Title: A World without Pythons would be so Simple

Speakers: Netta Engelhardt

Series: Quantum Fields and Strings

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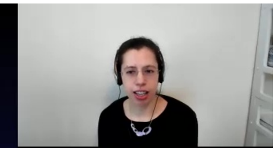
Abstract: We show that bulk operators lying between the outermost extremal surface and the asymptotic boundary admit a simple boundary reconstruction in the classical limit. This is the converse of the Python's lunch conjecture, which proposes that operators with support between the minimal and outermost (quantum) extremal surfaces - e.g. the interior Hawking partners - are highly complex. Our procedure for reconstructing this "simple wedge" is based on the HKLL construction, but uses causal bulk propagation of perturbed boundary conditions on Lorentzian timefolds to expand the causal wedge as far as the outermost extremal surface. As a corollary, we establish the Simple Entropy proposal for the holographic dual of the area of a marginally trapped surface as well as a similar holographic dual for the outermost extremal surface. We find that the simple wedge is dual to a particular coarse-grained CFT state, obtained via averaging over all possible Python's lunches. An efficient quantum circuit converts this coarse-grained state into a "simple state" that is indistinguishable in finite time from a state with a local modular Hamiltonian. Under certain circumstances, the simple state modular Hamiltonian generates an exactly local flow; we interpret this result as a holographic dual of black hole uniqueness.

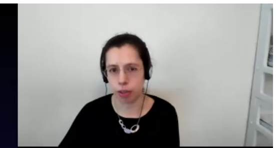


Simplicity in the Information Paradox

Netta Engelhardt

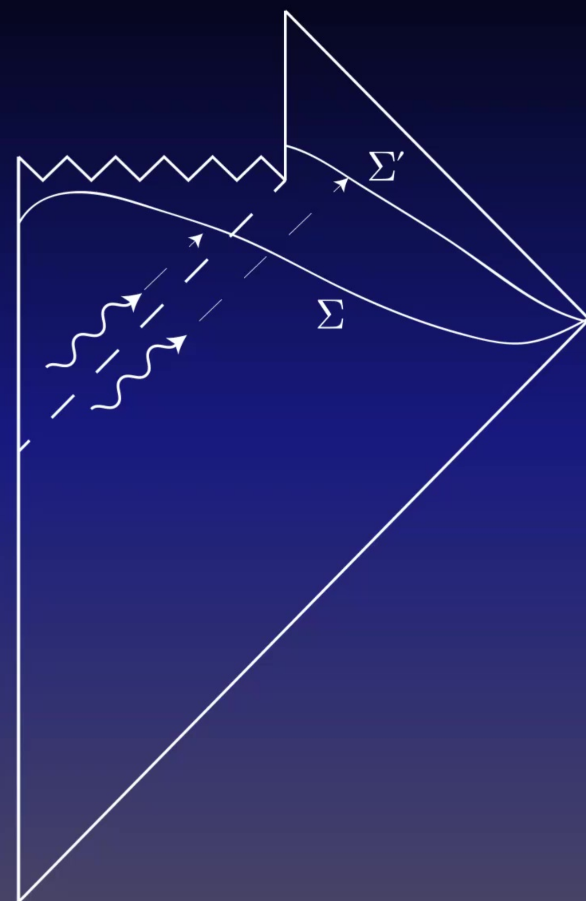
MIT

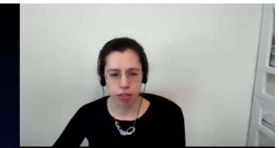
Based mostly on
2102.07774 and 2104.???? w/ Penington and Shahbazi-Moghaddam
1702.01748, 1706.02038, 1806.01281 w/ Wall



Some Recent New Insights

- The past (almost) two years have seen a lot of progress on the black hole information paradox
- Insights from these developments have started to teach us more about gravity in general
- This is of course why we are interested in the BH info paradox in the first place: to learn more about quantum gravity.
- But this is not (yet) a time for singing paeans: despite rapid progress, we have yet to actually resolve the paradox.



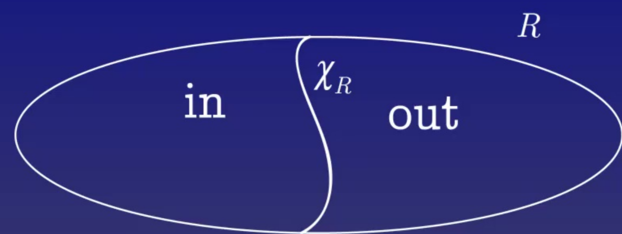


Where we stand now

- In 2019, we Almheiri, NE, Marolf, Maxfield; Penington computed the unitary Page curve from the QES formula NE, Wall '14:

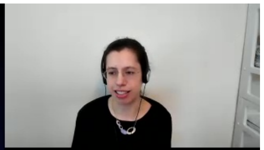
$$S_{vN}[\rho_R] = \frac{\text{Area}[\chi_R]}{4G\hbar} + S_{\text{out}}[\chi_R] = S_{\text{gen}}[\chi_R]$$

where χ_R is the minimal-S_{gen} surface that extremizes S_{gen}. (In the classical case we extremize just the area RT, HRT, FLM)



- In simple setups, this formula has since been derived from the gravitational path integral Penington et al; Almheiri et al

Some Questions

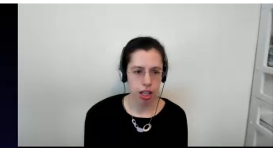


- This raised a few puzzles:

    2. How to get the QES result working in Hawking's setup; equivalently, what was Hawking's mistake?

It will turn out that the two puzzles are related.

Some Questions

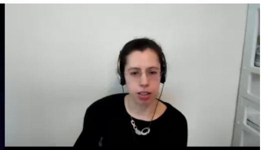


- This raised a few puzzles:

2. How to get the QES result working in Hawking's setup; equivalently, what was Hawking's mistake?
3. What is the geometric avatar of the exponential complexity of Hawking radiation, as predicted by Harlow-Hayden?

It will turn out that the two puzzles are related.

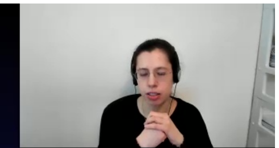
Some Questions



- This raised a few puzzles:
    1. How to explain the factorization problem in the gravitational path integral?

It will turn out that the two puzzles are related.

Where was Hawking's Mistake?

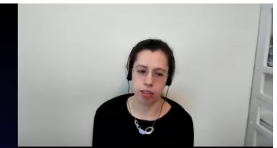


We have **two** ways of computing the entropy of Hawking radiation:

1. The QES formula, or equivalently the gravitational path integral
2. Hawking's calculation (via Bogoliubov transformations, etc.).

They give different answers; so to resolve the information paradox we must ask: **where do these two approaches diverge?**

Where Hawking went wrong: Pre-2019

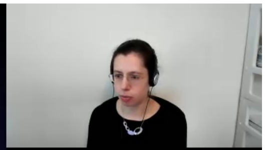


If you had asked me before 2019 where Hawking went wrong in his calculation, I probably would have said:

"Oh you know, Harlow-Hayden probably means that somewhere, somehow, Hawking's calculation lost track of any exponentially complex data in the state".

The logic here follows from Harlow-Hayden, Aaronson, Kim-Tang-Preskill, who showed that decoding the (post-Page time) Hawking radiation is exponentially complex.

