Title: Potentially realistic SO(10) GUTs and their phenomenology

Speakers: Michal Malinsky

Collection: Octonions and the Standard Model

Date: April 26, 2021 - 12:00 PM

URL: http://pirsa.org/21040029

Abstract: I will take a look at the SO(10) grand unified theories from the perspective of the typical phenomenology constraints imposed on their structure. The current status of the minimal potentially realistic models will be briefly commented upon.

Perimeter Institute workshop "Octonions and the Standard Model", April 26 2021

Potentially realistic SO(10) GUTs and their phenomenology

Michal Malinský
IPNP Charles University in Prague

Outline

How do we construct SO(10) GUTs?

Outline

What are GUTs good for?

How do we construct SO(10) GUTs?

The story of the minimal "reasonable" SO(10) GUT

What makes SO(10) so special?

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ **symmetry**

Georgi & Glashow 1974
Fritzsch & Minkowski 1975

Entire Standard Model matter family fits within one SO(10) irrep

$$16_F = (3,2,+\tfrac{1}{6}) \oplus (1,2,-\tfrac{1}{2}) \oplus (\bar{3},1,+\tfrac{1}{3}) \oplus (\bar{3},1,-\tfrac{2}{3}) \oplus (1,1,+1) \oplus (1,1,0)$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad d_R \quad u_R \quad e_R \quad \nu_R$$

Standard model + 3 RH neutrinos

	T^3_L	Y	Q	$(B-L)/2$	T³-like generator for RH fields!
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{1}{6}$	$+\frac{2}{3}$ $-\frac{1}{3}$	$+\frac{1}{6}$	0
u_R d_R	0 0	$+\frac{2}{3}$ $-\frac{1}{3}$	$+\frac{2}{3}$ $-\frac{1}{3}$	$+\frac{1}{6}$	$+\frac{1}{2}$ $-\frac{1}{2}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$+\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$	0 -1	$-\frac{1}{2}$	0
ν_R e_R	0 0	0 -1	0 -1	$-\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$

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u_R	0	$+\frac{2}{3}$	$+\frac{2}{3}$	$\frac{1}{6}$	$+\frac{1}{2}$
d_R					$-\frac{1}{2}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$+\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$	0 -1	$-\frac{1}{2}$	0
ν_R	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$
e_R	0	-1	-1		$-\frac{1}{2}$

$$Q = T_L^3 + T_R^3 + \frac{1}{2}(B-L)$$

Left-right symmetry perspective

$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ **symmetry**

recent review: G. Senjanovic, Riv. Nuovo Cim. 034, 2011

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\begin{matrix} u_R \\ d_R \end{matrix} \longrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$$

High-scale parity restoration!

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$\begin{matrix} \nu_R \\ e_R \end{matrix} \longrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

Pati-Salam perspective

$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ **symmetry**

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$SU(3)_c \otimes U(1)_{B-L} \subset SU(4)_C$$

$$\downarrow \qquad \downarrow$$

$$\begin{pmatrix} u & \nu_e \\ d & e \end{pmatrix}_L \qquad \begin{pmatrix} u & \nu_e \\ d & e \end{pmatrix}_R$$

First attempt to unify quarks and leptons... (even before the charm discovery!)

SU(5) perspective

VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

H. Georgi, S. Glashow, PRL 32, 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

Our starting point is the assumption that *weak and electromagnetic forces are mediated by the vector bosons of a gauge-invariant theory with spontaneous symmetry breaking.* A model describing the interactions of leptons using the gauge group SU(2)⊗U(1) was first proposed by Glashow, and was improved by Weinberg and Salam who incorporated spontaneous symmetry breaking.[1] This scheme can also describe hadrons, and is just one example of an infinite class of models compatible with observed weak-interaction phenomenology. If we assume that *there are as few fermion fields as possible* and, in particular, that there are no unobserved leptons, the Weinberg model becomes unique up to extensions of the gauge group: The observed leptons may be described by six left-handed Weyl fields (e_L^-, μ_L^-, ν_L, ν_L', e_L^+, μ_L^+) and their charge conjugates. If the gauge couplings do not mix leptons with quarks, these six fields must transform as a representation of the gauge group: one of the 23 subgroups of U(6) containing an SU(2)⊗U(1) subgroup in which the leptons behave as they do in the Weinberg model.

To include hadrons in the theory, we must use the Glashow-Iliopoulos-Maiani (GIM) mechanism and introduce a fourth quark p' carrying charm.[2] … of the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons* associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.[6] This insures that parity and hypercharge are conserved to order α,[7] and does not lead to any new anomalies, so that the theory remains renormalizable. The strongest binding forces are in color singlet states which may explain why observed hadrons lie in qqq and $q\bar{q}$ configurations.[8] And, it gives another important bonus: Since the strong interactions are associated with a non-Abelian theory, they may be asymptotically free.[9]

Thus, we see how attractive it is for strong, weak, and electromagnetic interactions to spring from a gauge theory based on the group $\mathfrak{F}$ = SU(3)⊗SU(2)⊗U(1). Alas, this theory is defective in one important respect: It does not truly unify weak and electromagnetic interactions. The SU(2)⊗U(1) gauge couplings describe two interactions with two independent coupling constants; a true unification would involve only one.

Electric charge is observed to be quantized. This has no natural explanation in the framework of conventional quantum electrodynamics, but it is necessarily true in any unified theory[10]—yet another reason to search for a true unification.

We must assume that the gauge group is larger than $\mathfrak{F}$. Suppose it is of the form SU(3)⊗$\mathcal{W}$ where $\mathcal{W}$ contains SU(2)⊗U(1) but has a unique gauge coupling constant. $\mathcal{W}$ must be simple, or the di-

$SU(5)$

$$\bar{5}: \quad \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu_e \end{pmatrix}$$

$$10: \quad \begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ . & 0 & u_1^c & u^2 & d^2 \\ . & . & 0 & u^3 & d^3 \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix}$$

$$1: \quad \nu_R$$

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Our starting point is the assumption that *weak and electromagnetic forces are mediated by the vector bosons of a gauge-invariant theory with* s[illegible]

Uniqueness of SU(5) at the rank=4 level

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$SU(5)$

$\overline{5}$
$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu_e \end{pmatrix}$$

10
$$\begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ . & 0 & u_1^c & u^2 & d^2 \\ . & . & 0 & u^3 & d^3 \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix}$$

1
$$\nu_R$$

What makes different unifications really different?

Extra degrees of freedom in SO(10) - a model building maze

- SO(10) models are often **ENTIRELY DEFINED** by their scalar sector

What makes different unifications really different?

Extra degrees of freedom - the minimal SU(5) GUT

Gauge sector: $24 = (8,1,0) \oplus (1,3,0) \oplus (1,1,0) \oplus (3,2,-\frac{5}{6}) \oplus (\overline{3},2,+\frac{5}{6})$

$G^\mu \quad \underbrace{A^\mu \quad B^\mu}_{W^\pm, Z, \gamma} \quad X^\mu$

Scalar sector:

SM Higgs: $SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_Q$

$\overline{5} = (1,\overline{2},+\frac{1}{2}) \oplus (\overline{3},1,-\frac{1}{3})$

$H \quad \Delta$

GUT-breaking scalars: $SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$

$24 = (1,1,0) \oplus (8,1,0) \oplus (1,3,0) \oplus (3,2,-\frac{5}{6}) \oplus (\overline{3},2,+\frac{5}{6})$

The choice of GUT-breaking scalars is very limited (rank=4)!

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$SU(4)_C \times SU(2)_L \times SU(2)_R$ classification

$$10 = (1,2,2) \oplus (6,1,1)$$
$$16 = (4,2,1) \oplus (\bar{4},1,2)$$
$$45 = (1,3,1) \oplus (1,1,3) \oplus (15,1,1) \oplus (6,2,2)$$
$$54 = (1,1,1) \oplus (1,3,3) \oplus (20',1,1) \oplus (6,2,2)$$
$$120 = (1,2,2) \oplus (10,1,1) \oplus (\overline{10},1,1) \oplus (6,3,1) \oplus (6,1,3) \oplus (15,2,2)$$
$$126 = (6,1,1) \oplus (10,3,1) \oplus (\overline{10},1,3) \oplus (15,2,2)$$
$$144 = (4,2,1) \oplus (\bar{4},1,2) \oplus (4,2,3) \oplus (\bar{4},3,2) \oplus (20,2,1) \oplus (\overline{20},1,2)$$
$$210 = (1,1,1) \oplus (15,1,1) \oplus (6,2,2) \oplus (15,3,1) \oplus (15,1,3) \oplus (10,2,2) \oplus (\overline{10},2,2)$$

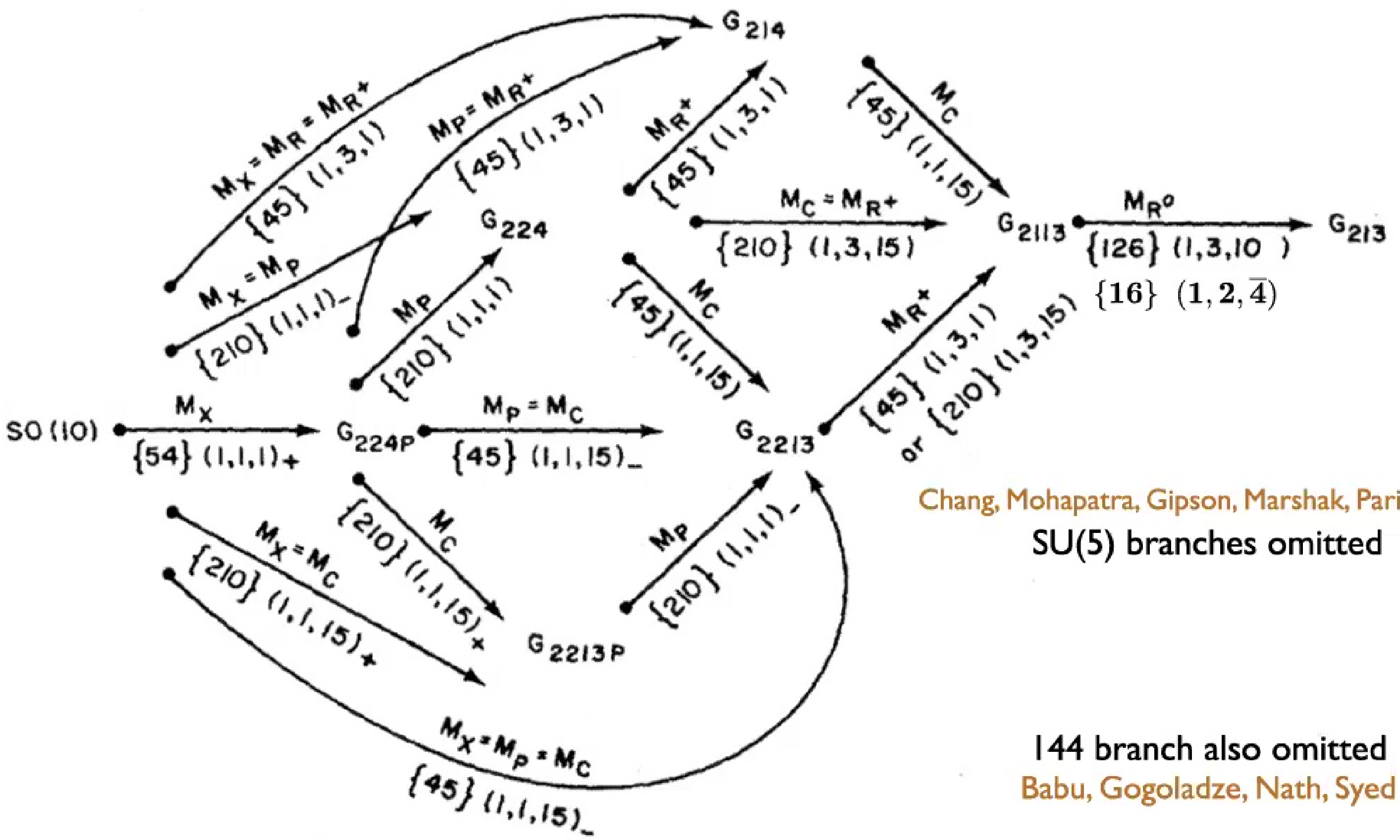
- SM singlets (red)
- SM Higgs-like doublets (blue)

