Title: Debate on Asymptotically Safe Quantum Gravity

Speakers: John Donoghue, Roberto Percacci

Series: Quantum Gravity

Date: April 15, 2021 - 2:30 PM

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Abstract: Asymptotically safe gravity is one of the most conservative approaches to quantum gravity. It relies on the framework of quantum field theory and the Wilsonian renormalization group. Recently, questions and open issues have been discussed both within and outside its community. This week, instead of a seminar, we will have a debate between John Donoghue ("A Critique of the Asymptotic Safety Program", arXiv:1911.02967) and Roberto Percacci ("Critical reflections on asymptotically safe gravity", arXiv:2004.06810), who will critically discuss the status of the field, and highlight its strengths and challenges.

Debate on Asymptotically Safe Quantum Gravity

Panelists:

John Donoghue and Roberto Percacci

15.04.2021

Chaired by Benjamin Knorr and Alessia Platania

PERIMETER INSTITUTE

Outline

1. **Introduction**
   - Basic Concepts in Asymptotic Safety
   - Why this debate?

2. **Brief Presentations**
   - John Donoghue, "*A critique of the Asymptotic Safety program*"
   - Roberto Percacci "*Critical Reflections on Asymptotically Safe Gravity*"

3. **Debate**
   - Debate between John and Roberto
   - Discussion including everybody

Asymptotic Safety in a nutshell

- **Quantum Field Theory + General Relativity**:
  - ultraviolet divergences that cannot be reabsorbed in a finite number of parameters
  - standard perturbative quantisation of gravity fails

- **Asymptotic Safety**:
  - physics idea: quantum realisation of scale symmetry in the ultraviolet
  - based on Quantum Field Theory concepts
  - conservative: only new ingredient is non-perturbative physics

Asymptotic Safety in a nutshell

Looking for Asymptotic Safety:

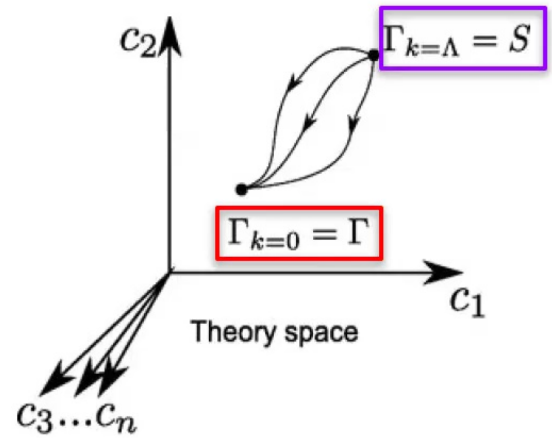
Does gravity make sense as a quantum field theory?
Is gravity asymptotically safe?

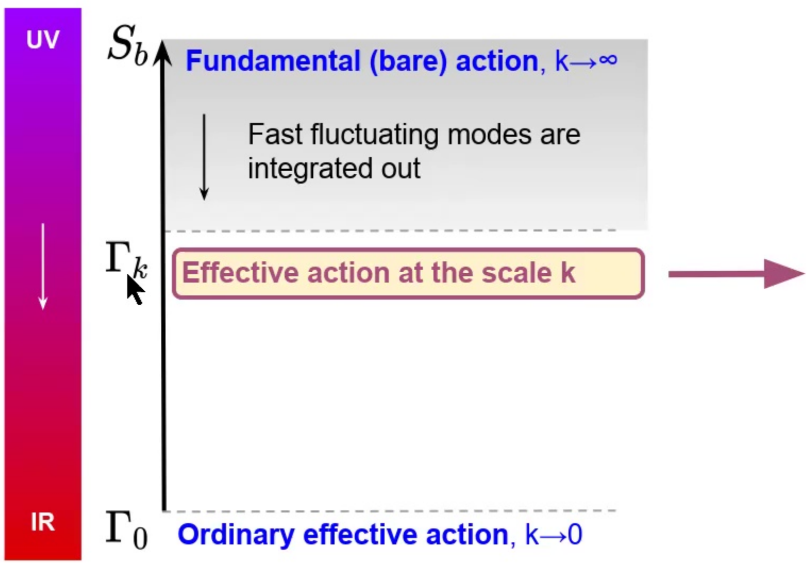
Asymptotic Safety in a nutshell

Looking for Asymptotic Safety:

Functional Renormalisation Group (FRG) equation:

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\,\partial_k \mathcal{R}_k\right\}$$





- **Ordering of momenta** from "high" to "low" requires Euclidean signature
- *k* is an **infrared cutoff** scale: only momenta p>k are integrated out
- **Flow towards the infrared** = Wilsonian shell-by-shell integration of fluctuations modes, flow interpolates between UV and IR
- In principle: flow ("**running**") of infinitely many couplings

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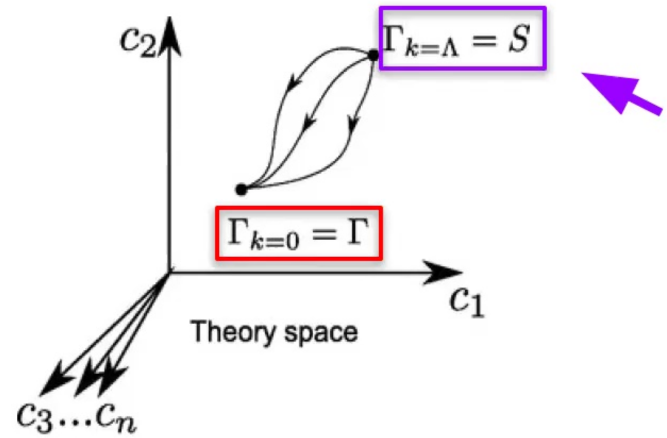

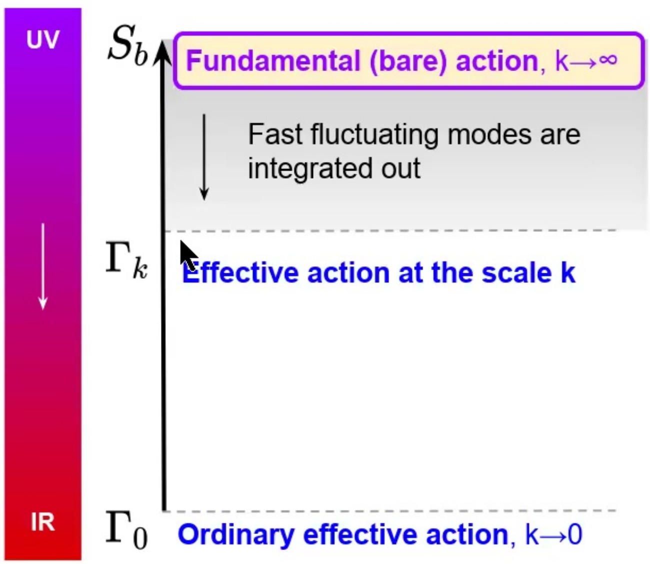

- Bare action(s) = **ultraviolet fixed point(s)** of the flow
- Wilsonian condition of **renormalizability**:
  - UV fixed point (⇔ UV completion, scale invariance)
  - Finite number N of relevant directions (⇒ IR physics depends on N parameters only)
- **Types of UV completions** / scale invariance:
  - **Free** theory: asymptotic freedom
  - **Interacting** theory: asymptotic safety

