

Title: Magnetism, Skyrmions and Superconductivity in Moiré Lattices

Speakers: Ashvin Vishwanath

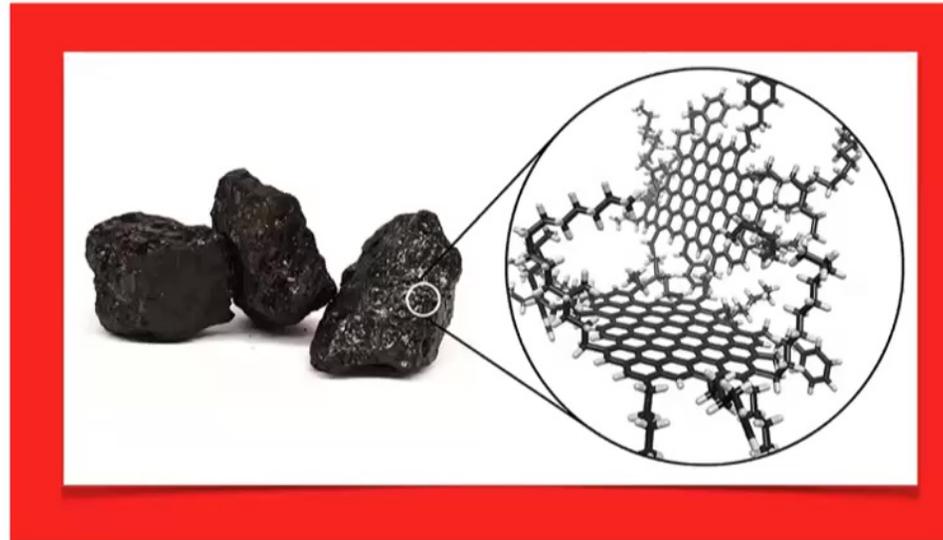
Series: Colloquium

Date: April 28, 2021 - 2:00 PM

URL: <http://pirsa.org/21040031>

Abstract: The remarkable properties of electrons moving through crystalline lattices continue to surprise us. Recently, electrons in artificial moiré lattices have emerged as an extraordinary new platform. The simplest such moiré material consists of a pair of graphene sheets twisted relative to one another. At a "magic" angle of about 1 degree, a variety of phenomena, including strong-coupling superconductivity, is observed. In this talk, I will review this rapidly moving field and describe our theoretical ideas that invoke the geometry of quantum states and topological textures like skyrmions. Finally, I will explain how these insights indicate a promising new family of moiré materials, twisted trilayer graphene, which is currently under active experimental study.

&nbsp;

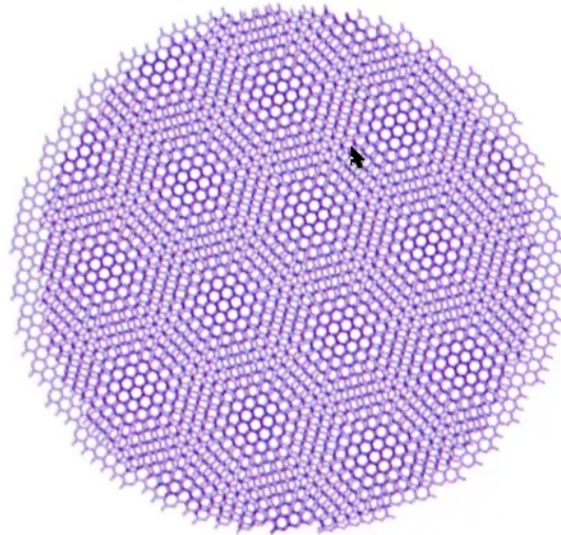


# Magnetism, Skyrmions and Superconductivity in Moiré Lattices

Ashvin Vishwanath  
Harvard University

# Graphene's Electrons in Moiré lattice

Electrons  
+  
Moiré lattice



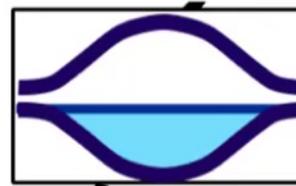
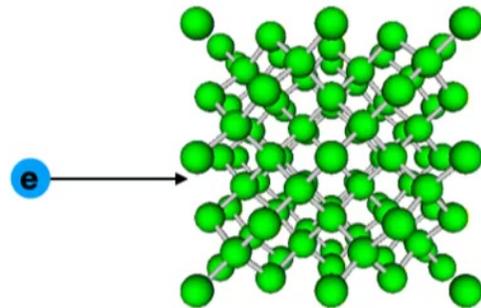
# Electronic States in a Crystal



Metals

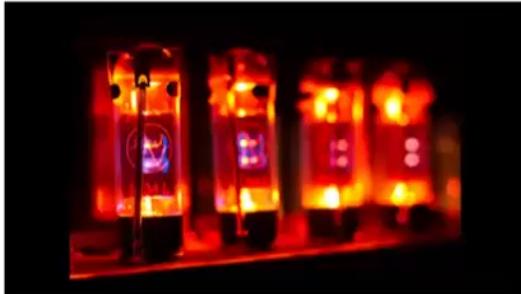


Insulators



Crystal => Artificial Vacuum for the electrons

# Crystals - Artificial Vacuum for Electrons



Vacuum Tubes <1960s

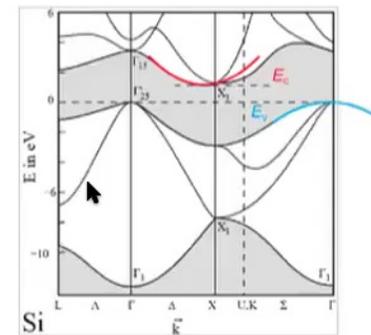


Transistor



"A sudden gasp filled the room when he flicked on an oscillator circuit, and it emitted a shrill tone instantaneously, with no warmup delay whatsoever."

