Title: From observations to complexity of quantum states: an unsupervised learning approach

Speakers: Zala Lenarcic

Series: Quantum Matter

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Abstract: The vast complexity is a daunting property of generic quantum states that poses a significant challenge for theoretical treatments, especially in non-equilibrium setups. Therefore, it is vital to recognize states which are locally less complex and thus describable with (classical) effective theories.

I will discuss how unsupervised learning can detect the local complexity of states. This approach can be used as a probe of scrambling and thermalization in chaotic quantum systems or to assign the local complexity of density matrices in open setups without knowing the corresponding Hamiltonian or Liouvillian. The analysis actually allows for the reconstruction of Hamiltonian operators or even noise-type that might be contaminating the measurements. Our approach is an ideal diagnostics tool for data obtained from (noisy) quantum simulators because it requires only practically accessible local observations. For example, it would be perfectly suited to detect the many-body localization (MBL) transition or integrability effects from the experimental snapshots obtained with cold atoms.

If time permits, I will mention other ways to detect properties of MBL transition in weakly open and driven setups and the advantages of such an unconventional approach.

M. Schmitt and Z. Lenarcic, arXiv:2102.11328.

Z. Lenarcic, O. Alberton, A. Rosch and E. Altman, PRL 125, 116601 (2020).

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From observations to complexity of quantum states: an unsupervised learning approach

Zala Lenarčič

Jozef Stefan Institute, Ljubljana, Slovenia

April 2021

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University of Cologne

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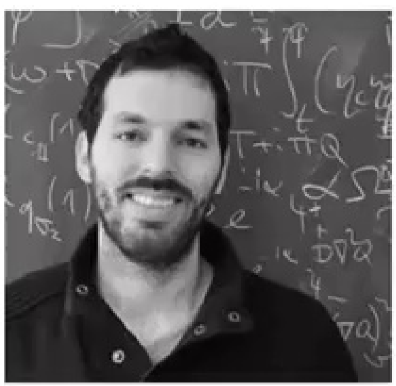
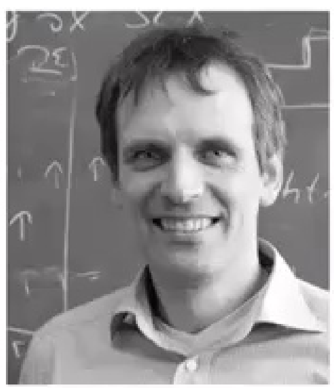
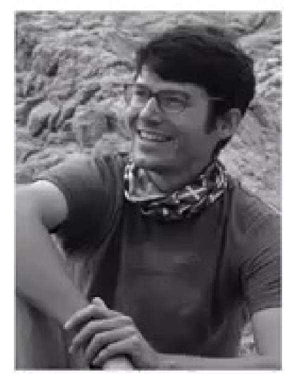


Ori Alberton



Achim Rosch

Ehud Altman



Markus Schmitt

Detect simple physical descriptions of complex quantum states

- A new measure for the local complexity of quantum many-body states
  - unsupervised learning of physical observations
- Local complexity:
  - detectable by local observables with a finite support
  - **number of parameters** needed to reproduce all **local observables**
- Experimentally friendly:
  - need measurements of local observables
- Applications:
  - detect stages of noneq dynamics
  - detect (lack of) thermalization
  - learn H from local observables
  - analyse and characterize quantum simulators
  - detect noise-type

M. Schmitt and Z. Lenarčič, arXiv:2102.11328.

Complexity dynamics: information bottleneck

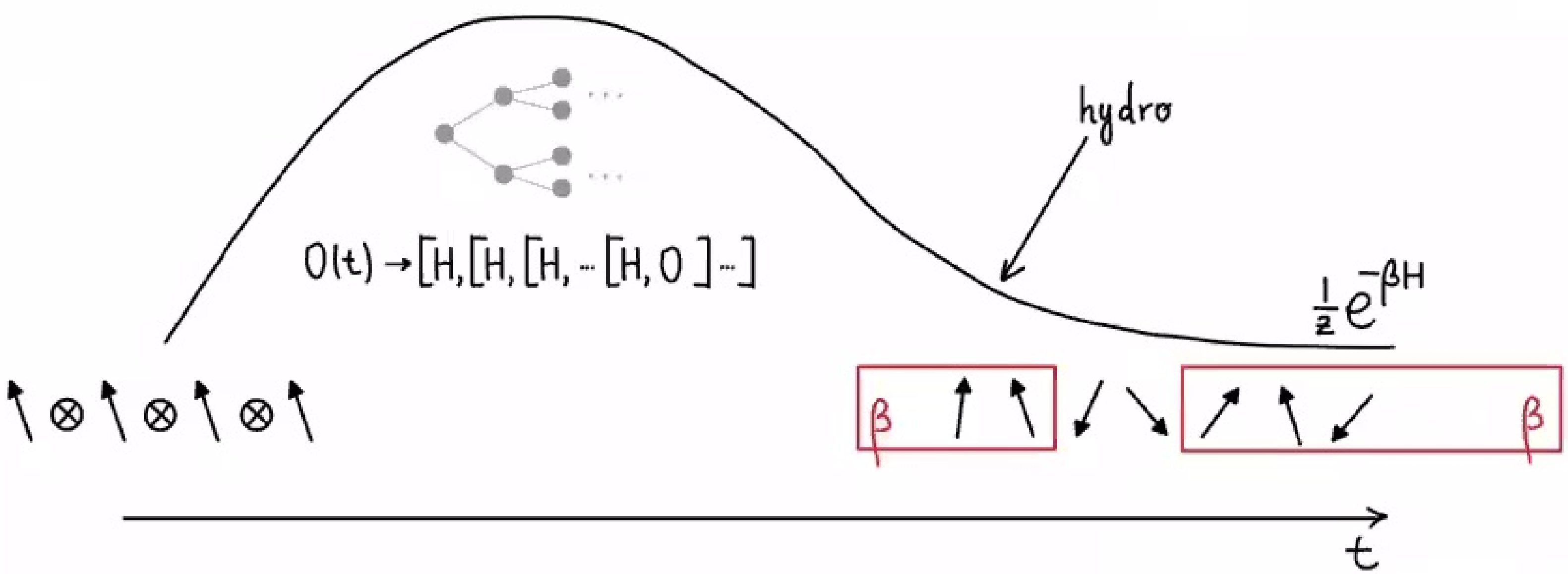
Short times:

- Inital product state
- Operator growth,
  Parker et al, PRX 2019

$$O(t) = \exp(\hat{\mathcal{L}}^\dagger t) O(0)$$

Long times:

- Emergent hydro description,
  Lux et al PRA 2014, Bohrdt et al NJP 2017
- Only conservation laws matter
- Statistic description



Complexity dynamics: different measures

- Entanglement entropy of states generically grows with time

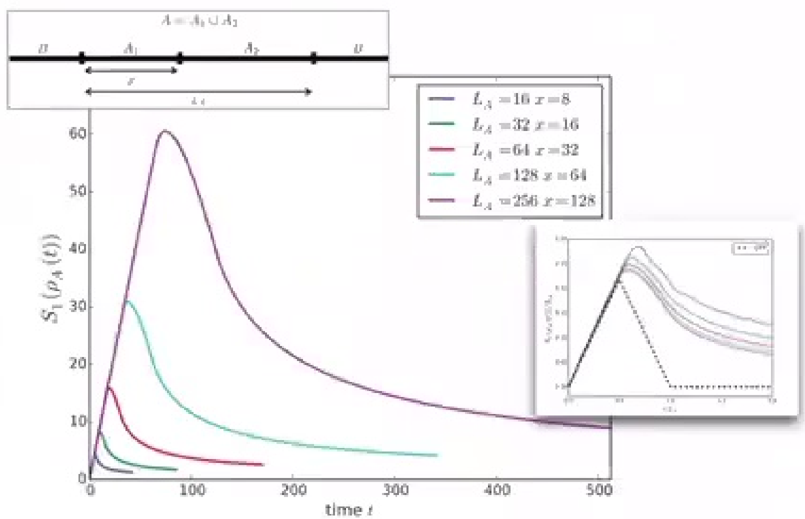
$$|\psi\rangle = \sum_j \sqrt{\gamma_j} |\psi_{j,A}\rangle \otimes |\psi_{j,B}\rangle \quad \to \quad S_{vN} = -\sum_j \gamma_j \ln(\gamma_j)$$

- Operator space entanglement entropy: $\rho \to \frac{|\rho\rangle}{\sqrt{\mathrm{tr}[\rho^\dagger \rho]}}$

  (Zanardi PRA 2001, Prosen et al PRA 2007)

  - Local measurements $\leftrightarrow$ reduced density matrix
  - initial product state: A-law
  - intermediate times: bond dimension blows up exponentially
  - final thermal $\rho_{th}(\beta)$: A-law

  Dubail, J. Phys. A: Math. Theor. 50 (2017); Wang&Zhou 2019; Noh et al, 2020



Dubail, J. Phys. A: Math. Theor. 50 (2017)

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Complexity dynamics: different measures

Find a measure that directly detects physically relevant degrees of freedom?