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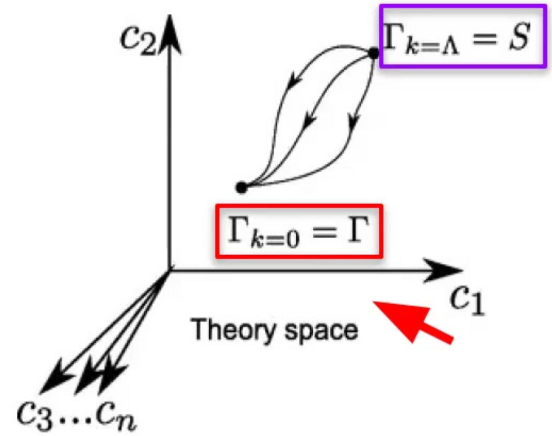

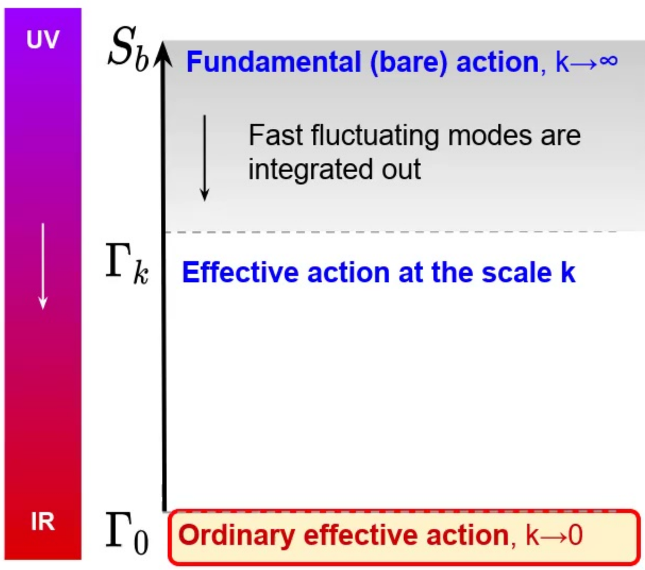

- **Infrared Limit k→0,** in principle (exact computation):
  ⇒ All quantum fluctuations are integrated out
  ⇒ **Ordinary effective action** of quantum field theory
- The effective action does not depend on *k*. But *dependence on the physical momenta p* (or, covariant derivatives)
- **Infinitely many terms**, but only some (= number N of relevant couplings) of them are free parameters
- In principle: Wick rotation to Lorentzian signature

Asymptotic Safety in a nutshell

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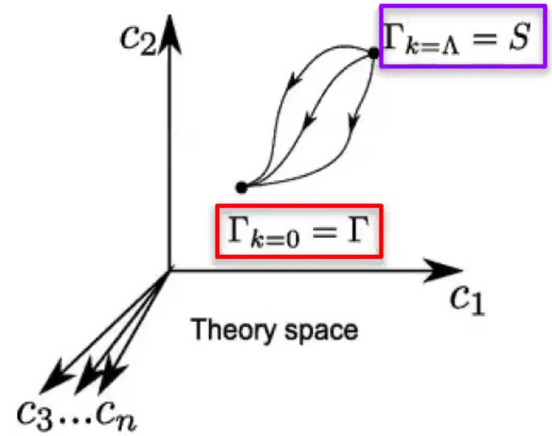

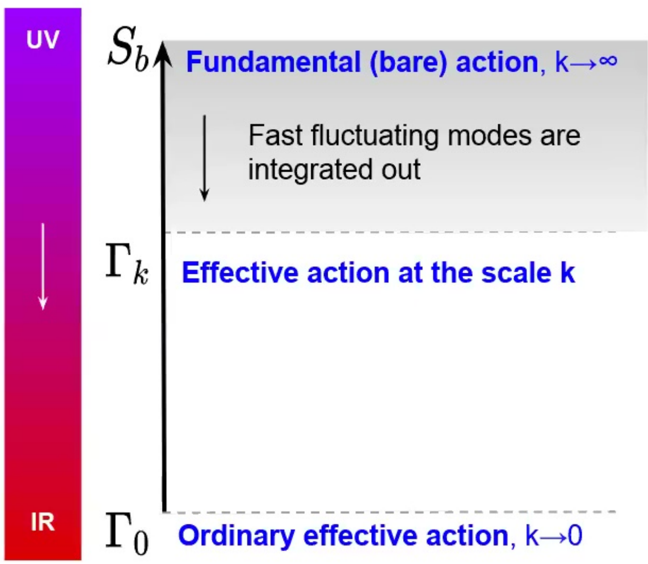

Two key warnings:

- Computations are performed within *Euclidean signature*
- **Approximations** ("truncations") involved

- **In principle**: solve the FRG equation for *all (infinitely many) couplings*
- **In practise**: "**truncate**" the action, taking a *finite number of operators/couplings only*

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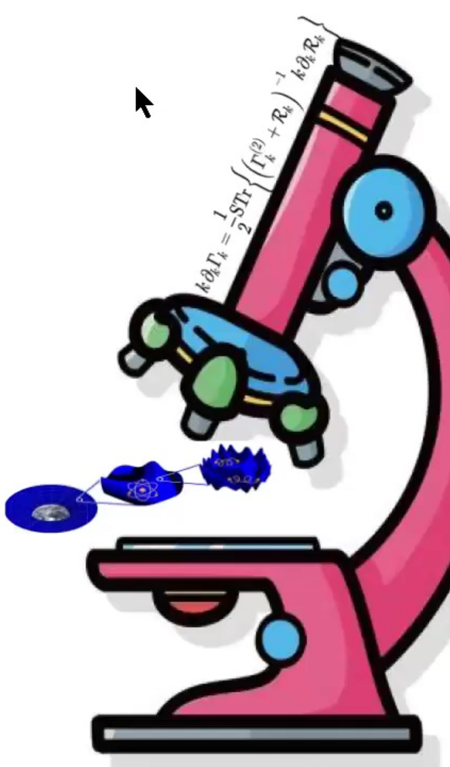
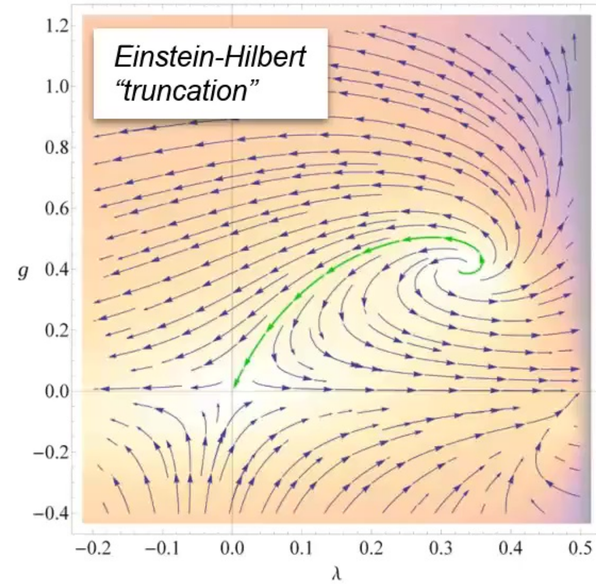


FRG



<u>Evidence so far</u> (within truncations and Euclidean signature)

- <u>Asymptotic safety of gravity</u> realized in all setups studied so far
- Computations point at: <u>2 or 3 relevant directions</u>

frontiers in Physics

ORIGINAL RESEARCH
published: 11 March 2020
doi: 10.3389/fphy.2020.00056

A Critique of the Asymptotic Safety Program

*John F. Donoghue**

Department of Physics, University of Massachusetts, Amherst, MA, United States

The present practice of Asymptotic Safety in gravity is in conflict with explicit calculations in low energy quantum gravity. This raises the question of whether the present practice meets the Weinberg condition for Asymptotic Safety. I argue, with examples, that the running of Λ and G found in Asymptotic Safety are not realized in the real world, with reasons which are relatively simple to understand. A comparison/contrast with quadratic gravity is also given, which suggests a few obstacles that must be overcome before the Lorentzian version of the theory is well behaved. I make a suggestion on how a Lorentzian version of Asymptotic Safety could potentially solve these problems.