Demonstration of the Transistor 1948  
From - Crystal Fire

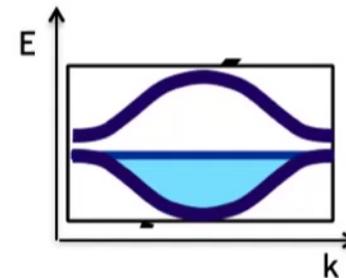


Modify properties of the electron  
Effective mass, electrons+holes etc.

## Qualitatively new effects?

- Semi-classical theory of electrons in a crystal

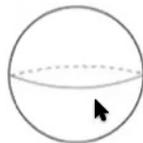
$$\dot{x} = \nabla_k \mathcal{E}(k) + \dot{k} \times \tilde{B}$$
$$\dot{k} = -\nabla_r \mathcal{V}(r) + e\dot{x} \times B$$



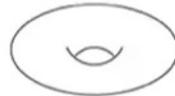
- Symmetry restored in a crystal - Berry Flux  $\tilde{B}$  leads to an anomalous velocity.
- Berry Flux is related to the Berry's phase acquired by states in the band. "Quantum Geometry" of bands.

# Topology and Geometry

- Topology: Robust aspects of shapes



genus 0



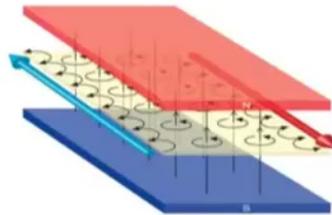
genus 1



genus 2

Topology of Surfaces:

Genus = # of holes



$$\oint K dS = 4\pi(1-g)$$

Gauss Formula

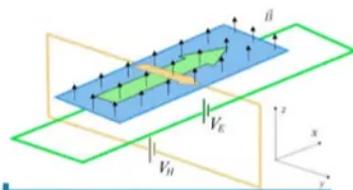
“Remarkable Theorem”

Integrate Berry Curvature

$$\oint \bar{B} d^2k = 2\pi n$$

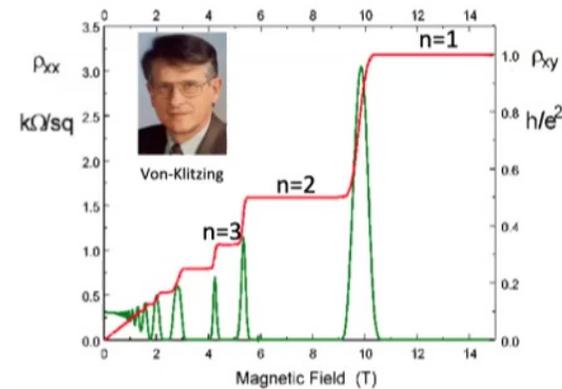
# of Edge States  
= Hall  
Conductance

# Quantized Hall effect in Insulators



Electrons in 2D in a magnetic field.

$$\rho_{xy} = \frac{h}{e^2} \frac{1}{n}$$

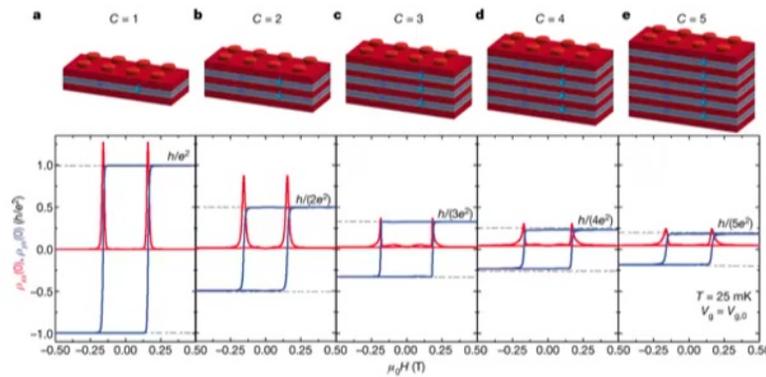


- Integer Quantum Hall States - (1980)
  - Different Integers 'n' - different phases.
  - topological distinction.
  - Accurate to 1 part in 10<sup>9</sup>!