Barring the singular 144, at least a pair is always needed, probably more…

What makes different unifications really different?

Extra degrees of freedom in SO(10) - a model building maze

- SO(10) admits multiple symmetry breaking patterns **and multiple scales**



Chang, Mohapatra, Gipson, Marshak, Parida 1985

SU(5) branches omitted

144 branch also omitted

Babu, Gogoladze, Nath, Syed

SO(10) model building constraints

Basic structural constraints = definition of the parameter space (practical x ideological)

- SUSY or non-SUSY?
- renormalizable x non-renormalizable?
- symmetry breaking pattern = selection of scalar irreps
- minimality (what does it mean?)
- GUT scale calculability
- doublet-triplet splitting
- naturalness

increasing cost

QFT consistency = discarding "obviously wrong" and/or "useless" p-space patches

- perturbativity (yes, we want to be able to calculate)
- local vacuum stability (no tachyons in the spectrum)
- global vacuum stability (insanely complicated)

Phenomenology = focusing on areas which can be OK, looking at numbers=doing physics

- masses and mixing patterns, weak mixing angle,… (reproducing SM)
- accommodation of known BSM signals (neutrinos, DM?) (addressing its shortcomings)
- calculability of proton decay (a gate to refutability)

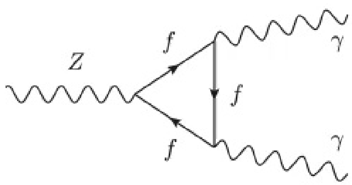
Charge quantization

Generators of simple non-Abelian Lie groups are discrete & traceless

⇩

charges obey non-trivial algebraic relations

Wait; anomalies quantize (hyper)charge in the SM too!?



Not if you do believe that neutrinos are Dirac

$$Y = Y_{SM} + \varepsilon(B - L)$$

Foot, Lew, Volkas, Mod.Phys.Lett. A5 (1990) 2721

Baryon/lepton number violation, flavour

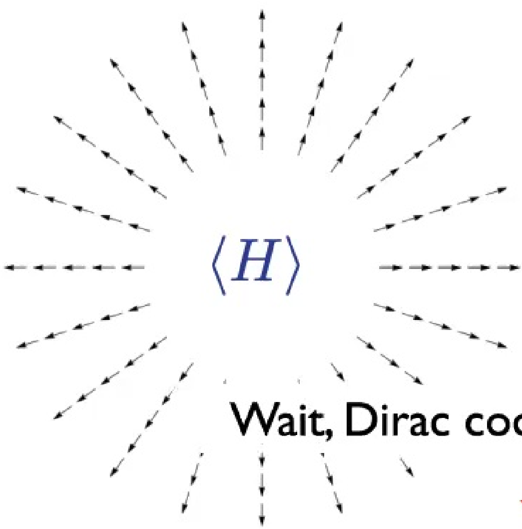
Crosstalk among quarks and leptons

Monopoles

Non-trivial vacuum manifold homotopy

⇩

heavy topologically stable finite-energy extended Higgs/gauge configurations

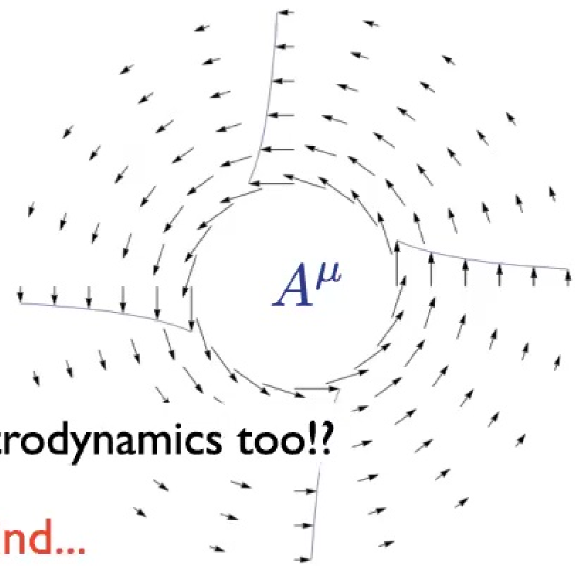


monopoles

vortices

domain walls

...



Wait, Dirac cooked a monopole in electrodynamics too!?

Yes, but of a different kind...

P. A. M. Dirac, Proc. Roy. Soc. A 33, 6, (1931)

Baryon/lepton number violation, flavour

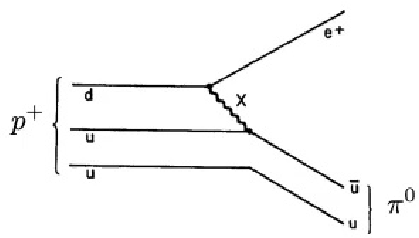
Crosstalk among quarks and leptons

- gauge bosons coupled to a universal charge ⇨ dynamical quark to lepton transitions
- Yukawas do not care about who is who either ⇨ flavour structure constraints

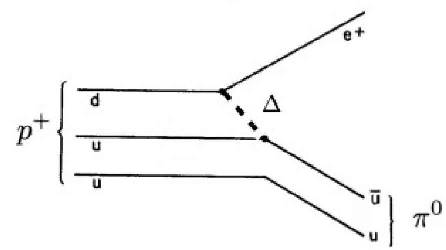
SU(5) example: $Y_5 \overline{5}_F 10_F \overline{5}_H + Y_{10} 10_F 10_F * \overline{5}_H^\dagger$ $\quad M_d = M_l^T \quad M_u = M_u^T$

Fundamental instability of matter! "Golden" mode: $p^+ \to \pi^0 \ell^+$

gauge-induced



scalar-induced



$\frac{f_1}{M_G^2}\overline{Q}u^c\overline{Q}e^c, \quad \frac{f_2}{M_G^2}u^c\overline{Q}d^c\overline{L}$ $\qquad \frac{f_3}{M_G^2}QQQL, \quad \frac{f_4}{M_G^2}u^c u^c d^c e^c$

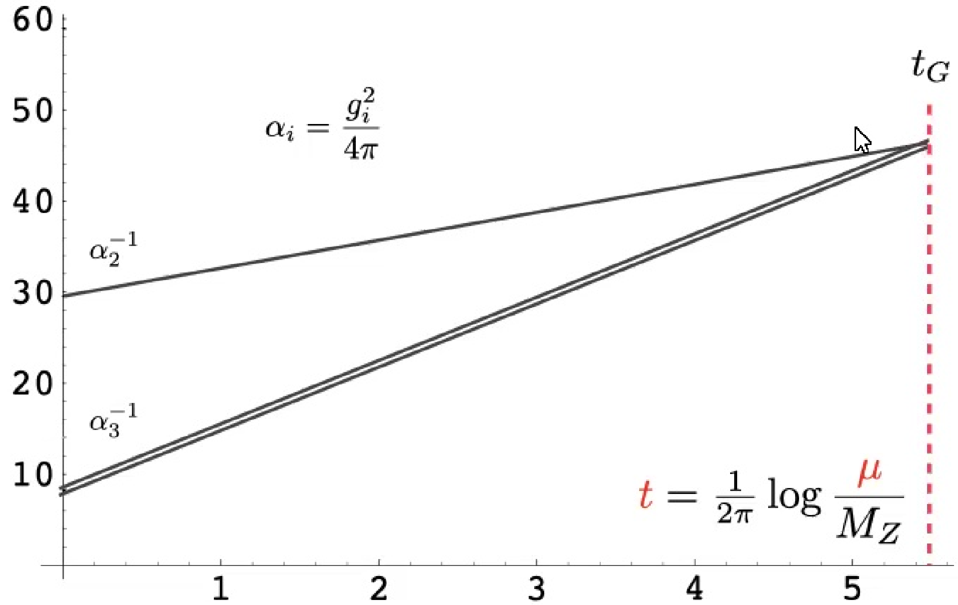
This is proton decay!

