Title: Getting hot without accelerating: vacuum thermal effects from conformal quantum mechanics

Speakers: Michele Arzano

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Abstract: In this talk I will discuss how the generators of radial conformal symmetries in Minkowski space-time are related to the generators of time evolution in conformal quantum mechanics. Within this correspondence I will show that in conformal quantum mechanics the state corresponding to the inertial vacuum for a conformally invariant field in Minkowski space-time has the structure of a thermofield double. The latter is built from a bipartite "vacuum state" corresponding to the ground state of the generators of hyperbolic time evolution, which cover only a portion of the time domain. When such generators are the ones of conformal Killing vectors mapping a causal diamond in itself and of dilations, the temperature of the thermofield double reproduces, respectively, the diamond temperature and the Milne temperature perceived by observers whose constant proper time hyper-surfaces define a hyperbolic slicing of the future cone. I will point out how this result indicates that, for conformal invariant fields, the fundamental ingredient for vacuum thermal effects in flat-space time is the non-eternal nature of the lifetime of observers rather than their acceleration.

Getting hot without accelerating:

vacuum thermal effects from conformal quantum mechanics

Michele Arzano

Università di Napoli Federico II
and
INFN Sezione di Napoli

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In quantum field theory the notion of **vacuum state** has no universal meaning

"Before the 70s nobody thought very much about "for whom" the vacuum state appears devoid of "stuff"..."

(P.C.W. Davies, *"Particles Do Not Exist,"* In Christensen, (Ed.): Quantum Theory Of Gravity, 66-77 (1984))

This rather simple observation is **directly** related to **one of the most significant insights** semiclassical gravity provides on quantum gravity

Puzzles of BH thermodynamics

- **Bekenstein** (Phys. Rev. D7, 2333 (1973)) suggested that BH carry **entropy**:

$$S_{BH} \sim \frac{A}{L_p^2}$$

- Spectacularly confirmed by **Hawking** (Nature **248**, 30 (1974)): *static observers* far away from the BH detect **thermal radiation** at temperature

$$T_H = \frac{1}{8\pi GM}$$

direct consequence of **Hartle-Hawking vacuum** (free falling observers) being perceived as a **thermal state**

To date two issues remain unanswered:

⇒ The **enigmatic nature** of the **degrees of freedom** that S_{BH} is **counting**

⇒ Fate of **unitarity** ("unitarity crisis") in BH quantum *evaporation*:

do BH **evolve pure states into mixed states**?

Looking deeper: (free quantum fields)

- **vacuum state** ambiguity = different possible choices of **time-like Killing vectors**

  needed to decompose the **space of solutions** of the equations of motion into **positive and negative energy subspaces**

  used to define the **one-(anti)particle Hilbert space, Fock space and vacuum**

For **conformally invariant fields** the range of choices extends to (time-like) **conformal Killing vectors**

E.g. use **dilations** as generators of time evolution in the **future cone** of 2d Minkowski space-time: "Milne quantization" of a massless field **(Wald, Phys. Rev. D 100 (2019), 065019)**

Orbits of a conformal Killing vector are worldlines of observers within a **causal diamond**: **diamond temperature? (Martinetti and Rovelli, Class. Quant. Grav. 20, 4919 (2003))**

This talk: give a unified, group-theoretical description of **Milne and diamond temperature** using a correspondence between **radial conformal symmetries** in Minkowski space-time and **time evolution in conformal quantum mechanics**

MA, JHEP **05**, 072 (2020) [arXiv:2002.01836 [gr-qc]], arXiv:2103.07228 [hep-th]

Radial conformal motions in Minkowski space-time

Minkowski metric in spherical coordinates

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

radial vector field

$$\xi = A(t, r, \theta, \phi)\,\partial_t + B(t, r, \theta, \phi)\,\partial_r$$

imposing that

$$\mathcal{L}_\xi \eta_{\mu\nu} \propto \eta_{\mu\nu}$$

implies that ξ is independent of θ and ϕ and it has the general form

$$\xi = \left(a(t^2 + r^2) + bt + c\right)\,\partial_t + r(2at + b)\,\partial_r$$

with a, b, c real constants **(Herrero and Morales, J. Math. Phys. 40, 3499 (1999))**