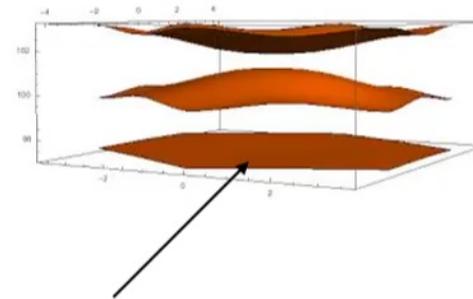
# Quantized Hall effect without Magnetic Fields

## Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,<sup>(a)</sup> M. P. Nightingale, and M. den Nijs  
*Department of Physics, University of Washington, Seattle, Washington 98195*  
(Received 30 April 1982)



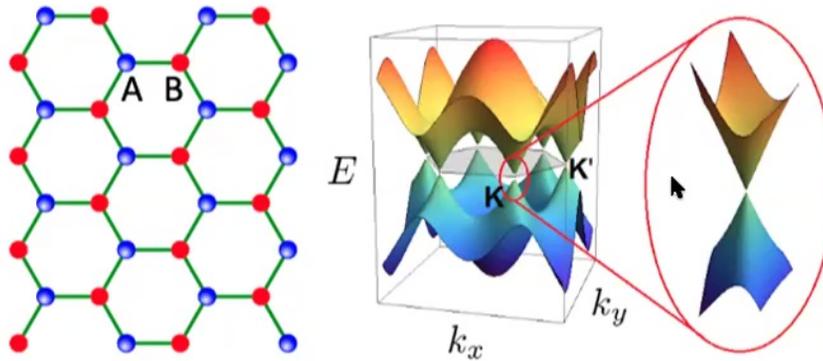
(Bi,Sb)<sub>2-x</sub>Cr<sub>x</sub>Te<sub>3</sub> Zhao et al. Nature 2020



Bands with Chern Number = C

# Qualitatively new effects in band structures

## Graphene band structure



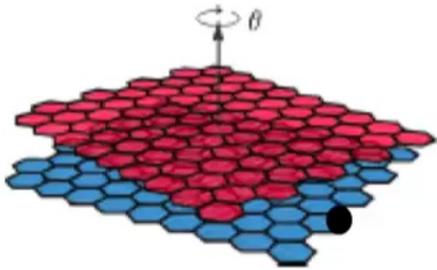
Electrons governed by 2+1D *massless Dirac equation!*

$$\Psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

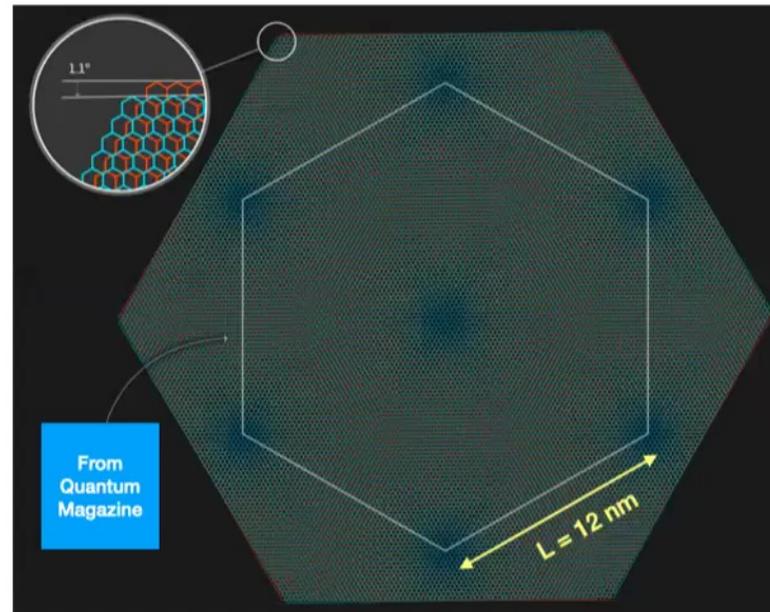
$$H = v_F \hat{\alpha} \cdot p$$

# MAGIC ANGLE GRAPHENE

$$\theta \sim 1/60 \text{ radians} \quad L \sim a/\theta$$



**MAGIC ANGLE  $\sim 1.1^\circ$ :**  
Tunneling time =  
Lattice Moire  
time



# Continuum model

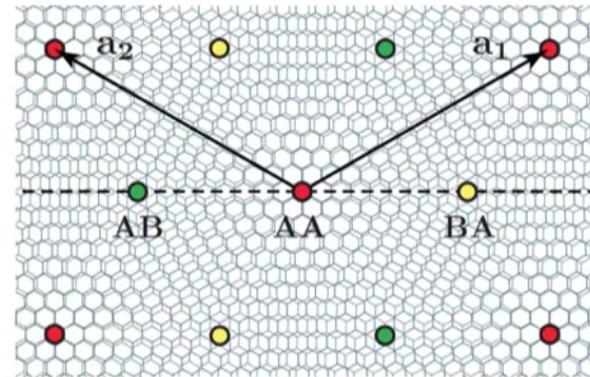
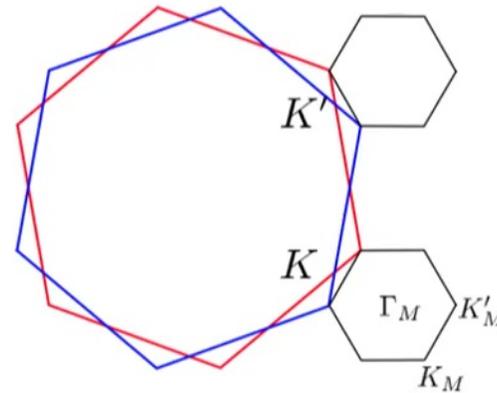
- Larger unit cell  $\rightarrow$  smaller BZone
- Bistrizer-Macdonald (BM) model (2011)

$$\mathcal{H}_K = \begin{pmatrix} -iv_F \boldsymbol{\sigma}_{\theta/2} \cdot \nabla & T(\mathbf{r}) \\ T^\dagger(\mathbf{r}) & -iv_F \boldsymbol{\sigma}_{-\theta/2} \cdot \nabla \end{pmatrix}_{12},$$