...many open issues

frontiers in Physics

REVIEW
published: 03 August 2020
doi: 10.3389/fphy.2020.00269

Critical Reflections on Asymptotically Safe Gravity

***Alfio Bonanno*[1], *Astrid Eichhorn*[2,3]*, *Holger Gies*[4], *Jan M. Pawlowski*[3], *Roberto Percacci*[5], *Martin Reuter*[6], *Frank Saueressig*[7] *and Gian Paolo Vacca*[8]**

[1] INAF, Osservatorio Astrofisico di Catania, Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Catania, Catania, Italy, [2] CP3-Origins, University of Southern Denmark, Odense, Denmark, [3] Institute for Theoretical Physics, Heidelberg University, Heidelberg, Germany, [4] Theoretisch-Physikalisches Institut, Abbe Center of Photonics, Helmholtz Institute Jena, Friedrich-Schiller-Universität Jena, Jena, Germany, [5] SISSA, Istituto Nazionale di Fisica Nucleare (INFN), Trieste, Italy, [6] Institute of Physics (THEP), University of Mainz, Mainz, Germany, [7] Institute for Mathematics, Astrophysics and Particle Physics (IMAPP), Radboud University, Nijmegen, Netherlands, [8] Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Bologna, Bologna, Italy

Asymptotic safety is a theoretical proposal for the ultraviolet completion of quantum field theories, in particular for quantum gravity. Significant progress on this program has led to a first characterization of the Reuter fixed point. Further advancement in our understanding of the nature of quantum spacetime requires addressing a number of open questions and challenges. Here, we aim at providing a critical reflection on the state of the art in the asymptotic safety program, specifying and elaborating on open questions of both technical and conceptual nature. We also point out systematic pathways, in various stages of practical implementation, toward answering them. Finally, we also take the opportunity to clarify some common misunderstandings regarding the program.

Asymptotic Safety

A discussion with Roberto Percacci

See also

A Critique of the Asymptotic Safety Program 1911.02967
JFD and M. Anber, On the running of the gravitational constant 1111.2875
JFD and M. Anber, Running couplings and operator mixing in gravitational corrections to coupling constants 1011.3229
The cosmological constant and the use of cutoffs 2009.00728
Also work on quadratic gravity with Gabriel Menezes

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

John F. Donoghue
April 15, 2021
Hosted by PI

Shared understandings

1) **Nonlinear sigma model / Chiral perturbation theory**

$$\mathcal{L} = aTr(m(U+U^\dagger)) + \frac{F^2}{4}Tr(\partial_\mu U \partial^\mu U^\dagger) + L_i \left[Tr(\partial_\mu U \partial^\mu U^\dagger)\right]^2 + ... \qquad U = e^{i\tau\cdot\phi/F}$$

- Non-gravitational model for AS and EFT
- Advantage is that there are many contacts with experiment – explicit results
- **No power-law running** in practice

2) **Effective field theory of general relativity**
- systematic expansion valid below M_P
- **overlap** with region of applicability of AS

3) **Quadratic gravity / Quadratic truncation**
- $\Lambda + R + R^2$ - renormalizeable theory by itself

4) **Dimensional regularization vs cutoff**
- I have also used cutoffs in ChPTh - obtains same results as dim-reg

5) **Lorentzian vs Euclidean**

My points:

AS practice is not AS as defined by Weinberg
- what is it and what is its rationale?

Running couplings of AS are not applicable in physical processes

Finding Euclidean UV fixed points is not enough

Case has not yet been made for when and why these are useful

AS must give up some of the axioms of axiomatic QFT
- which ones, and to what effect?

The many layers of problems with higher derivative theories

May be fundamental obstacles to Euclidean/Lorentzian relation in a higher derivative quantum gravity

Is the AS program really Asymptotic Safety?

Proposed as running couplings in physical reactions
- requires power-law running

This fails for gravitational physics below Planck scale

Moreover, it does not appear to be the present interpretation of the AS program in practice

Weinberg's vision does not work in practice:

There are many reactions which we can study
- with multiple kinematic scales $E \to s, t$ ... of both signs, $g(s)$ or $g(t)$?

Try to operationally define running couplings ala Weinberg
- can always be done for any one reaction

But no definition works for other reactions
- not even for crossed versions of the same reaction

And certainly the AS $G(k)$ does not work (k is not a momentum after all)

Power-law running couplings are not seen in 4D Lorentzian reactions

The reasons are clear:

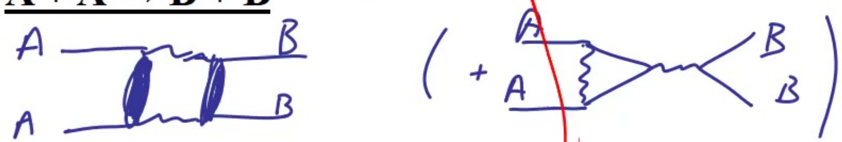
1) Power-law corrections are not renormalizations of the original coupling
   - generate higher order effects in the momenta $Gs, \quad Gt$
2) These corrections are not universal
3) The kinematic come with both signs $s > 0, \quad t < 0$

Example: Two different types of massless particles:

A + B → A + B

$$\mathcal{M}_{\text{total}} = \frac{i\kappa^2 E^2}{2}\left[1 - \frac{\kappa^2 E^2}{10(4\pi)^2}\left((19 + 10\ln 2)\ln\left(\frac{E^2}{\mu^2}\right) + 5(\pi^2 - (\ln 2 - 1)\ln 2)\right)\right].$$

A + A → B + B



$$\mathcal{M}_{\text{total}} = \frac{i\kappa^2 E^2}{8}\left[1 + \frac{\kappa^2 E^2}{10(4\pi)^2}\left(9\ln\left(\frac{E^2}{\mu^2}\right) - 5\pi^2 + (19 + 5\ln 2)\ln 2\right)\right].$$

Both crossing problem and universality problem

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Don't use FRG running couplings as physical running couplings

The logic of Weinberg's vision was clear, but…

Present AS practice is **something different** than Weinberg's AS vision

Instead produce Lagrangian by running from FP to $k \to 0$

This Lagrangian hopefully has some special properties – **Why?**

Effective action

- parameterized by coefficients of local terms
- action also has non-local effects

$$\mathcal{L} = \sqrt{-g}\left[-\Lambda_{vac} - \frac{1}{16\pi G}R + c_1 R^2 + c_2 C_{\mu\nu\alpha\eta}C^{\mu\nu\alpha\eta} + d_1 R^3 + d_2 R\Box R + ...\right]$$

In Euclidean world with IR cutoff *k,* FRG fixed points

- all **dimensionful couplings diverge** in particular way

$$\Lambda(k) \sim \Lambda_{expt} - g_\lambda k^4$$

$$\frac{1}{16\pi G} \to M(k)^2 \sim M_P^2 + g_G k^2$$

$$d_1 \to \frac{1}{M_*^2} \qquad M_*^2(k) \sim M_0^2 + g_* k^2$$

Here k is a cutoff, not a renormalization point (à la Weinberg)

Why are dimensionless couplings more relevant than the physical ones?

- why divide Λ by (cutoff)^4 ?

What is the physics which makes this result special?