Complexity and Decoding the Radiation



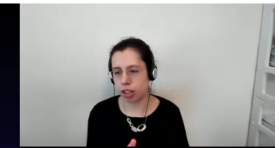
The idea here is that the Hawking radiation after the Page time is "pseudorandom": it walks like a thermal state and quacks like a thermal state, but is not in fact a thermal state. (If it were a thermal state it would of course have no information about the infalling matter)

But to see that it is not a thermal state – i.e. that it in fact contains information about the infalling matter – it takes an exponentially complex operation.

If you can only make simple measurements on the radiation, it will look thermal.

So Hawking's calculation could potentially be coarse-graining over the outcomes of high complexity operations.

Where Hawking went wrong: Post-2019



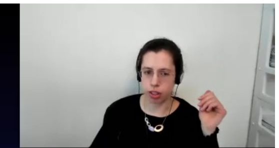
If you ask people now, most would probably say something along the lines of:

"Isn't it obvious? He used the wrong saddle in the gravitational path integral!"

The "wrong" saddle corresponds to a subdominant QES coming from the disconnected topology.

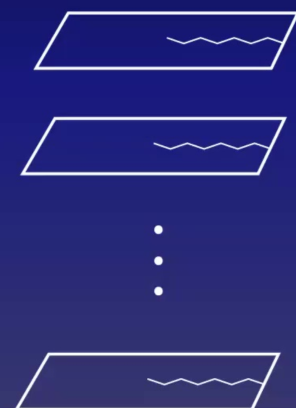
(Of course, it's not clear where in Hawking's calculation he started to indiscriminately impose ignorance of high-complexity data. And he certainly *didn't* use the gravitational path integral. But we're trying to bridge the gap between two calculations here.)

Review: the S_{vN} Replica Trick



In Euclidean signature, where we can write the state in terms of the Euclidean partition function:

$$S_{\mathrm{vN}}[\rho_R] = \lim_{n\to 1} \frac{1}{1-n} \left(\ln Z(B_n) - n \ln Z(B)\right)$$

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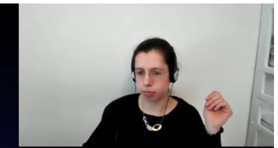
Review: $S_{\rm vN}$ Trick from the GPI



Getting back to gravity, the idea is (again, sticking to the Euclidean case) Lewkowycz-Maldacena: replace Z by the gravitational path integral:

$$Z(B) \to \mathcal{P}(B) = \int_{\partial M = B} Dg e^{-S}$$

and

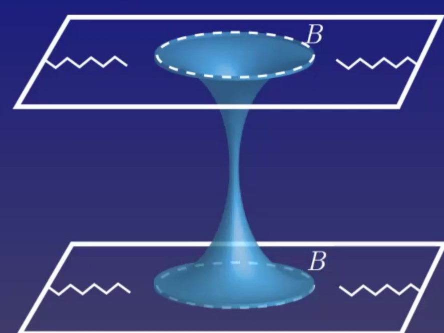
$$Z(B_n) \to \mathcal{P}(B_n) = \int_{\partial M = B_n} Dg e^{-S}$$

Subsequent Developments: Deriving the QEI

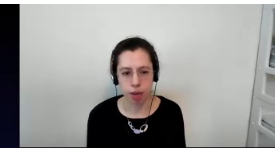
In general we have

$$\mathcal{P}(B^n) = \mathcal{P}(B)^n + \sum_{\substack{\text{connected}\\ \text{topologies}}} ,$$

It was found by Almheiri et al, Penington et al that in certain toy models, after the Page time, the contribution from connected topologies dominates. This corresponds to the new QES after the Page time.



Where Hawking Went Wrong

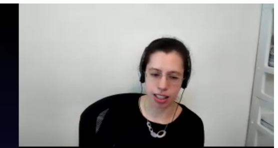


How are these two perspectives – complexity and saddle – compatible? By understanding the way in which these two mistakes are one and the same, we can make progress on resolving the paradox.

What does it mean in terms of the Lorentzian bulk geometry to (a) implement ignorance of high complexity data, and (b) use the wrong saddle in the GPI?

Can these be packaged into one unified statement?

Complexity of the State



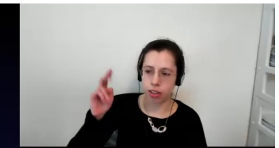
The second puzzle was about geometrization of complexity. We have a nice proposal by Lenny, further elucidated upon by this group at Perimeter, that

$$C \sim \mathcal{V}$$

where C is the computational complexity of the state, and $\mathcal{V}$ is the maximum volume slice of the bulk dual (at some time). similar proposal for the action, but this is Perimeter, so you all already know that

(**Aside** shameless self-advertising: upcoming paper with my student, Aasmund Folkestad: $\mathcal{V}$ in *all* spacetimes satisfying the WEC is universally bounded from below by $\mathcal{V}$ of pure AdS. This can be arbitrarily badly violated in spacetimes that satisfy the NEC (and BF bound) but violate WEC.)

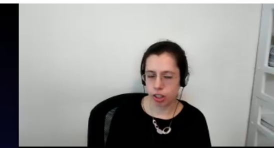
Complexity of the State



But $\mathcal{V}$ doesn't agree with Harlow-Hayden. Who is wrong?

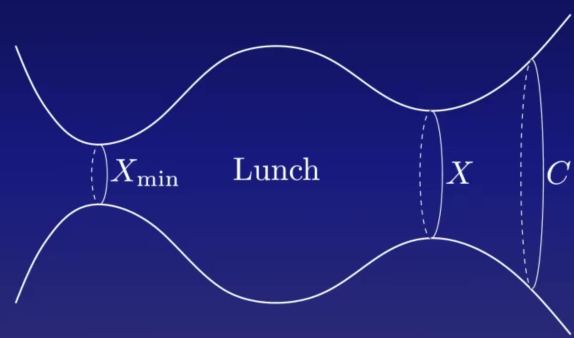
Python's Lunch Proposal: Brown et al both are right, because they are computing different quantities. $C = \mathcal{V}$ is computing the unrestricted complexity; Harlow-Hayden is asking you to decode the Hawking radiation having restricted your access to only the black hole exterior.

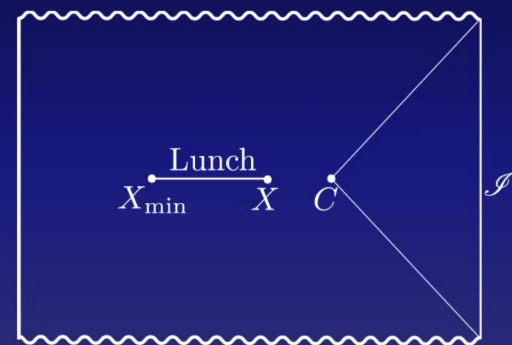
They proposed that to get the restricted complexity, you must take into account the *nonminimal QESs.*



Python's Lunch Proposal

Motivated by tensor network models, Brown et al proposed that whenever there exists a nonminimal QES in the entanglement wedge of the boundary, reconstruction of the region behind the nonminimal QES is exponentially complicated.