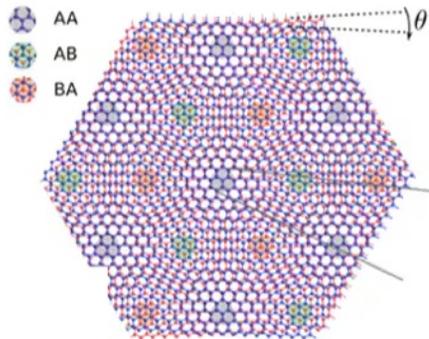
- Moire “potential”

$$T(\mathbf{r}) = \begin{pmatrix} w_0 U_0(\mathbf{r}) & w_1 U(\mathbf{r}) \\ w_1 U^*(-\mathbf{r}) & w_0 U_0(\mathbf{r}) \end{pmatrix}_{AB}$$

- Lattice relaxation: AB stacking favored to AA stacking (Carr *et al.* 2019, Nam, Koshino 2017)  
 $\implies w_0/w_1 \approx 0.7$

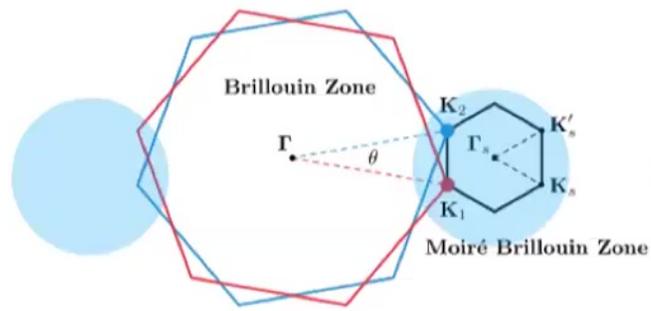
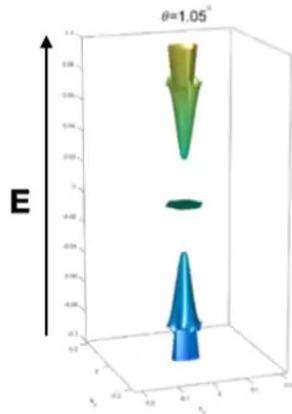


# MAGIC ANGLE GRAPHENE



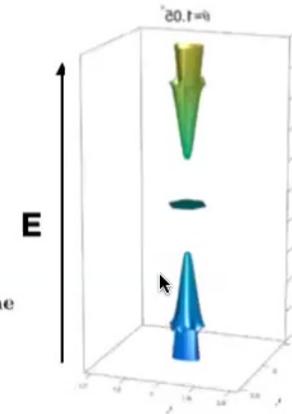
$$T(\mathbf{r}) = \begin{pmatrix} w_0 U_0(\mathbf{r}) & w_1 U_1(\mathbf{r}) \\ w_1 U_1^*(-\mathbf{r}) & w_0 U_0(\mathbf{r}) \end{pmatrix} \begin{matrix} A \\ B \\ A' \\ B' \end{matrix}$$

Valley K



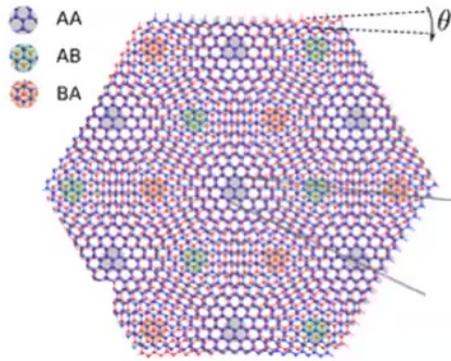
Momentum Space

Valley K'



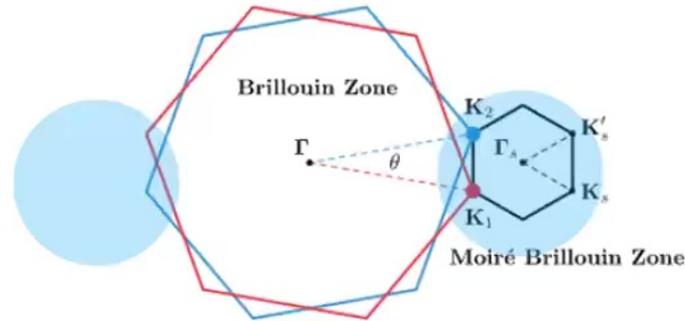
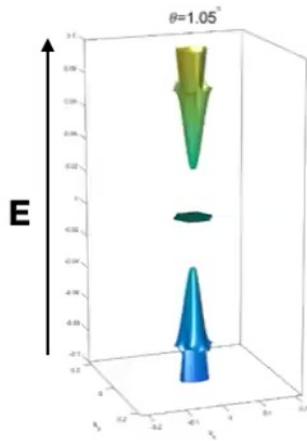
Y. Cao... (2018)