AS to one loop –quadratic truncation

Codello
Percacci

$$S = \int d^4x\sqrt{-g}\left[-\Lambda + \frac{1}{16\pi G}R + \frac{1}{2\lambda}C^2 - \frac{\omega^2}{3\lambda}R^2 + \frac{\theta}{\lambda}E\right]$$

This is the same action as quadratic gravity
The running of the dimensionless term is the same – logarithmic running

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25 + 1098\omega + 200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56 - 171\theta)}{90}\lambda$$

Fixed point for $\omega = -0.23$
Weyl-squared term λ is asymptotically free (so is $\frac{\omega}{\lambda}$)
Gauss-Bonnet term θE does not contribute

Dimensionful running comes from the tadpole diagram

$$\int \frac{d^4p}{(2\pi)^4} F_k(p) \qquad \int \frac{d^4p}{(2\pi)^4} \frac{1}{k^2} F_k(p)$$

$$\beta_\Lambda = -\frac{1}{4\pi^2} k^4 \quad .$$

$$\Lambda(k) = \Lambda|_{expt} - \frac{1}{16\pi^2} k^4$$

$$\beta_{G^{-1}} = \frac{83}{18\pi} k^2$$

$$\frac{1}{G}(k) = \frac{1}{G}|_{expt} + \frac{83}{36\pi} k^2$$

This does have UV fixed point, but is physically meaningless

- no dynamics, no momentum flow
- in applications, would have to add in contribution up to k (not a renorm. point)
- vanishes in dim reg (i.e. does not contribute to physical processes)

Existence of UV fixed point is not enough

Technical point: Moreover, $\Lambda \sim k^4$ running is not really correct

There is a contact interaction which cancels this

Fradkin Vilkovisky (1973)
JFD (2020)

$$\Delta \mathcal{L} = i\frac{1}{8}\delta(0)\log(-\tilde{g})$$

- arises from interaction H with two deriviatives
  - or in PI measure

The $\delta(0)$ term cancels all $(\text{cutoff})^4$ tadpole terms
  - vanishes in dim reg.

This needs to be generalized and taken into account in AS

But this is not the full one-loop content

In fact, **non-local / non-analytic** terms are the interesting quantum content

In physical observables these give dominant effects

For example, Barvinsky-Vilkovisky "expansion in curvature"

$$S_{curv} \sim \int d^4x\sqrt{-g}\ \ldots + c(\mu)R^2 + dR\log(\Box/\mu^2)R + R^2\frac{1}{\Box}R + \ldots + R^{n+1}\frac{1}{\Box^n}R + \ldots$$

and very complicated "third order in the curvature"

This is not an objection to studying the local terms

But:

- they are not the most interesting or important
- in background spacetimes, **local and non-local can be confused**
- analytic continuation of non-local terms requires special care

194 pages of dense results, such as these:

$$\int dx\, g^{1/2}\, \mathrm{tr}\, \hat{a}_4(x,x) = \int dx\, g^{1/2}\, \mathrm{tr}\, \{ \frac{\Box_2{}^2}{120}\hat{P}_1\hat{P}_2 + \frac{\Box_2{}^2}{1260}\hat{P}_1 R_2 + \frac{\Box_2{}^2}{1680}\hat{\mathcal{R}}_{1\mu\nu}\hat{\mathcal{R}}_2^{\mu\nu}$$

$$+ \frac{\Box_2{}^2}{15120} R_{1\,\mu\nu} R_2^{\mu\nu}\hat{1} + \frac{\Box_3}{24}\hat{P}_1\hat{P}_2\hat{P}_3 - \frac{\Box_3}{630}\hat{\mathcal{R}}_1{}^{\mu}{}_{\alpha}\hat{\mathcal{R}}_2{}^{\alpha}{}_{\beta}\hat{\mathcal{R}}_3{}^{\beta}{}_{\mu}$$

$$+ (\frac{\Box_1}{180} + \frac{\Box_2}{180} + \frac{\Box_3}{90})\hat{\mathcal{R}}_1^{\mu\nu}\hat{\mathcal{R}}_{2\,\mu\nu}\hat{P}_3 + (\frac{\Box_1}{7560} - \frac{\Box_3}{15120}) R_1 R_2 \hat{P}_3$$

$$+ (\frac{\Box_1}{1680} + \frac{\Box_1{}^2}{1680\Box_2} + \frac{\Box_3}{2520} + \frac{\Box_1\Box_3}{1680\Box_2} - \frac{\Box_3{}^2}{336\Box_2} + \frac{\Box_3{}^3}{1120\Box_1\Box_2}) R_1^{\mu\nu} R_{2\,\mu\nu}\hat{P}_3$$

$$+ \frac{\Box_3}{720}\hat{P}_1\hat{P}_2 R_3 + (\frac{13\Box_1}{30240} - \frac{\Box_3}{15120}) R_1 \hat{\mathcal{R}}_2^{\mu\nu}\hat{\mathcal{R}}_{3\,\mu\nu}$$

$$+ (\frac{\Box_1}{840} + \frac{\Box_3}{210} + \frac{\Box_2\Box_3}{210\Box_1} + \frac{\Box_3{}^2}{210\Box_1}) R_1^{\alpha\beta}\hat{\mathcal{R}}_{2\,\alpha}{}^{\mu}\hat{\mathcal{R}}_{3\,\beta\mu}$$

$$+ (\frac{\Box_1{}^2}{25200\Box_3} + \frac{\Box_1\Box_2}{50400\Box_3} - \frac{\Box_3}{25200} + \frac{\Box_3{}^3}{50400\Box_1\Box_2}) R_1 R_2 R_3 \hat{1}$$

$$+ (- \frac{\Box_1{}^2}{9450\Box_3} - \frac{\Box_1\Box_2}{18900\Box_3} - \frac{\Box_3}{12600} + \frac{\Box_3{}^3}{12600\Box_1\Box_2}) R^{\mu}_{1\,\alpha} R^{\alpha}_{2\,\beta} R^{\beta}_{3\,\mu}\hat{1}$$

$$+ (\frac{\Box_1}{151200} - \frac{\Box_1{}^2}{151200\Box_2} + \frac{\Box_3}{25200} + \frac{\Box_1\Box_3}{18900\Box_2} - \frac{13\Box_3{}^2}{151200\Box_2}$$

$$+ \frac{\Box_3{}^3}{50400\Box_1\Box_2}) R_1^{\mu\nu} R_{2\,\mu\nu} R_3 \hat{1} + \frac{1}{252}\hat{\mathcal{R}}_1^{\alpha\beta}\nabla^{\mu}\hat{\mathcal{R}}_{2\mu\alpha}\nabla^{\nu}\hat{\mathcal{R}}_{3\nu\beta}$$

$$+ \frac{1}{60}\hat{\mathcal{R}}_1^{\mu\nu}\nabla_{\mu}\hat{P}_2\nabla_{\nu}\hat{P}_3 + \frac{1}{180}\nabla_{\mu}\hat{\mathcal{R}}_1^{\mu\alpha}\nabla^{\nu}\hat{\mathcal{R}}_{2\,\nu\alpha}\hat{P}_3 - \frac{1}{1890} R_1^{\mu\nu}\nabla_{\mu} R_2 \nabla_{\nu}\hat{P}_3$$

$$+ (\frac{1}{630} + \frac{\Box_1}{420\Box_2} + \frac{\Box_3}{210\Box_2} - \frac{\Box_3{}^2}{280\Box_1\Box_2})\nabla^{\mu} R_1^{\nu\alpha}\nabla_{\nu} R_{2\,\mu\alpha}\hat{P}_3$$

