Title: Entanglement entropy in expanding spacetimes

Speakers: Wilke van der Schee

Series: Quantum Fields and Strings

Date: April 27, 2021 - 2:00 PM

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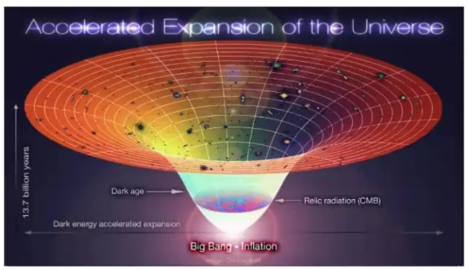
Abstract: This talk will be about entanglement entropy in empty 4-dimensional de Sitter spacetime of a non-conformal QFT [1]. I will first briefly describe the set-up and show how a hydrodynamic plasma dilutes and falls out of equilibrium due to expansion towards empty de Sitter spacetime. Interestingly, in the empty setting we can show that extremal surfaces in the holographic dual of spherical entangling regions on the boundary QFT probe beyond the dual event horizon if and only if the entangling region is larger than the cosmological horizon.

[1] Jorge Casalderrey-Solana, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space, 2011.08194

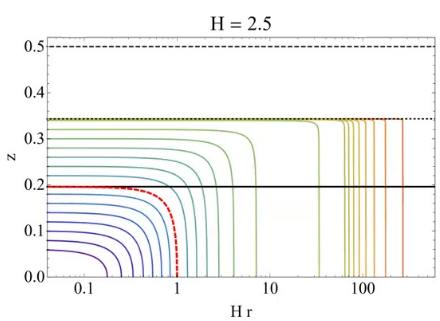
ENTANGLEMENT ENTROPY IN EXPANDING SPACETIMES

HOLOGRAPHIC QFTS ON CURVED BACKGROUNDS



With Jorge Casalderrey-Solana, Christian Ecker and David Mateos

Reference: 2011.08194 (JHEP)



Wilke van der Schee

String seminar Perimeter Institute
Waterloo (virtual), 27 April 2021

OUTLINE

Set-up, hydrodynamics and empty de Sitter space-time

- Non-conformal model
- Expansion driven decay towards empty de Sitter

Entanglement in de Sitter space-time

- Event and apparent horizons
- From boundary cosmological horizon to entanglement horizon/shadow

Recent results / outlook on backreacted de Sitter (semi-classical)

NON-CONFORMAL MODEL ON DE SITTER$_4$

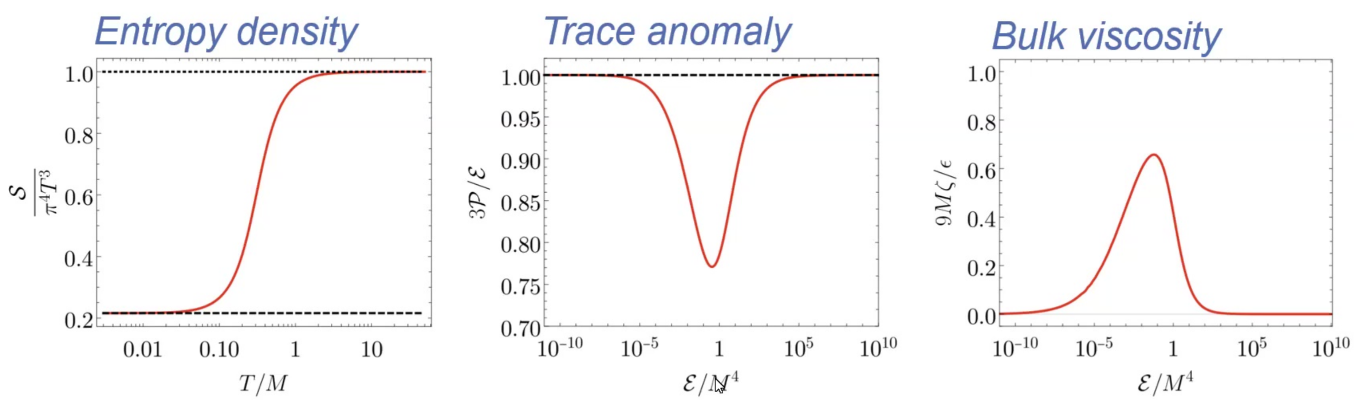
De Sitter is conformally flat: almost trivial for CFT