1st key observation: this conformal Killing vector can be written as

$$\xi = aK_0 + bD_0 + cP_0\,,$$

where K_0, D_0 and P_0 generate, respectively, special conformal transformations, dilations and time translations

Families of radial conformal Killing vectors

The generators K_0, D_0 and P_0

$$P_0 = \partial_t\,, \qquad D_0 = r\,\partial_r + t\,\partial_t\,, \qquad K_0 = 2tr\,\partial_r + (t^2 + r^2)\,\partial_t$$

close the $\mathfrak{sl}(2,\mathbb{R})$ Lie algebra

$$[P_0, D_0] = P_0\,, \qquad [K_0, D_0] = -K_0\,, \qquad [P_0, K_0] = 2D_0$$

General RCKV can be classified according to the sign of $\Delta = b^2 - 4ac$

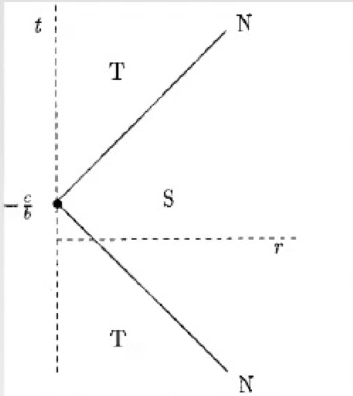
- $\Delta < 0$: **elliptic transformations** ($\mathfrak{sl}(2,\mathbb{R}) \simeq \mathfrak{so}(2,1) \rightarrow$ rotations)

$$R_0 = \frac{1}{2}\left(\alpha P_0 + \frac{K_0}{\alpha}\right)$$

- $\Delta = 0$: **parabolic transformations** (null rotations): P_0 and K_0.
- $\Delta > 0$: **hyperbolic transformation** (Lorentz boosts): D_0 and

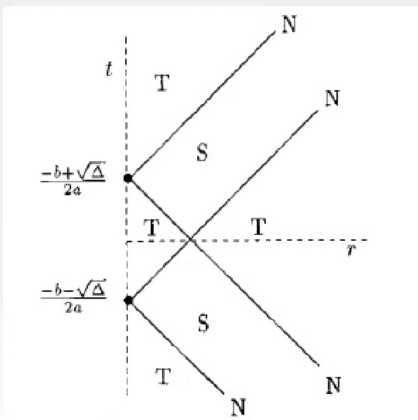
$$S_0 = \frac{1}{2}\left(\alpha P_0 - \frac{K_0}{\alpha}\right)$$

Domains of causality



(i) Case $a = 0$ and $b \neq 0$

$$D_0 \longrightarrow a = 0\,, b = 1\,, c = 0$$



(iii) Case $a \neq 0$ and $\Delta > 0$

$$S_0 \longrightarrow a = -\frac{1}{2\alpha}\,, b = 0\,, c = \frac{\alpha}{2}$$

(Pictures taken from Herrero and Morales, J. Math. Phys. 40, 3499 (1999))

Milne and diamond times

In the **future cone** D_0 generates conformal time evolution in a Milne universe

$$ds^2 = -d\bar{t}^2 + \bar{t}^2 \left(d\chi^2 + \sinh\chi^2 d\Omega^2\right)$$

with $t = \bar{t} \cosh\chi$ and $r = \bar{t} \sinh\chi$ (notice similarity with Rindler coordinates...)