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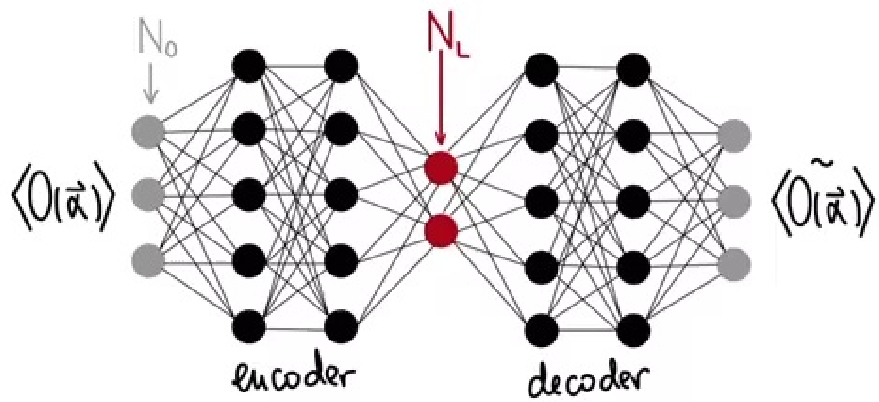
Autoencoders → efficient data encoding (compression)



Input x: expectation values $\mathrm{tr}[O(\alpha)\rho]$ of local operators

$$O(\alpha) = \sigma_1^{\alpha_1} \ldots \sigma_{|S|}^{\alpha_{|S|}}, \qquad \boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_{|S|}) \in \{0, x, y, z\}^{|S|}$$

Data sets:

- one data set: N_O operators
- need N_{real} sets for learning and training

Dimensional reduction by encoder:

- N_O-dim to N_L-dim

Bootleneck: N_L neurons

Output $f_\theta(x)$: network reproduction of local observables

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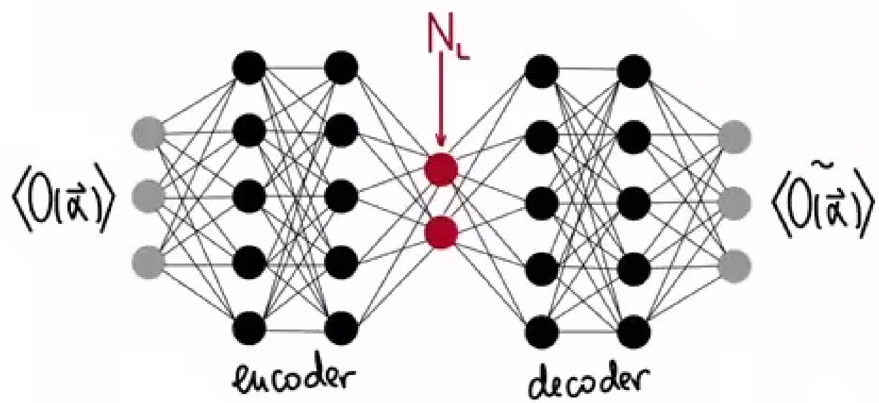
Autoencoders → efficient data encoding (compression)



Input x: expectation values $\mathrm{tr}[O(\boldsymbol{\alpha})\rho]$ of local operators

$$O(\boldsymbol{\alpha}) = \sigma_1^{\alpha_1} \dots \sigma_{|\mathcal{S}|}^{\alpha_{|\mathcal{S}|}}, \qquad \boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_{|\mathcal{S}|}) \in \{0, x, y, z\}^{|\mathcal{S}|}$$

Bootleneck: N_L neurons

Output $f_\theta(x)$: network reproduction of local observables

1. Initialize the network by training on a subset of realizations

$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} \left(f_\theta(x) - x\right)^2$$

2. How well unseen $\langle O(\boldsymbol{\alpha})\rangle$ can be reproduced by the network: Test error

Complexity dynamics: information bottleneck

Short times:

- Inital product state
- Operator growth,
  Parker et al, PRX 2019

$$O(t) = \exp(\hat{\mathcal{L}}^\dagger t) O(0)$$

Long times:

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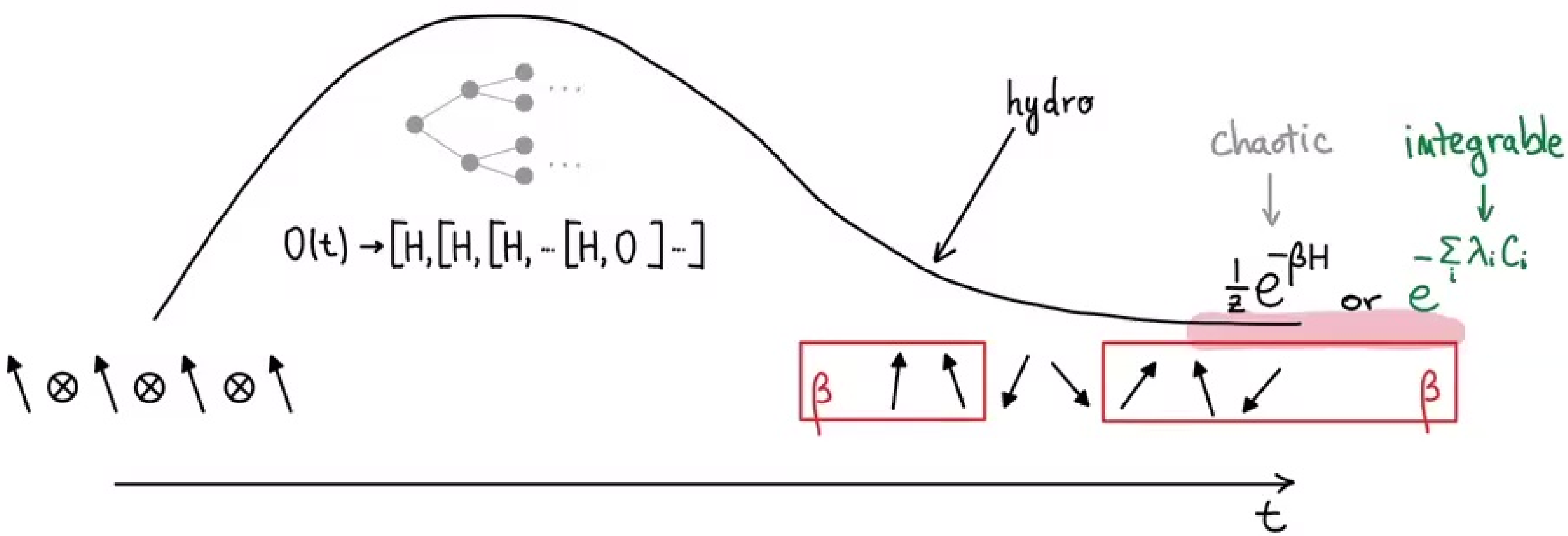
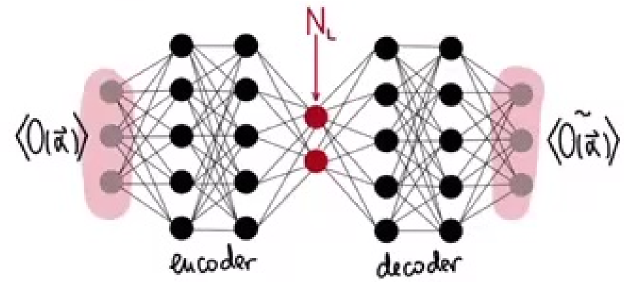


Detecting (generalized) Gibbs ensemble

Hamiltonian: integrable t-Ising

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x \quad \rightarrow \quad [C_i, H] = 0$$



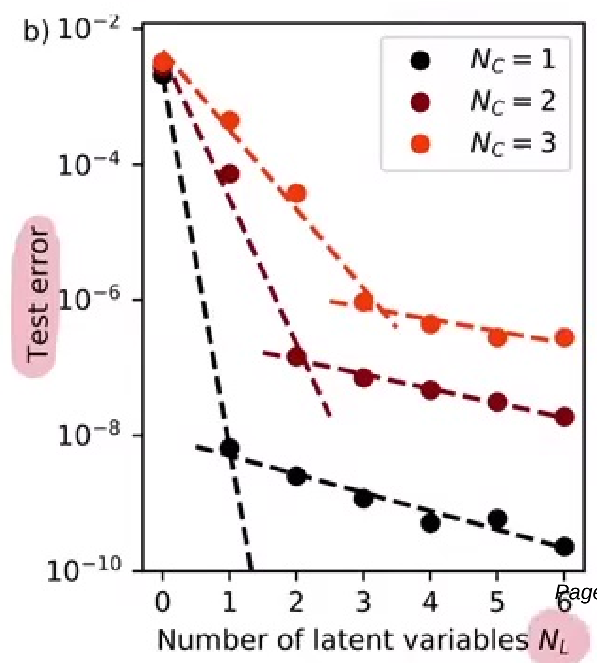
Data set: 'synthetic' GGE

$$\langle O(\alpha)\rangle = \mathrm{tr}[O(\alpha)\rho_{GGE}], \quad \rho_{GGE} = \frac{1}{Z}e^{\sum_{i=1}^{N_c}\lambda_i C_i}, \quad C_0 = H$$

with random $\lambda_i \in [-2, 2]$

Test error:

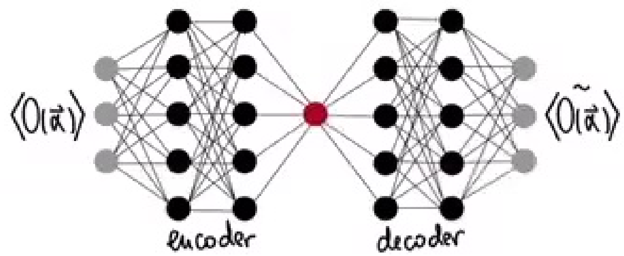
$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|}\sum_{x\in\mathcal{D}_T}\left(f_\theta(x) - x\right)^2$$