- Break scale invariance by V(Φ) with source M=1:

$$S = \frac{2}{8\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{4}R[g] - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$$L^2 V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

- Leads to non-trivial EOS and bulk viscosity (no shear considered):



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HOLOGRAPHIC RENORMALISATION

$$ds_{\rm b}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 . \quad S_0(t) = e^{Ht}$$

$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2 , \quad \phi = \phi(r,t) ,$$

Action needs (scheme-dependent) counter-terms:

$$S_{\rm ct} = \frac{1}{8\pi G}\int \mathrm{d}^4x\sqrt{-\gamma}\left[\left(-\frac{1}{8}R[\gamma] - \frac{3}{2} - \frac{1}{2}\phi^2\right) + \frac{1}{2}(\log\rho)\,\mathcal{A} + \left(\alpha\mathcal{A} + \beta\phi^4\right)\right] ,$$

$$\mathcal{A} = \mathcal{A}_g + \mathcal{A}_\phi , \qquad \mathcal{A}_g = \frac{1}{16}(R^{ij}R_{ij} - \frac{1}{3}R^2) , \qquad \mathcal{A}_\phi = -\frac{\phi^2}{12}R$$

Leads to an ambiguity in the stress-tensor:

$$\begin{aligned}\mathcal{E}(t) = &-\frac{3a_{(4)}(t)}{4} - M\bar{\phi}_{(2)}(t) + \frac{3S_0'(t)^4}{16S_0(t)^4} + M^2\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{2S_0''(t)}{3S_0(t)}\right)\\ &- M^2\alpha\frac{S_0'(t)^2}{2S_0(t)^2} - M^4\left(\beta - \frac{7}{36}\right) ,\end{aligned}$$

$$\begin{aligned}\mathcal{P}(t) = &-\frac{a_{(4)}(t)}{4} + \frac{1}{3}M\bar{\phi}_{(2)}(t) + \frac{S_0'(t)^2\left(S_0'(t)^2 - 4S_0(t)S_0''(t)\right)}{16S_0(t)^4}\\ &- \frac{M^2}{3}\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{13S_0''(t)}{12S_0(t)}\right) + M^2\alpha\left(\frac{S_0'(t)^2}{6S_0(t)^2} + \frac{S_0''(t)}{3S_0(t)}\right) + M^4\left(\beta - \frac{5}{108}\right)\end{aligned}$$

α and β encode scheme dependencies (cosmological constant); fixed such that late time solution has vanishing energy

Ambiguities come in at order H^2

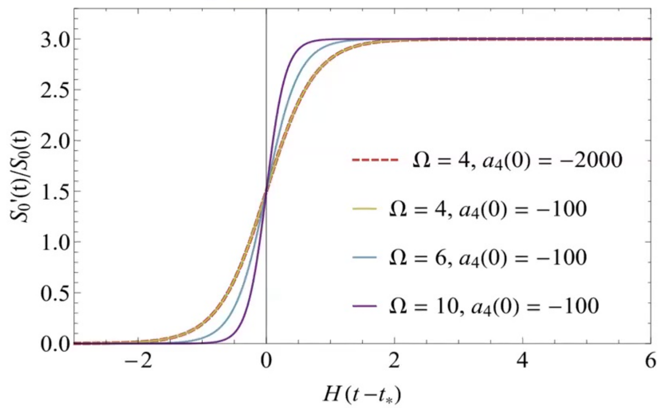
HOW WE SET UP A STATE

Non-trivial boundary metric: $ds_b^2 = -dt^2 + S_0(t)^2 d\vec{x}^2 . \quad S_0(t) = e^{Ht}$