The true beauty of GUTs

SM gauge couplings encode the new dynamics scale

Evolution of gauge couplings in the SM:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3}\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}_{gauge} + 2\begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix}_{ferm.} + \frac{1}{3}\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}_{scal.}$$

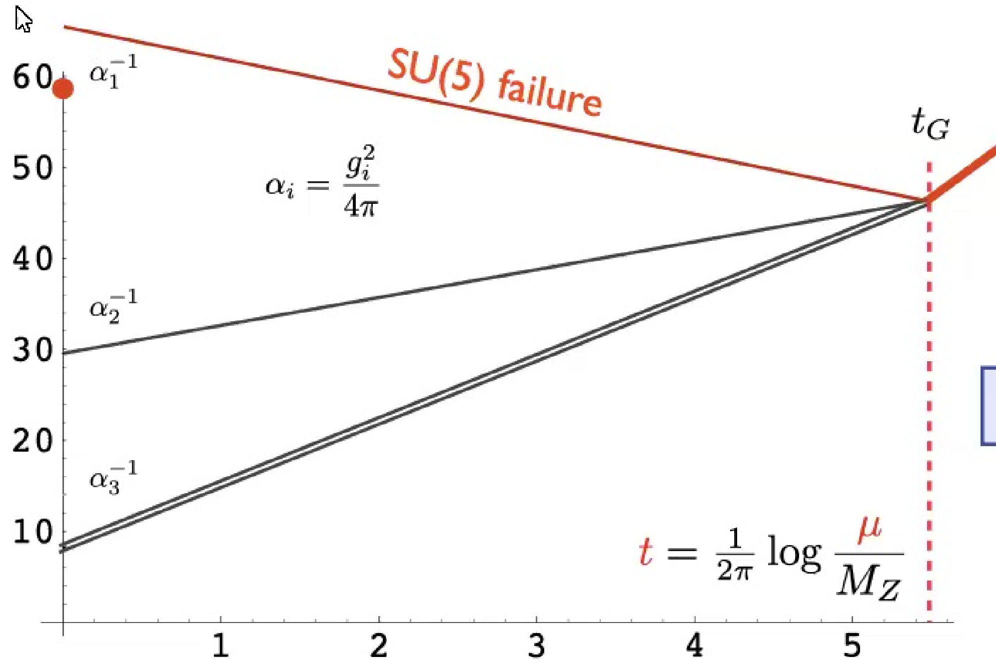


The true beauty of GUTs

SM gauge couplings encode the new dynamics scale

Evolution of gauge couplings in SU(5) = SM $+X+\Delta\ \ldots$

$$\begin{pmatrix} \frac{3}{5}b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3}\begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}_{gauge} + 2\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}_{ferm.} + \frac{1}{3}\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}_{scal.}$$



simple gauge dynamics!

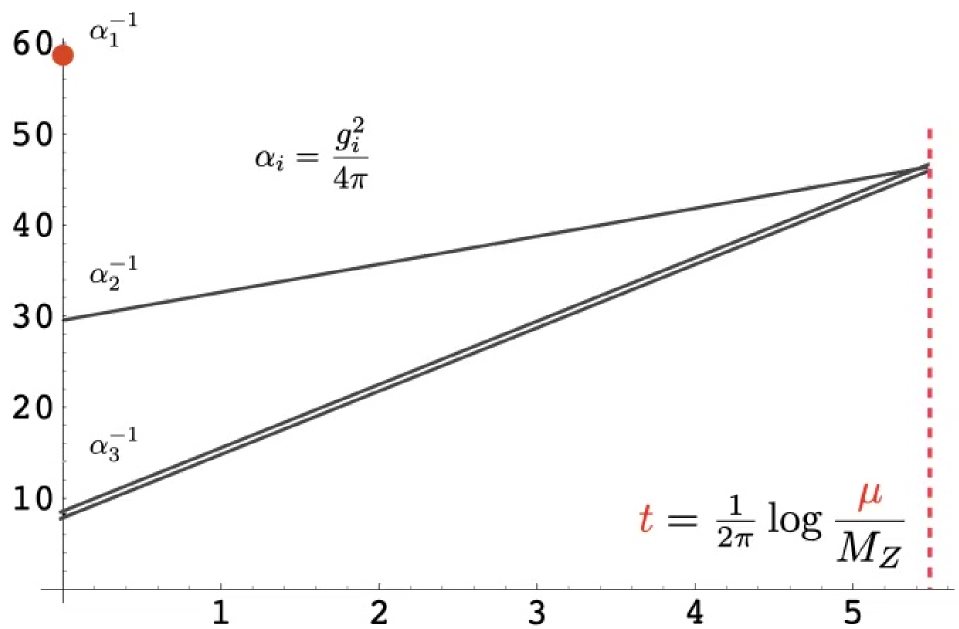
this makes sense only for

$M_G \sim 10^{16}\mathrm{GeV}$

Maybe the SM gauge couplings encode two scales !?

Evolution of gauge couplings in SO(10) GUT with intermediate LR

$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$



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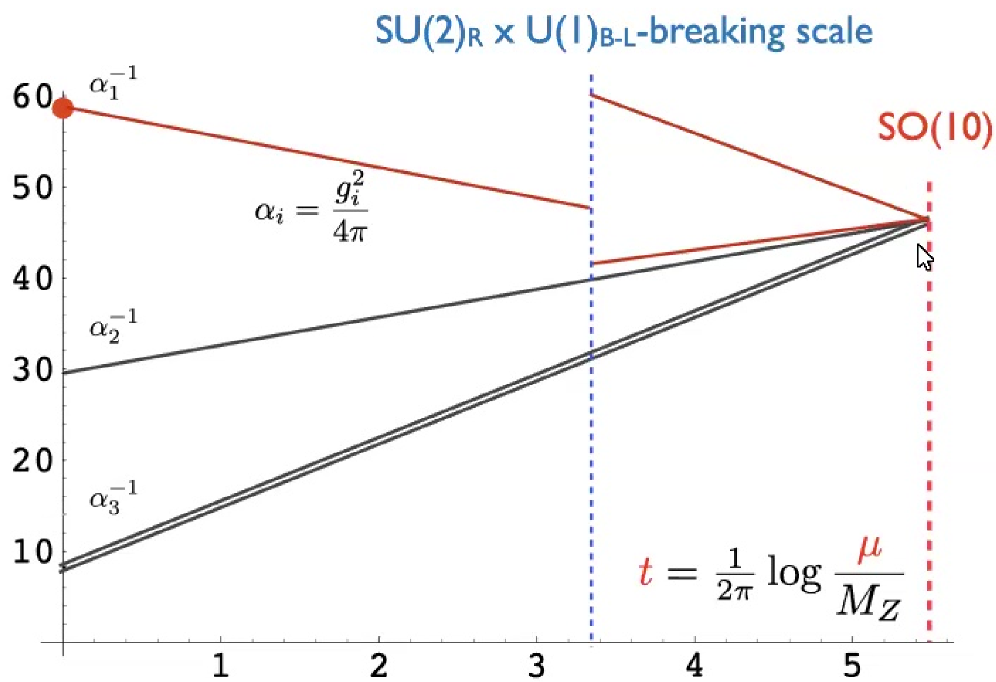


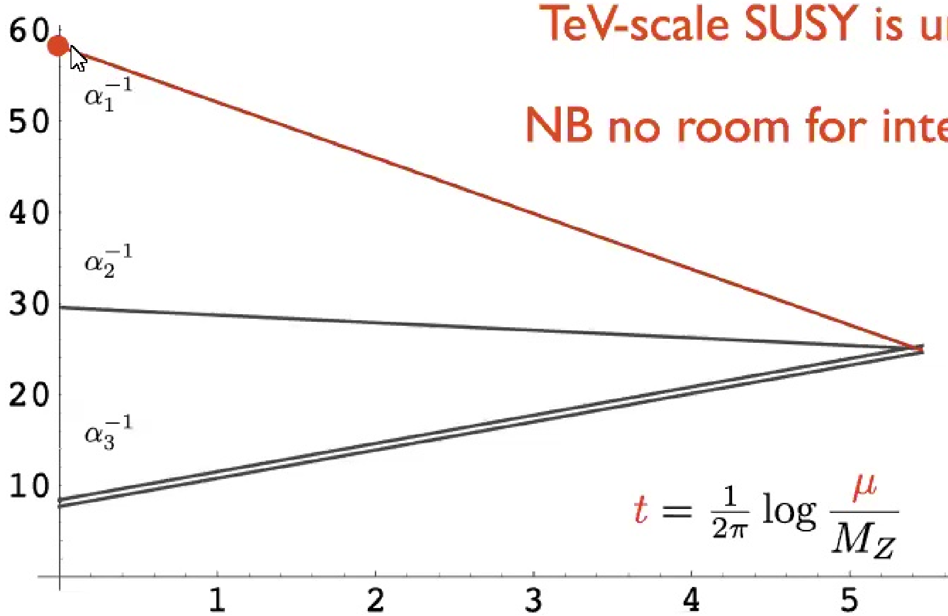
…or they may indicate a low-scale supersymmetry(?)

Evolution of gauge couplings in the MSSM (with TeV-scale SUSY)

+ gauginos

+ higgsinos

+ squarks and sleptons



TeV-scale SUSY is unlikely though…

NB no room for intermediate stages!

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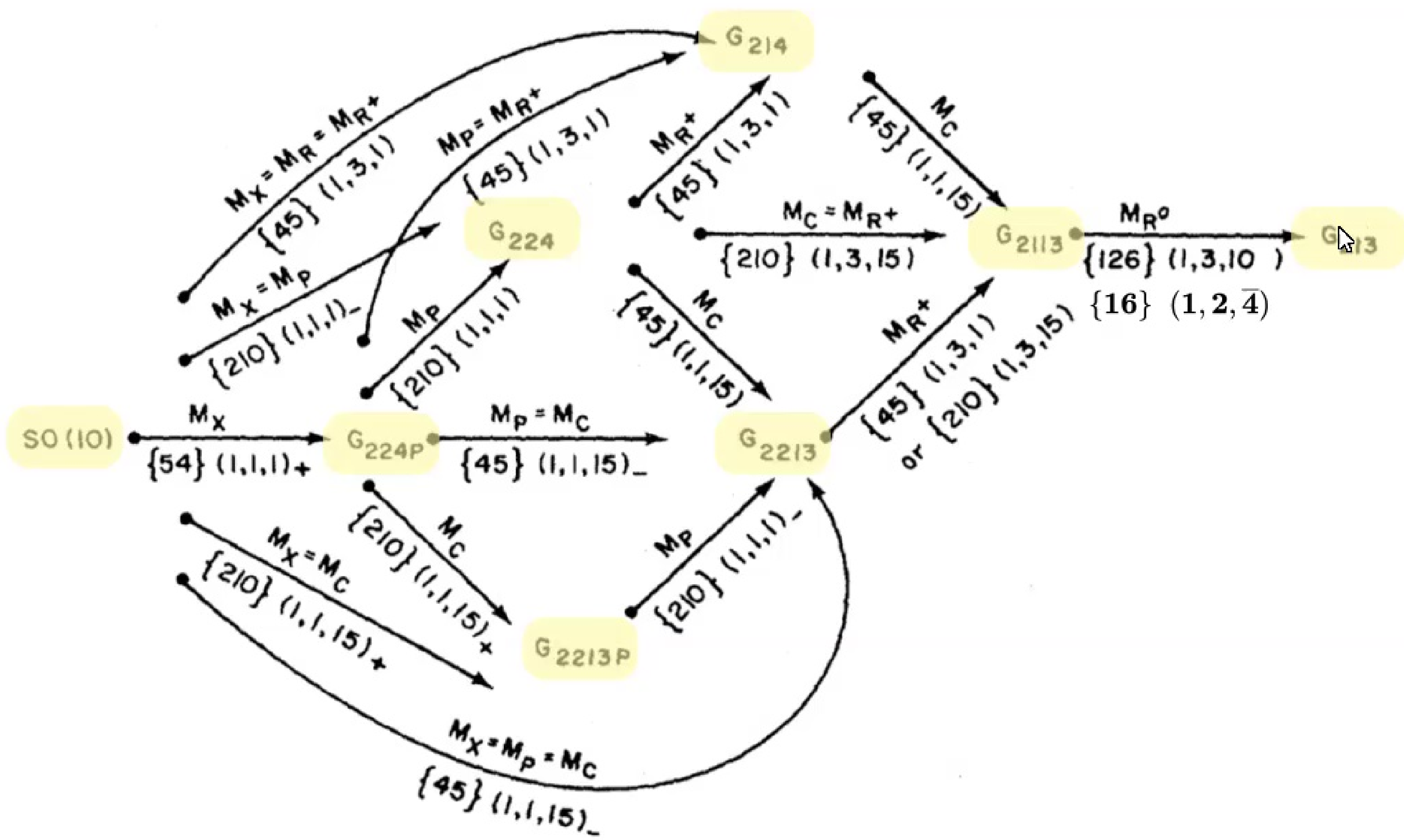
SO(10) symmetry breaking maze