$$+ \frac{1}{180} R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\hat{P}_2\hat{P}_3 + (\frac{1}{1260} + \frac{\Box_3}{105\Box_1}) R_{1\,\alpha\beta}\nabla_{\mu}\hat{\mathcal{R}}_2^{\mu\alpha}\nabla_{\nu}\hat{\mathcal{R}}_3^{\nu\beta}$$

$$+ (- \frac{1}{1260} - \frac{\Box_3}{210\Box_1}) R_1^{\alpha\beta}\nabla_{\alpha}\hat{\mathcal{R}}_2^{\mu\nu}\nabla_{\beta}\hat{\mathcal{R}}_{3\,\mu\nu} - \frac{1}{7560} R_1 \nabla_{\alpha}\hat{\mathcal{R}}_2^{\alpha\mu}\nabla^{\beta}\hat{\mathcal{R}}_{3\,\beta\mu}$$

$$+ (\frac{1}{630} + \frac{\Box_2}{105\Box_1} + \frac{\Box_3}{105\Box_1}) R_1^{\mu\nu}\nabla_{\mu}\nabla_{\lambda}\hat{\mathcal{R}}_2^{\lambda\alpha}\hat{\mathcal{R}}_{3\,\alpha\nu} + (\frac{1}{226800} - \frac{\Box_1}{8400\Box_2}$$

$$- \frac{\Box_1{}^2}{10080\Box_2\Box_3} + \frac{\Box_3}{25200\Box_1} - \frac{\Box_3}{25200\Box_2} - \frac{\Box_3{}^2}{25200\Box_1\Box_2}) R_1^{\alpha\beta}\nabla_{\alpha} R_2 \nabla_{\beta} R_3 \hat{1}$$

$$+ (- \frac{\Box_1}{37800\Box_2} + \frac{\Box_3}{5400\Box_2} - \frac{\Box_3{}^2}{12600\Box_1\Box_2})\nabla^{\mu} R_1^{\nu\alpha}\nabla_{\nu} R_{2\,\mu\alpha} R_3 \hat{1}$$

$$+ (- \frac{1}{9450} - \frac{\Box_1}{12600\Box_2} + \frac{\Box_1{}^2}{8400\Box_2\Box_3} - \frac{\Box_3}{6300\Box_2}) R_1^{\mu\nu}\nabla_{\mu} R_2^{\alpha\beta}\nabla_{\nu} R_{3\,\alpha\beta}\hat{1}$$

$$+ (- \frac{1}{3150} - \frac{\Box_1}{9450\Box_2} - \frac{\Box_1{}^2}{6300\Box_2\Box_3} - \frac{\Box_3}{3150\Box_1} + \frac{\Box_3}{9450\Box_2} + \frac{\Box_3{}^2}{3150\Box_1\Box_2})$$

$$\times R_1^{\mu\nu}\nabla_{\alpha} R_{2\,\beta\mu}\nabla^{\beta} R^{\alpha}_{3\,\nu}\hat{1} + (\frac{1}{420\Box_2} + \frac{\Box_3}{280\Box_1\Box_2})\nabla_{\alpha}\nabla_{\beta} R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu} R_2^{\alpha\beta}\hat{P}_3$$

$$+ (- \frac{1}{3780\Box_2} - \frac{1}{6300\Box_3} - \frac{\Box_1}{4200\Box_2\Box_3} + \frac{\Box_3}{25200\Box_1\Box_2})\nabla_{\alpha}\nabla_{\beta} R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu} R_2^{\alpha\beta} R_3 \hat{1}$$

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$$+ 29\Box_2{}^4\Box_3{}^2 - 10\Box_1{}^3\Box_3{}^3 - 6\Box_1{}^2\Box_2\Box_3{}^3 + 78\Box_1\Box_2{}^2\Box_3{}^3$$

$$- 30\Box_2{}^3\Box_3{}^3 + 10\Box_1{}^2\Box_3{}^4 - 2\Box_1\Box_2\Box_3{}^4 + 16\Box_2{}^2\Box_3{}^4$$

$$- 5\Box_1\Box_3{}^5 - 5\Box_2\Box_3{}^5 + \Box_3{}^6)$$

$$+ \frac{\ln(\Box_1/\Box_3)}{9D^3\Box_2\Box_3}(- \Box_1{}^5\Box_2 + 5\Box_1{}^4\Box_2{}^2 - 10\Box_1{}^3\Box_2{}^3$$

$$+ 10\Box_1{}^2\Box_2{}^4 - 5\Box_1\Box_2{}^5 + \Box_2{}^6 - 2\Box_1{}^5\Box_3$$

$$- 21\Box_1{}^4\Box_2\Box_3 + 36\Box_1{}^3\Box_2{}^2\Box_3 - 6\Box_1{}^2\Box_2{}^3\Box_3 - 2\Box_1\Box_2{}^4\Box_3$$

$$- 5\Box_2{}^5\Box_3 + 10\Box_1{}^4\Box_3{}^2 - 6\Box_1{}^3\Box_2\Box_3{}^2 - 162\Box_1{}^2\Box_2{}^2\Box_3{}^2$$

$$+ 78\Box_1\Box_2{}^3\Box_3{}^2 + 16\Box_2{}^4\Box_3{}^2 - 20\Box_1{}^3\Box_3{}^3 + 66\Box_1{}^2\Box_2\Box_3{}^3$$

$$- 36\Box_1\Box_2{}^2\Box_3{}^3 - 30\Box_2{}^3\Box_3{}^3 + 20\Box_1{}^2\Box_3{}^4 - 25\Box_1\Box_2\Box_3{}^4$$

$$+ 29\Box_2{}^2\Box_3{}^4 - 10\Box_1\Box_3{}^5 - 13\Box_2\Box_3{}^5 + 2\Box_3{}^6)$$

$$+ \frac{\ln(\Box_2/\Box_3)}{9D^3\Box_2\Box_3}(\Box_1{}^5\Box_2 - 5\Box_1{}^4\Box_2{}^2 + 10\Box_1{}^3\Box_2{}^3$$

$$- 10\Box_1{}^2\Box_2{}^4 + 5\Box_1\Box_2{}^5 - \Box_2{}^6 - \Box_1{}^5\Box_3$$

$$+ 42\Box_1{}^3\Box_2{}^2\Box_3 - 72\Box_1{}^2\Box_2{}^3\Box_3 + 23\Box_1\Box_2{}^4\Box_3 + 8\Box_2{}^5\Box_3$$

$$+ 5\Box_1{}^4\Box_3{}^2 - 42\Box_1{}^3\Box_2\Box_3{}^2 + 114\Box_1\Box_2{}^3\Box_3{}^2 - 13\Box_2{}^4\Box_3{}^2$$

$$- 10\Box_1{}^3\Box_3{}^3 + 72\Box_1{}^2\Box_2\Box_3{}^3 - 114\Box_1\Box_2{}^2\Box_3{}^3 + 10\Box_1{}^2\Box_3{}^4$$

$$- 23\Box_1\Box_2\Box_3{}^4 + 13\Box_2{}^2\Box_3{}^4 - 5\Box_1\Box_3{}^5 - 8\Box_2\Box_3{}^5 + \Box_3{}^6)$$

$$+ \frac{\ln(\Box_1/\Box_2)}{(\Box_1 - \Box_2)}\frac{1}{3\Box_3}$$

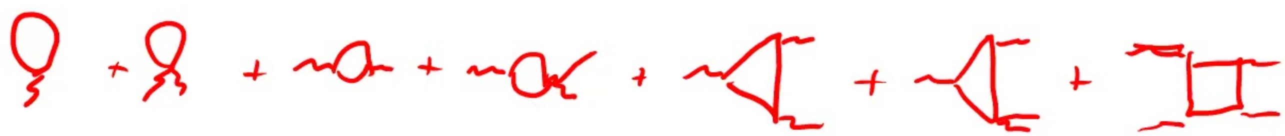
$$+ \frac{\ln(\Box_1/\Box_3)}{(\Box_1 - \Box_3)}\frac{1}{3\Box_2}$$

$$+ \frac{1}{3D^2}(16\Box_1{}^2 - 12\Box_1\Box_2 - 4\Box_2{}^2 - 12\Box_1\Box_3 + 8\Box_2\Box_3 - 4\Box_3{}^2),$$