(Wald, Phys. Rev. D 100 (2019), 065019)

The conformal Killing vector

$$S_0 = \frac{1}{2}\left(\alpha P_0 - \frac{K_0}{\alpha}\right)$$

maps a **causal diamond of radius** α centered at the origin into itself

(Jacobson, Phys. Rev. Lett. 116 (2016), 20)

> S_0 generates evolution in **diamond time**:
> the proper time of **uniformly accelerated observers with finite lifetime**

Worldline conformal time evolution

Along $r = \text{const}$ worldlines and on the light cones $u = t - r = \text{const}$, $v = t + r = \text{const}$

$$\xi = \left(a\tau^2 + b\tau + c\right)\partial_\tau$$

$\Longrightarrow$ the generator of **conformal transformations of the real (time) line**

- $P_0 = \partial_\tau$ generates translations in **"inertial time"** τ covering the **entire time line**
- $D_0 = \tau\partial_\tau$ generates translation in **"Milne time"** ν: $D_0/\alpha = \partial_\nu$

  $$\tau = \pm 2\alpha \exp\frac{\nu}{\alpha}$$

  covering **half time line** ($\tau > 0$ or $\tau < 0$)
- $S_0 = \frac{1}{2\alpha}\left(\alpha^2 - \tau^2\right)\partial_\tau$ generates translation in **"diamond time"** σ: $S_0/\alpha = \partial_\sigma$

  $$\tau = \alpha \tanh \sigma/2\alpha$$

  covering the region $|\tau| < \alpha$: the "diamond"

Conformal quantum mechanics

As it turns out

$$G = i\xi = i\left(a\,\tau^2 + b\,\tau + c\right)\partial_\tau$$

is the most general **generator of time evolution in "conformal quantum mechanics"**.

A $0+1$-dimensional QFT invariant under $SL(2,\mathbb{R})$

(de Alfaro, Fubini and Furlan, Nuovo Cim. A 34, 569 (1976));

Starting from the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\dot{q}(t)^2 + \frac{g}{q(t)^2}\right), \qquad g > 0$$

the $\mathfrak{sl}(2,\mathbb{R})$ algebra can be canonically realized

$$H = iP_0 = \frac{1}{2}\left(p^2 + \frac{g}{q^2}\right)$$

$$D = iD_0 = t\,H - \frac{1}{4}(pq + qp)$$

$$K = iK_0 = -t^2\,H + 2t\,D + \frac{1}{2}q^2$$

Conformal quantum mechanics as a CFT_1

Tha dAFF model can be interpreted as CFT_1

(Chamon, Jackiw, Pi and Santos, Phys. Lett. B 701, 503 (2011); Jackiw and Pi, Phys. Rev. D 86, 045017 (2012))

Two-point functions are built from the kets $|\tau\rangle$ first introduced by dAFF

$$H\,|\tau\rangle = -i\,\partial_\tau\,|\tau\rangle$$

One starts from **irreps of** $\mathfrak{sl}(2,\mathbb{R})$: define ladder operators

$$L_\pm = \tfrac{1}{2}\left(\tfrac{K}{\alpha} - \alpha\,H\right) \pm i\,D\,, \qquad L_0 = \tfrac{1}{2}\left(\tfrac{K}{\alpha} + \alpha\,H\right)$$

with $[L_-, L_+] = 2L_0\,,\quad [L_0, L_\pm] = \pm L_\pm$, irreps are given by kets $|n\rangle$

$$L_0\,|n\rangle = r_n\,|n\rangle\,, \qquad r_n = r_0 + n\,, \qquad r_0 > 0\,, n = 0, 1\dots$$

$$\mathcal{C}\,|n\rangle = \left(\tfrac{1}{2}\,(KH + HK) - D^2\right)\,|n\rangle = r_0(r_0 - 1)\,|n\rangle$$

CFT_1 two-point function

The $|\tau\rangle$ states can be characterized by their overlap with $|n\rangle$ states

$$\langle \tau | n \rangle = (-1)^n \left[\frac{\Gamma(2r_0 + n)}{n!} \right]^{\frac{1}{2}} \left(\frac{\alpha - i\tau}{\alpha + i\tau} \right)^{r_n} \left(1 + \frac{\tau^2}{\alpha^2} \right)^{-r_0}$$

from which one obtains the inner product

$$\langle \tau_1 | \tau_2 \rangle = \frac{\Gamma(2r_0)\, \alpha^{2r_0}}{[2i(\tau_1 - \tau_2)]^{2r_0}}$$

which Jackiw, Pi et al. interpret as **the two-point function** of the CFT_1

For $r_0 = 1$: two-point function of a **massless scalar field in Minkowski space-time**, evaluated along the **worldline of an inertial observer** sitting at the origin

