Title: Subsystem-Symmetry protected phases of matter

Speakers: Fiona Burnell
Date: April 05, 2021 - 12:30 PM

URL: http://pirsa.org/21040009

Abstract: We know that different systems with the same unbroken global symmetry can nevertheless be in distinct phases of matter. These different "symmetry-protected topological" phases are characterized by protected (gapless) surface states. After reviewing this physics in interacting systems with global symmetries, I will describe how a different class of symmetries known as subsystem symmetries, which are neither local nor global, can also lead to protected gapless boundaries. I will discuss how some of these subsystem-symmetry protected phases are related (though not equivalent) to interacting higher-order topological insulators, with protected gapless modes along corners or hinges in higher dimensional systems.

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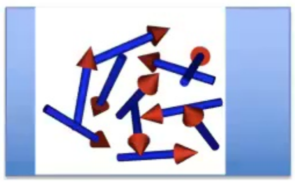
Subsystem-Symmetry protected phases of matter

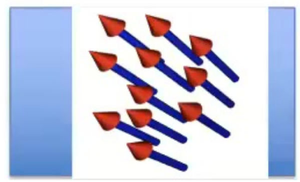
F. J. Burnell
University of Minnesota

How do symmetries distinguish phases of matter? Conventional approach:

- Landau paradigm



High temperature

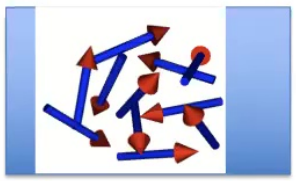


Low temperature

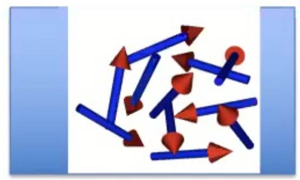
- Different phases = different symmetry groups (related by spontaneous symmetry breaking)

How do symmetries distinguish phases of matter? A new possibility

- Symmetry-Protected Topological (SPT) phases



High temperature



Low temperature

- 2 phases with same unbroken symmetry can be distinct (cannot get from one to the other without a phase transition)

- … but how? Locally, they look the same!

Symmetry-protected phases: a brief history

- Pre-2004: a few examples known (e.g. SSH, Haldane/AKLT)
- 2004-2009: Topological band insulators in 1,2, and 3 dimensions
- 2009 : Interacting symmetry-protected phases
- 2017 : Higher-order topological band insulators
- 2018: Subsystem symmetry protected topological phases

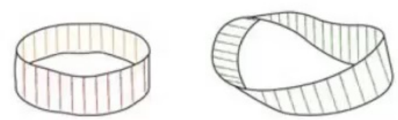
SPT phases of free fermions: topological band structures

(Kane, Mele; Fu, Kane, Mele; Moore, Balents; Roy)

- Free fermions: (crystal) momentum is conserved.

- Crystal momentum is periodic, and the phase of the wave function can ``twist" in the Brillouin zone.

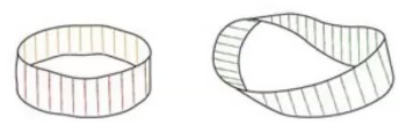
- This twisting leads to quantized topological invariants that characterize different phases with the same symmetry



(Figure: Manoharan, '10)

Topological band structures: Examples

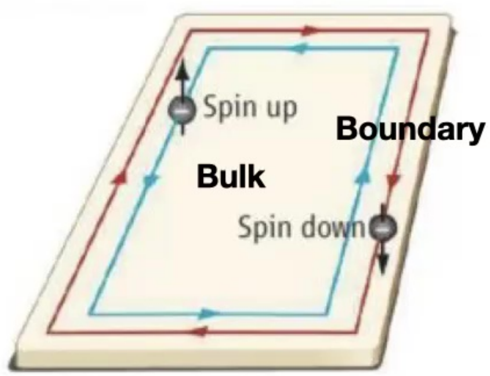
- Integer quantum Hall effect
  (Thouless, Kohmoto, Nightingale, den Nijs)
- Topological insulators in 2D and 3D
  (Kane Mele; Fu, Kane, Mele; Moore, Balents; Roy
- Topological superconductors
  (Read, Green; Kitaev; Ryu,Schnyder, Furasaki, Ludwig)
- Higher-order topological band insulators
  (Benalcazar, Bernevig, Hughes)