But this is not the full one-loop content

In fact, **non-local / non-analytic** terms are the interesting quantum content

In physical observables these give dominant effects



For example, Barvinsky-Vilkovisky "expansion in curvature"

$$S_{curv} \sim \int d^4x\sqrt{-g}\ \ ...+c(\mu)R^2+dR\log(\Box/\mu^2)R+R^2\frac{1}{\Box}R+...+R^{n+1}\frac{1}{\Box^n}R+...$$

and very complicated "third order in the curvature"

This is not an objection to studying the local terms
But:

- they are not the most interesting or important
- in background spacetimes, **local and non-local can be confused**
- analytic continuation of non-local terms requires special care

And also other problems

When used in Lorentzian signature, this particular result has both a tachyon and ghost

Spin 0 tachyon –(pole at space-like momenta)
- also present in Euclidean theory

Spin 2 ghost (negative metric pole)

Tachyon may not be present when treated more fully — Falls, Ohta, Percacci

Ghost will be present in Lorentzian world
- but may not be fatal — JFD, Menezes

But, ghost is an obstacle to continuation from Euclidean — Anselmi

But UV fixed point has not added any useful physics here.

When and Why would UV fixed point be useful?

Euclidean/Lorentzian problems

Why is Euclidean description of Lorentzian theory possible?

- **magical** in standard theories (i.e. with two derivatives at most)
- analyticity properties (from causality and $i\epsilon$)

Eg. Källén–Lehmann representation of two point functions (axiomatic)

$$\Delta(p) = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\epsilon}$$

with positive definite $\rho(\mu^2)$

- cannot fall faster than $1/p^2$
- **violated in AS and other higher derivative theories**

Special danger with finite cutoffs

Lorentzian on-shell states $p^2 = 0$ are below any cutoff

Euclidean $p_E^2 \neq 0$ can be above cutoff

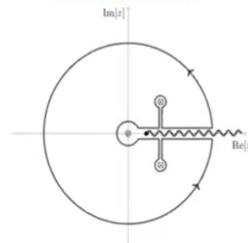
Or the reverse can happen $\frac{s}{p_E^2} \to \frac{s}{p^2}$

But, analyticity properties change with more derivatives

- for example quadratic gravity

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}} + \frac{1}{\pi}\int_{4m_j^2}^{\infty} ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Coleman



- changes the analyticity/causality structure of the theory!
- poles prevent standard rotation

Three, four, five… point functions are more complicated

Cutkosky et al (CLOP)

This only gets more difficult with yet more derivatives

- unknown analyticity properties

Problem is even yet worse in gravity – spacetime changes

Some of the axioms of QFT are violated in higher derivative theories

Hypothesis: Quantum gravity with higher derivatives is fundamentally different (and disconnected) in Euclidean vs Lorentzian

Many layers of higher derivative problems

Tachyons
Ghosts
Analyticity
Stability (Ostrogradsky)
Unitarity
Causality

Some of these are hidden when looking at Euclidean theory

AS will deviate from standard field theory
- which of these changes and how does the theory survive?

UV fixed point does not by itself solve these

What about the AS procedure helps with these problems?

My points:

AS practice is not AS as defined by Weinberg
- what is it and what is its rationale?

Running couplings of AS are not applicable in physical processes

Finding Euclidean UV fixed points of dimensionally reduced couplings is not enough

Case has not yet been made for when and why these are useful

AS must give up some of the axioms of axiomatic QFT
- which ones, and to what effect?

The many layers of problems with higher derivative theories

May be fundamental obstacles to Euclidean/Lorentzian relation in a higher derivative quantum gravity

Debate on Asymptotic safety

Roberto Percacci

SISSA, Trieste

PI, April 15, 2021

Based on

[A. Bonanno, A. Eichhorn, H. Gies, J.M. Pawlowski, R. Percacci, M. Reuter, F. Saueressig, G.P. Vacca, “Critical reflections on asymptotically safe gravity”, Front.in Phys. 8 (2020) 269, e-Print: 2004.06810 [gr-qc]]

and ongoing work with A. Baldazzi and L. Zambelli

Key contrasts

- Euclidean vs. Lorentzian
- powers vs. logarithms
- cutoffs vs. dimensional regularization

"The present practice of Asymptotic Safety in gravity is in conflict with explicit calculations in low energy quantum gravity".

There cannot be any conflict between calculations done using the FRG and calculations in EFT as long as one uses the same approximations, in the same regime.

If there is an apparent conflict, it is because one uses different approximations, or studies different regimes, or both.

The low energy EFT

Example: the low energy EA of gravity contains nonlocal terms

$$\Gamma \sim \frac{1}{32\pi^2}\int d^4x\sqrt{|g|}\left[\frac{1}{60}R\log\left(\frac{-\Box}{\mu^2}\right)R+\frac{7}{10}R_{\mu\nu}\log\left(\frac{-\Box}{\mu^2}\right)R^{\mu\nu}\right],$$

These contribute to scattering processes.

Main points

It reproduces the vacuum polarization contribution to the one-loop potential between heavy scalars.

$$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2 c^3} + \dots\right] .$$

$$\frac{41}{10} = \frac{43}{30} - \frac{21}{3} + \frac{47}{3} - 28 + 22$$

[A. Satz, A. Codello, F.D. Mazzitelli, "Low energy Quantum Gravity from the Effective Average Action" Phys.Rev.D 82 (2010) 084011, e-Print: 1006.3808 [hep-th]]

see also

[A. Codello, R. P., Lesław Rachwał, A. Tonero, "Computing the Effective Action with the Functional Renormalization Group" Eur.Phys.J.C 76 (2016) 4, 226, e-Print: 1505.03119 [hep-th]]

Core question: physical meaning of running couplings

- "the running of G and Λ with k is unphysical"
- "only dimensionless couplings can run and the running is logarithmic"

Semantics

What is a running coupling?

- In perturbatively renormalizable theory, dependence of a dimensionless coupling on the renormalization point.
- In the nonperturbative RG, the dependence of a coupling on a cutoff.