Chang, Mohapatra, Gipson, Marshak, Parida 1985

SU(5) branches omitted



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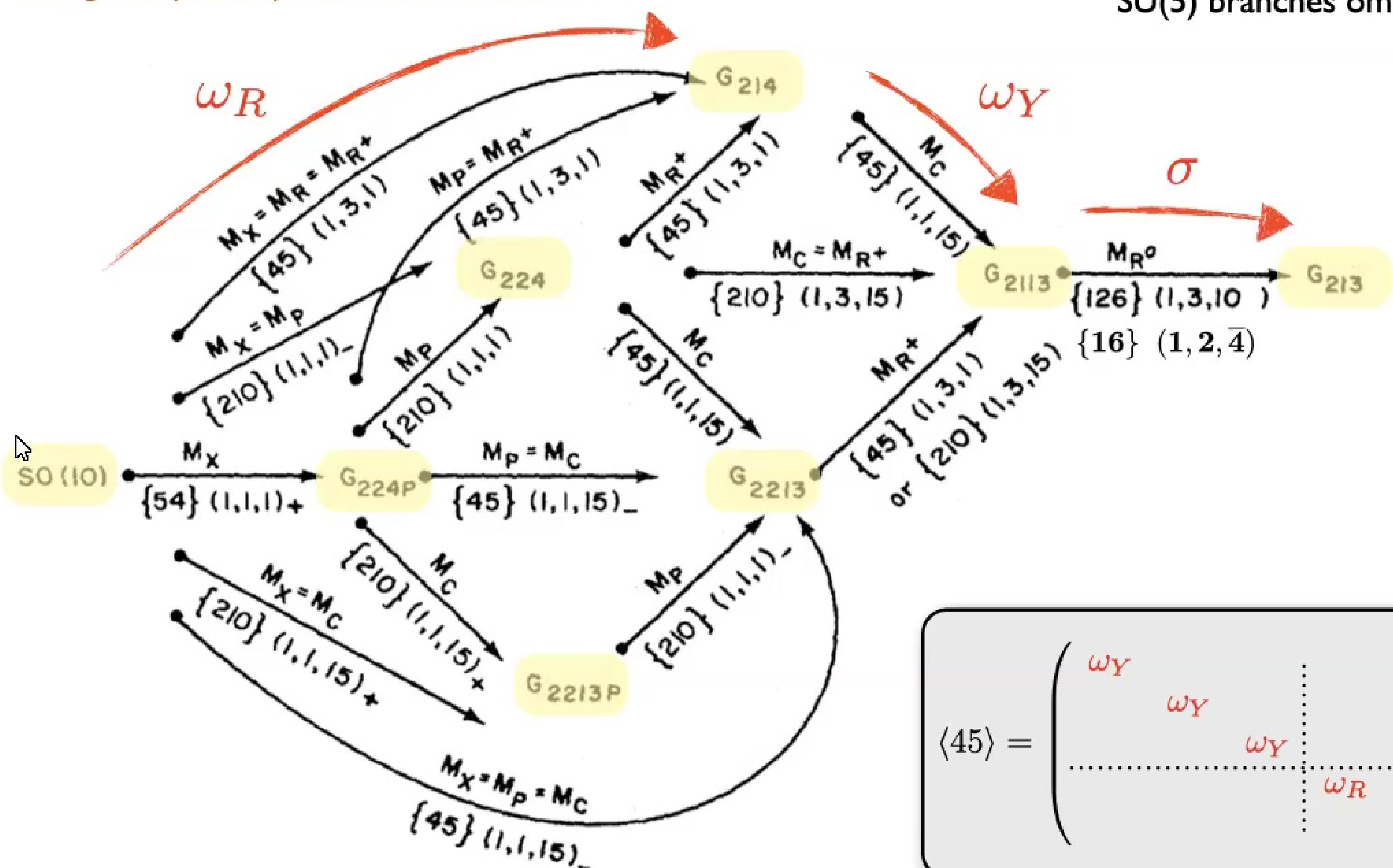

$$\langle 45\rangle = \begin{pmatrix} \omega_Y & & & & \\ & \omega_Y & & & \\ & & \omega_Y & & \\ & & & \omega_R & \\ & & & & \omega_R \end{pmatrix} \otimes \tau_2$$

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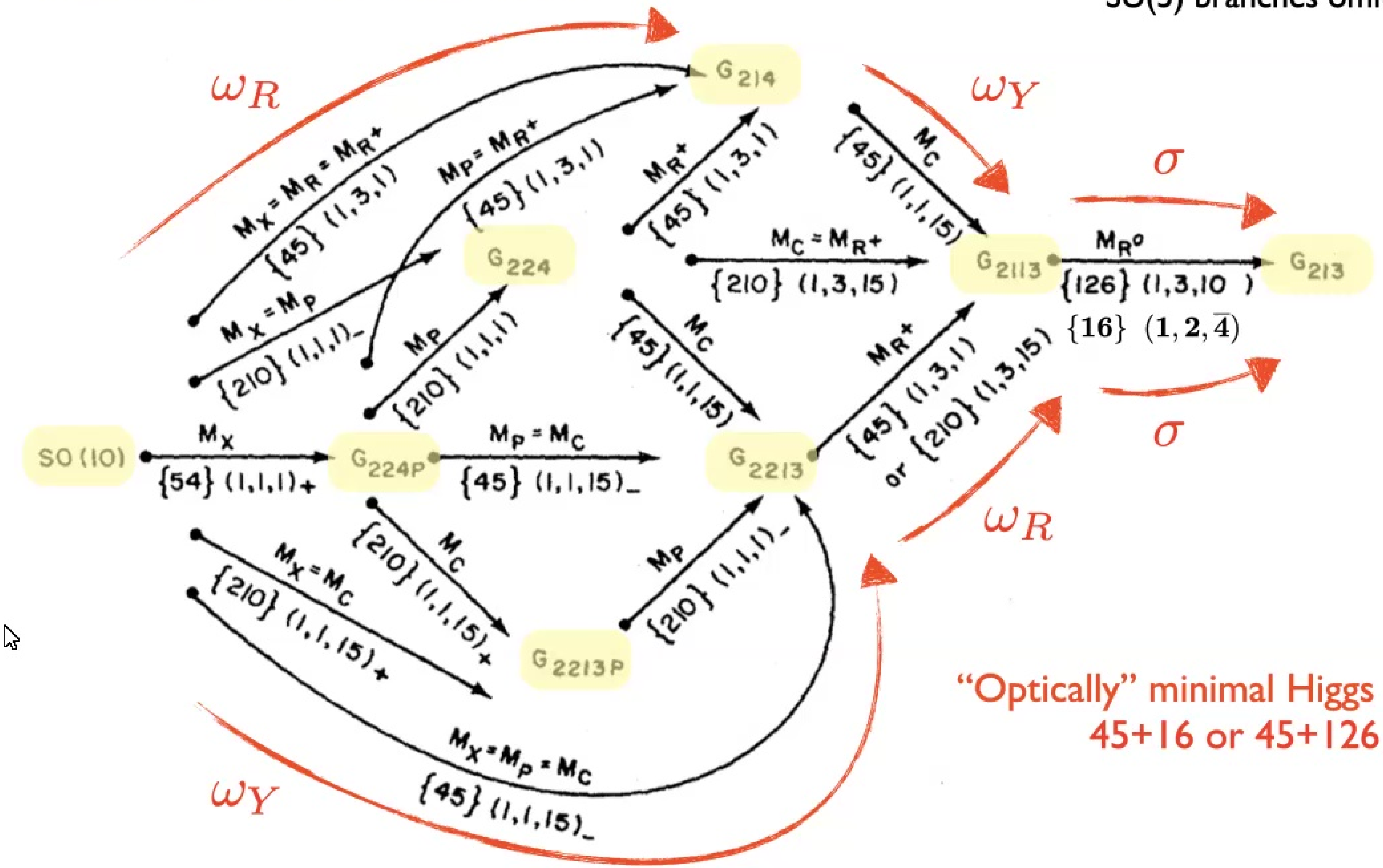

"Optically" minimal Higgs models:
45+16 or 45+126

Intermezzo: Seesaw mechanism for neutrino masses

Weinberg's d=5 operator:

S. Weinberg, PRL 43, 1566 (1979)

$$\mathcal{L}_{eff} \ni \frac{c}{\Lambda} LHLH$$

U(1)$_{B-L}$ must be broken

LOWER

- Neutrino oscillations:

$$\Delta m^2_{\odot} = (8.0 \pm 0.3) \times 10^{-5} \mathrm{eV}^2$$

$$|\Delta m^2_{A}| = (2.5 \pm 0.3) \times 10^{-3} \mathrm{eV}^2$$

UPPER

- Cosmology (structure): $\sum_i m_i \lesssim 1\mathrm{eV}$
- Beta decay experiments: $\langle m^{ee} \rangle \lesssim 1\mathrm{eV}$

$$\Lambda \sim (10^{12} - 10^{14})\ \mathrm{GeV}$$

U(1)$_{B-L}$ breaking should happen below the GUT scale !

45 + (16 or 126)?