Detecting Gibbs ensemble

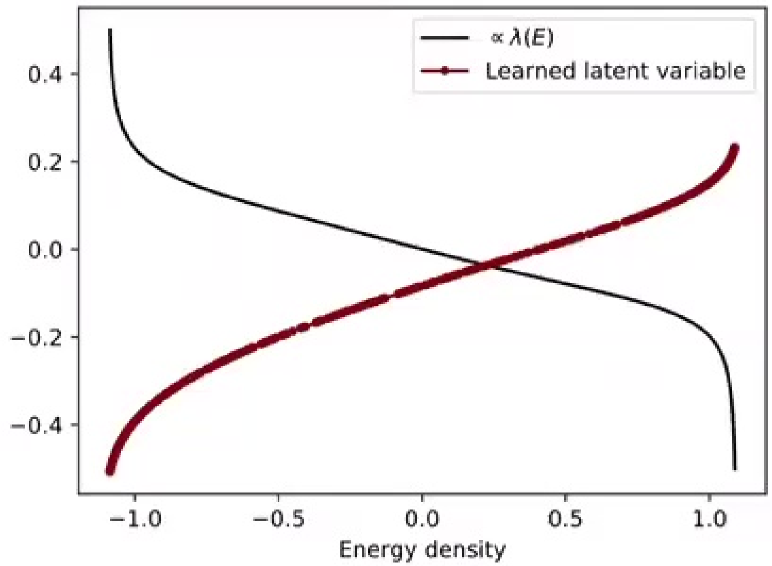
Thermal states

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x, \quad \rho_{GGE} = \frac{1}{Z}e^{\lambda_0 C_0},$$

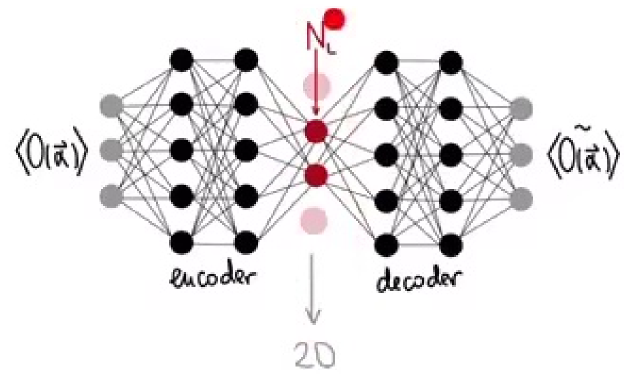


Towards interpretable learning: What does it learn?

- temperature, energy density?
- 1st latent is an invertible function of energy



Detecting Gibbs ensemble



Thermal states

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x, \quad \rho_{GGE} = \frac{1}{Z}e^{\lambda_0 C_0},$$

Towards interpretable learning: What does it learn?

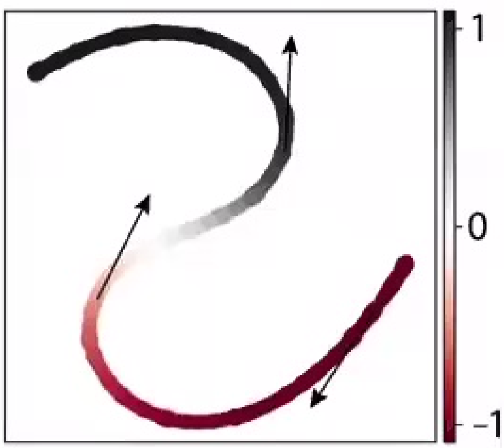
- projection of N_L dimensional latent space to 2D (t-SNE)
- color by $\langle H\rangle/N \rightarrow$ monotonic function of energy

Reconstructing H from measurements in Gibbs ensemble

Consider $\rho_{GE} = \frac{1}{Z} e^{\lambda_0 C_0}$ of

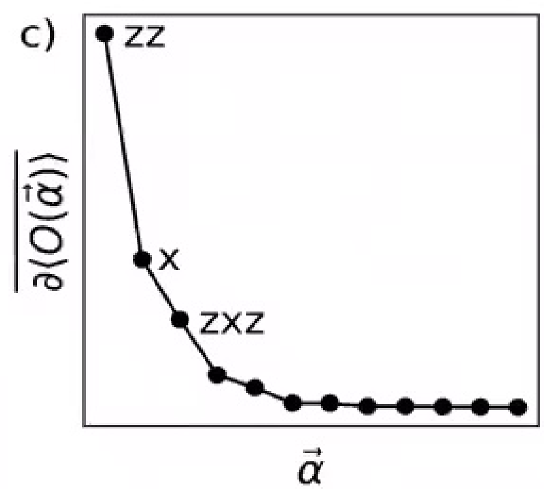
$$H = \sum_j J\sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x$$

t-SNE projection of latent space



Reconstruction of H from $\langle O(\alpha)\rangle$:

0. t-sne, test error → expect $\rho \sim e^{-\beta H}$
1. Find operators $\langle O(\alpha)\rangle$ with largest gradient along the line:



2. Newtons method → get $\tilde{h}_x = h_x/J$

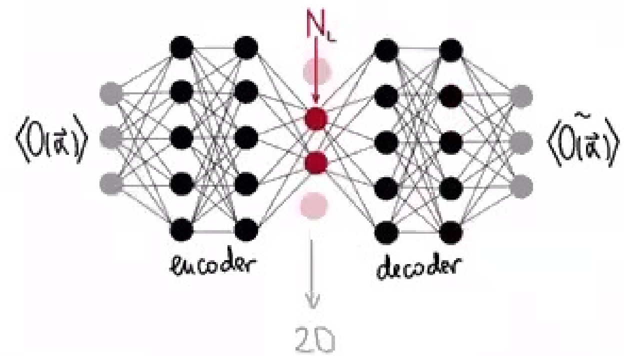
$$\mathrm{tr}[\sigma_j^z \sigma_{j+1}^z e^{-\beta(\sigma_j^z \sigma_{j+1}^z + \tilde{h}_x \sigma_j^x)}] - \langle \sigma_j^z \sigma_{j+1}^z \rangle_{mes} = 0$$

$$\mathrm{tr}[\sigma_j^x e^{-\beta(\sigma_j^z \sigma_{j+1}^z + \tilde{h}_x \sigma_j^x)}] - \langle \sigma_j^x \rangle_{mes} = 0$$

Detecting (generalized) Gibbs ensemble

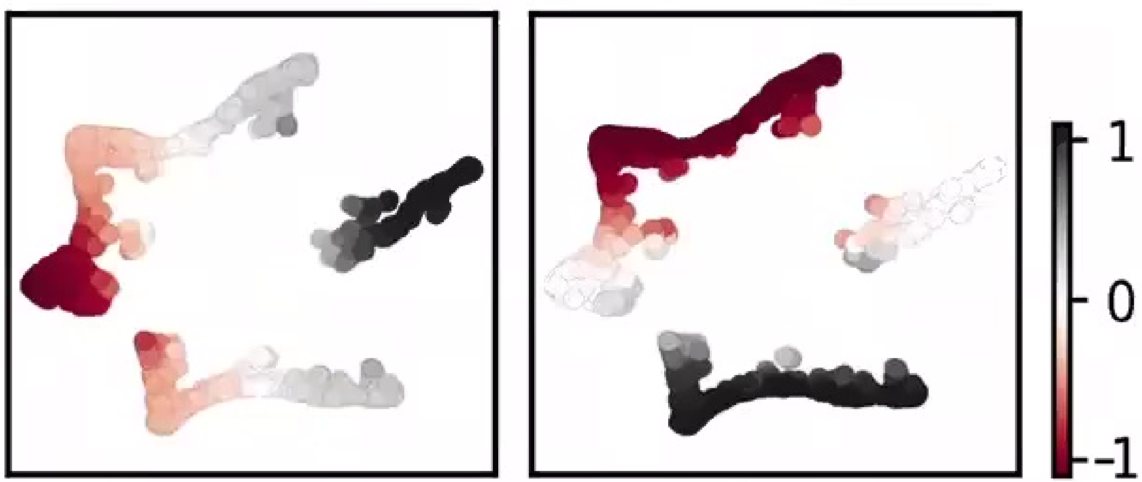
Consider integrable t-Ising Hamiltonian

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x, \quad \rho_{GGE} = \frac{1}{Z}e^{\lambda_0 C_0 + \lambda_1 C_1},$$



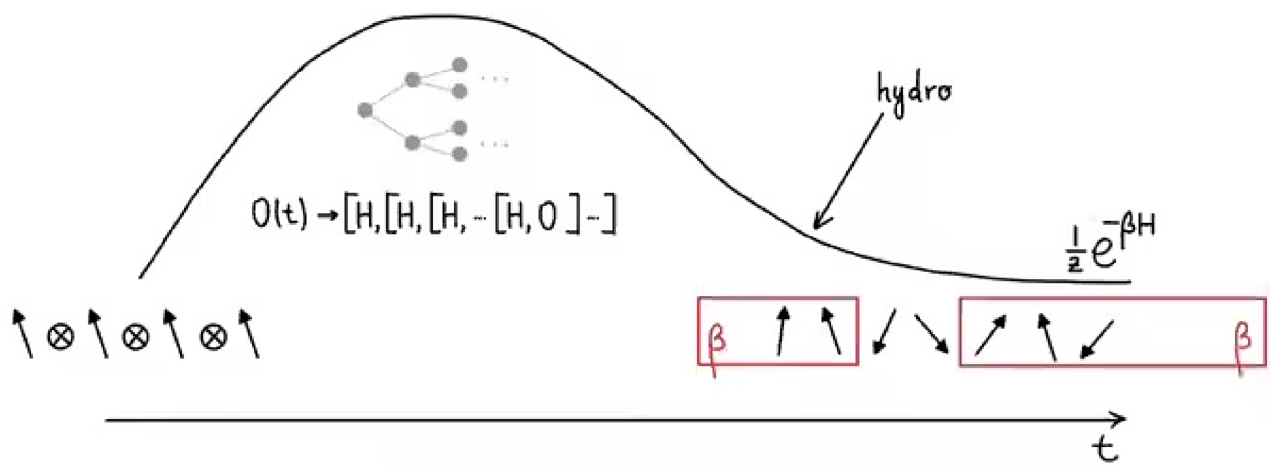
Towards interpretable learning: What does it learn?