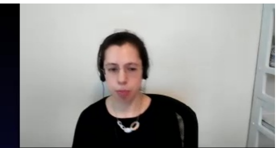




And that the *restricted* complexity is given by:

$$C \sim \mathcal{V} \exp\left[\frac{1}{2}\left(S_{\rm gen}[\gamma_{\rm bulge} - S_{\rm gen}[X]\right)\right]$$

In this talk...



Will discuss the gap between the "wrong saddle" and the coarse-graining over complexity in Hawking's original calculation. In doing so we will propose a stronger form of the Python's Lunch:

Strong Python's Lunch NE, Penington, Shahbazi-Moghaddam

Nonminimal quantum extremal surfaces are the *only* source of exponential complexity.

We prove this in the classical limit in our paper already on the arXiv. We are finishing up a demonstration of this in the presence of significant quantum corrections (i.e. where the QES is nonperturbatively different from the classical extremal surface).

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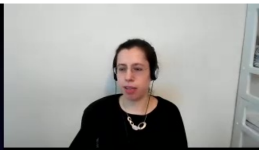
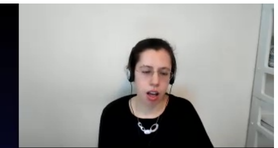



It is no bad thing
to celebrate
a simple life.
–J.R.R. Tolkien

The Simple Entropy Engelhardt, Wall '17, '18



Motivated by a separate question on black hole thermodynamics, we defined a dual CFT entropy that coarse-grains over high complexity using the Jaynes coarse-graining procedure:

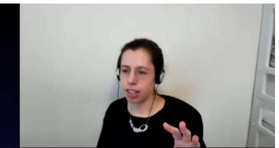
Jaynesian Coarse-Graining

Given a set of the expectation values of some operators $\{\mathcal{O}\}$, define an entropy that coarse-grains over all other data:

$$S^{\text{Jaynes}}[\{\mathcal{O}\}] = \max_{\rho \in \mathcal{H}} S_{\text{vN}}[\rho]$$

where $\mathcal{H}$ is the set of all ρ with the given expectation values of $\{\mathcal{O}\}$.

The Simple Entropy Engelhardt, Wall '17, '18



The Simple Entropy:

$$\boxed{S^{\text{simple}}[\rho_1, t_i, t_f] = \max_{\rho \in \mathcal{H}} S_{\text{vN}}[\rho]}$$

$\mathcal{H}$ = the set of ρs indistinguishable from the state ρ_1 if we only measure 1-point functions (between time t_i and t_f) but allow ourselves to turn on arbitrary "simple" sources.
In terms of equations:

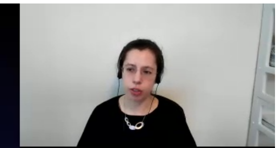
$$\left\langle E\mathcal{O}E^\dagger \right\rangle_{\rho_1} = \left\langle E\mathcal{O}E^\dagger \right\rangle_{\rho}$$

where $\mathcal{O}$ is a local operator,

$$E = \mathcal{T}exp\left[-i\int_{t_i}^{t_f} dt'\ J(t')\mathcal{O}_J(t')\right]$$

$J(t)$ is a simple source, and $\mathcal{O}_J$ is its corresponding simple operator.

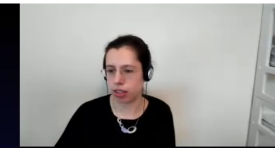
Simple Sources?



What exactly do we mean by simple sources?

This is difficult to define in general, so here we'll restrict ourselves to holographic QFTs and give a definition in terms of the dual bulk.

Using the intuition that precursors are highly complex, we define *simple sources* as causally propagating in the dual bulk.



Dual of the Simple Entropy

It turns out that the simple entropy computes the area of apparent horizons* in the dual AdS bulk!

By *apparent horizon**, I really mean compact surfaces satisfying the following criteria:

- Marginally trapped:

$$\theta_k \propto \frac{d\text{Area}}{d\lambda_k} = 0 \qquad \theta_\ell \propto \frac{d\text{Area}}{d\lambda_\ell} \leq 0$$

- Homologous to the asymptotic boundary.
- Is the outermost marginally trapped surface on some Cauchy slice.
- $\theta_{\ell,k} \leq 0$, with equality only if $\theta_\ell = 0$.

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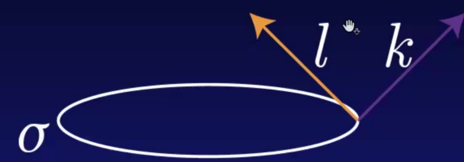

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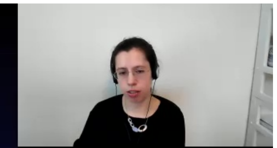
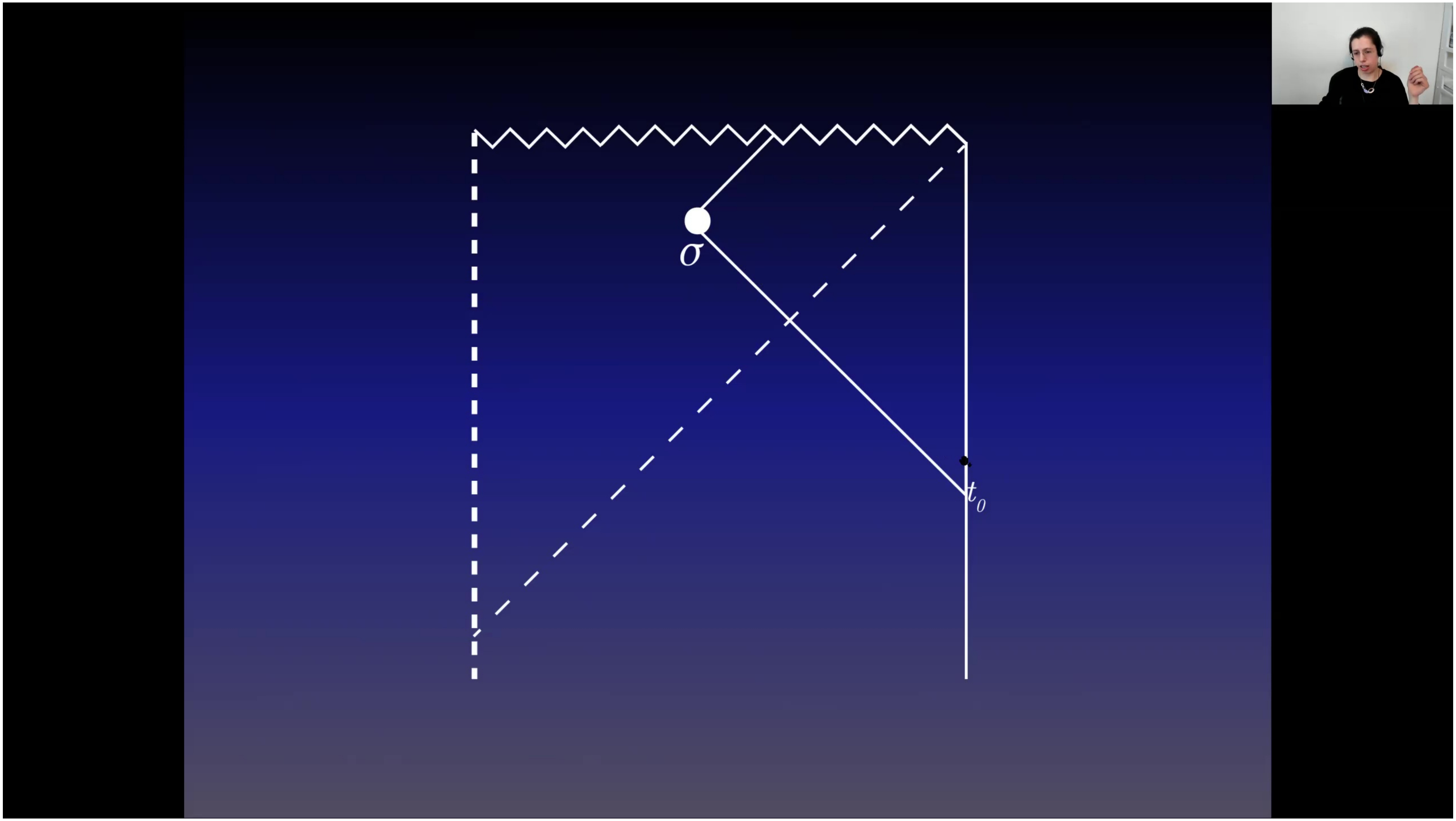
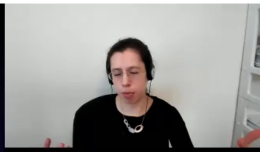