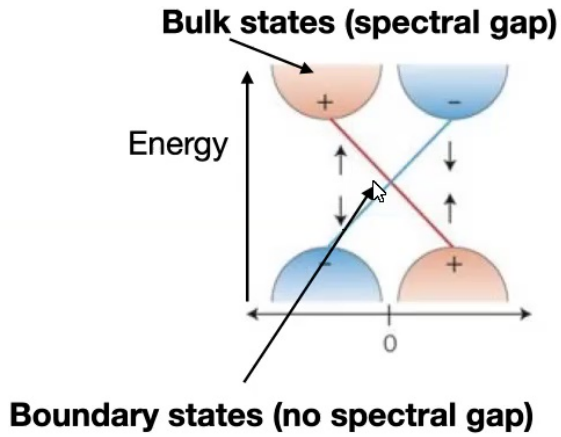


(Figure: Manoharan, '10)

We know how to predict the band structures of many materials pretty well.
Lots of success in identifying non- (weakly-) interacting SPT phases

Physical properties of SPT phases: Symmetry- protected gapless boundary modes

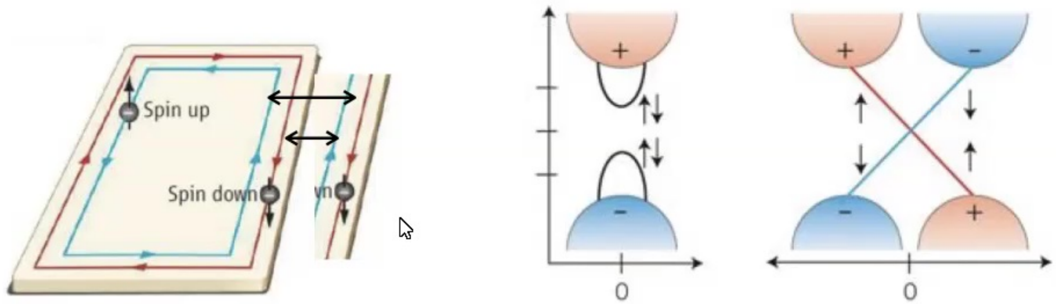




Reason: Kramers' degeneracy (required by time reversal) forbids it!

Physical properties of SPT phases: Symmetry- protected gapless boundary modes

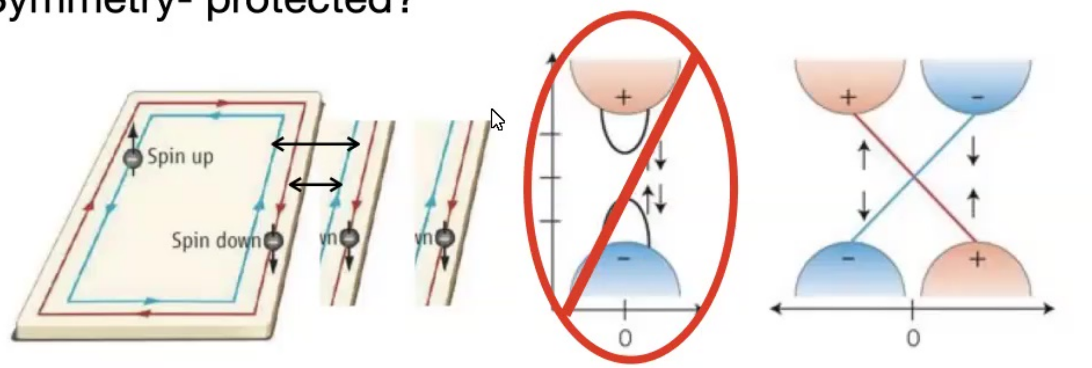
- Symmetry- protected?



- If we could add only one copy of the edge, we could back-scatter and open a gap without breaking time-reversal symmetry

Physical properties of SPT phases: Symmetry- protected gapless boundary modes

- Symmetry- protected?