- projection of N_L dimensional latent space to 2D (t-SNE)
- color by $\langle H\rangle/N, \langle C_1\rangle/N$

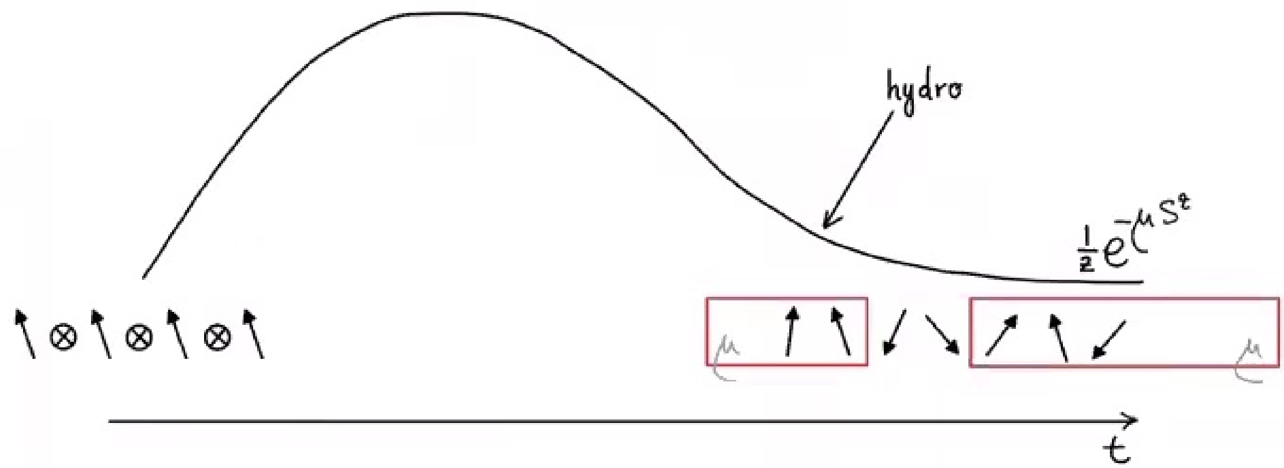


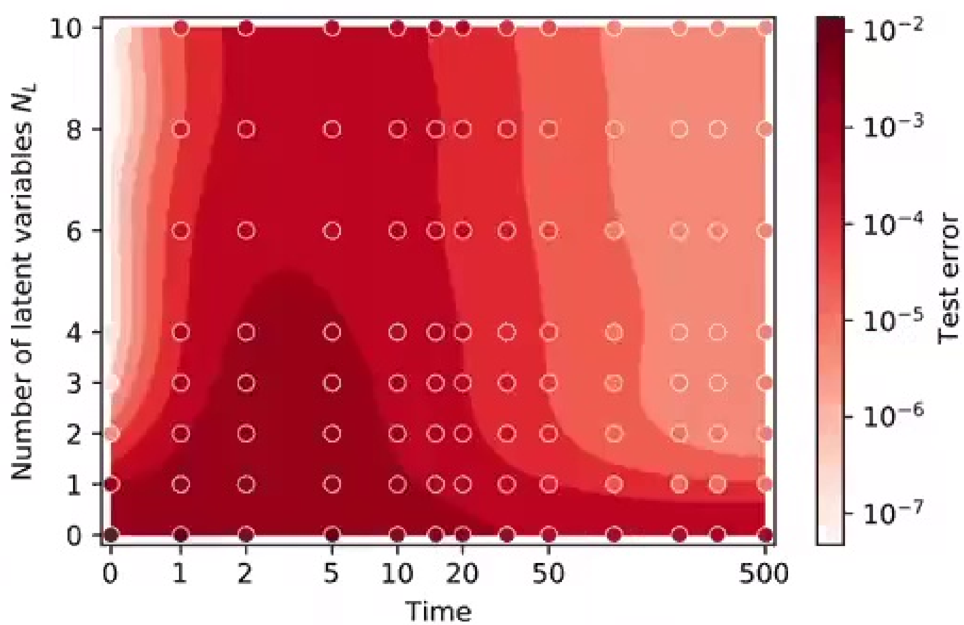
Partial summary: steady states of closed systems

1. Distinguish chaotic from integrable models
2. Reconstruct H from local observables



Complexity growth in random circuits with S_z conservation





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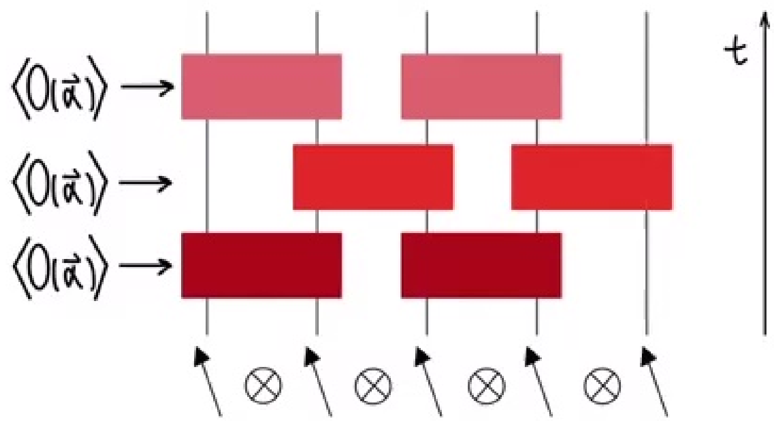

- Start with a product state → need 2 variables
- Propagate with **random unitaries with magnetization conservation** →
- Measure local observables at different times

Complexity dynamics: different measures

- Entanglement entropy of states generically grows with time

$$|\psi\rangle = \sum_j \sqrt{\gamma_j}|\psi_{j,A}\rangle \otimes |\psi_{j,B}\rangle \quad \rightarrow \quad S_{vN} = -\sum_j \gamma_j \ln(\gamma_j)$$

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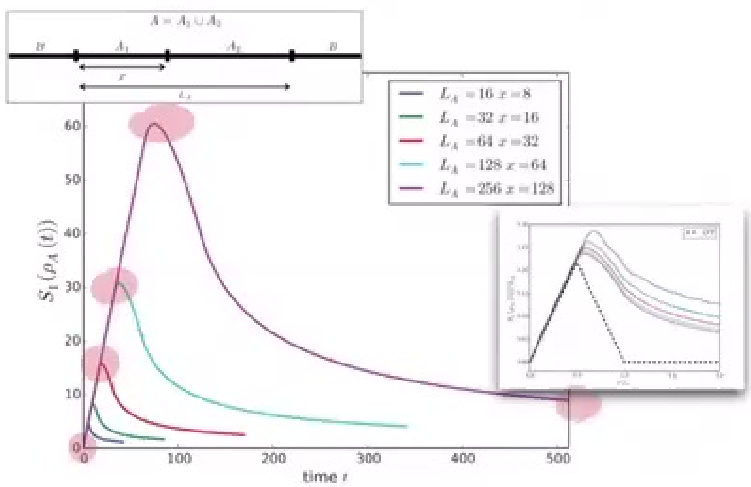

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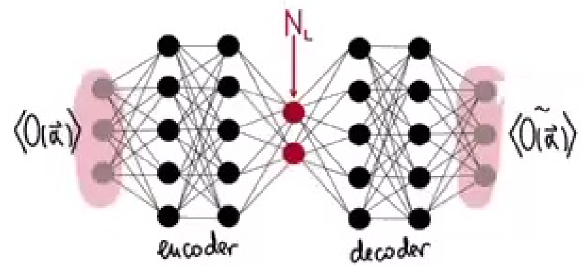

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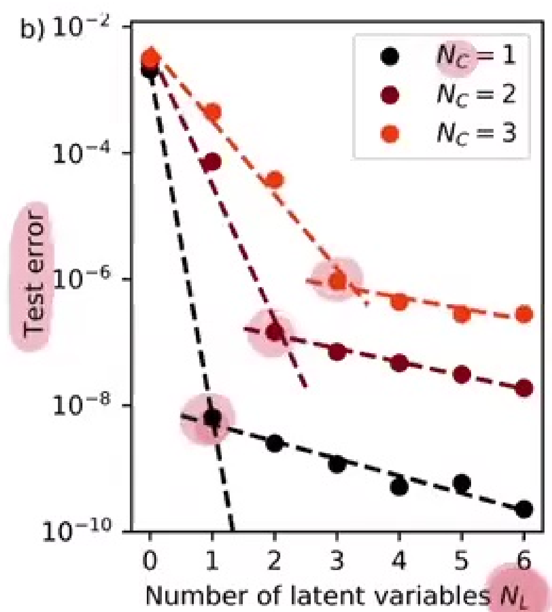
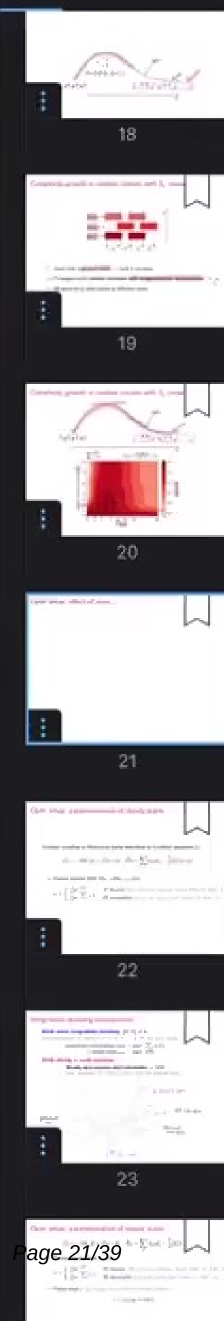