σ
t_0

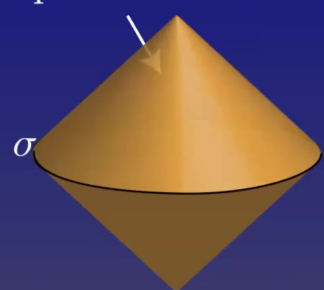
Dual of the Simple Entropy Engelhardt, Wall '17, '18



We proved that:

$$\frac{\text{Area}[\sigma]}{4G\hbar} = \max_{(M,g)\in\mathcal{B}} \frac{\text{Area}[X_{(M,g)}]}{4G\hbar} = S^{\text{outer}}[\sigma] = S_{\text{vN}}[\rho_{\text{coarse}}]$$

where $\mathcal{B}$ consists of all classical spacetimes that are indistinguishable outside of the apparent horizon α, and $X_{(M,g)}$ is the HRT surface of (M, g).

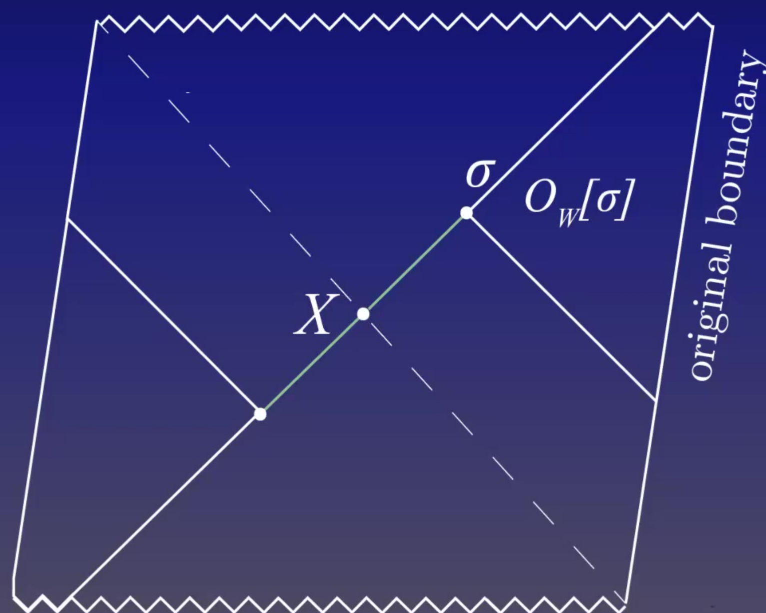


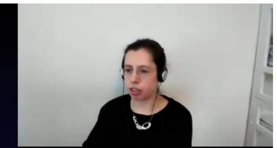
That is, the area of an apparent horizon is the maximum entropy that is consistent with its exterior.

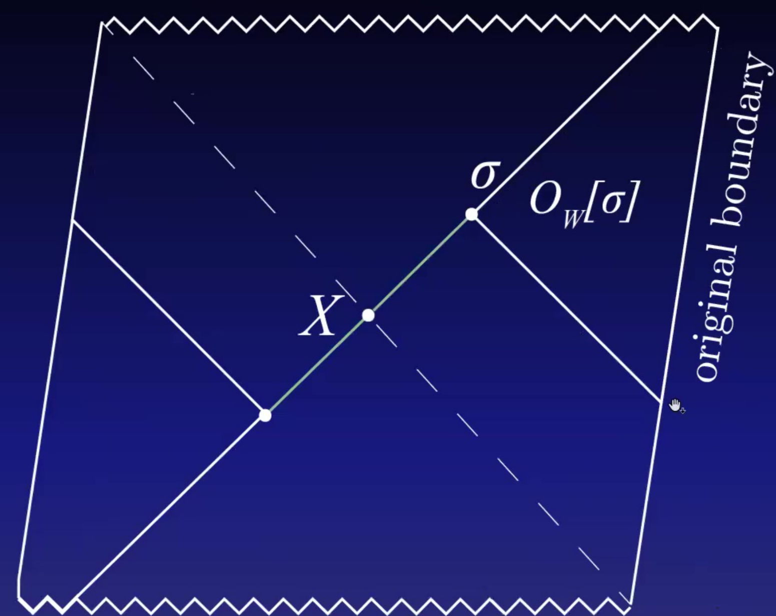
Argument

The argument is actually important for later parts of the talk, so I'll briefly go through it.

- It's easy to prove using focusing arguments that the Area of σ is an upper bound on the maximum entropy compatible with what we called $O_W[\sigma]$ – the exterior "wedge" of σ.
- To prove that this bound is saturated we constructed the saturating spacetime.







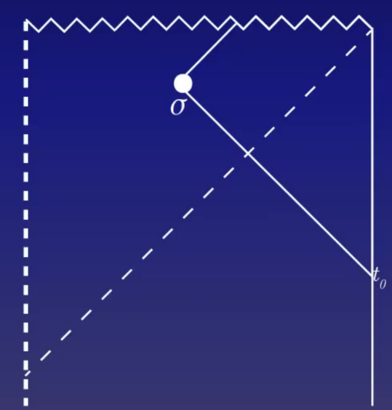
The green hypersurface here is stationary: $\theta_k = 0$ on all of it. Both X (by definition of extremality) and σ (by definition of marginality) have $\theta_k = 0$. The focusing theorem requires $\partial_k \theta_k \leq 0$. So stationary of the green hypersurface is crucial in allowing σ and X to be null-separated.

Holographic Complexity Coarse-Graining

Aron and I conjectured that

$$S^{\text{simple}}[\rho_{\text{bdy}}, t_0, t_f] = S^{\text{outer}}[\sigma]$$

Coincides with, where t_0 is the time at which the null hypersurface from σ hits the boundary and t_f is taken to be large.