This is reminiscent of the $SL(2,\mathbb{R})$-invariant **wordline quantum mechanics** for **static patch observers** in de Sitter space-time

(Anninos, Hartnoll and Hofman, Class. Quant. Grav. 29, 075002 (2012))

A bi-partite vacuum state

As shown by Jackiw, Pi et al. we can re-write the CFT_1 two-point function as

$$G_2(\tau_1, \tau_2) \equiv \langle \tau_1 | \tau_2 \rangle = \langle \tau = 0 | e^{-i(\tau_1 - \tau_2)H} | \tau = 0 \rangle$$

where

$$|\tau = 0\rangle = \exp(-L_+)|n = 0\rangle \quad \text{(we set } r_0 = 1\text{)}$$

Crucial observation:
$L_\pm$ and L_0 can be realized in terms of creation and annihilation operators

$$L_+ = a_L^\dagger a_R^\dagger , \quad L_- = a_L a_R \, , \quad L_0 = \frac{1}{2}\left(a_L^\dagger a_L + a_R^\dagger a_R + 1\right)$$

and thus

$$|\tau = 0\rangle = \exp\left[-a_L^\dagger a_R^\dagger\right] |0\rangle_L \otimes |0\rangle_R$$

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and thus

$$|\tau = 0\rangle = \exp\left[-a_L^\dagger a_R^\dagger\right] |0\rangle_L \otimes |0\rangle_R$$

so that

$$|n = 0\rangle = |0\rangle_L \otimes |0\rangle_R$$

the vacuum state $|n = 0\rangle$ has a **bi-partite structure**!

CFT_1 vacua

Notice now that the Lie algebra

$$[L_-, L_+] = 2L_0\,, \quad [L_0, L_\pm] = \pm L_\pm$$

can be realized via another combination of H, D and K, namely

$$L_0 = iS\,, \qquad L_+ = \frac{1}{2}(D - R)\,, \qquad L_- = 2(D + R)$$

we have two **vacuum-like states**...

- $|n = 0\rangle$ "Boulware vacuum":

  the **ground state** of the generator of diamond time evolution S

- $|\tau = 0\rangle$ "Hartle-Hawking vacuum": the **"inertial vacuum"** from which we build

$$G_2(\tau_1, \tau_2) = \langle \tau = 0| e^{-i(\tau_1 - \tau_2)H} |\tau = 0\rangle$$

as in the "real world" the Hartle-Hawking vacuum is a **thermofield double state** built on the bi-partite Boulware vacuum

The thermofield double of CFT_1

With simple manipulations

$$|\tau = 0\rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(a_L^\dagger a_R^\dagger\right)^n |0\rangle_L \otimes |0\rangle_R = \sum_{n=0}^{\infty} (-1)^n |n\rangle_L \otimes |n\rangle_R$$

$$= -\sum_{n=0}^{\infty} e^{i\pi L_0} |n\rangle_L \otimes |n\rangle_R$$

and thus

$$|\tau = 0\rangle = -\sum_{n=0}^{\infty} e^{-\pi S} |n\rangle_L \otimes |n\rangle_R$$

mini-detour: given a set eigenstates $H|n\rangle = E_n|n\rangle$ for a quantum system, the **thermofield double state** is built by "doubling" the system

$$|TFD\rangle = \frac{1}{Z(\beta)} \sum_{n=0}^{\infty} e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

tracing over the degrees of freedom of one copy $\Rightarrow$ thermal density matrix at $T = 1/\beta$

$$Tr_L(|TFD\rangle\langle TFD|) = e^{-\beta H}$$

Diamond temperature

The inertial vacuum

$$|\tau = 0\rangle = -\sum_{n=0}^{\infty} e^{-\pi S} |n\rangle_L \otimes |n\rangle_R$$