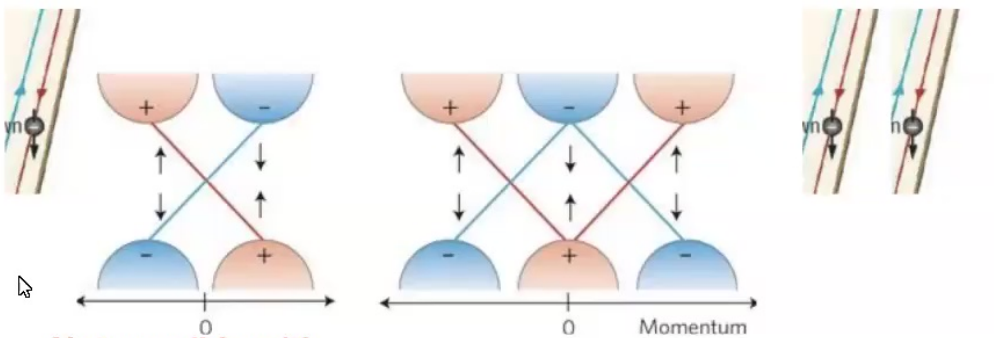
- If we could add only one copy of the edge, we could back-scatter and open a gap without breaking time-reversal symmetry

- But 1D systems with time reversal must contain an even number of copies!

Symmetry- protected gapless boundary modes

- Cannot do this: boundary state is incompatible with symmetry without the bulk

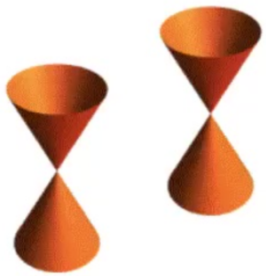
- *anomalous boundary states*



Not possible with TRS in 1D (protected)

Possible with TRS in 1D (NOT protected)

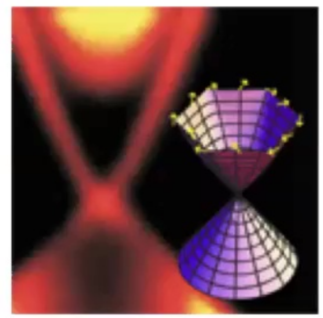
Symmetry- protected gapless boundary modes in 3D (Witten anomaly)



2 D: pairs of Dirac cones

Regulator for 1 Dirac cone breaks T (or charge conservation)

Witten '82; Redlich '84; Haldane '88



2D surface of 3D TI:

3 D bulk= T-invariant regulator

Michael Mulligan and FJB, '13

SPT phases of interacting bosons (and fermions)

- Insight: Something about the (d-1 dimensional) boundary is not possible without the (d dimensional) bulk. i.e. no d-1 dimensional lattice model with this symmetry and particle content (*'t Hooft anomaly*)
- This leads to symmetry-protected gapless boundaries (1d, 2d, or 3d) or topologically ordered boundaries with anomalous symmetry (3d)
- Let's quickly review how this works …

Anomalous boundaries in 1D: Haldane/ AKLT chain

- Spin 1 chain (3 spin states per site, SO(3) spin rotation symmetry)

- Hamiltonian (AKLT): penalizes states where two adjacent spins are in a net spin 2 representation

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3}(\vec{S}_j \cdot \vec{S}_{j+1})^2$$

- Ground state: project out spin-2 representations

Anomalous boundaries in strongly interacting systems: 1D

- Spin 1 chain (3 spin states per site, SO(3) spin rotation symmetry)