# MAGIC ANGLE GRAPHENE



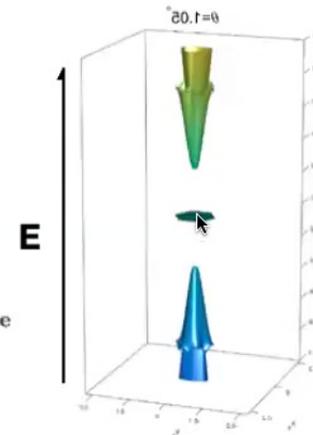
$$T(\mathbf{r}) = \begin{pmatrix} w_0 U_0(\mathbf{r}) & w_1 U_1(\mathbf{r}) \\ w_1 U^*(-\mathbf{r}) & w_0 U_0(\mathbf{r}) \end{pmatrix} \begin{matrix} A \\ B \\ A' \\ B' \end{matrix}$$

Valley K



Momentum Space

Valley K'



Y. Cao... (2018)

Transitions

No Transition Effect

Add an Effect

Start Transition

Delay

On Click

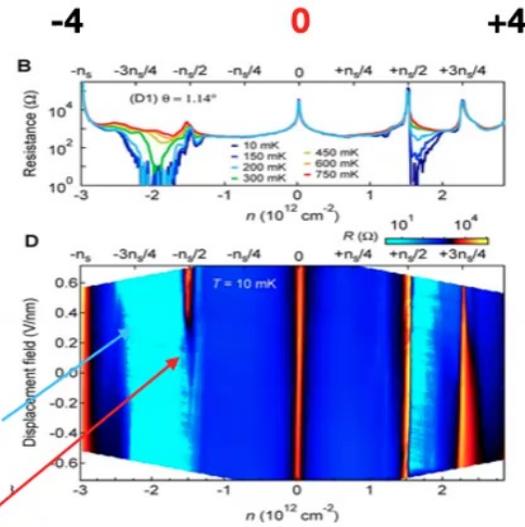
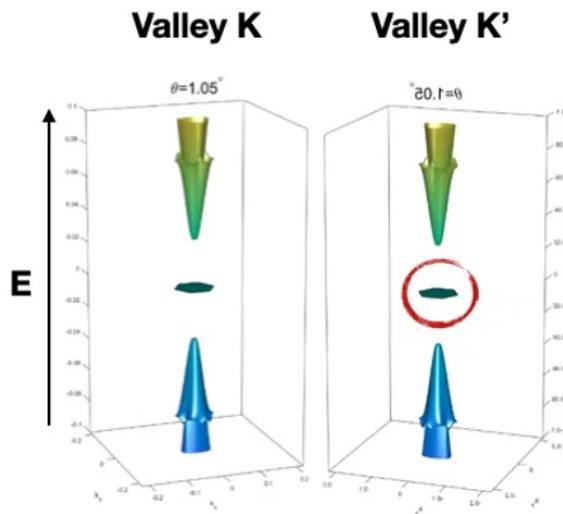
0.50 s

Build Order

# Experiments



Pablo Jarillo-Herrero's group (MIT)  
 Cao *et al.* Nature 556, 80 (2018)  
 Cao *et al.* Nature 556, 43 (2018)



Superconductor

Insulators

Yankowitz *et al.* Science

Transitions

No Transition Effect

Add an Effect

Start Transition

Delay

On Click

0.50 s

Build Order

# Chiral Model

Tarnopolski, Kruchkov, AV  
PRL 2019

$$T(\mathbf{r}) = \begin{pmatrix} w_0 \cancel{U}(\mathbf{r}) & w_1 U(\mathbf{r}) \\ w_1 U^*(-\mathbf{r}) & w_0 \cancel{U}(\mathbf{r}) \end{pmatrix}$$