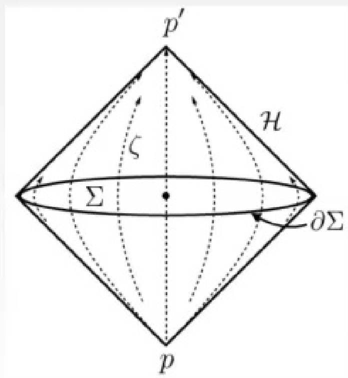
can be seen as a thermofield double at temperature

$$T_S = \frac{1}{2\pi\alpha}$$

for the Hamiltonian S/α which generates **diamond time evolution**

This is just the **diamond temperature** for diamond observers at the origin

(Su and Ralph, Phys. Rev. D 93, no.4, 044023 (2016))



(from Jacobson and Visser, SciPost Phys. 7, no.6, 079 (2019))

From the diamond to Milne

S and D belong to the same class of generators of **hyperbolic time evolution**

$\downarrow$

one can find a $SL(2,\mathbb{R})$ transformation **mapping one into another**

such map $\tau \to \tau'$ is easily found by requiring that $S(\tau) \equiv D(\tau')$

$$\tau' = -2\alpha\,\frac{\tau+\alpha}{\tau-\alpha}$$

note: this is the map from the **causal diamond to the Rindler wedge** used to derive the **diamond modular Hamiltonian** from the Rindler one (in light-cone coordinates)

(Casini, Huerta and Myers, JHEP 05, 036 (2011))

The Milne temperature

The conformal map

$$\tau' = -2\alpha \frac{\tau + \alpha}{\tau - \alpha}$$

leads to the following identification for the ladder operators

$$L_0 = iD \,, \qquad L_+ = -\alpha H \,, \qquad L_- = \frac{K}{\alpha}$$

$|n = 0\rangle$ is seen as the CFT_1 analogue of the vacuum state associated to the generator of Milne time evolution D

the "inertial" vacuum $|\tau = 0\rangle$ is the thermofield double for the Hamiltonian D/α at the **Milne temperature** **(Olson and Ralph, PRL 106, 110404 (2011), arXiv:1003.0720)**

$$T_D = \frac{1}{2\pi\alpha}$$

Getting hot without accelerating

Observers whose worldlines are **integral curves of time-like RCKV**

$$\xi = aK_0 + bD_0 + cP_0$$

are **accelerated** (Herrero and Morales, J. Math. Phys. 40, 3499 (1999))

$$|\mathbf{a}| = \frac{2|a|}{\sqrt{\omega - \Delta}}$$

where $\Delta = b^2 - 4ac$ and $\omega = \frac{a(t^2 - a^2) + bt + c}{r}$

- For integral curves of D (worldlines of Milne observers) $a = c = 0 \Longrightarrow \boxed{|\mathbf{a}| = 0}$
- For integral curves of S (diamond observers) $b = 0$, $a = -\frac{1}{2\alpha}$, $c = \frac{\alpha}{2}$ at $r = 0$ we have $\omega = \infty$ and thus $\boxed{|\mathbf{a}| = 0}$

You can get hot without accelerating!
(if you enjoy conformal symmetry...)

Conclusions

I showed that CFT_1 is rich enough to reproduce **vacuum thermal effects** related to the freedom in the choice of time evolution in QFT

Evidence for the existence of the **diamond and Milne temperatures** has appeared in bits and pieces in the literature (and only for 1+1-d Minkwoski space-time)

Correspondence between **radial conformal flows in Minkowski space-time** and **time evolution in conformal quantum mechanics**

$\downarrow$

group-theoretic evidence for existence of Milne and diamond temperatures

the inertial vacuum is a thermal state for observers whose **time evolution is not eternal**

$\Longrightarrow$ **thermodynamic** properties of the **Milne "patch"** and of **causal diamonds** are deeply connected...
new tools for studying **entanglement in Minkowski space-time?**

APPENDIX

- AdS_2: H and S as different generators of time evolution

(Järvelä, Keränen, Keski-Vakkuri, Phys. Rev. D**93**, no.4, 046002 (2016) [arXiv:1509.01092 [hep-th]])

vacua of AdS_2 black holes

(Spradlin and Strominger, JHEP **11**, 021 (1999), [arXiv:hep-th/9904143 [hep-th]])


THANK YOU!