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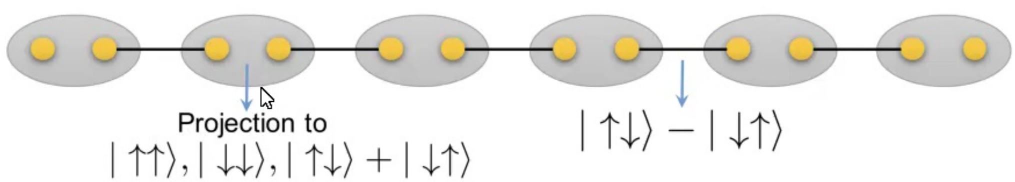
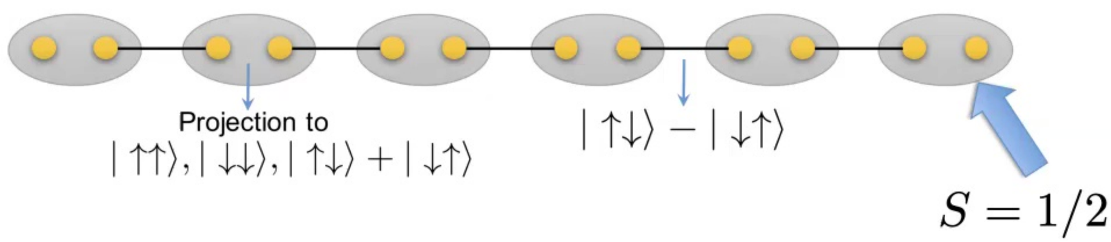
- Ground state: project out spin-2 representations



Anomalous boundaries in strongly interacting systems: 1D



- Free spin-1/2 at each end of the spin 1 chain!

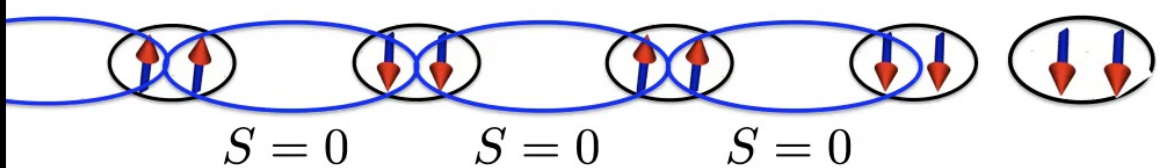
What's "anomalous" about s=1/2?

(Pollmann, Berg, Turner, Oshikawa; Chen, Gu, Liu, Wen)

- Projective representations: $(i.e.\ H^2(G, U(1))\ )$

Symmetry	S= 1/2	S=1
$\mathcal{T}^2$	-1	$+1$
$(R_\pi^{x,y,z})^2$	-1	$+1$
$R_\pi^x R_\pi^y R_\pi^x R_\pi^y$	-1	$+1$

- Action of symmetry is different from what can be realized in a 0D, integer spin system.



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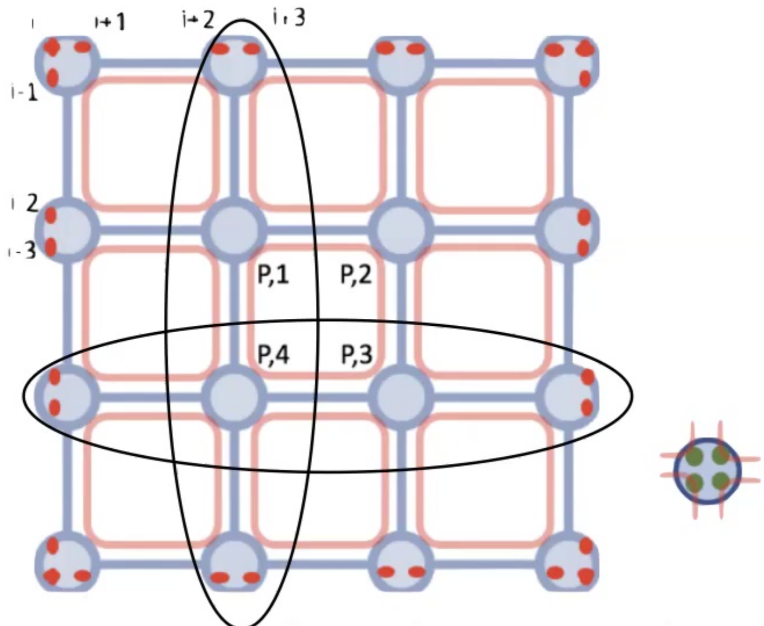
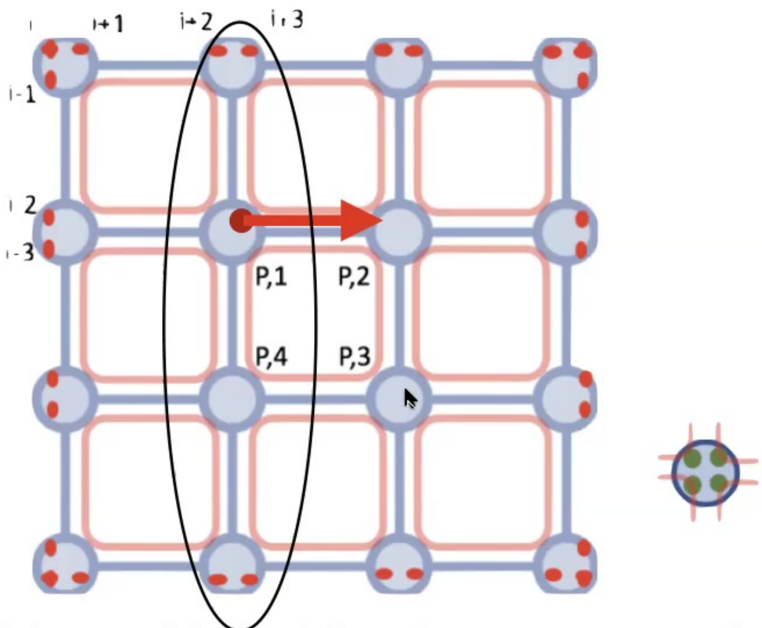
- 2017 : Higher-order topological band insulators
- 2018: Subsystem symmetry protected topological phases (d=1 sub-systems, i.e. symmetry acts along lines)