GUT - seesaw scale separation prefers **renormalizable seesaw structure**

$\langle 16 \rangle$: non-renormalizable seesaw

$$M_R \sim \langle 16 \rangle^2 / M_P$$

- predictivity & scale problems

$\langle 126 \rangle$: renormalizable seesaw

$$M_R \sim \langle 126 \rangle$$

+ predictive, seesaw OK

The minimal SO(10) Higgs model

SO(10) broken by 45, rank reduced by 126

Scalar potential: $V = V_{45} + V_{126} + V_{\rm mix}$

$$V_{45} = -\frac{\mu^2}{2}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2\,,$$

$$\begin{aligned} V_{126} = &-\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 \\ &+ \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \\ &+ \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda_4'}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \\ &+ \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2\,, \end{aligned}$$

$$\begin{aligned} V_{\rm mix} = &\frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 \\ &+ \frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \frac{\beta_4'}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \\ &+ \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2\,. \end{aligned}$$

$$(\phi\phi)_0(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}\phi_{kl}\phi_{kl}$$

$$(\phi\phi)_2(\phi\phi)_2 \equiv \phi_{ij}\phi_{ik}\phi_{lj}\phi_{lk}$$

$$(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}\,,\quad (\Sigma\Sigma^*)_0 \equiv \Sigma_{ijklm}\Sigma^*_{ijklm}$$

$$(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 \equiv \Sigma_{ijklm}\Sigma^*_{ijklm}\Sigma_{nopqr}\Sigma^*_{nopqr}$$

$$(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \equiv \Sigma_{ijklm}\Sigma^*_{ijkln}\Sigma_{opqrm}\Sigma^*_{opqrn}$$

$$(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 \equiv \Sigma_{ijklm}\Sigma^*_{ijkno}\Sigma_{pqrlm}\Sigma^*_{pqrno}$$

$$(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \equiv \Sigma_{ijklm}\Sigma^*_{ijkno}\Sigma_{pqrln}\Sigma^*_{pqrmo}$$

$$(\Sigma\Sigma)_2(\Sigma\Sigma)_2 \equiv \Sigma_{ijklm}\Sigma_{ijkln}\Sigma_{opqrm}\Sigma_{opqrn}$$

$$(\phi)_2(\Sigma\Sigma^*)_2 \equiv \phi_{ij}\Sigma_{klmni}\Sigma^*_{klmnj}$$

$$(\phi\phi)_0(\Sigma\Sigma^*)_0 \equiv \phi_{ij}\phi_{ij}\Sigma_{klmno}\Sigma^*_{klmno}$$

$$(\phi\phi)_4(\Sigma\Sigma^*)_4 \equiv \phi_{ij}\phi_{kl}\Sigma_{mnoij}\Sigma^*_{mnokl}$$

$$(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \equiv \phi_{ij}\phi_{kl}\Sigma_{mnoik}\Sigma^*_{mnojl}$$

$$(\phi\phi)_2(\Sigma\Sigma)_2 \equiv \phi_{ij}\phi_{ik}\Sigma_{lmnoj}\Sigma_{lmnok}$$

$$(\phi\phi)_2(\Sigma^*\Sigma^*)_2 \equiv \phi_{ij}\phi_{ik}\Sigma^*_{lmnoj}\Sigma^*_{lmnok}$$

The minimal SO(10) ~~Higgs model~~ nightmare

SO(10) broken by 45, rank reduced by 126

Scalar potential: $V = V_{45} + V_{126} + V_{\rm mix}$

$$V_{45} = -\frac{\mu^2}{2}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2\,,$$

$$\begin{aligned} V_{126} = &-\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 \\ &+ \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \\ &+ \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda'_4}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \\ &+ \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2\,, \end{aligned}$$

$$\begin{aligned} V_{\rm mix} = &\frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 \\ &+ \frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \frac{\beta'_4}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \\ &+ \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2\,. \end{aligned}$$

???

$$(\phi\phi)_0(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}\phi_{kl}\phi_{kl}$$

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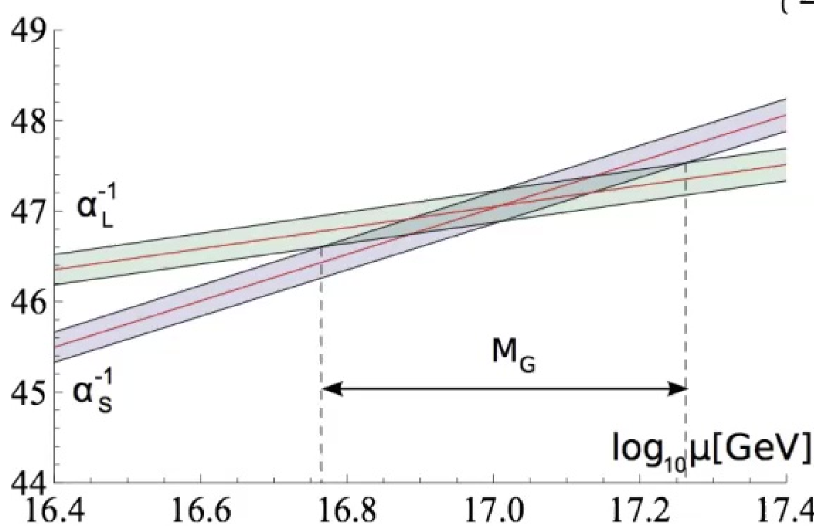
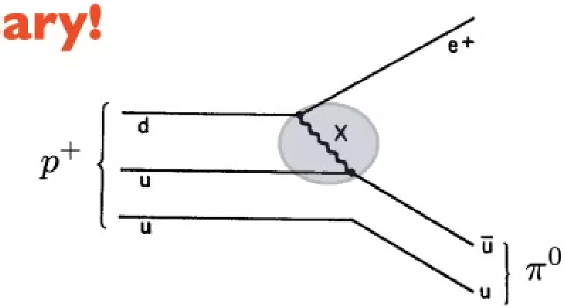
Intermezzo: GUT scale calculability

Two-loop running necessary!





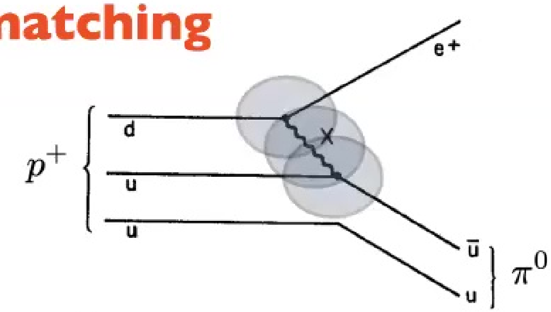
Full GUT-scale spectrum information needed...

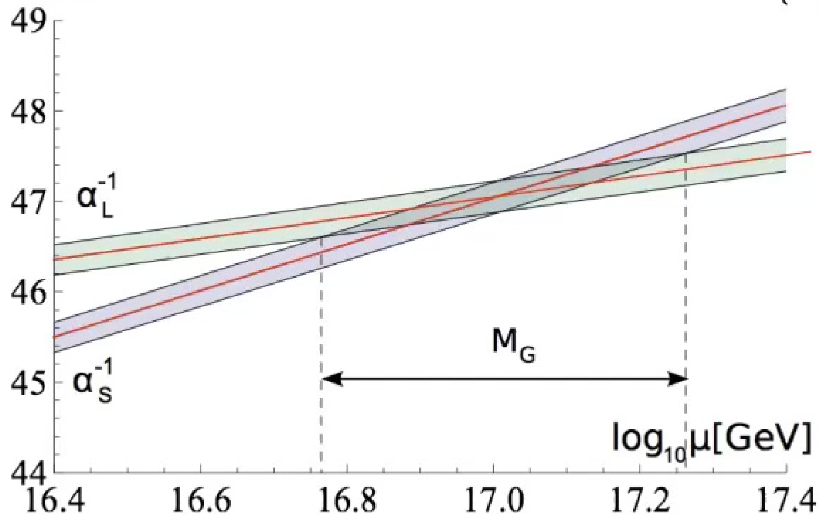
Intermezzo: GUT scale calculability

Planck-scale effects in gauge matching

Larsen, Wilczek, NPB 458, 249 (1996)
G. Veneziano, JHEP 06 (2002) 051
Calmet, Hsu, Reeb, PRD 77, 125015 (2008)
G. Dvali, Fortsch. Phys. 58 (2010) 528-536

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$





Intermezzo: GUT scale calculability

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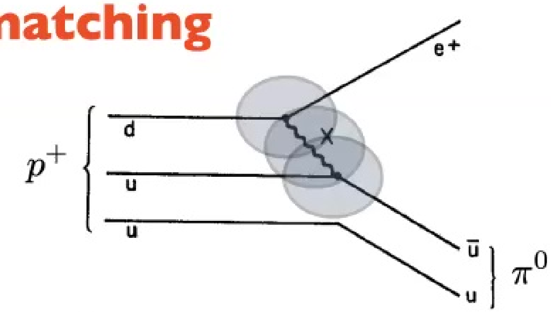

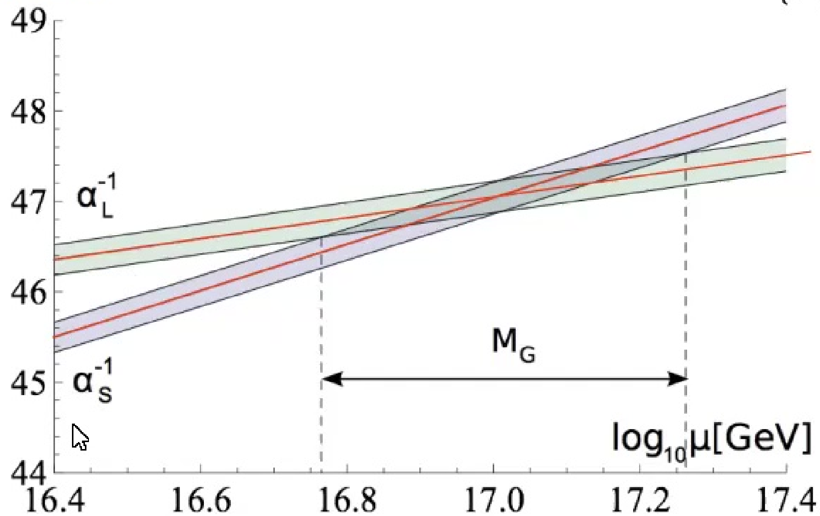

out-of-control inhomogeneous shifts in matching $\Delta\alpha_i^{-1} \sim 1$

Intermezzo: GUT scale calculability

Planck-scale effects in gauge matching

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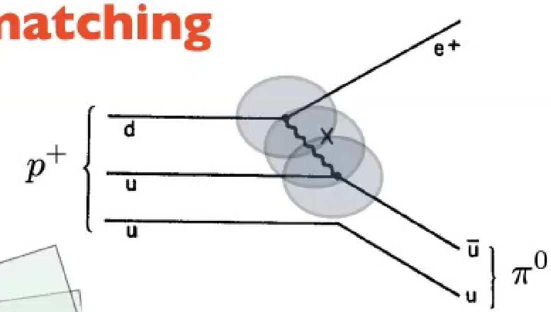