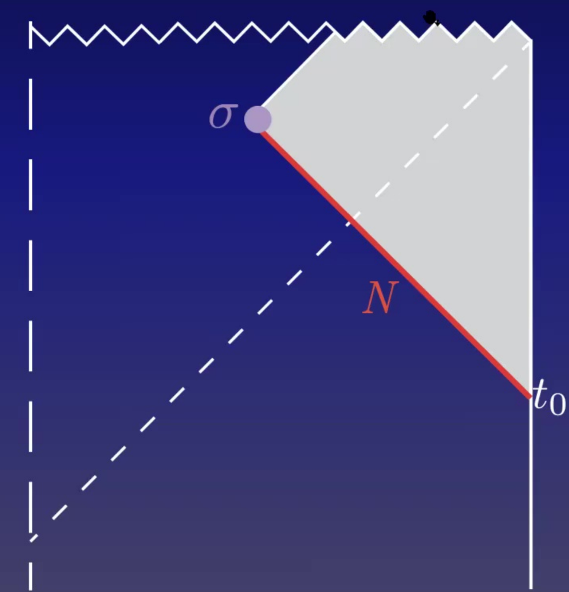
This idea is crucial for the recent paper with Geoff and Arvin, so I'll briefly explain it.

Boundary Description

Can we compute S^{outer} from the CFT?

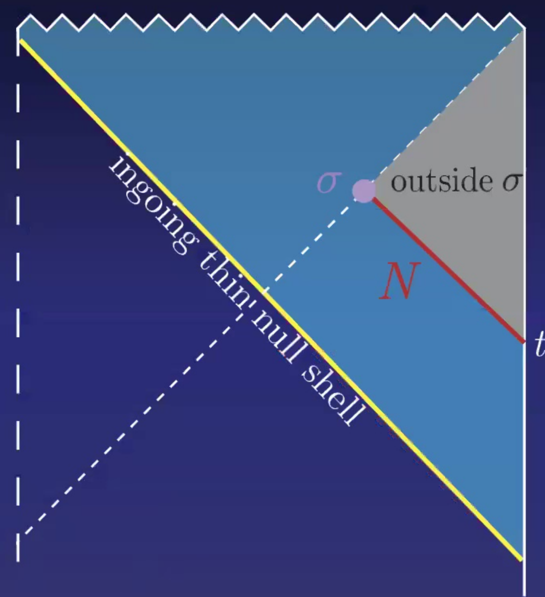
To do so, we need to find a way of fixing the outer wedge from boundary data.

This boils down to reconstructing the data on N from the CFT:



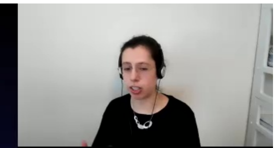
Boundary Description

If the black hole is in equilibrium after N, then this is the same as reconstructing the data in the causal wedge.



$$S^{coarse}[s] = S^{\text{simple}}[\rho_{bdy}, t_0]$$

Also works if black hole is approximately in equilibrium - i.e. after it settles down.

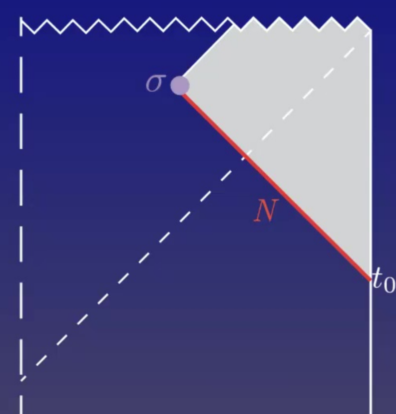


Boundary Description Out of Equilibrium

Black hole not in equilibrium $\Rightarrow$ there is infalling matter.

Idea

Suppose that by turning on certain local sources (that propagate causally in the bulk) σ, we can remove this matter. Can reconstruct causal wedge in new state from the one point functions $\langle O \rangle_\sigma$. Then we can turn σ off. This gives us the data on N.



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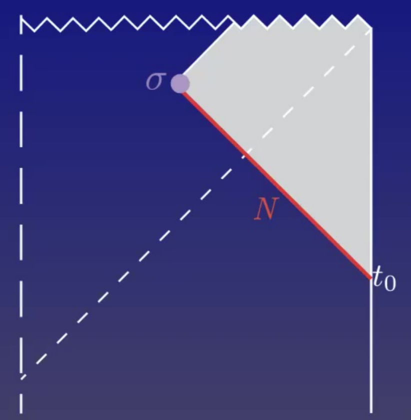
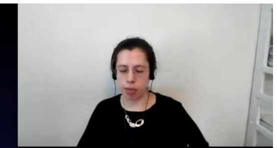
Idea

Suppose that by turning on certain local sources (that propagate causally in the bulk) σ, we can remove this matter. Can reconstruct causal wedge in new state from the one point functions $\langle O \rangle_\sigma$. Then we can turn σ off. This gives us the data on N.



Boundary Description Out of Equilibrium



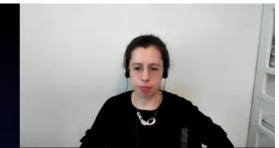
But we don't necessarily know which sources correspond to removing the infalling matter.

Theorem: We can never recover *more* than $O_W[\mu]$ no matter which sources we turn on. Hawking

$$S^{\text{outer}}[\sigma] \leq S^{\text{simple}}[\rho_{bdy}, t_0]$$

If the infalling matter can be removed by some local bulk-causal source, then it is an equality.

Coarse-Graining over Complexity: Summary



We have a procedure that forgets about high complexity operators after a certain time t_0, and which has a conjectured geometric dual in the area of an apparent horizon null-separated to t_0.

This conjectured duality is proven perturbatively around equilibrium at late times.

Important: locally minimal extremal surfaces are a *special instance* of apparent horizons with $t_0 \to -\infty$. So this prescription in particular applies to the "appetizer" surface.

Also note: while I focused on the classical construction here, it was generalized to include quantum corrections and quantum apparent horizons by Bousso, Chandrasekaran, Shahbazi-Moghaddam

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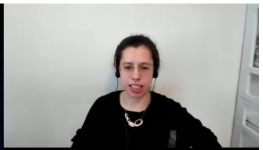
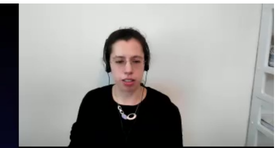
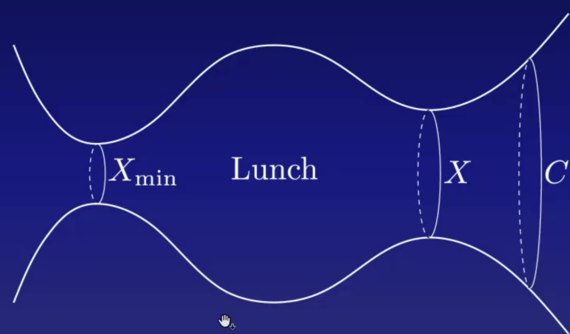
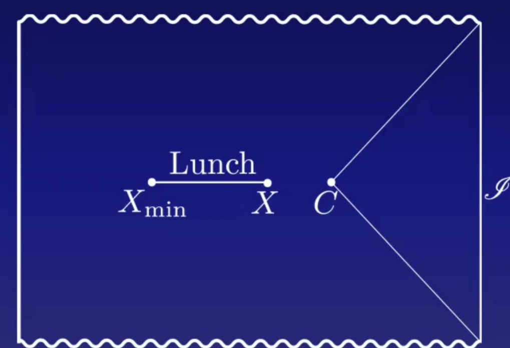


Python's Lunch Proposal Brown et al

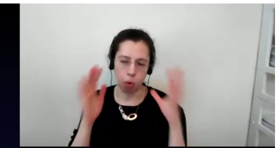


Recall:





Simple Entropy and the Wrong Saddle The next few slides are based on Engelhardt, Penington, Shahbazi-Moghaddam '21



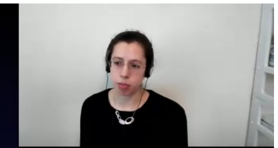
If we are to interpret the simple entropy and wrong saddle calculations as identical, then it must be the case that a strong Python's lunch proposal holds:

Strong Python's Lunch Proposal

Nonminimal QESs are the *only* source of exponential complexity.