Test error:

$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} \left(f_\theta(x) - x\right)^2$$

Open setup: effect of noise



Open setup: parametrization of steady states

Consider coupling to Markovian baths described by Lindblad operators L_k

$$\hat{\mathcal{L}}\rho = -i[H,\rho] + \epsilon\hat{\mathcal{D}}\rho = 0, \quad \hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

- Known simpler limit $\lim_{\epsilon\to 0}\lim_{t\to\infty}\rho(t)$

$$\rho = \begin{cases} \frac{1}{Z}e^{-\beta H}, & H \text{ chaotic; ZL, [Alberton] Altman, Rosch, PRL '18 [PRL '20],} \\ \frac{1}{Z}e^{-\sum_i \lambda_i C_i}, & H \text{ integrable; Lange, ZL, Rosch, Nat Comm '17, PRB '18} \end{cases}$$

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Integrability breaking perturbations

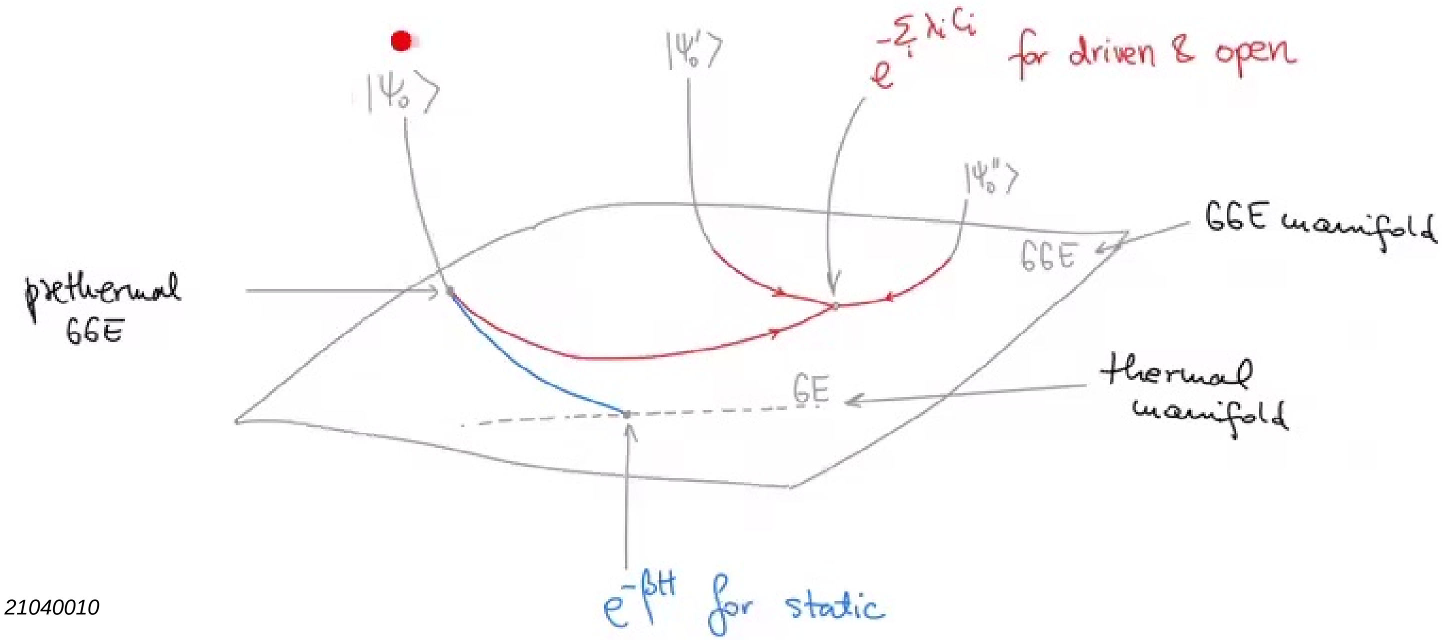
Weak static integrability breaking: $[H, C_i] \neq 0$

Moeckel&Kehrein '11; Kollar et al '11; Bertini et al '15, Ben Lev's group '17

- prethermal intermediate $\rho_{GGE} \sim \exp(-\sum_i \lambda_i C_i)$
- $\rightarrow$ steady-state $\rho_{GE} \sim \exp(-\beta H)$

Weak driving + weak openness

- **Weakly open systems don't thermalize $\rightarrow$ GGE**
- Nat. Commun. 8, 15767 (2017), PRB 97, 165138 (2018).



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- Finite small ϵ (ZL, Lange, Rosch, PRB 97, 024302 (2018))

$$\rho = \rho_{(G)GE} + \delta\rho(\epsilon)$$

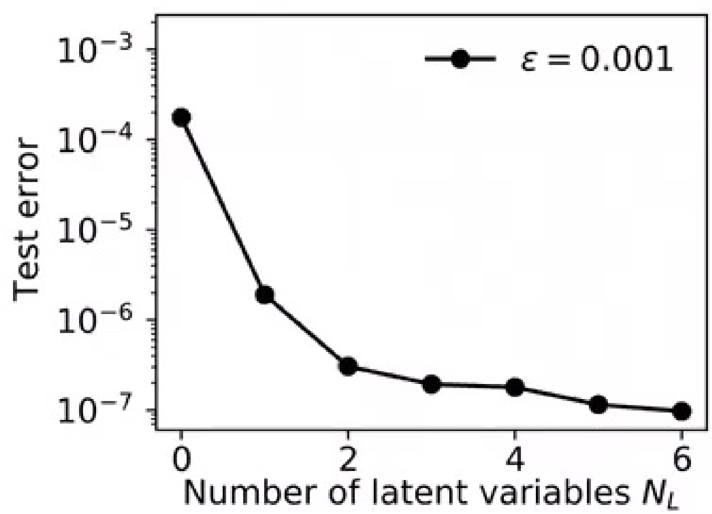
- Setup studied here: randomly rotated 1-site and 2-site Lindblads

$$L_i^{(1)} = \left(R_z(\theta)R_y(\phi)S_i^- R_y^{-1}(\phi)R_z^{-1}(\theta)\right) P_{i+1}^\downarrow, \quad L_i^{(2)} = R_z(\theta')R_y(\phi')P_{i+1}^\uparrow R_y^{-1}(\phi')R_z^{-1}(\theta')$$

Open setup: weak coupling to baths $\epsilon \ll 1$

Chaotic $H \to \rho \approx \frac{1}{Z} e^{-\beta H}$,

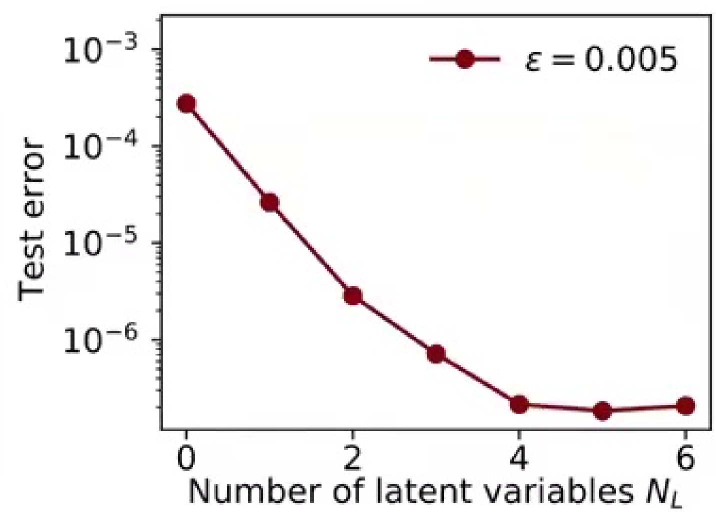
$$H = \sum_j J\sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x + h_z \sigma_j^z$$



1st latent var. $\to$ energy
2nd latent var. $\to$ noise

Integrable $H \to \rho \approx \frac{1}{Z} e^{-\sum_i \lambda_i C_i}$,

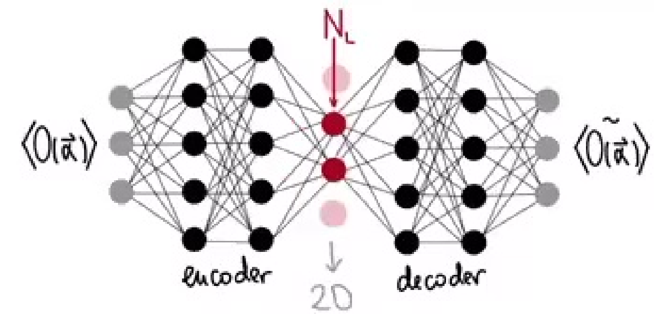
$$H = \sum_j J\sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x$$



more latent vars. $\to$ cons. laws C_i
additional latent var. $\to$ noise

What is learned?