Only AB coupling

Chiral Symmetry

$$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$$

↑  
sublattice

Transitions

No Transition Effect

Add an Effect

Start Transition

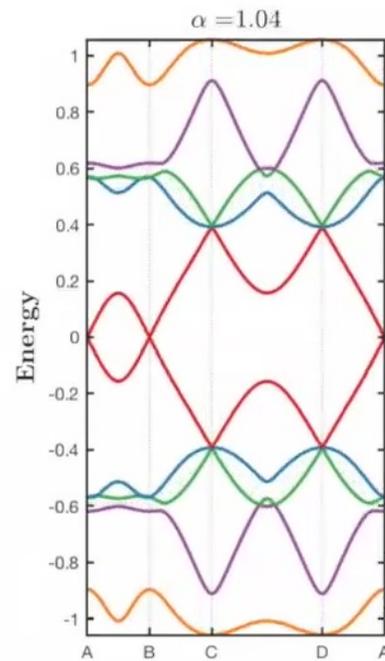
Delay

On Click

0.50 s

Build Order

# Perfectly Flat Bands in the Chiral Model



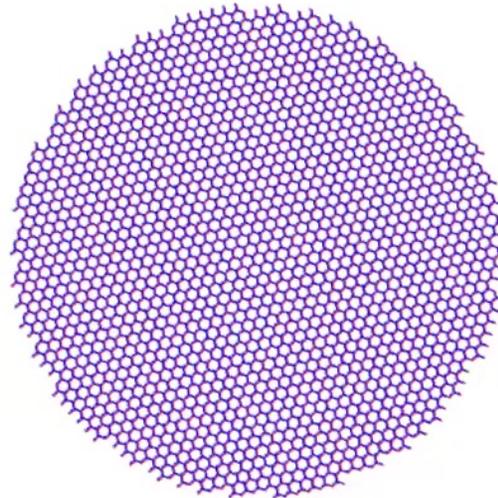
$$\alpha = \frac{0.6}{\theta^0}$$

## Graphene's Electrons in Moiré lattice

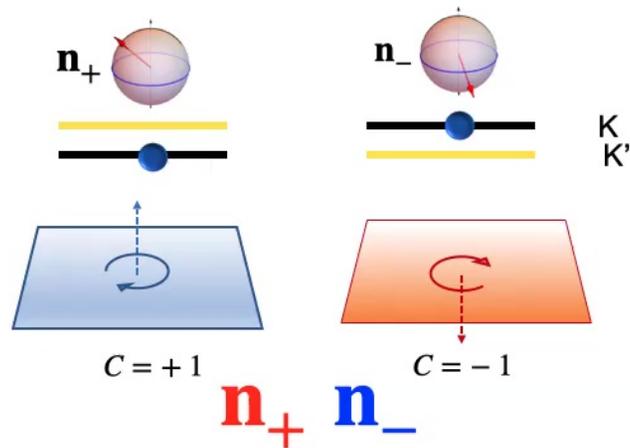
Graphene Electrons

+ Moire lattice

+ INTERACTIONS



# Insulators

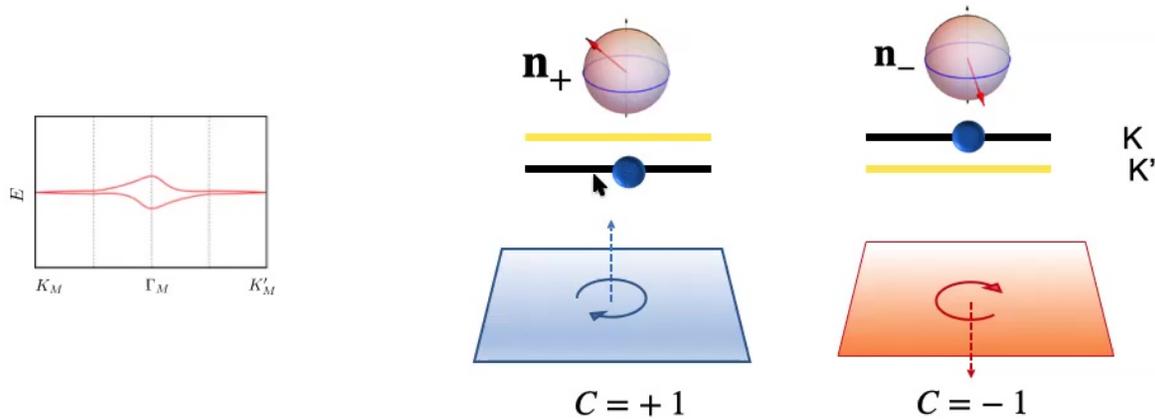


- Topological sigma model - show topological textures in order parameter carry electric charge. “(Pseudo) spin-charge entanglement”
- Topological mechanism of superconductivity?

also: Abanov and Wiegman, Grover & Senthil, Wang, Wang. C...Assad.

Here microscopic theory *with* Coulomb repulsion. Khalaf, Chatterjee, Bultnick, Zaletel, AV arXiv:2004.00638

# Landau Level Picture of Flat bands

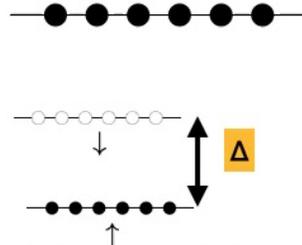


$n_+$   $n_-$

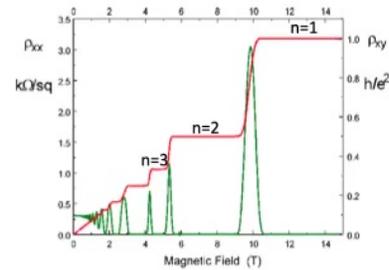
- Fill 2 out of 4 bands.
- $\sigma$  Model - but topological features.

San Jose et al PRL (2012), Tarnopolsky et al, PRL (2019), J. Liu et al, PRB (2019)

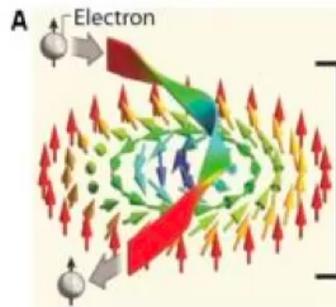
# Quantum Hall Ferromagnet and Skyrmions



A spin-degenerate Landau level



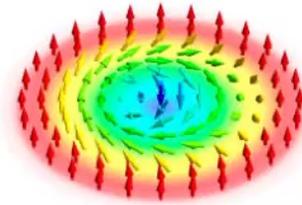
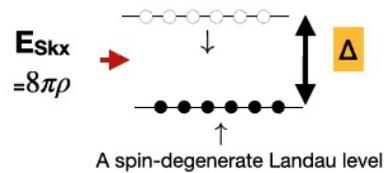
- Anything interesting combining symmetry breaking + band topology?