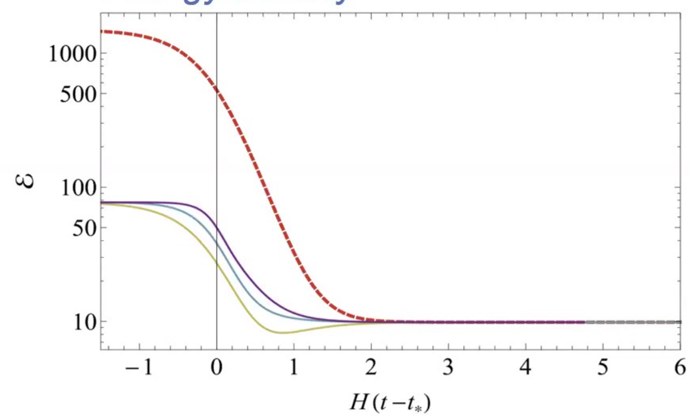
Start with thermal (high-temperature) state in flat space

- Quench system by suitable fast tanh to constant Hubble parameter
- Energy density decreases towards final `vacuum energy' (VE)
- Final (Bunch-Davis)-VE is ambiguous → chose scheme with zero VE

Hubble parameter



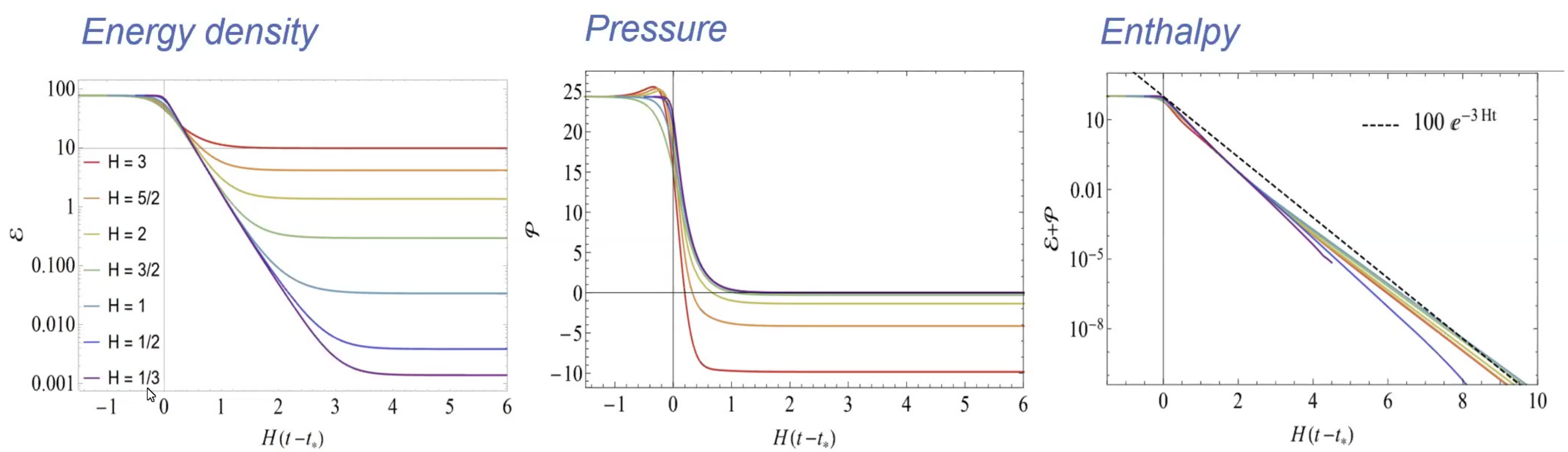
Energy density



TIME EVOLUTION OF THE PROTOCOL

Evolution of stress tensor for different Hubble constants

- Energy density decreases towards VE (can be renormalised to zero)
- Pressure decreases, changes sign and becomes –VE
- Enthalpy is scheme independent; decays due to expansion



THE APPROACH TOWARDS HYDRODYNAMICS

$$ds_b^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 . \quad S_0(t) = e^{Ht}$$

Comparing with the hydrodynamic constituent relations:

$$T_\perp^{\mu\nu} = P(\varepsilon)\Delta^{\mu\nu} - \eta(\varepsilon)\sigma^{\mu\nu} - \zeta(\varepsilon)\Delta^{\mu\nu}(\nabla\cdot u) ,$$
$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Symmetric set-up: only non-trivial part is bulk viscosity:

$$\Delta\mathcal{P}^{\text{hydro}}(t) \equiv \Delta p_{\text{eq}}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2) ,$$

A subtlety: EOS and viscosity computed in flat space;
what is the energy density in de Sitter space?