Otherwise, we could have Hawking's calculation not corresponding in a nonminimal QES.

Simple Entropy and the Wrong Saddle The next few slides are based on Engelhardt, Penington, Shahbazi-Moghaddam '21



If we are to interpret the simple entropy and wrong saddle calculations as identical, then it must be the case that a strong Python's lunch proposal holds:

Strong Python's Lunch Proposal

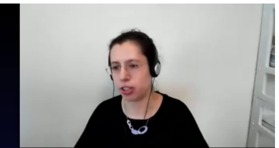
Nonminimal QESs are the *only* source of exponential complexity.

Otherwise, we could have Hawking's calculation not corresponding in a nonminimal QES.

This boils down to showing that in the absence of QESs, reconstruction is always simple. Because the causal wedge admits a simple reconstruction, this means that the region between the event horizon and the outermost QES always admits reconstruction from simple operators.

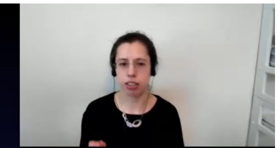
Strong Python's Lunch

Engelhardt, Penington, Shahbazi-Moghaddam '21



In the recent paper with Geoff and Arvin, we established this result for *classical* extremal surfaces.
The basic idea is as follows

1. Take to heart the simple entropy proposal that the only reason for the apparent horizon not to lie on the event horizon is if some matter falls across the event horizon.
2. "Remove" this matter by prescribing new initial data on the event horizon.
3. Use HKLL to evolve this new initial data to the asymptotic boundary. This requires assuming that HKLL always works outside of event horizons. This shows that there are simple operators we can turn on to remove the requisite matter.
4. Now we have a stationary future horizon, but what we actually want is a stationary bifurcation surface.
5. So we evolve backwards in time and repeat this procedure.

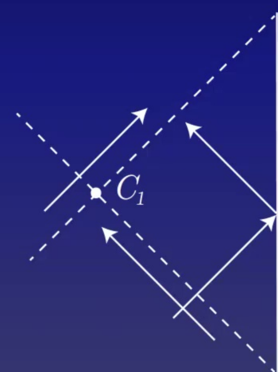


Two-Dimensional Illustration

Let's work with JT + massless scalar. The horizon will not be stationary, so the bifurcation surface will in general also not be stationary:

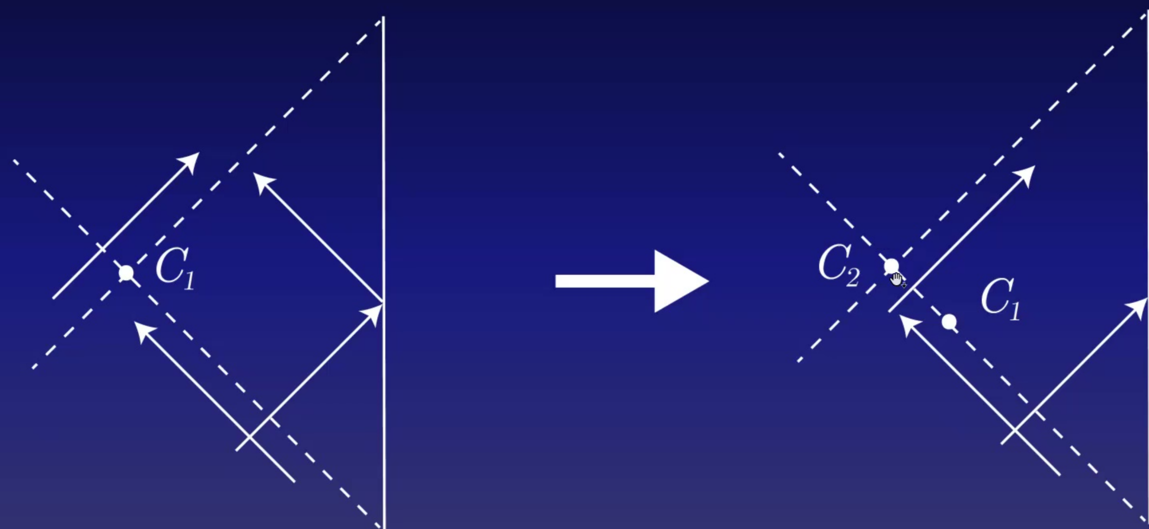
$$\partial_n \Phi|_{C_1} \neq 0$$

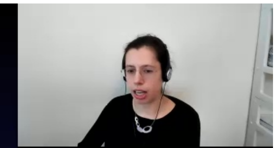
For any normal vector n^a.



In JT+massless scalar, this can only be a result of focusing from the scalar.

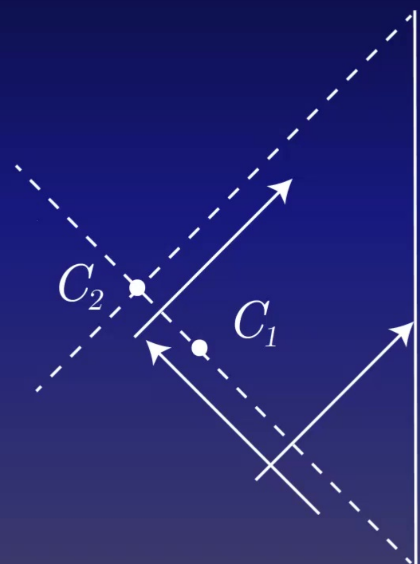
We can remove the source of focusing from the future horizon by absorbing boundary conditions for the right movers.

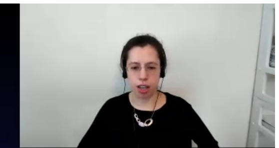




A new, deeper surface is now the bifurcation surface. This surface is marginally trapped:

$$\partial_\ell \Phi|_{C_2} < 0 \qquad \partial_k \Phi|_{C_2} = 0$$

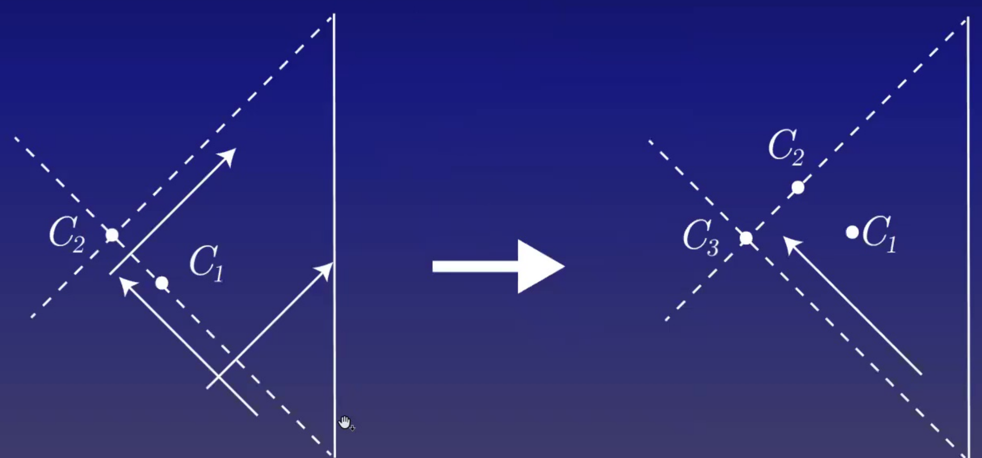


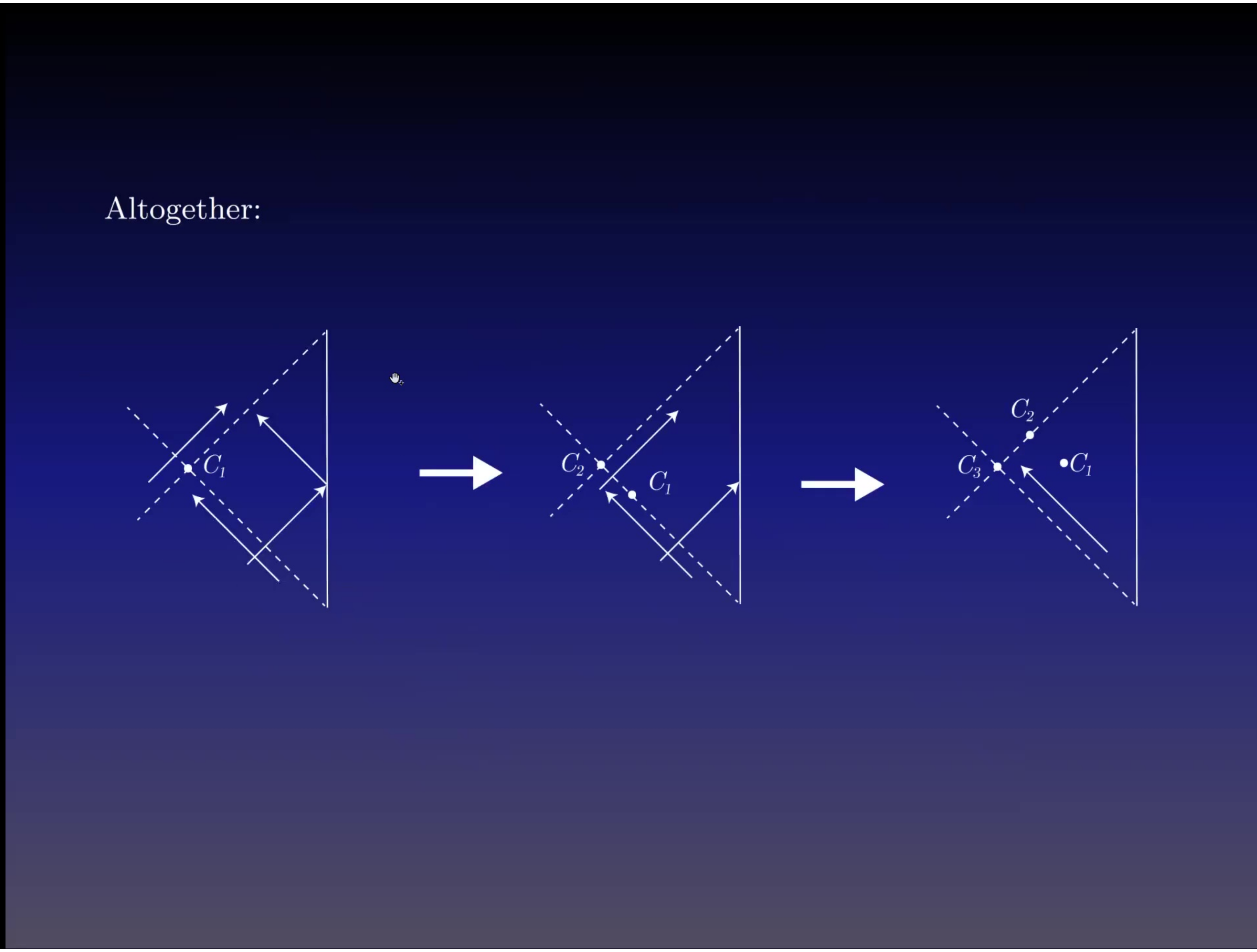


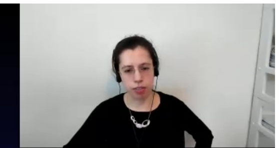
Now we evolve backwards in time, imposing boundary conditions that remove the right-movers.

This reveals a deeper bifurcation surface, which is now marginally anti-trapped:

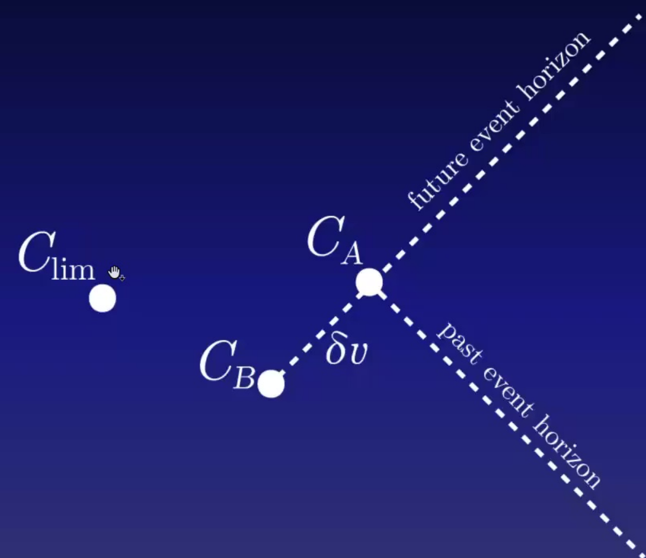
$$\partial_\ell \Phi|_{C_3} = 0 \qquad \partial_k \Phi|_{C_3} > 0$$




Altogether:
C_1
C_2
C_1
C_2
C_3
C_1



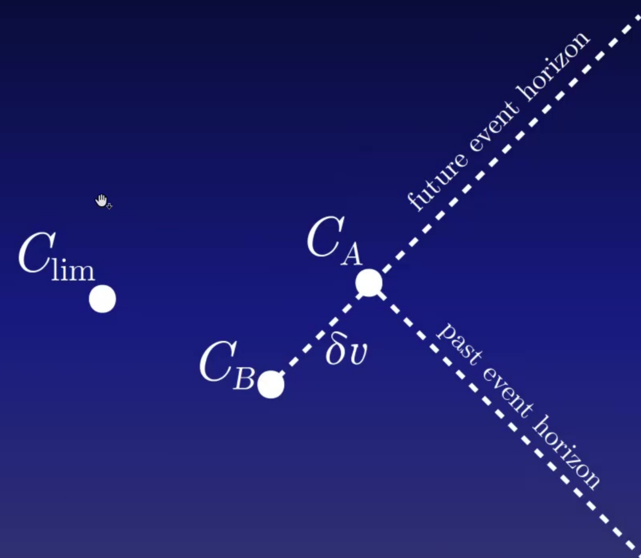
This procedure is bounded by the outermost extremal surface, but does it reach the outermost extremal surface?