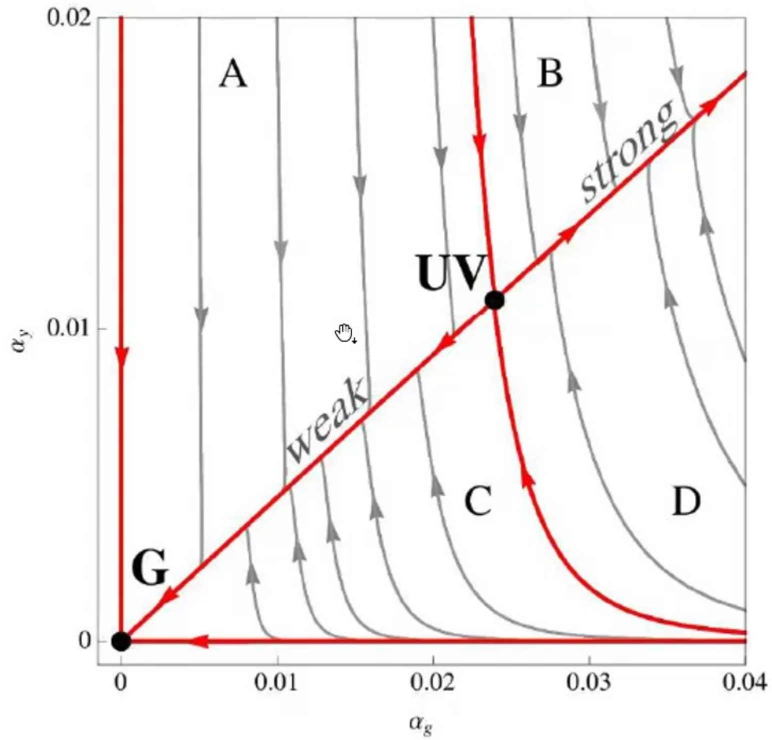
John would like to use the terms "renormalization group", "running couplings" etc only in the former case and the terms "incomplete integration", "incomplete coupling constant" etc. in the latter.

In perturbative treatment of renormalizable theories, where the couplings are dimensionless, the "non-perturbative beta functions" calculated from the FRG reproduce the perturbative ones.
The ambiguities that are inherent in the definition of the coarse-graining automatically disappear in these cases.

"The lesson of the renormalization group, that in order to avoid large logarithms we should take μ to be of the order of the energy E typical of the process being studied, is a special case of a broader principle, that in order to do calculations at a given energy we should first get rid of the degrees of freedom of much higher energy."

[S. Weinberg. "The quantum theory of fields", vol.2]

Asymptotically safe gauge-Yukawa theories



[D.F. Litim and F. Sannino, "Asymptotic safety guaranteed" JHEP 1412 (2014) 178]

Physics question #1

Does $\lambda(k)$ have any physical meaning?

Specific running couplings can acquire physical meaning when a system is characterized by a single scale and that scale behaves like a mass in the two point function.
Then we can identify k with that scale.

Example: Coleman-Weinberg potential

In a massless scalar theory with quartic interactions, expanding around $\bar{\phi}$, the propagator goes like

$$\frac{1}{-q^2 + \lambda\bar{\phi}^2}$$

In this case it is justified to identify the cutoff $k = \bar{\phi}$ and the effective potential is

$$V = \frac{1}{4!}\lambda(\bar{\phi})\bar{\phi}^4$$

Example: running of quartic coupling

Define $\lambda(k) = \Gamma^{(4)}|_{s=t=u=-k^2}$

$$i\Gamma^{(4)} = -i\lambda + \frac{\lambda^2}{2}\int_{|q|=\Lambda} \frac{d^4q}{(2\pi)^4}\left\{\frac{1}{((q+p_1+p_2)^2-m^2)(q^2-m^2)} + (p_2 \to -p_3) + (p_1 \to -p_4)\right\}$$

$$= -i\lambda + i\frac{\lambda^2}{32\pi^2}\int_0^1 dx\left(\log\frac{\Lambda^2}{m^2 - x(1-x)s} - 1\right) + (s \to t) + (s \to u)$$

where $s = (p_1+p_2)^2,\ t = (p_1-p_3)^2,\ u = (p_1-p_4)^2$

Example: thermal partition function

Thermal partition function of a scalar boson gas in d dimensions at temperature T= Euclidean partition function on $\mathbb{R}^d \times S^1$ with a periodic coordinate of period $1/T$

The 1-loop effective action is

$$\Gamma^{(1)} = \frac{V}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^d q}{(2\pi)^d} \log\left[q^2 + (2\pi T n)^2\right]$$

For each n T appears as an effective mass and therefore can be seen as an IR cutoff.

Taking the derivative with respect to T yields:

$$T\partial_T\Gamma^{(1)} = V\sum_{n=-\infty}^{\infty}\int\frac{d^dq}{(2\pi)^d}\frac{E_n^2}{E_n^2+q^2} = 2\pi^{d/2}\,\Gamma\left(1-\frac{d}{2}\right)\zeta(-d)VT^d$$

(In even dimension the Gamma function has a simple pole which is compensated by the simple zero of the Riemann Zeta function.)
Putting $d=3$ we get the usual result

$$T\partial_T\Gamma^{(1)} = -\frac{\pi^2VT^3}{30} \quad\Longrightarrow\quad \log Z = -\Gamma^{(1)} = \frac{\pi^2VT^3}{90}$$

The same result comes from the FRG by identifying $k=T$ *independent of details of the cutoff.*

[A. Baldazzi, R.P., V. Skrinjar, "Quantum fields without Wick rotation", Symmetry (2019) 11(3), 373]

Example: curved space generalization

A scalar field coupled to a static metric on $\mathbb{R} \times \Sigma$ gives:

$$\Gamma^{(1)} = -\int d^3x \sqrt{g} \left(\frac{\pi^2 T^3}{90} + \frac{T}{144} R \right)$$

We have an induced Hilbert term.
Also in this case $T\partial_T \Gamma$ is the same function as $k\partial_k \Gamma$ with the identification $k = T$ (Again independent of details of the cutoff).

Physics question #2

If the running of G with k is (generally) unphysical, why should we impose that its beta function is zero?

Quantum scale invariance

Classical scale invariance

$$\delta_\epsilon x^\mu = \epsilon x^\mu \ , \qquad \delta_\epsilon \phi = -\frac{d-2}{2}\epsilon\phi \ , \qquad \delta_\epsilon \lambda_i = 0$$

Classical scale invariance

$$0 = \delta_\epsilon S = \epsilon \int d^4 x T^\mu{}_\mu$$

implies

$$V(\phi) = \frac{\lambda}{4!}\phi^4 \, ,$$

The anomaly in perturbatively renormalizable QFT

The WI of scale transformations is anomalous

$$\delta_\epsilon \Gamma = \epsilon \int_x \langle T^\mu{}_\mu \rangle \equiv -\mathcal{A}(\epsilon)$$

$$\mathcal{A}(\epsilon) = \epsilon\,\beta \int d^4x \frac{1}{4!}\phi^4 \ ,$$

$$\beta = \frac{3\lambda^2}{16\pi^2} \,.$$

For a general theory

Assuming S is scale invariant, in the presence of an IR cutoff k

$$\delta_\epsilon \Gamma_k = -\mathcal{A}(\epsilon) + \epsilon k \partial_k \Gamma_k \,,$$

For

$$\Gamma_k = \sum_i \lambda_i(k)\, \mathcal{O}_i \,,$$

this leads to

$$\mathcal{A}(\epsilon) = \epsilon \sum_i \tilde{\beta}_i \, \tilde{\mathcal{O}}_i$$

where $\tilde{\lambda} = \lambda k^{-d_\lambda}$, $\tilde{\mathcal{O}} = \mathcal{O} k^{-\Delta}$.
The anomaly vanishes at a FP.

[T.Morris, R.P. "Trace anomaly and infrared cutoffs", Phys.Rev. D99 (2019) 105007 arXiv:1810.09824 [hep-th]]

However Γ_k at a FP is not scale invariant according to the original definition of $\delta\epsilon$.

e.g. if

$$\Gamma_k = \sum_i \lambda_i(k)\, \mathcal{O}_i \ ,$$

we have

$$\delta_\epsilon \Gamma_k = -\epsilon \sum_i \Delta_i\, \lambda_i\, \mathcal{O}_i$$

where Δ_i is the dimension of $\mathcal{O}_i$.