We decided to fix α such that late time solution has zero energy density

(in any case: ambiguity is order H^2)
(also, note that α depends on $S_0(t)$ / H for our choice)

THE APPROACH TOWARDS HYDRODYNAMICS

$$\Pi = -\zeta\,(\nabla\cdot u) + \zeta\tau_\Pi D\,(\nabla\cdot u) + \xi_1\sigma^{\mu\nu}\sigma_{\mu\nu} + \xi_2\,(\nabla\cdot u)^2 + \xi_3\Omega^{\mu\nu}\Omega_{\mu\nu} + \xi_4\nabla_\mu^\perp \ln s\,\nabla_\perp^\mu \ln s + \xi_5 R + \xi_6 u^\alpha u^\beta R_{\alpha\beta}\,.$$

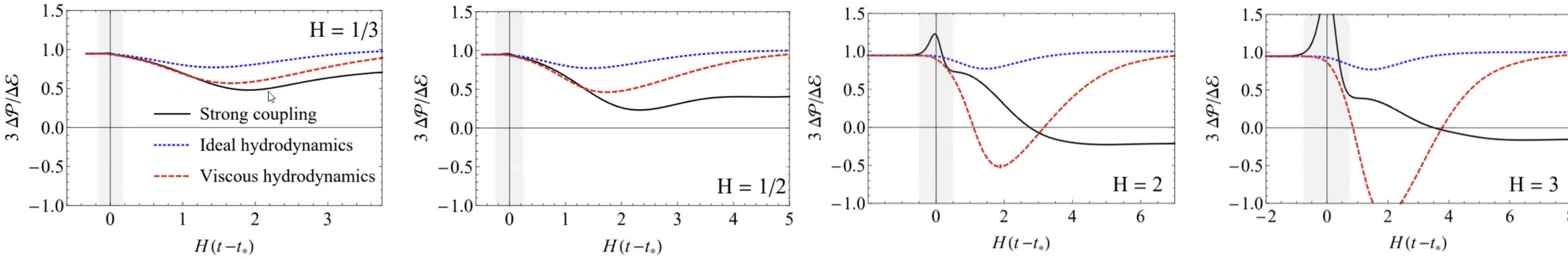
Non-trivial hydrodynamic prediction

$$\Delta\mathcal{P}^{\rm hydro}(t) \equiv \Delta p_{\rm eq}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2)\,,$$

- Conjecture: ambiguities cancelled by higher order transport coefficient ξ_5

Results

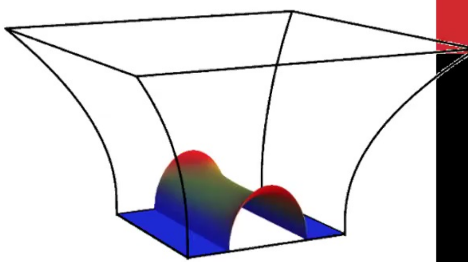
- Viscous hydro works for small H (gradients). Negative \`EOS' for large H.



Paul Romatschke, Relativistic Viscous Fluid Dynamics and Non-Equilibrium Entropy (2009)

BLACK HOLE THERMODYNAMICS

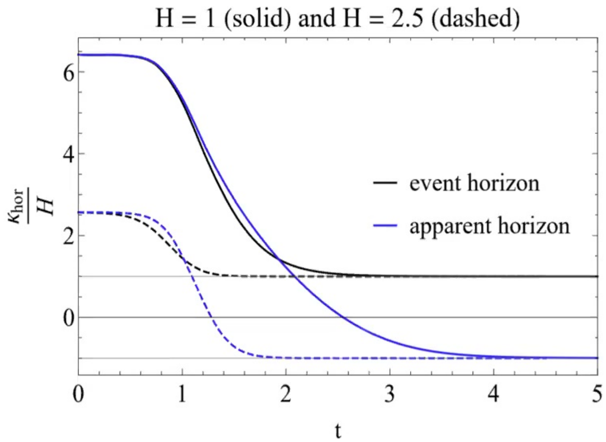
$$ds^2 = -A(r,t)dt^2 + 2drdt + S(r,t)^2 d\vec{x}^2$$