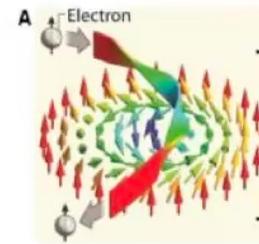
**Electric charge carried by Skyrmions.**

Theory: Sondhi, Karlhede, Kivelson, Rezayi; Lee & Kane  
Experiments: NMR - Barrett et al.

# Quantum Hall Ferromagnet and Skyrmions



Schütte&Garst

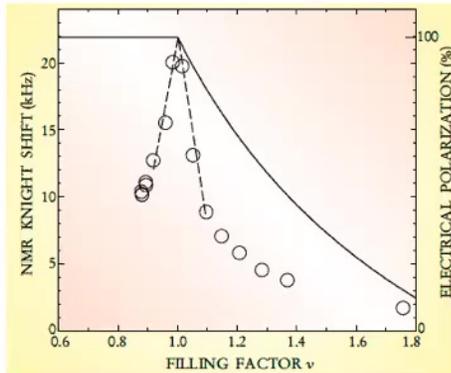


A. Rosch

- Quantum Hall Ferromagnet

## Electric charge carried by Skyrmions.

Theory: Sondhi, Karlhede, Kivelson, Rezayi; Lee & Kane  
 Experiments: NMR - Barrett et al.



[topological  $\sigma$ -model]

$$\rho_c = \sigma_{xy} \frac{e}{4\pi} \hat{n} \cdot \partial_x \hat{n} \times \partial_y \hat{n}$$

↑  
1

$$E_{s_{kx}} = 8\pi\rho_s = \frac{\Delta}{2}$$

# What is Fundamental?

Electron -> ferromagnet

Ferromagnet -> electron

# Tony Skyrme



## A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME †

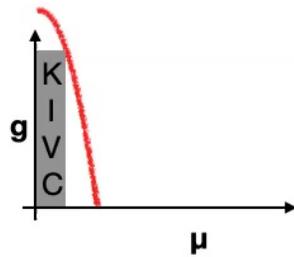
*A.E.R.E., Harwell, England*

*Nuclear Physics* **31** (1962) 556—569;

† Now at Department of Mathematics, University of Malaya, Pantai Valley, Kuala Lumpur, Malaya.

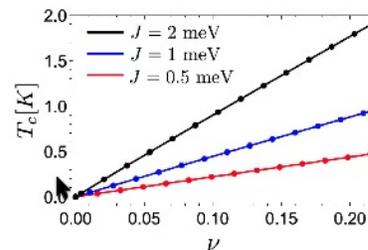
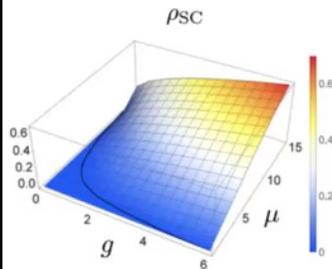
We are indebted to Mr. A. J. Leggatt for carrying out the calculations reported in sect. 3, while a vacation student at A.E.R.E.

# Phase Diagram & Stiffness



Sigma model -  
 ORDERED- Insulator  
 DISORDERED - Superconductor  
**Solve using large-N CP<sub>N</sub>**

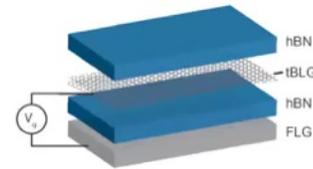
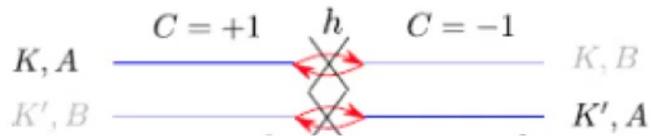
$$\mathcal{L} = \frac{\Lambda}{g} |(\partial_\mu - ia_\mu)z|^2 + \mu \frac{b}{\pi}$$



EK, Chatterjee, Bultinck, Zaletel, Vishwanath arXiv:2004.00638

## Essential Ingredients for Superconductivity?

- $C_2\mathcal{T}$  symmetry helps superconductivity in this setup
- Sublattice potential *suppresses* the coupling  $J$



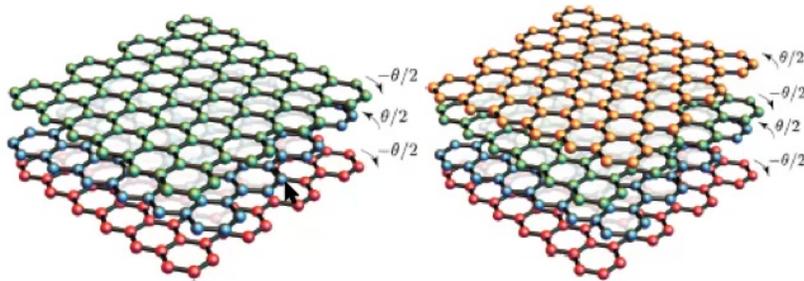
- Twisted bilayer graphene aligned on hBN  
→ no superconductivity  
(Sharpe et al. Science 19, Serlin et al. Science 20)

- Relatively *few* Moire materials apart from magic angle graphene with this symmetry.

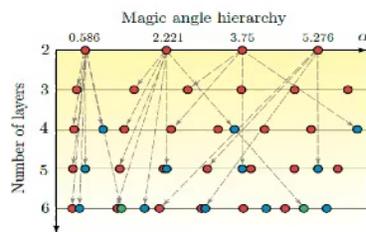
# New platforms for Superconductivity?