Do 2D projection of the latent space



Chaotic $H \to \rho \approx \frac{1}{Z} e^{-\beta H}$,

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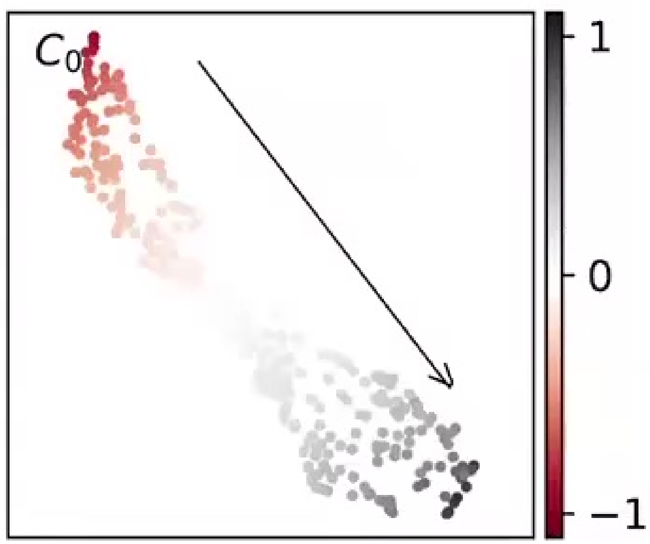
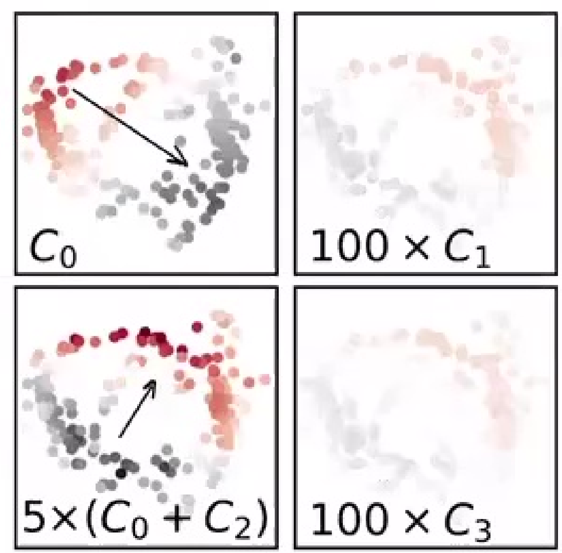


→ energy

Integrable $H \to \rho \approx \frac{1}{Z} e^{-\sum_i \lambda_i C_i}$,

$$H = \sum_j J\sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x$$



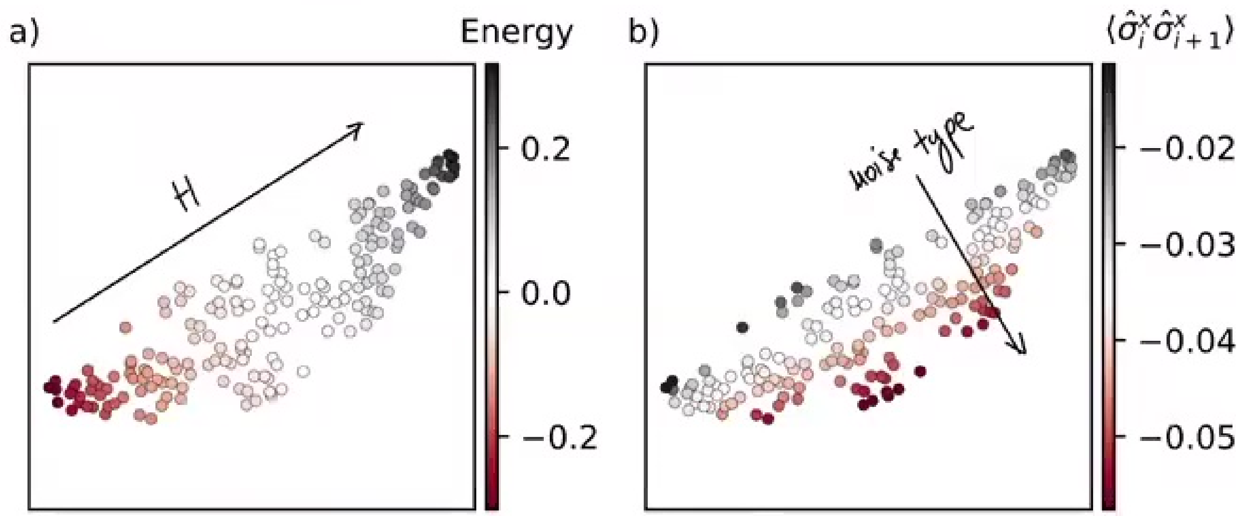
→ most local parity-even cons. laws

Open setup: noise-type reconstruction

$$H = \sum_i J_z S_i^z S_{i+1}^z + h_x S_i^x + h_z S_i^z \tag{1}$$

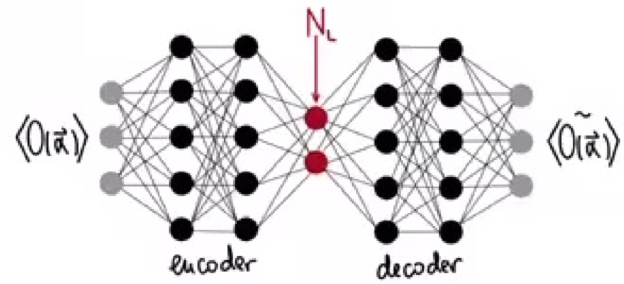
with Lindblad operators that favour AFM $\sigma_i^x \sigma_{i+1}^x$ correlations

$$\begin{aligned} L_i^{(1a)} &= S_i^{+,x} P_{i+1}^{\downarrow,x}, \quad L_i^{(1b)} = P_i^{\downarrow,x} S_{i+1}^{+,x}, \\ L_i^{(2a)} &= S_i^{-,x} P_{i+1}^{\uparrow,x}, \quad L_i^{(2b)} = P_i^{\uparrow,x} S_{i+1}^{-,x}, \\ L_i^{(3)} &= S_i^z \end{aligned} \tag{2}$$



Effective thermal description at larger coupling to baths?

$$\hat{\mathcal{L}}\rho = -i[H,\rho] + \epsilon\hat{\mathcal{D}}\rho = 0,$$
$$\hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$



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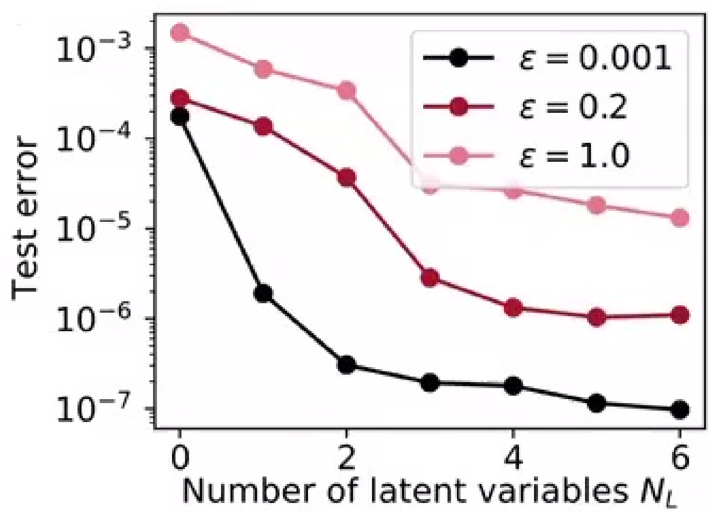
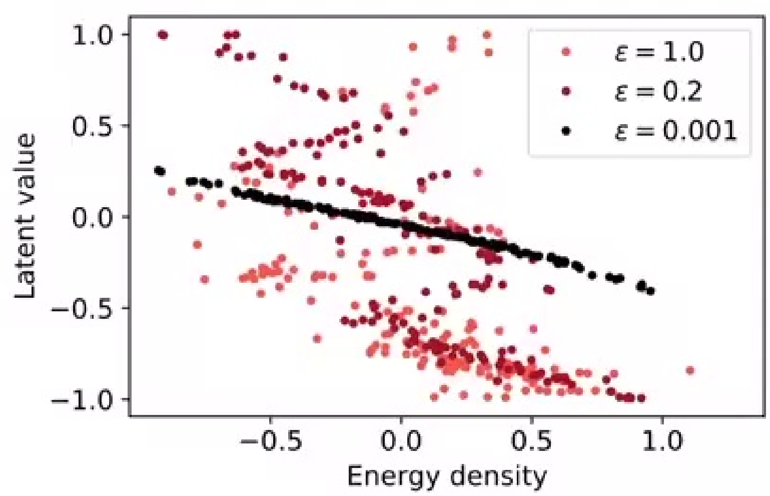
- Can $\rho \overset{?}{=} \frac{1}{Z}e^{-\beta' H'}$ be effectively thermal at large ϵ? $\rightarrow$ No

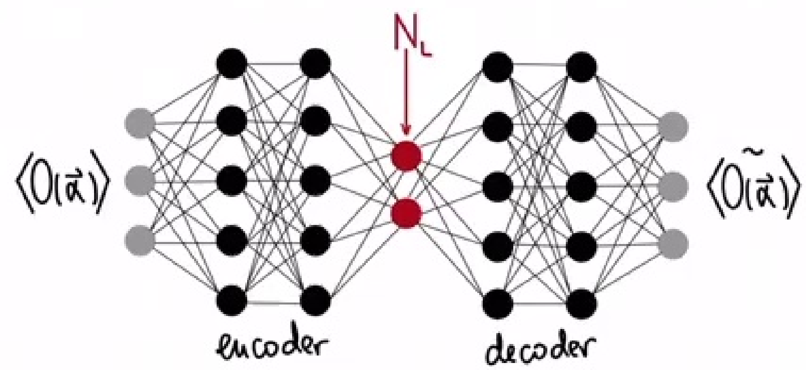


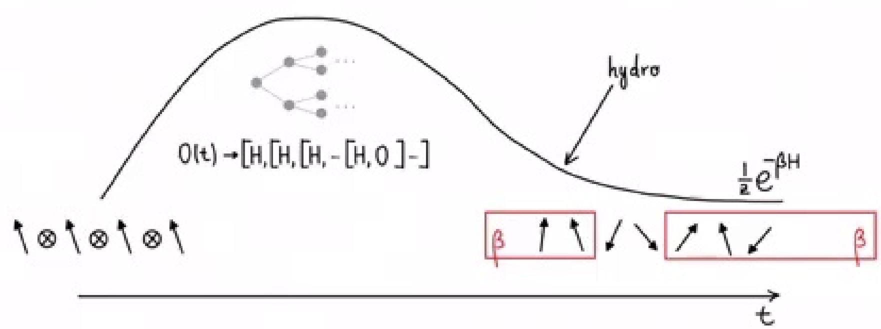


Conclusions

Detect low-dimensional physical represenation

- Closed systems
  - Distinguish chaotic from integrable system
  - Reconstruction of H from measurements
- Open systems
  - Detect the complexity of density matrix
  - Reconstruction of noise-type
- Advantages
  - Suitable for experimental data
  - Ansatz-free measure of local complexity