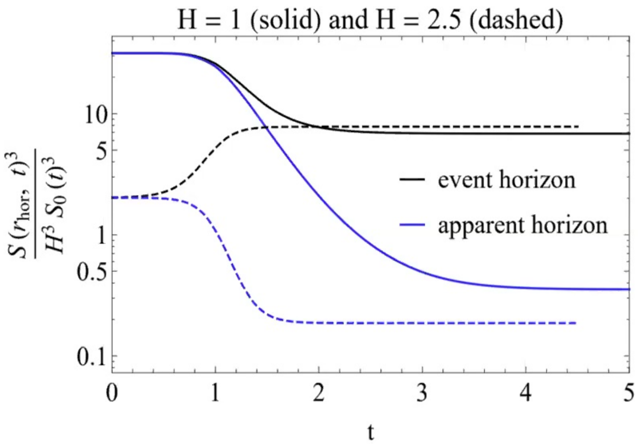
Keep track of bulk event and apparent horizons (EF coordinates)

- Dynamical setting: horizons not coincide at late times:
- Surface gravities can be shown analytically: $\kappa_{\rm EH} = -\kappa_{\rm AH} = H$
  EH confirms Hawking's temperature in de Sitter
- Area density apparent horizon vanishes for conformal theory

Surface gravity

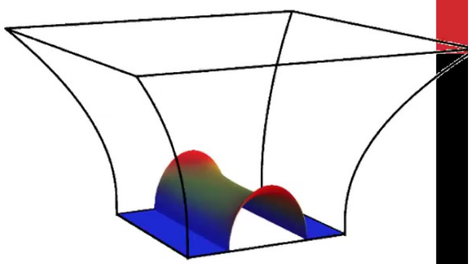


Area densities



Willy Fischler, Sandipan Kundu and Juan Pedraza, Entanglement and out-of-equilibrium dynamics in holographic models of de Sitter QFTs (2013)
Alex Buchel, Entanglement entropy of N = 2* de Sitter vacuum (2019)

BLACK HOLE THERMODYNAMICS

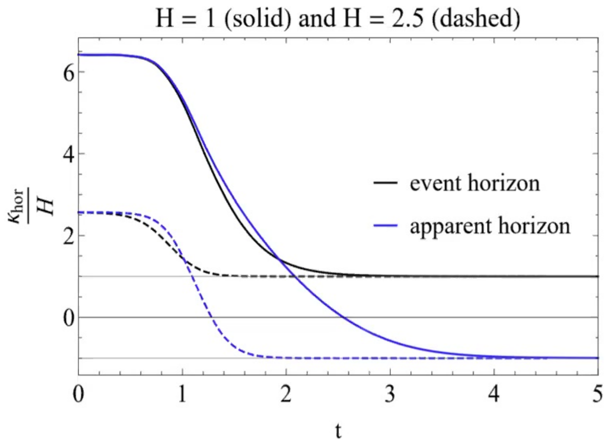


Several interpretational issues

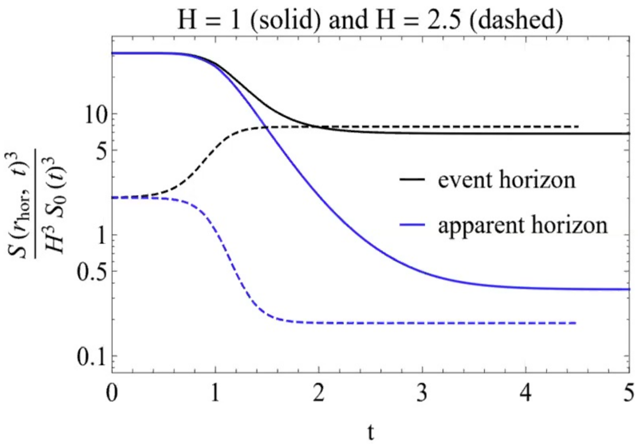
- Expanding space: mapping boundary to bulk horizon not clear
- Apparent horizon: time slicing dependent
- In general: no volume law entropy density expected

