Title: Baby Universes and Worldline Field Theories

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Abstract: The quantum gravity path integral involves a sum over topologies. Superficially, this feature is similar to Feynman diagrams of quantum field theory and the genus expansion of worldsheet string theory. There are however some key differences. While the standard construction leads to the non-abelian algebra of quantum fields, the quantum gravity path integral has been argued to define an abelian algebra associated with partition function type observables. We will discuss these issues and argue that one needs to make certain discrete choices to construct a Hilbert space from path integrals that sum over topologies. In particular, the natural choices for quantum gravity differ from those used to construct QFTs as we shall illustrate in the context of one-dimensional models.

Baby Universes & worldline field theories

w/ Eduardo Casali, Henry Maxfield, Don Marolf

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I. Motivation

II. General features of $\mathcal{H}_{BU}$

III. Worldline path integrals

a) ingredients

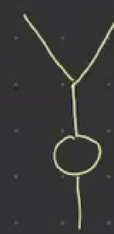
b) discrete choices

IV. Why $QG \neq QFT$?

V. Open Questions.

I. Motivation

Structural similtarities $\qquad \Sigma$ topologies



QG / Euclidean wormholes

$\longrightarrow$ Third Quantization

$\rightarrow$ Factorization

$\rightarrow$ superselection sectors / α-vacua

QG : Abelian algebra of superselected observables. <u>not QFT</u>

Analyze the rules for constructing Hilbert spaces + observables.

$\rightarrow$ make explicit basic ingredients

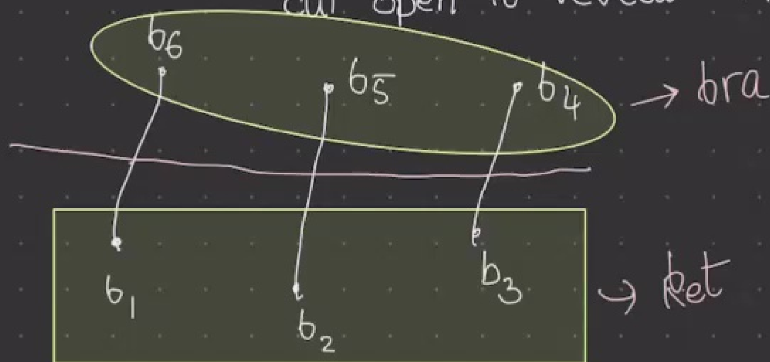
$\rightarrow$ assumptions & choices we need to make

Focus: on 1d worldline theories:

Baby universe Hilbert space:

Path Integrals: BCs $\longrightarrow$ Amplitudes

↳ cut open to reveal $|\rangle$, $\langle|$



$$\langle b_1, b_2, b_3, \dots, b_6 \rangle = \langle b_2, b_3, b_6, \dots \rangle = \text{perms.}$$

state space: $|a_1, \dots a_m\rangle$ indexed by bc.

Boundary creation ops $\quad b\,|a_1 \dots a_m\rangle = |b\, a_1 \dots a_m\rangle$

$$[b_1, b_2] = 0$$

imagine all states being created from some no bdy state $|HH\rangle$

$b^\dagger$ $\quad\langle a_1 \dots a_m | b_1 \dots b_n\rangle = \langle HH | a_1^\dagger \cdots a_m^\dagger \, b_1 \cdots b_n | HH\rangle$

$$= \langle a_1^\dagger, \dots a_m^\dagger, b_1, \dots b_n\rangle$$

- check b's preserve null states
- simultaneously diagonalizable $\quad [b, b^\dagger] = 0$

$$b|\alpha\rangle = b_\alpha |\alpha\rangle \qquad \alpha \cdot \text{states.}$$

II. Ingredients for worldline 'gravity'.

$$\sum \text{1d worldlines} \rightarrow \int dT$$

$\searrow$ proper time

$\langle b_1, \dots, b_n\rangle$: wick contraction of pairings.

$$\mathcal{A}(b_1, b_2) = \int_{\mathcal{D}} dT \int_{b_1}^{b_2} \mathcal{D}x \; e^{\eta \, S_{\text{matter}}[x, T]}$$

- (*) worldline signature $\quad \eta \in \{i, -1\}$
- (*) Domain of T integration
- (*) S_{matter} spec. $\qquad S_{\text{matter}} = \int d\tau \left[\frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \dots \right]$
- (*) allowed bc for S_{matter}
- (*) a $\dagger$ operation.

Choice 1: Euclidean statistical systems [abelian superselection]

Choice 2: Euclidean QFTs [no such thing].

Choice 1: Euclidean Statistical Theories:

- matter amplitudes: $\langle x | e^{-HT} | y \rangle$ $\quad x, y$ - label $\mathcal{M}_{target}$
- $T \in \mathbb{R}_+$ $\quad \mathcal{A}(x,y) = \langle x | \frac{1}{H} | y \rangle$
- unoriented worldlines $\quad x^{\dagger} = x$

$$\langle x_1, \dots x_n \rangle \propto \int \mathcal{D}\phi \; \phi(x_1) \cdots \phi(x_n) \; e^{-I}$$

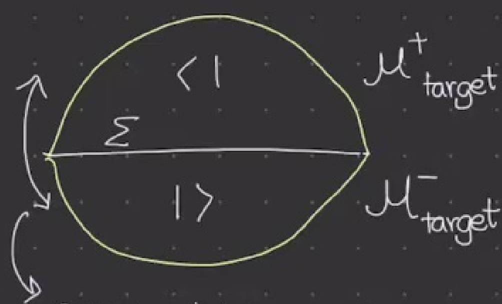
$$I = \int dx \left[\frac{1}{2} (\partial \phi(x))^2 + \dots \right]$$

single universe state $\quad |x\rangle$

superposition $\quad \int dx \, F(x) \, |x\rangle \; : \; |F\rangle$

$$\langle G | F \rangle = \int \mathcal{D}\phi \; G^*(\phi) \, F(\phi) \, e^{-I(\phi)}$$

Choice 2 : Euclidean QFT



CPT involution σ

$|x\rangle \quad x \in \mathcal{M}^-_{\text{target}}$

$\langle y| \quad y \in \mathcal{M}^+_{\text{target}}$

$\mathcal{M}_{\text{target}} = \mathbb{R}_E \times \Sigma$

σ has ± 1 eigenvalues

so baby universe inner product is not positive semi definite

$$\langle y_1 \dots y_4 | x_1 \dots x_4 \rangle = \int \mathcal{D}\Phi \; \Phi(\sigma(y_1))\, \Phi(\sigma(y_2)) \cdots \Phi(\sigma(y_4)) \; \Phi(x_1) \cdots \Phi(x_4)\, e^{-I}$$

$$\langle G | F \rangle = \int \mathcal{D}\Phi \; G^*[\Phi \circ \sigma]\, F[\Phi]\, e^{-I}$$