What is subsystem symmetry?



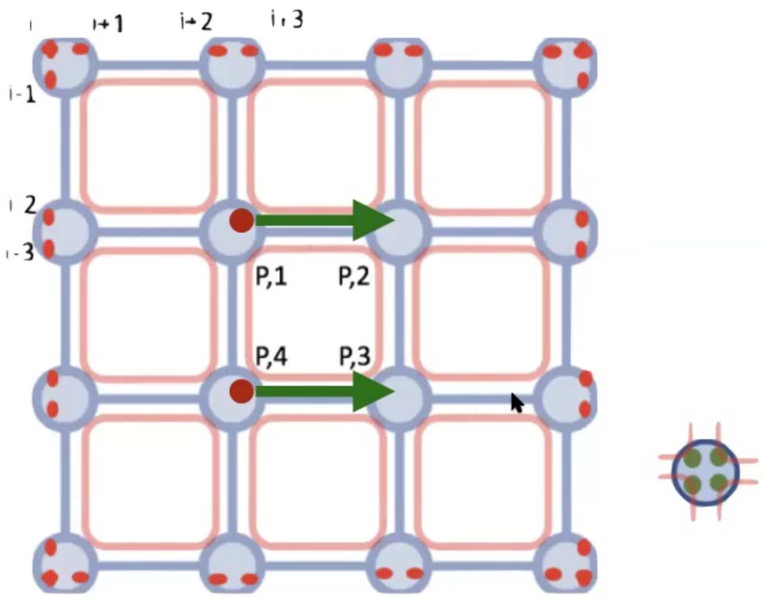
- Subsystem symmetry: independent symmetries along different lines (or planes, or fractals)

What is subsystem symmetry?



- Individual "charges" immobile: charge conserved on each line

What is subsystem symmetry?



- "Dipoles" mobile perpendicular to their direction of separation

Why subsystem symmetry?

- Connection to fracton orders (in 3D)
- Applications in quantum information (cluster states)
- Because it's different in some important (qualitative) ways, that can teach us about interacting HOTI