Resolution → entanglement entropy is well defined

Surface gravity



Area densities



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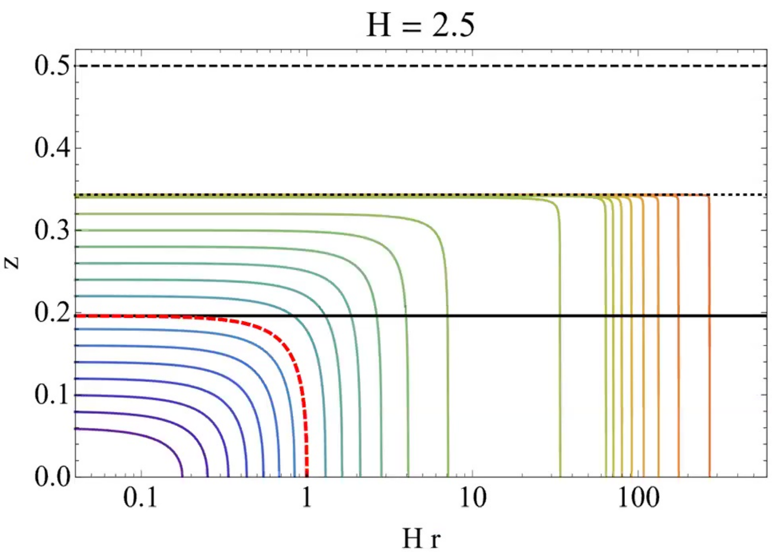
ENTANGLEMENT IN DE SITTER

Extremal surfaces dual to spherical entangling regions:

- Large entangling regions probe beyond event horizon
- A new `entanglement horizon' forms, between AH and EH, with zero surface gravity

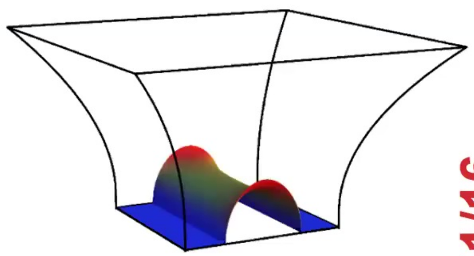


Cosmological horizon

Apparent horizon: $T = -\frac{H}{2\pi}$

Entanglement horizon/shadow: $T = 0$

Event horizon: $T = \frac{H}{2\pi}$

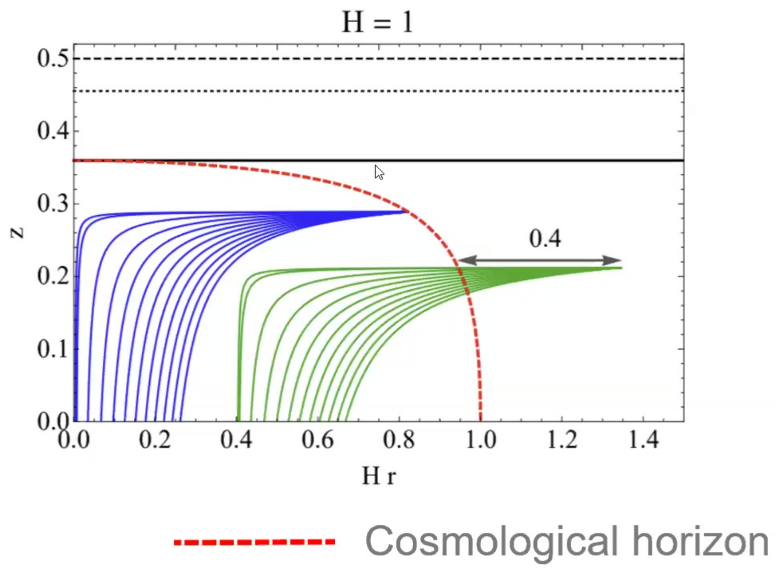


Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

ENTANGLEMENT IN DE SITTER

Extremal surface dual to cosmological horizon:

- Separates points in the bulk from which light can reach the (boundary) origin
- *Boundary cosmological horizon → full bulk cosmological horizon*