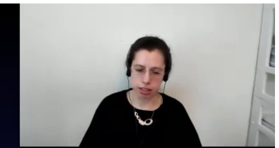
By construction, $\partial_k \Phi_{C_A} = 0$, and $\partial_\ell \Phi_{C_A} < 0$.

In these coordinates we can give a lower bound to δv:

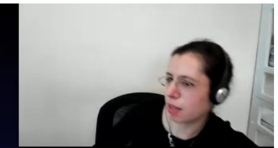
$$\delta v \geq \frac{\partial_u \Phi}{\partial_v \partial_u \Phi|_{\max}}$$



Similarly for δu for the next iteration.
Assuming that $\partial_u \partial_v \Phi$ is bounded from below, δv and δu go to zero no slower than the expansions of the surface approach zero: so the limiting surface is indeed extremal.



Higher Dimensions



- Higher dimensions are harder!
- But we can still do it. The basic idea is to work perturbatively: consider some deformation δg to the event horizon.
- This perturbation needs to "open up" the lightcones so as to push the event horizon deeper in. This means, on the future horizon:

$$\delta g_{kk} \leq 0$$

- The trick is to show that it is possible to find such a perturbation that satisfies the null constraint equation on the event horizon.
- We show this, but it's ugly!

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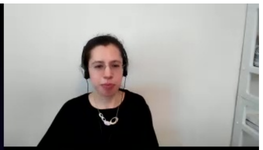


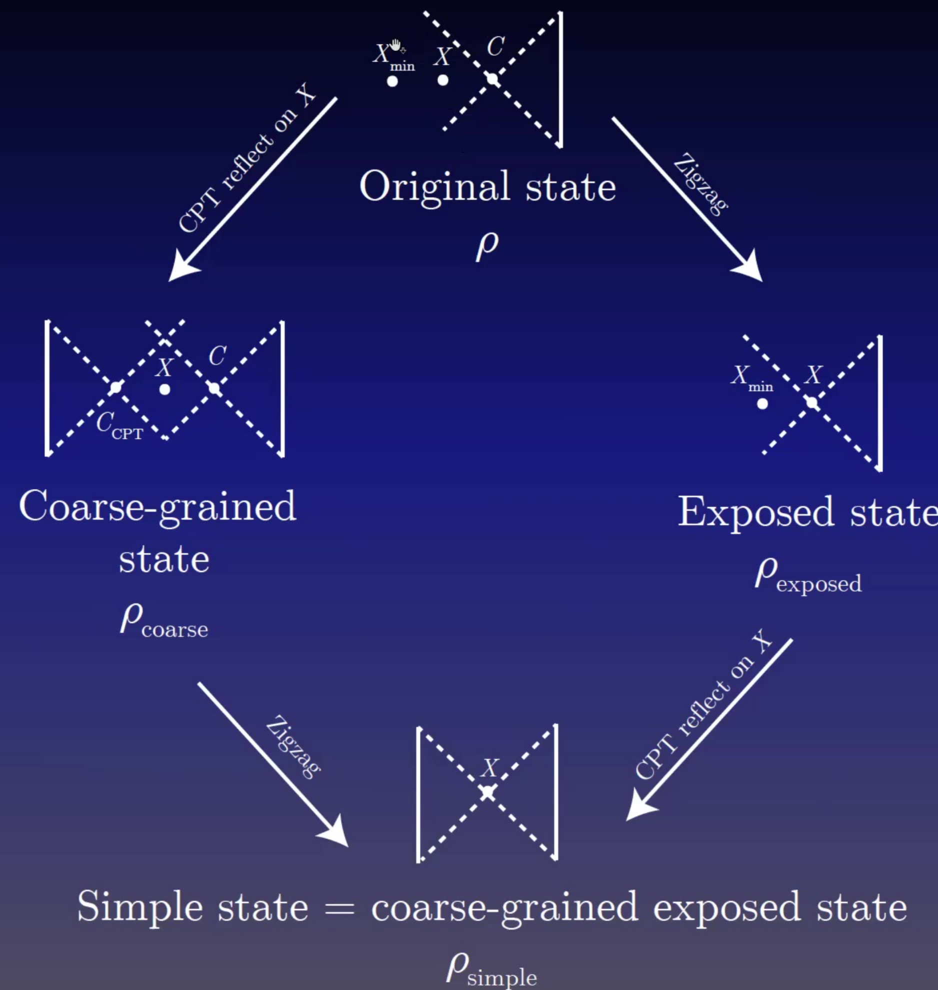
Coarse-Graining over Complexity

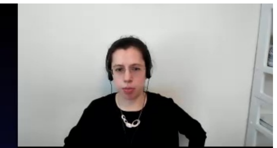
Using the wrong Saddle

A Simple Synthesis

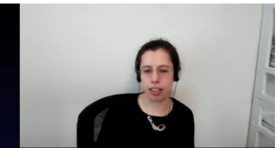
Applications and Extensions

The Simple State





Black Hole Uniqueness



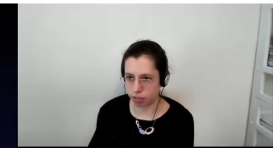
In some cases, our procedure makes X exactly coincide with the bifurcation surface (as opposed to asymptoting to it). Using the reconstruction procedure of Faulkner, Lewkowycz, we can prove the following theorem:

Coincidence of Causal and Entanglement Wedges NE Geoff and Arvin

The causal and entanglement wedges coincide exactly if and only if the dual boundary modular Hamiltonian generates a geometric flow.

This, coupled with our other results, gives a CFT dual to black hole uniqueness: **Stationary black holes in AdS are in a one-to-one correspondence with states whose modular Hamiltonian generates an exactly local flow.**

Quantum Corrections upcoming paper NE Geoff and Arvin



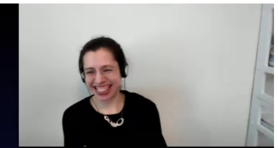
This proves everything we want in the classical regime.

What about quantum corrections? We already know that QESs don't always behave the same way that classical extremal surfaces do. Plus, there are "obvious" examples where we don't see a QES but there is exponential complexity!

For example, if we consider a a *nonevaporating* black hole with submaximal entanglement across the horizon at some late time, the blueshift a scrambling time in the past results in highly nontrivial quantum gravity effects there – we expect exponential complexity.

Our upcoming work shows that even though at first glance it doesn't look like there's a QES in this spacetime, a highly quantum QES (dominated mostly by bulk entropy) does in fact appear as a result of the blueshift.

Big Picture



- We want to understand the omission in Hawking's calculation in the language of Hawking's calculation.
- One way of going about this is to understand how omission of complex data (pre-2019 perspective) and omission of relevant saddles in the GPI (post-2019 perspective) agree.
- The Python's lunch prescription proposes that everything behind a nonminimal QES is highly complex.
- The time-folded simple entropy procedure shows that everything outside of a nonminimal classical extremal surface is highly complex; we will give fairly conclusive evidence (though not a proof) that this remains true under inclusion of quantum corrections in an upcoming paper.
- We are now in position to potentially identify the simple state with the Hawking state. To do this definitively and rigorously, it would be useful to have an understanding of the coarse-graining procedure in terms of quantum error correction and operator algebras work in progress w/ Akers and Harlow.