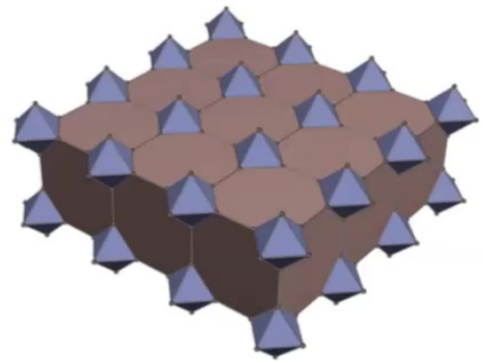


Image credit: Asavanant et. Al., Science '19

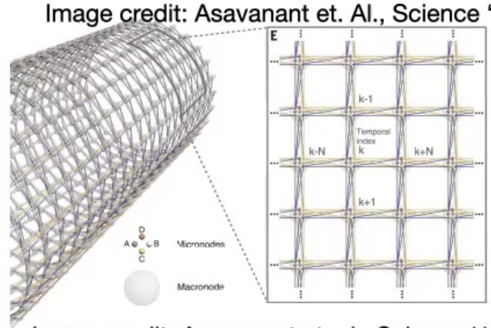
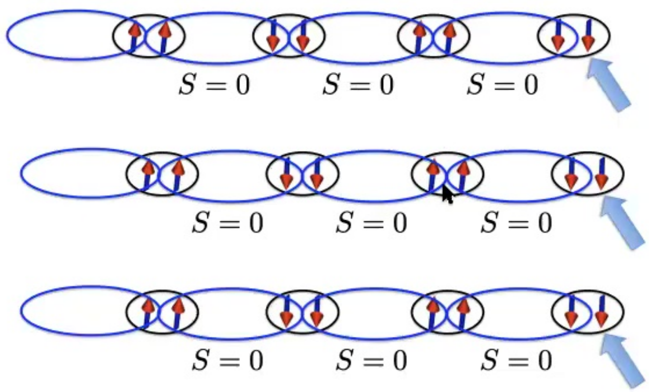


Image credit: Asavanant et. al., Science '19

Subsystem symmetry protected topological (SSPT) phases

You, Devakul, F. J. B., Sondhi;
Devakul, Williamson, You



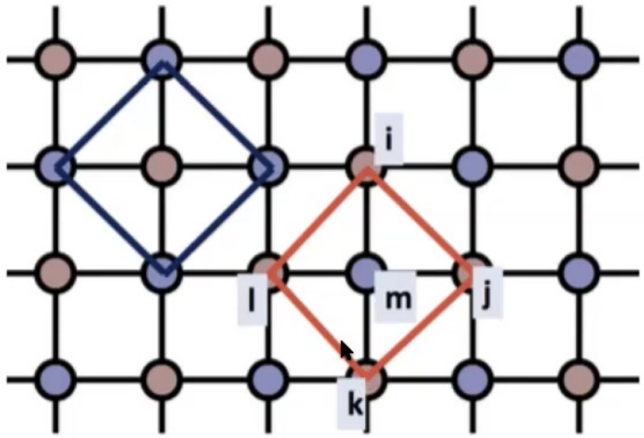
- Stack of decoupled lower-dimensional SPT's: $(H^2(G, U(1)))^N$
- All possibilities:

$$(H^2(G^N, U(1)))$$

- Boring: a stack of decoupled 1D systems
- For some G, this is not all
- "Strong" SSPT phases

Strong SSPT phases: the 2D cluster state

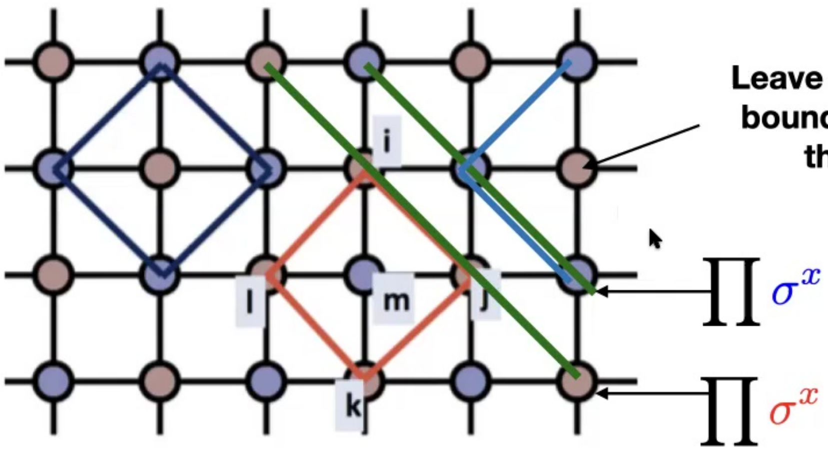
(You, Devakul, FJB, Sondhi)



$$H = \sum \prod_{\text{Red plaquette}} \sigma^z \sigma^x + \prod_{\text{blue plaquette}} \sigma^z \sigma^x$$