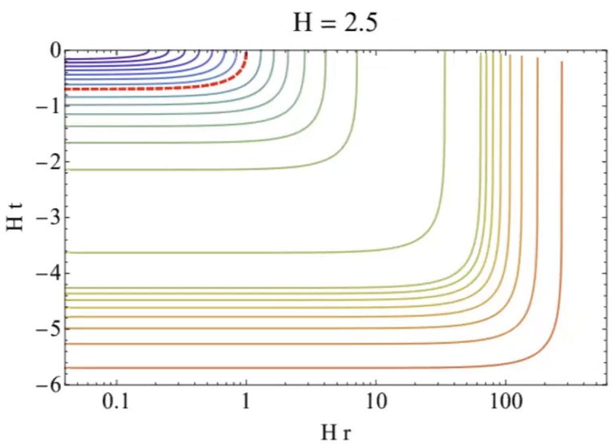


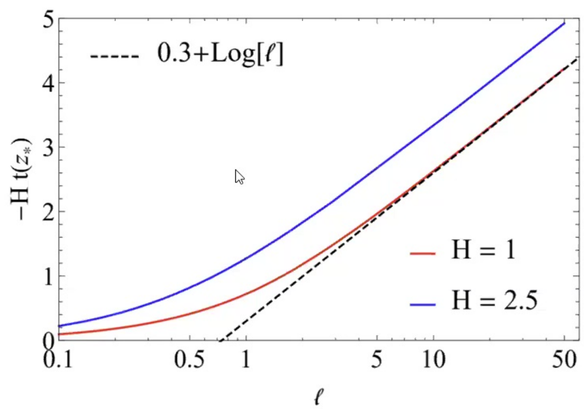
Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

ENTANGLEMENT IN DE SITTER

Extremal surfaces go backward in time

- Time at the deepest point grows exactly as log(l) for large l
- *Implies that `entanglement horizon' contribution has a constant instead of volume law contribution*
- Standard `area law' divergence still applies





Cosmological horizon

OUTLOOK: BOUNDARY GRAVITY

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

Future work: study dynamics including semi-classical gravity:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G_{N,4} \langle T_{\mu\nu} \rangle$$

- Stress-tensor includes possible cosmological constant
- NB: renormalisation counterterms are now physical
- We treat the boundary Newton constant as a (small) parameter

Dynamics of scale factor $S_0(t)$ is now consequence of Friedmann equations

Still working on the best way to initialise the dynamics

- Starting with an after-quench de Sitter result as presented above
  - Not guaranteed that Friedmann implies the same Hubble rate
  - Could tune Λ such that it does
- Start with thermal Minkowski profile and small boundary $G_{N,4}$

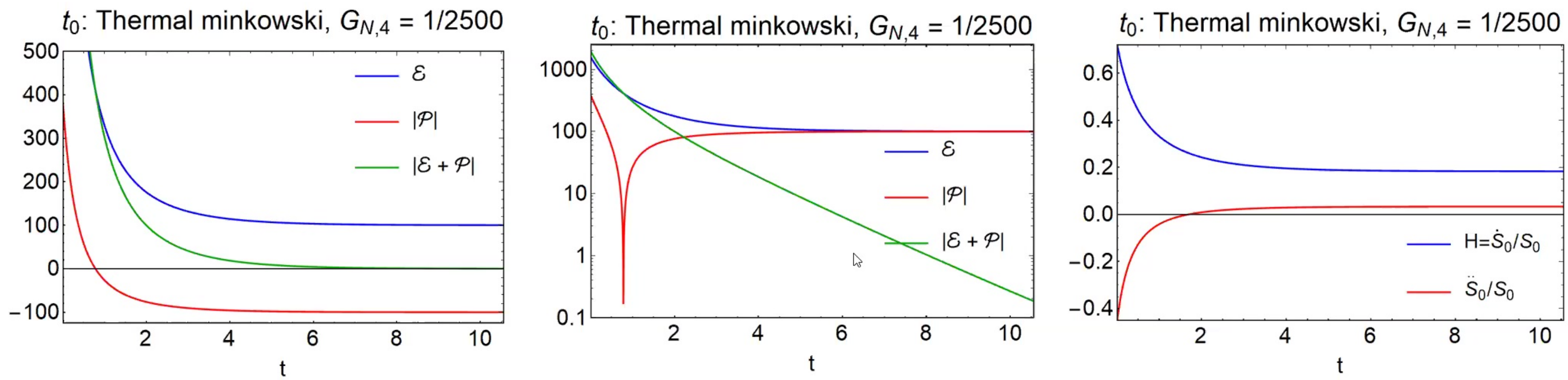
Jewel Ghosh, Elias Kiritsis, Francesco Nitti and Lukas Witkowski, Back-reaction in massless de Sitter QFTs: holography, gravitational DBI action and f(R) gravity (2020)
Paul Chesler and Abraham Loeb, Holographic duality and mode stability of de Sitter space in semiclassical gravity (2020)