M. Schmitt and Z. Lenarčič, arXiv:2102.11328.

Integrability breaking perturbations

$\rho(t) = \rho_{GGE}(t) + \boxed{\delta\rho(\varepsilon)}$

$\lambda_i(t) \sim 1/\varepsilon$

Weak static integrability breaking: $[H, C_i] \neq 0$

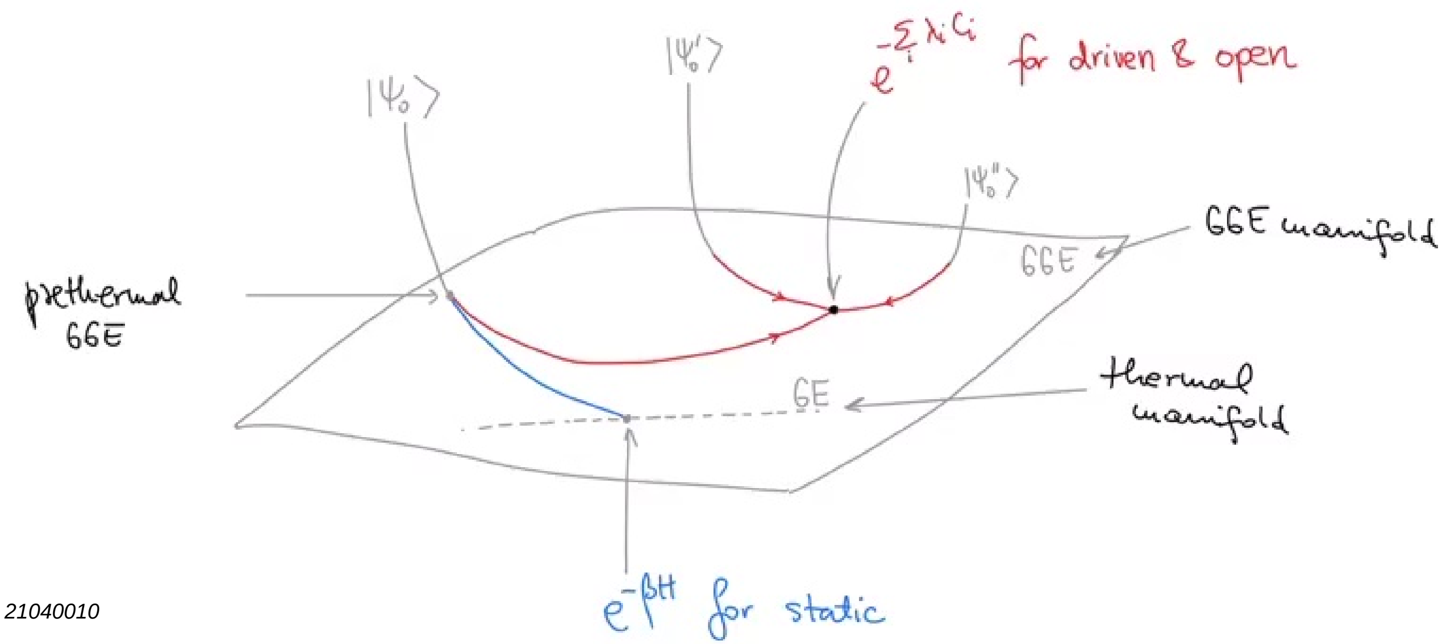
Moeckel&Kehrein '11; Kollar et al '11; Bertini et al '15, Ben Lev's group '17

- prethermal intermediate $\rho_{GGE} \sim \exp(-\sum_i \lambda_i C_i)$
- $\rightarrow$ steady-state $\rho_{GE} \sim \exp(-\beta H)$

$\lim_{\varepsilon \to 0} \lim_{t \to \infty} \rho = \rho_{GGE}$

Weak driving + weak openness

- **Weakly open systems don't thermalize $\rightarrow$ GGE**
- Nat. Commun. 8, 15767 (2017), PRB 97, 165138 (2018).



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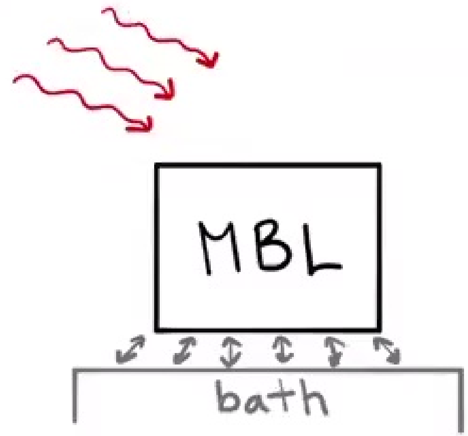


Ergodic:

- only H
- **thermal** steady-state

$$\rho \sim \frac{1}{Z} e^{-\beta(\theta) H} + \delta\rho(\epsilon, \theta)$$

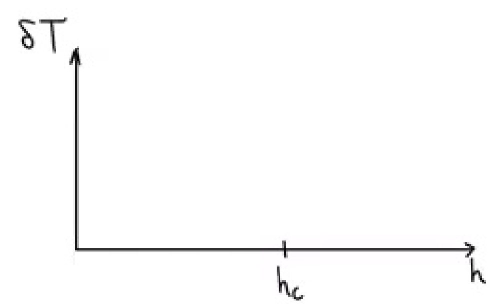
MBL:

- more conservation laws τ_i^z
- **non-thermal** steady-state

$$\rho \sim \frac{1}{Z} e^{\Sigma_i \lambda_i(\theta) \tau_i^z + \dots} + \delta\rho(\epsilon, \theta)$$

Order parameter:

$$\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle\langle \mathrm{Var}(T_i) \rangle\rangle}}{\langle\langle \mathbb{E}(T_i) \rangle\rangle}.$$



PRL 121, 267603 (2018), PRL 125, 116601 (2020)

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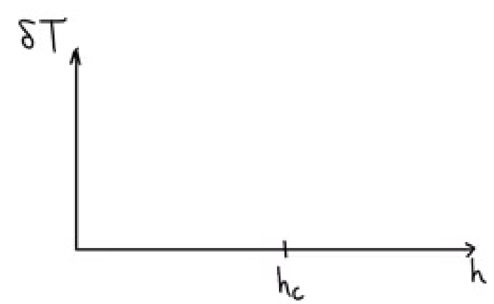

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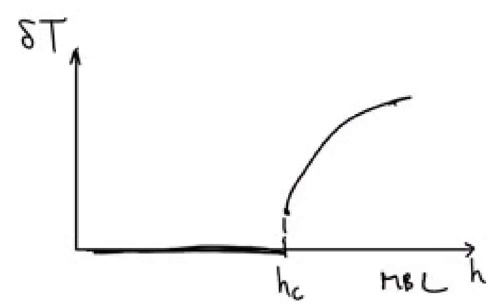
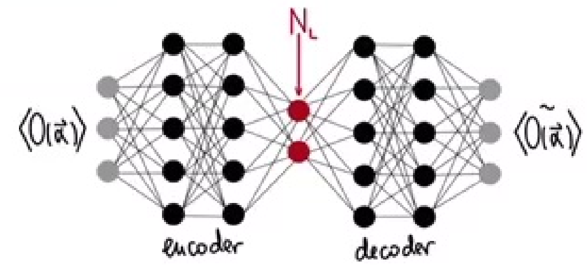


PRL 121, 267603 (2018), PRL 125, 116601 (2020)

Effective thermal description at larger coupling to baths?

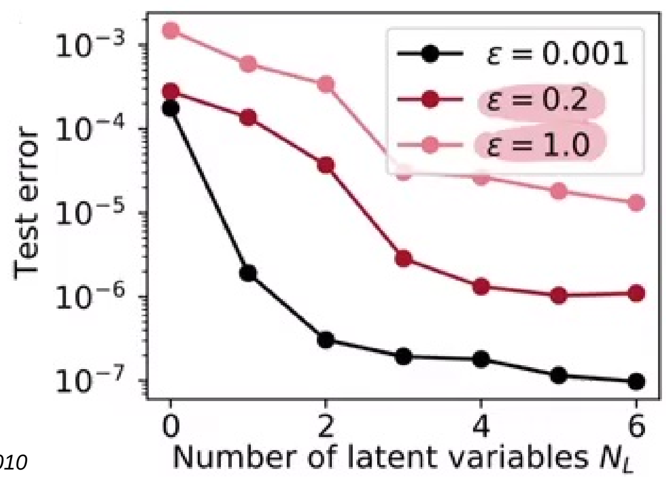
$$\hat{\mathcal{L}}\rho = -i[H,\rho] + \epsilon\hat{\mathcal{D}}\rho = 0,$$
$$\hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

