2011.08213
✉️ kianna@stanford.edu