However, define $\hat{\hat{\delta}}_\epsilon$

$$\hat{\hat{\delta}}_\epsilon k = -\epsilon\, k \ ,$$

and the same as the action of δ_ϵ on all other quantities. Then

$$\hat{\hat{\delta}}_\epsilon \Gamma_k = \delta_\epsilon \Gamma_k - \epsilon k \partial_k \Gamma_k = \mathcal{A}(\epsilon)$$

The anomaly is the Wilsonian RG

This implies that at a fixed point one has scale invariance *in the sense of* $\hat{\hat{\delta}}_\epsilon$.

Main conclusion

The (non-perturbative) RG is a device that allows us to scan theory space in search of quantum scale invariant theories.

All couplings (in particular also $\tilde{G}$) must go to a FP in order to have quantum scale invariance.

This is independent of the physical meaning of this running.

Further, (a) the running of $\tilde{G}$ will affect scaling exponents that may be related to some observable and (b) the relation between $\tilde{G}_*$ and $\tilde{G}(0)$ is necessary to calculate the relations between the low energy couplings due to AS.

Next steps

Having obtained a FP, one can integrate the flow along a (safe) RG trajectory down to $k = 0$.

How do we know that the resulting EA will exhibit good high momentum behavior?

General arguments are given in

> [S. Weinberg, "Critical phenomena for field theorists", in the proceedings of the International School of Subnuclear Physics, Ettore Majorana Center for scientific culture, Erice, July 24-26, 1976.]

Some results

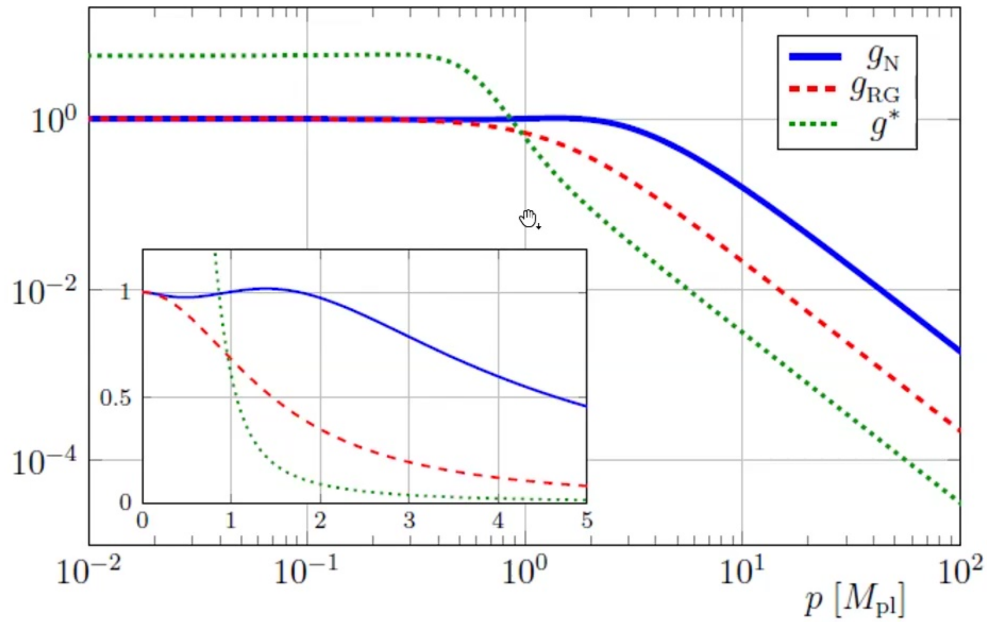


Figure: 3-point function of spin-2 at symmetric point, for $k = 0$, as function of p^2 (blue) for $p = 0$, as function of k^2 (red,dashed).

[A. Bonanno, T. Denz, J.M. Pawlowski, M. Reichert, "Reconstructing the graviton", e-Print: 2102.02217 [hep-th]]

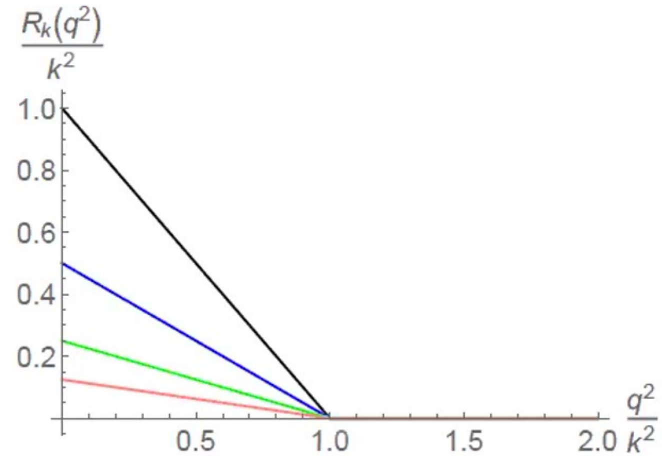
Dimreg vs. cutoffs

Everything that is physically meaningful can be seen with dimensional regularization.

Try to emulate dimreg with a suitable choice of (pseudo)-regulator.

A family of regulators

Additive IR suppression term

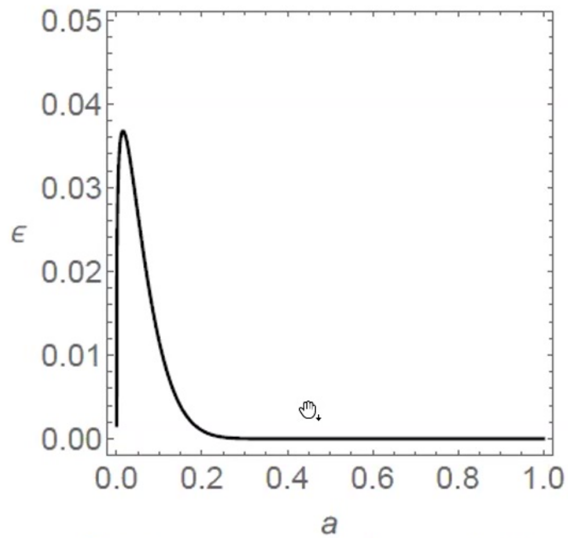


$$R_k(z, \epsilon, 0) = a(k^2 - q^2)\theta(k^2 - q^2)$$

Study limit $a \to 0$

A two parameter family of regulators

$$R_k(z,\epsilon,a) = \left[a\,k^2 + (1-a)\left(\frac{k^2}{\mu^4}\right)^{\epsilon} z^{1+\epsilon} - z\right]\theta\left(k^2 - \frac{a}{a+\epsilon}z\right)$$



We note that

$$\begin{aligned} R_k(z,0,1) &= R_k^{opt}(z) \\ R_k(z,\epsilon,0) &= R_k^{dim}(z) \end{aligned}$$

[A. Baldazzi, R.P, L. Zambelli, "Functional renormalization and MSbar", Phys. Rev. in print, e-Print: 2009.03255 [hep-th]. "Vanishing regulators", in preparation.]

Preliminary results for gravity

First $a \to 0$ then $\epsilon \to 0$. No running of G and Λ.

First $\epsilon \to 0$ then $a \to 0$.

$$\tilde{G}_* \sim \frac{1}{a \log a} \to \infty$$
$$\tilde{\Lambda}_* \sim \frac{1}{\log a} \to 0$$

Interesting questions regarding the R^2 couplings...

My conclusions

- AS attractive solution of the UV problems in QFT
- The functional/non-perturbative RG is a useful tool to do calculations in quantum gravity
- significant technical issues not discussed here
- complementary to other approaches, in particular "lattice" gravity
- separate physical from unphysical information
- focus on observables