- Other systems with C2 symmetry

## Alternating twist sandwich



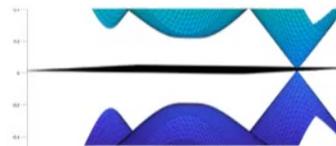
Eslam Khalaf



$$\varphi = \frac{1 + \sqrt{5}}{2}$$

Larger magic angles!

Magic angle =  $\sqrt{2} 1.1^\circ \sim 1.55^\circ$



Flat band+Dirac

Carr et al. stability `19

Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

# Matthias' Rules for Superconductivity

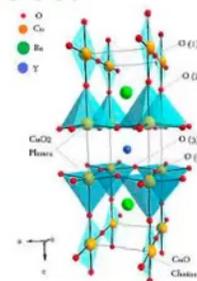
**Bernd Matthias (1918-1980)**

1. high symmetry is good, cubic symmetry is the best
2. high density of electronic states is good
3. stay away from oxygen
4. stay away from magnetism
5. stay away from insulators
6. stay away from theorists.

# Matthias' Rules for Superconductivity

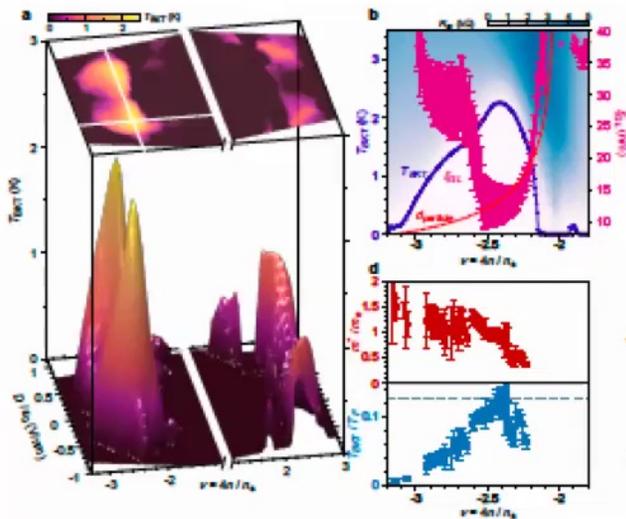
Bernd Matthias (1918-1980)

1. high symmetry is good, cubic symmetry is the best
2. high density of electronic states is good
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4. stay away from magnetism
5. stay away from insulators
6. stay away from theorists.



# Alternating twist trilayer -EXPT

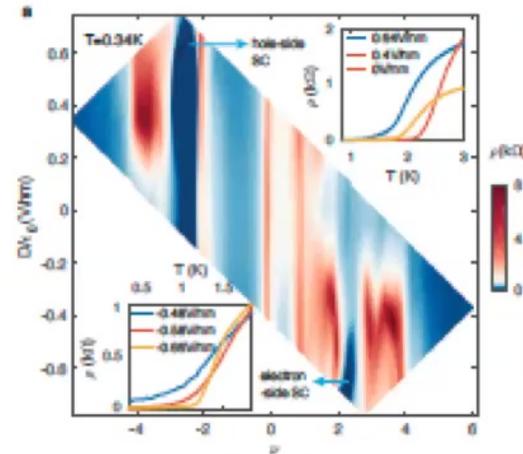
Strong coupling superconductivity  
Tuning by displacement field.



Park et al.

Tunable Phase Boundaries and Ultra-Strong Coupling Superconductivity in Mirror Symmetric Magic-Angle Trilayer Graphene

Jeong Min Park,<sup>1,\*</sup> Yuan Cao,<sup>1,\*†</sup> Kenji Watanabe,<sup>2</sup>  
Takashi Taniguchi,<sup>2</sup> and Pablo Jarillo-Herrero<sup>1,1</sup>



Hao et al.

Electric field tunable unconventional  
superconductivity in alternating twist magic-angle  
trilayer graphene

Zeyu Hao<sup>1†</sup>, A. M. Zimmerman<sup>1†</sup>, Patrick Ledwith<sup>1</sup>, Eslam Khalaf<sup>1</sup>,  
Danial Haie Najafabadi<sup>1</sup>, Kenji Watanabe<sup>2</sup>, Takashi Taniguchi<sup>2</sup>,  
Ashvin Vishwanath<sup>1</sup> & Philip Kim<sup>1\*</sup>

# Conclusions & Future Directions

- **Skymions** gives a natural mechanism for superconductivity in 2-3 layer graphene.
  - A soluble strong coupling theory - not local DOS related.
  - associated with  $\nu=2$
  - Connection to  $C_2T$  symmetry, accounts for robust Sc platforms.
  - Predicts new platforms.
- **Future Directions**
  - measure collective modes to find  $J, \rho_p, \lambda$ . Quantitative relation between normal state and predictions of skymions Sc.
  - We want to incorporate fermions into the theory (coexisting fermions + Skx).
  - Connection to weak coupling theories?
  - Distinct superconductors without flavor polarization?

Lecture Notes: <https://scholar.harvard.edu/avishwanath/teaching>

## Collaborators



Eslam Khalaf

Harvard



Nick Bultinck



Shubhayu Chatterjee

Berkeley



Mike Zaletel



Shang Liu

Harvard



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