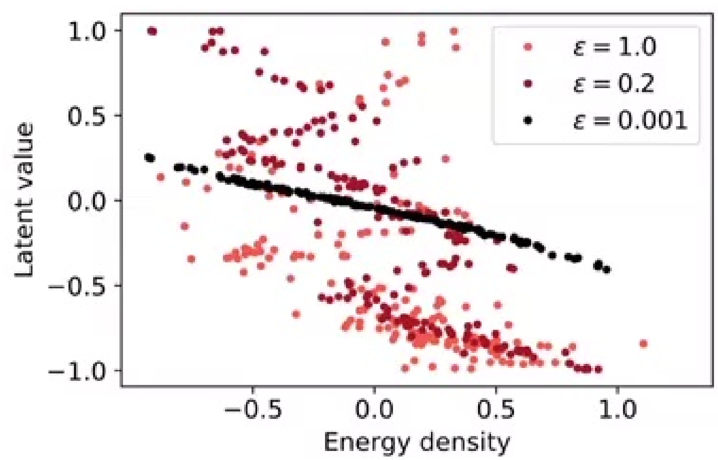


- Setup studied here: randomly rotated 1-site and 2-site Lindblads

$$L_i^{(1)} = \left(R_z(\theta)R_y(\phi)S_i^- R_y^{-1}(\phi)R_z^{-1}(\theta)\right) P_{i+1}^{\downarrow}, \quad L_i^{(2)} = R_z(\theta')R_y(\phi')P_{i+1}^{\uparrow}R_y^{-1}(\phi')R_z^{-1}(\theta')$$

- Can $\rho \overset{?}{=} \frac{1}{Z}e^{-\beta' H'}$ be effectively thermal at large ϵ? $\rightarrow$ No

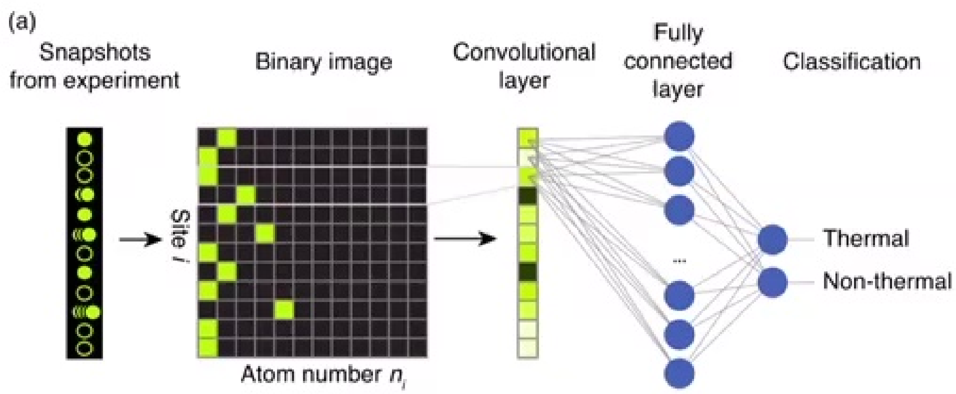




Outlook: detect MBL phase transition

Greiner's group at Harvard: Bohrt et al 2012.11586 (2020)

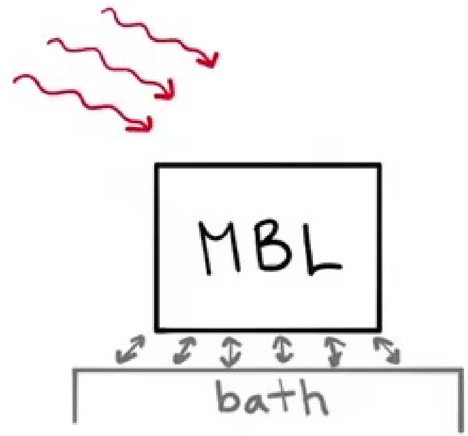
- Look at experimental snap-shots
- use superviser or unsupervised learning to detect the transition



Bohrt et al., arxiv 2012.11586 (2020)

- supervised: train on ED data
- unsupervised: confusion learning

- Bottleneck: slow relaxation to the thermal state on ergodic side?



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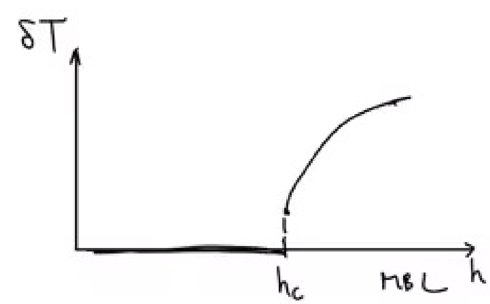

PRL 121, 267603 (2018), PRL 125, 116601 (2020)

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TEBD calculations for coupling to Markovian baths

Steady state

$$\hat{\mathcal{L}}\rho_\infty = -i[H, \rho_\infty] + \epsilon\hat{\mathcal{D}}\rho_\infty = 0$$

Dominant Hamiltonian part

$$H = \sum_i S_i \cdot S_{i+1} + h(\alpha_i^z S_i^z + \alpha_i^x S_i^x), \quad \alpha_i^{x,z} \in [-1, 1]$$

Coupling to Markovian baths

$$\hat{\mathcal{D}} = \sum_\alpha \hat{\mathcal{D}}^{(\alpha)}, \quad \hat{\mathcal{D}}^{(\alpha)}\rho = \sum_i L_i^{(\alpha)}\rho(L_i^{(\alpha)})^\dagger - \frac{1}{2}\{(L_i^{(\alpha)})^\dagger L_i^{(\alpha)}, \rho\}$$

Lindblad operators

$$L_i^{(1a)} = S_i^+\left(\frac{1}{2}\mathbb{1}_{i+1} - S_{i+1}^z\right), \quad L_i^{(1b)} = \left(\frac{1}{2}\mathbb{1}_i - S_i^z\right)S_{i+1}^+,$$

$$L_i^{(2a)} = S_i^-\left(\frac{1}{2}\mathbb{1}_{i+1} + S_{i+1}^z\right), \quad L_i^{(2b)} = \left(\frac{1}{2}\mathbb{1}_i + S_i^z\right)S_{i+1}^-,$$

$$L_i^{(3)} = S_i^z$$

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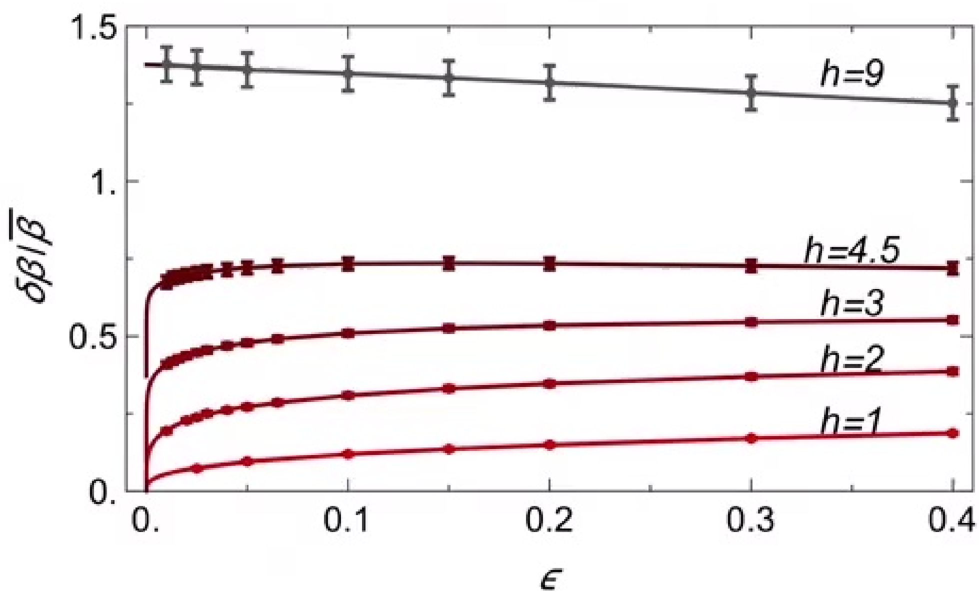
Detect the underlying transition from ϵ dependence

- Two regimes

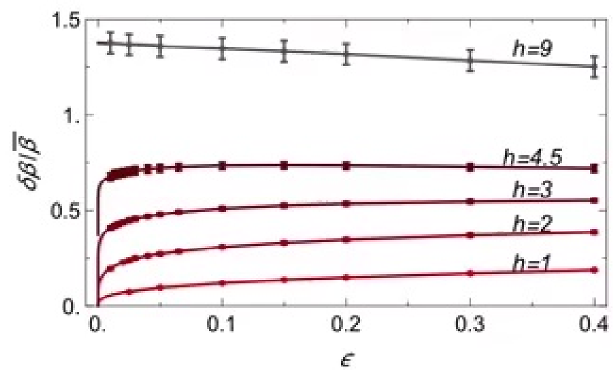
$$\frac{\delta\beta}{\overline{\beta}}(\epsilon) \sim \begin{cases} 0 & + \epsilon^{1/2z}, \quad \text{ergodic} : h < h_c, \\ \frac{\delta\beta}{\overline{\beta}}\Big|_{\epsilon\to 0} & - b\,\epsilon + \mathcal{O}(\epsilon^2), \quad \text{MBL} : h \geq h_c \end{cases}$$



Conclusions open MBL

- **Complexity of steady state can detect the MBL transition**
- Measurement of dynamical exponent $z(h)$ and critical disorder

$$\frac{\delta T}{\bar{T}}(\epsilon) \sim \begin{cases} 0 \qquad\quad + \epsilon^{1/2z}, & h < h_c, \\ \frac{\delta T}{\bar{T}}\Big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & h \geq h_c \end{cases}$$



- Level spacing is out of question!

$\epsilon \to 0$: Lenarčič, Altman, Rosch, PRL 121, 267603 (2018),
$\epsilon > 0$: Lenarčič, Alberton, Rosch, Altman, PRL 125, 116601 (2020)