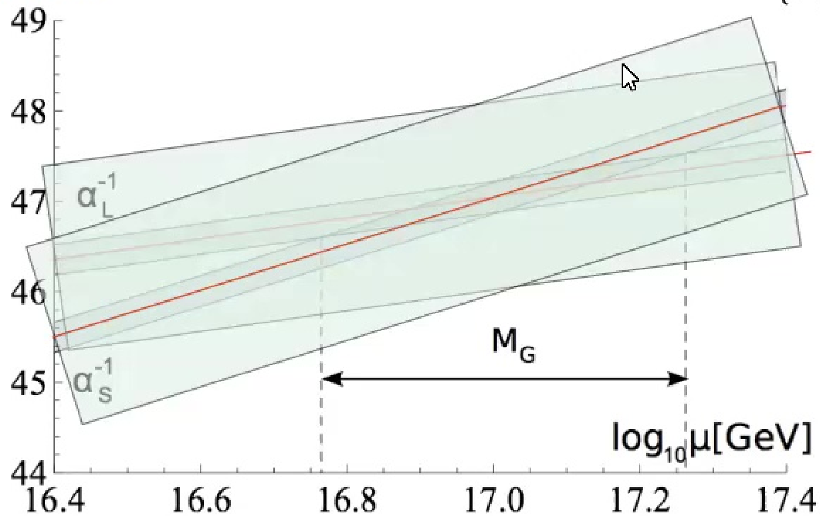

out-of-control inhomogeneous shifts in matching $\Delta\alpha_i^{-1} \sim 1$

More than order of magnitude uncertainty in M_G!

Intermezzo: GUT scale calculability

Not a problem @ d=5 if Φ is not in $(Adj. \otimes Adj.)_{sym}$

SU(5) GUTs: $(24 \otimes 24)_{sym} = 24 \oplus 75 \oplus 200$

SO(10) GUTs: $(45 \otimes 45)_{sym} = 54 \oplus 210 \oplus 770$

These, however, are the **"usual suspects"**…

Leading Planck-scale effects are absent in 45-broken SO(10) models !

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle 45 \rangle F_{\mu\nu} = 0$$

GUT scale under control in the minimal SO(10) with 45!

SO(10) model building constraints

Basic structural constraints = definition of the parameter space (practical x ideological)

- SUSY or non-SUSY?
- renormalizable x non-renormalizable?
- symmetry breaking pattern = selection of scalar irreps
- minimality (what does it mean?)
- GUT scale calculability
- doublet-triplet splitting
- naturalness

increasing cost

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- **local vacuum stability (no tachyons in the spectrum)**
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The minimal SO(10) ~~Higgs model~~ nightmare^2

Ruled out in 1980's

$$m^2_{(8,1,0)} = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y)$$
$$m^2_{(1,3,0)} = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)$$

Yasuè 1981, Anastaze, Derendinger, Buccella 1983, Babu, Ma 1985

$$\langle 45 \rangle = \begin{pmatrix} \omega_Y & & & & \\ & \omega_Y & & & \\ & & \omega_Y & & \\ & & & \omega_R & \\ & & & & \omega_R \end{pmatrix} \otimes \tau_2$$

$\omega_Y \gg \omega_R$

$$SO(10) \underset{\omega_Y}{\overset{45}{\rightarrow}} SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \underset{\omega_R}{\overset{45}{\rightarrow}} SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \overset{16}{\rightarrow} SM$$

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$$
\begin{aligned}
m^2_{(8,1,0)} &= 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y) \\
m^2_{(1,3,0)} &= 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)
\end{aligned}
$$

Yasuè 1981, Anastaze, Derendinger, Buccella 1983, Babu, Ma 1985

$$
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$$

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$$

$\omega_R \gg \omega_Y$

$$
SO(10) \underset{\omega_R}{\overset{45}{\rightarrow}} SU(4)_C \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \underset{\omega_Y}{\overset{45}{\rightarrow}} SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \overset{16}{\rightarrow} SM
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$$
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$$

Aaarrrggh... tachyonic spectrum unless $\frac{1}{2} < |\omega_Y/\omega_R| < 2$

SU(5)-like vacua only, not far from the single-step breaking regime!

The minimal SO(10) ~~Higgs model~~ nightmare^2

No-go!

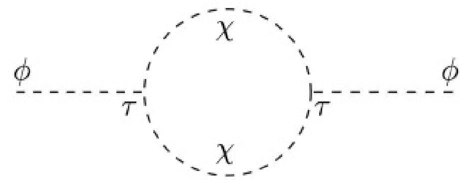
"Do not trust arguments based on the lowest order of perturbation theory."

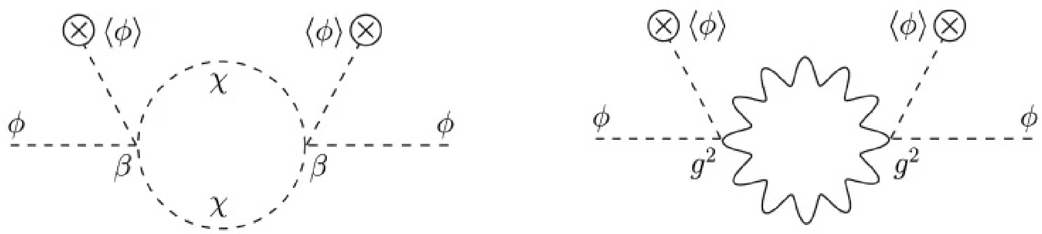
S.Weinberg , "Why RG is a good thing"
in "Asymptotic Realm of Physics", MIT press 1983

The minimal SO(10) ~~Higgs model~~ nightmare²

Quantum-level salvation in 2010

One-loop effective potential:





$$\Delta m^2_{(1,3,0)} = \frac{1}{4\pi^2}\left[\tau^2 + \beta^2(2\omega_R^2 - \omega_R\omega_Y + 2\omega_Y^2) + g^4\left(16\omega_R^2 + \omega_Y\omega_R + 19\omega_Y^2\right)\right] + \text{logs}\,,$$

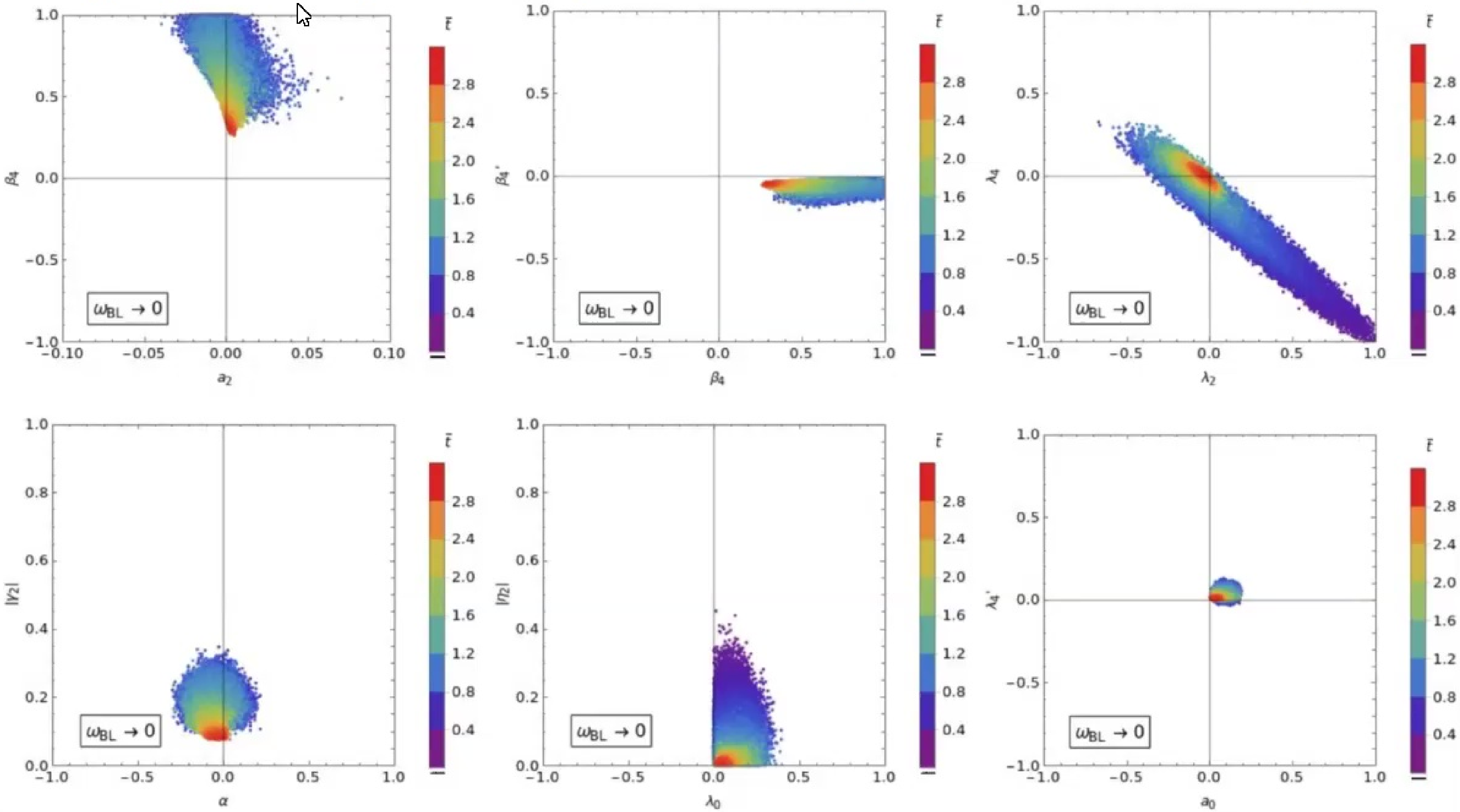
$$\Delta m^2_{(8,1,0)} = \frac{1}{4\pi^2}\left[\tau^2 + \beta^2(\omega_R^2 - \omega_R\omega_Y + 3\omega_Y^2) + g^4\left(13\omega_R^2 + \omega_Y\omega_R + 22\omega_Y^2\right)\right] + \text{logs}\,,$$

Bertolini, Di Luzio, MM, PRD 81, 035015 (2010)

The minimal SO(10) ~~Higgs model~~ nightmare²

Ongoing detailed perturbativity studies:

K. Jarkovská, MM, T. Mede, V. Susic, to appear soon



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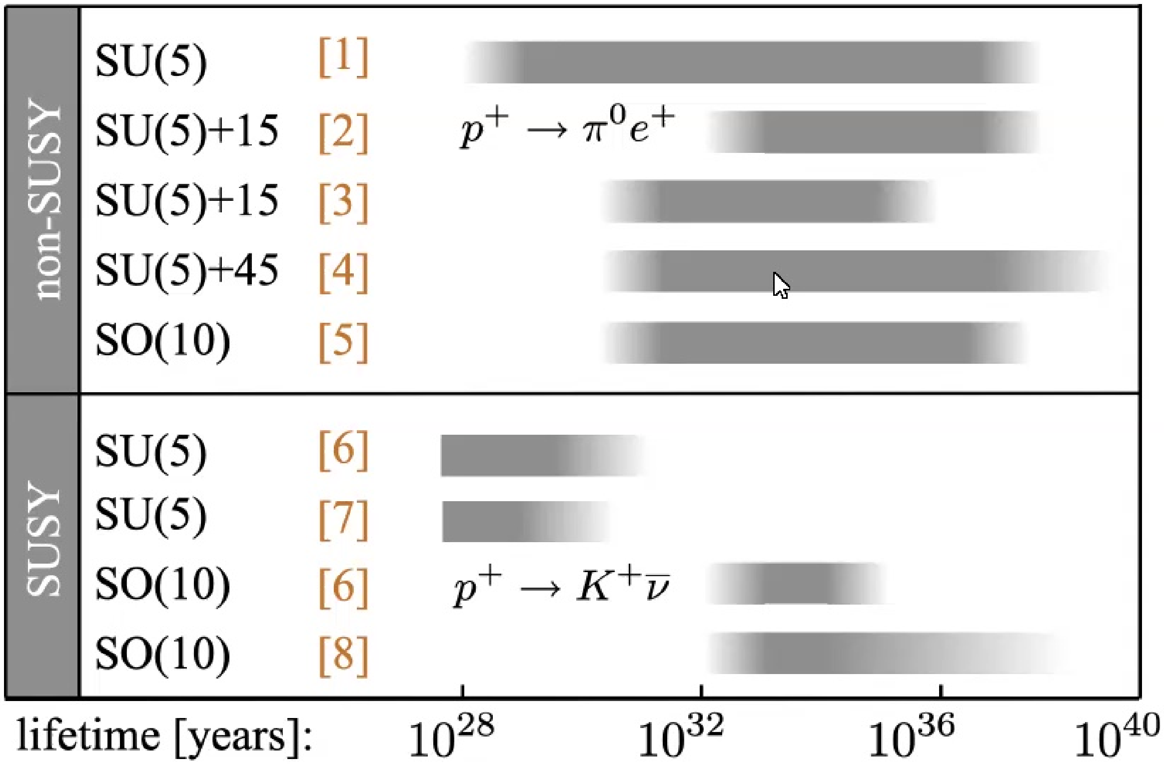
Suppose proton decay was found...

Days 3, 4, 5, 6, 7, 8, 9, ...

Are we in a position to discriminate among different GUTs ?

Existing proton lifetime estimates are really poor

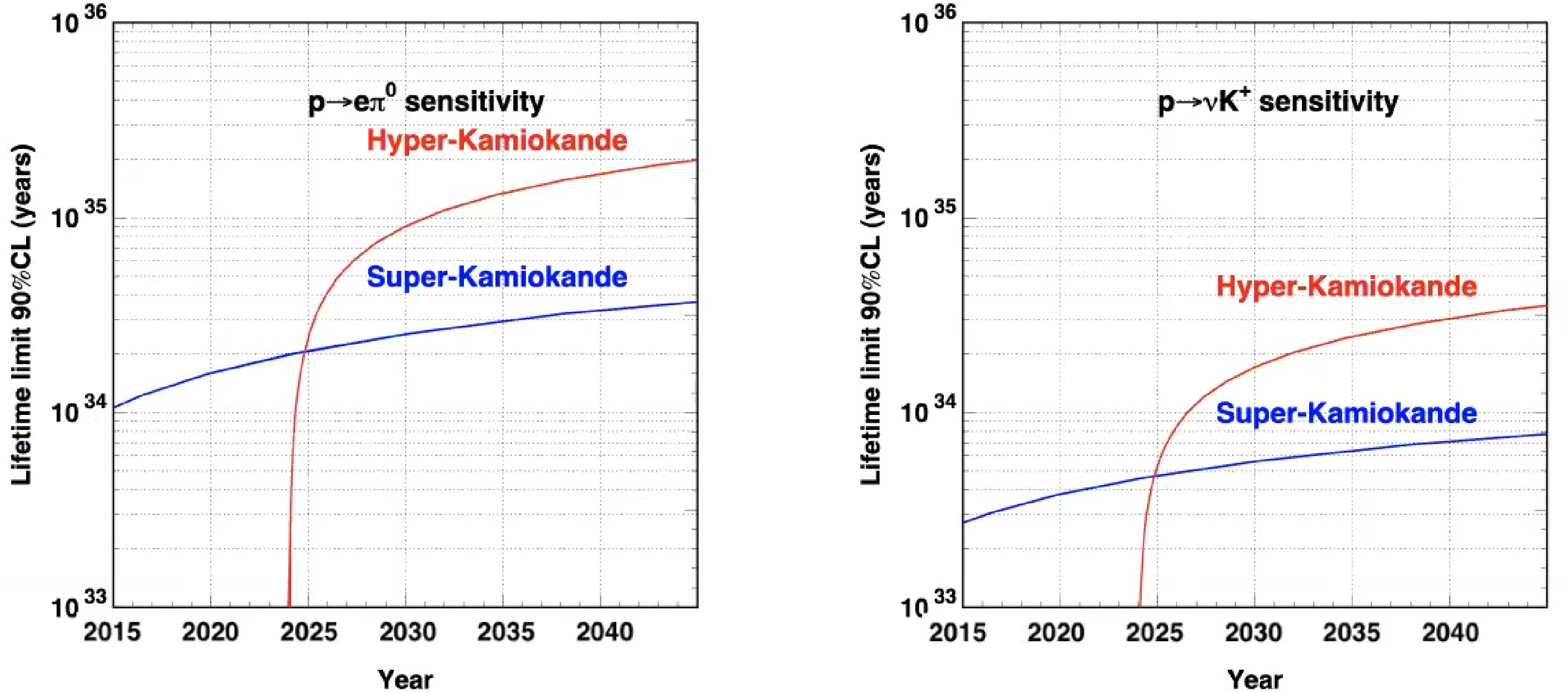


[1] Georgi, Quinn, Weinberg, PRL 33, 451 (1974)
[2] Dorsner, Fileviez Perez, NPB 723, 53 (2005)
[3] Dorsner, Fileviez Perez, Rodrigo, PRD75, 125007 (2007)
[4] Dorsner, Fileviez Perez, PLB 642, 248 (2006)
[5] Lee, Mohapatra, Parida, Rani, PRD 51 (1995)
[6] Pati, hep-ph/0507307
[7] Murayama, Pierce, PRD 65. 055009 (2002)
[8] Dutta, Mimura, Mohapatra, PRL 94, 091804 (2005)
... and many more.

Sensitivity of current and future p-decay searches

p-decay sensitivity projection (HK in 2025)

Abe et al., arXiv:1109.3262 [hep-ex]



about 1 o.o.m. "improvement window"

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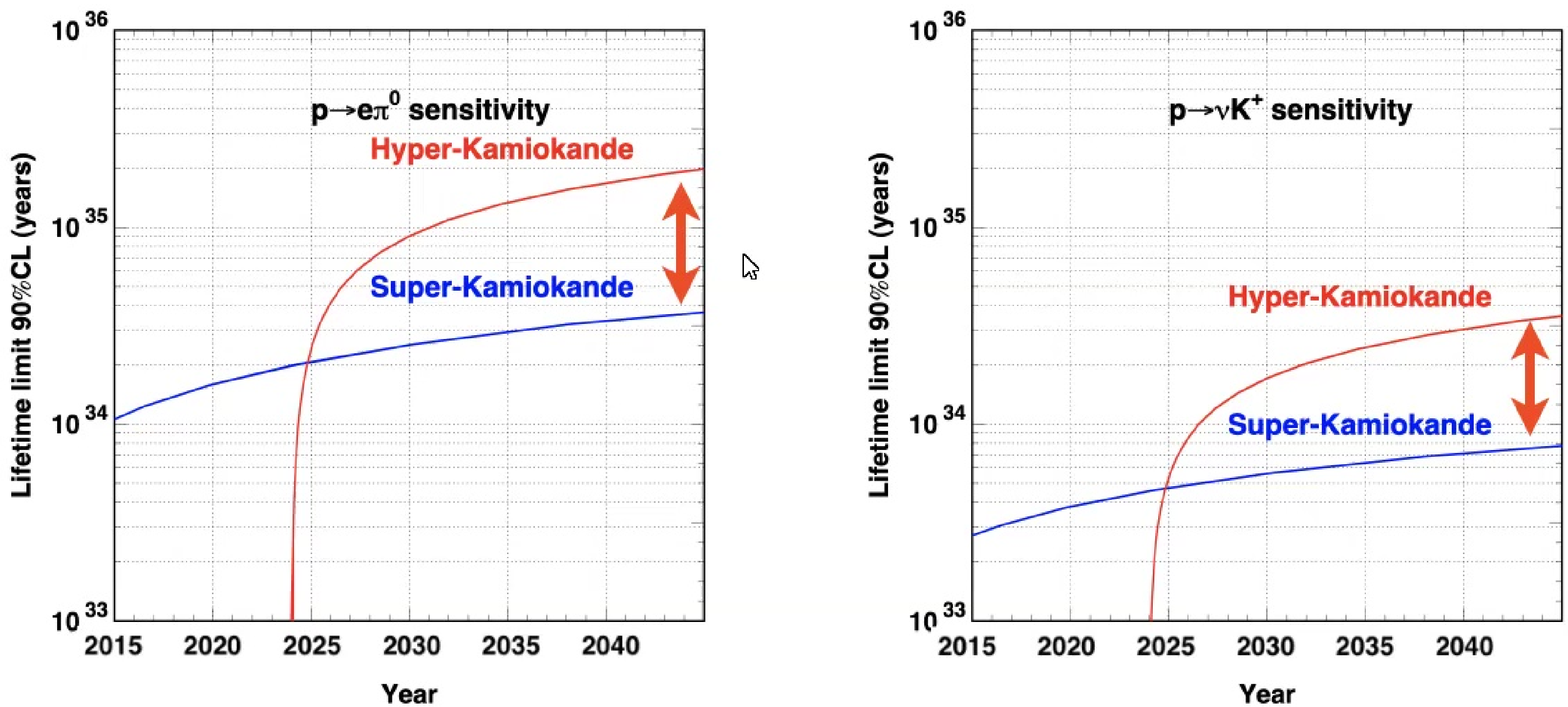
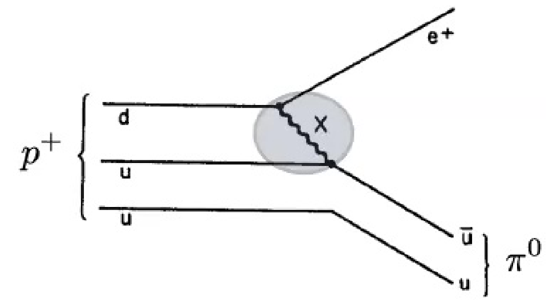


about 1 o.o.m. "improvement window"