Wilke van der Schee, CERN

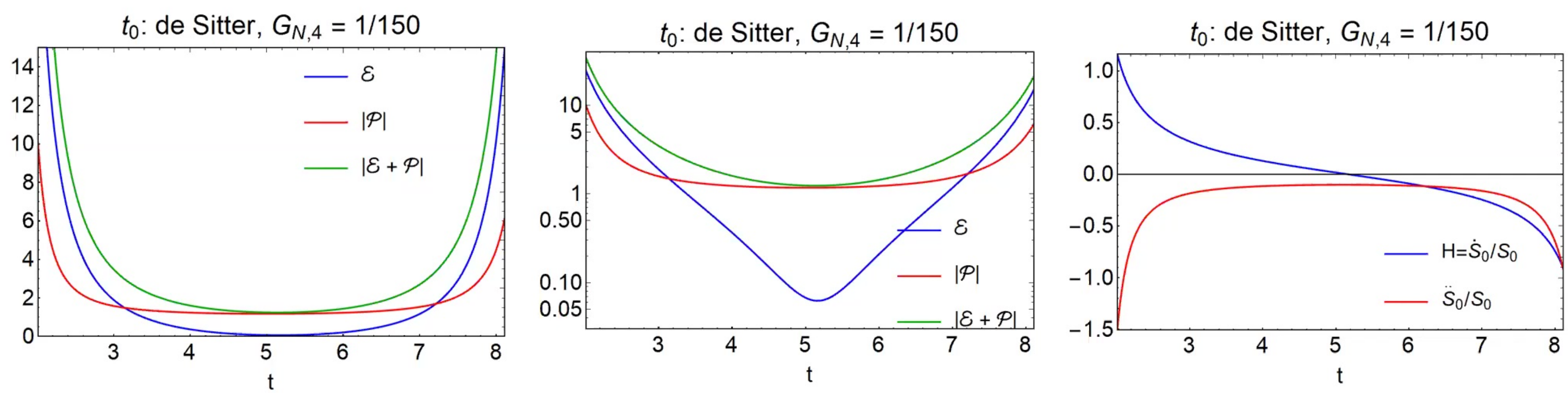
OUTLOOK: BOUNDARY GRAVITY

$$ds_b^2 = -dt^2 + S_0(t)^2 d\vec{x}^2$$

Preliminary results, thermal Minkowski: `standard' result



Preliminary results, de Sitter + matter: big crunch geometry



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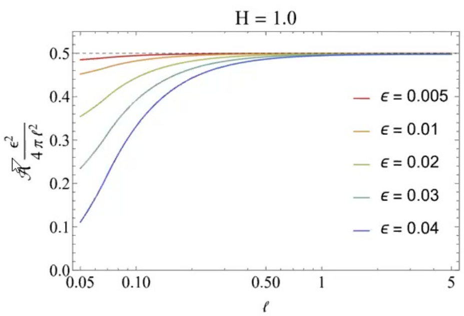
DISCUSSION

Hydrodynamics and Entanglement in de Sitter

- Viscous hydrodynamics works for small gradients
- Event and apparent horizon differ: negative temperature AH (?)
- Extremal surfaces beyond cosmological horizon probe behind EH
- Extremal surface cosmological horizon extends into bulk as bulk CH

Numerically hard to extract 'interesting' piece of the EE: area HRT surface:



Phenomenologically not realistic: physics at a vacuum with $e < T^4$, with T about 10^{-30} K..

Outlook: backreacted de Sitter. In principle nothing stops us from relaxing symmetries?

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