Proton lifetime calculability

Mediator mass determination

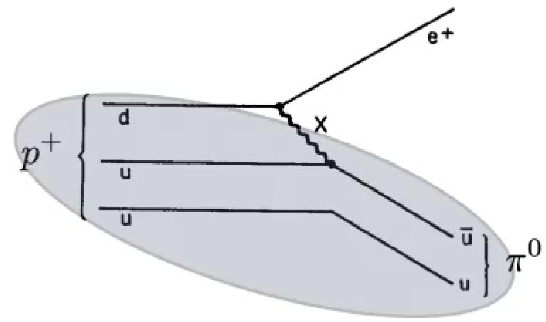
- Planck-scale proximity
- scalar spectrum & threshold effects
- higher-loop running



← **THE MAIN SHOW-STOPPERS**
(except for special scenarios like 45-driven SO(10) models)

Proton lifetime calculability

- **Mediator mass determination**
  - Planck-scale proximity
  - scalar spectrum & threshold effects
  - higher-loop running



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- **Hadronic matrix elements**

← **ANOTHER TRADITIONAL SHOW-STOPPER**
(until recently)

Proton lifetime calculability

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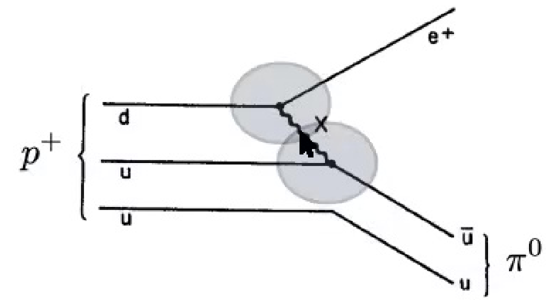

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- **Hadronic matrix elements**

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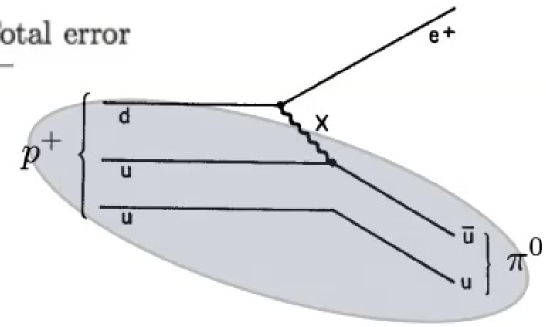
(until recently)

- **Flavor structure of the B&L violating currents**
  - Yukawa sector structure/fits

Hadronic matrix elements

Matrix element	$W_0(\mu = 2\text{GeV})$ GeV^2	(%) Total error
$\langle \pi^0 \vert (ud)_R u_L \vert p \rangle$	−0.103 (23) (34)	40
$\langle \pi^0 \vert (ud)_L u_L \vert p \rangle$	0.133 (29) (28)	30
$\langle \pi^+ \vert (ud)_R d_L \vert p \rangle$	−0.146 (33) (48)	40
$\langle \pi^+ \vert (ud)_L d_L \vert p \rangle$	0.188 (41) (40)	30
$\langle K^0 \vert (us)_R u_L \vert p \rangle$	0.098 (15) (12)	20
$\langle K^0 \vert (us)_L u_L \vert p \rangle$	0.042 (13) (8)	36
$\langle K^+ \vert (us)_R d_L \vert p \rangle$	−0.054 (11) (9)	26
$\langle K^+ \vert (us)_L d_L \vert p \rangle$	0.036 (12) (7)	39
$\langle K^+ \vert (ud)_R s_L \vert p \rangle$	−0.093 (24) (18)	32
$\langle K^+ \vert (ud)_L s_L \vert p \rangle$	0.111 (22) (16)	25
$\langle K^+ \vert (ds)_R u_L \vert p \rangle$	−0.044 (12) (5)	30

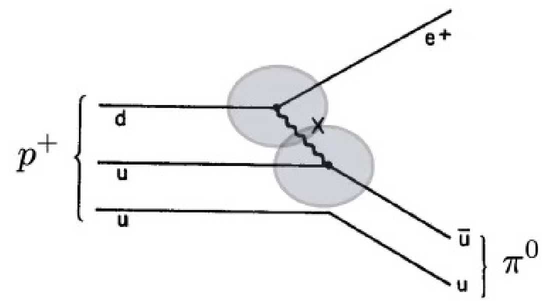


...

Y. Aoki, E. Shintani, A. Soni, Phys.Rev. D89 (2014) 014505 (lattice)

Flavor structure of BLNV charged currents

Example: $$\frac{g^2}{M_{1/6}^2} C_{ijk}\,\overline{u^c}\gamma^\mu d_i\,\overline{d_j^c}\gamma_\mu \nu_k$$



$$C_{ijk} \equiv (U_C^\dagger D)_{1i}(D_C^\dagger N)_{jk}$$

RH rotations - simple Yukawa sector desirable

some channels may be more "robust" than others

$$\Gamma(p\to\pi^+\overline{\nu}) \propto \sum_{\ell=1}^{3} |k_1^2(U_C^\dagger D)_{11}(D_C^\dagger N)_{1\ell} + k_2^2(D_C^\dagger D)_{11}(U_C^\dagger N)_{1\ell}|$$

$$k_{1,2} = \frac{g}{M_{X_{1,2}}}$$

Flavor structure of BLNV charged currents

Special scenarios with fully calculable (partial) **p-widths**

Nath, Fileviez-Perez, Phys.Rept.441
Dorsner, Fileviez-Perez, PLB605

SU(5): **symmetric up-quark mass matrix**

$$\Gamma(p \to \pi^+ \overline{\nu}) \propto k_1^4 \left|(V_{CKM})_{11}\right|^2$$
$$\Gamma(p \to K^+ \overline{\nu}) \propto k_1^4 \left(B_1^2 \left|(V_{CKM})_{12}\right|^2 + B_2^2 \left|(V_{CKM})_{11}\right|^2\right)$$

SO(10): **both quark mass matrices symmetric**

$$\Gamma(p \to \pi^+ \overline{\nu}) \propto k_1^4 \left|(V_{CKM})_{11}\right|^2 + k_2^4 + 2k_1^2 k_2^2 \left|(V_{CKM})_{11}\right|^2$$
$$\Gamma(p \to K^+ \overline{\nu}) \propto k_1^4 \left(B_1^2 \left|(V_{CKM})_{12}\right|^2 + B_2^2 \left|(V_{CKM})_{11}\right|^2\right)$$

$$B_1 = \frac{2m_p}{3m_B} D$$
$$B_2 = \frac{m_p}{3m_B}(D+3F)+1$$

Fully symmetric renormalizable SO(10) flavour structure

$$16_F = (4,2,1) \oplus (\overline{4},1,2)$$

Renormalizable Yukawa couplings: $16 \otimes 16 = 10 \oplus 120 \oplus 126$

SM doublet(s)
(homogeneous)

$$10 = (1,2,2) \oplus (6,1,1)$$

antisymmetric Yukawa

~~$120 = (1,2,2) \oplus (10,1,1) \oplus (\overline{10},1,1) \oplus (6,3,1) \oplus (6,1,3) \oplus (15,2,2)$~~

$$\overline{126} = (6,1,1) \oplus (\overline{10},3,1) \oplus (10,1,3) \oplus (15,2,2)$$

SM triplet with
small induced VEV
(type-II seesaw)

intermediate scale
SM singlet, large VEV,
gives mass to RH neutrino
(type-I seesaw)

SM doublet(s)
(inhomogeneous!)

Fully symmetric renormalizable SO(10) flavour structure

$$\mathcal{L} \ni 16_F Y_1^{10} 16_F 10_S + 16_F Y_2^{10} 16_F 10_S^* + 16_F Y^{126} 16_F \overline{126}_S + \text{h.c.}$$

Effective GUT-scale quark and lepton mass matrices in realistic SM-like scenarios

$$M_u = Y_2^{10} v_d^{10*}$$
$$M_d = Y_1^{10} v_d^{10} + Y^{126} v_d^{126}$$
$$M_l = Y_1^{10} v_d^{10} - 3\, Y^{126} v_d^{126}$$
$$m_\nu = c_1\, Y^{126} v_\Delta - M_\nu^D (M_\nu^M)^{-1} M_\nu^D$$

$$M_\nu^D = Y_2^{10} v_d^{10*}$$
$$M_\nu^M = c_2\, Y^{126} V_{B-L}$$

$$V_{B-L} \sim \langle (1,1,0)_{\overline{126}} \rangle$$
$$v_{u,d}^{10,126} \sim \langle (1,2,\mp 1)_{10,\overline{126}} \rangle$$
$$v_\Delta \sim \langle (1,3,+1)_{\overline{126}} \rangle$$

Subject to running, a rather nontrivial multi-parameter game…

H.S. Goh, R.N. Mohapatra, S-P Ng, PRD68, N. Oshimo, PRD66, S. Bertolini, M. Frigerio, MM, PRD70, L. Lavoura, H. Kuhbock, W. Grimus, NPB754, A.S. Joshipura and K. M. Patel, PRD83, A. Dueck, W. Rodejohann, JHEP09, G. Altarelli, D. Meloni, JHEP08, D. Meloni, T. Ohlsson, S. Riad, JHEP12, P. Nath and P. Perez, Phys.Rep.441, …

Conclusions, recapitulation

GUTs are theories of baryon/lepton number violation

Main signals: monopoles, proton decay

Notoriously difficult to calculate (Planck scale, perturbativity)

SO(10) GUTs are “phenomenologically optimal”

Conclusions, recapitulation

GUTs are theories of baryon/lepton number violation

Main signals: monopoles, proton decay

Notoriously difficult to calculate (Planck scale, perturbativity)

SO(10) GUTs are “phenomenologically optimal”

The 45-broken SO(10) is arguably the most promising GUT