Strong SSPT phases: the 2D cluster state

(You, Devakul, FJB, Sondhi)



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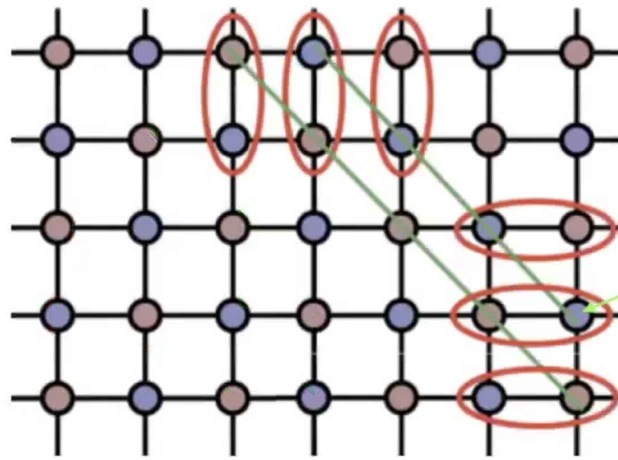


No X terms on boundary spins:

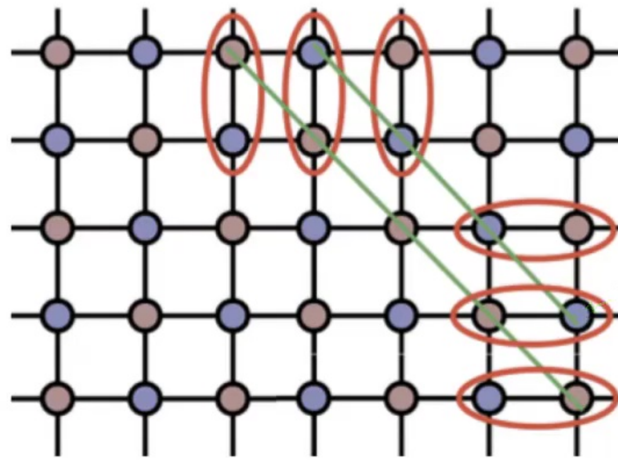
$$\pi^z = \sigma^z_B$$

$$\pi^x = \sigma^x_B \sigma^z_R$$

commute with H

Strong SSPT phases: the 2D cluster state

(You, Devakul, FJB, Sondhi)

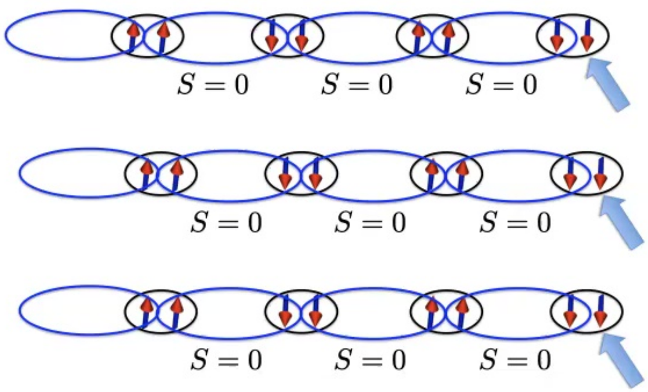


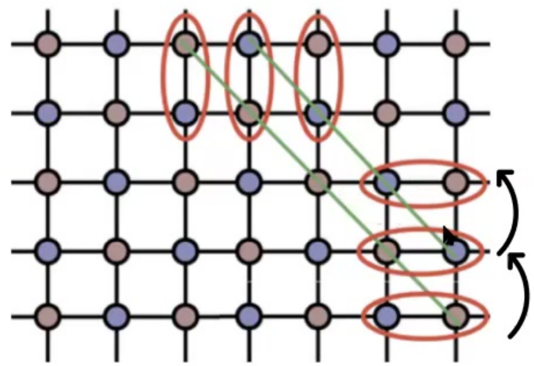
$$\pi^z = \sigma^z_B$$
$$\pi^x = \sigma^x_B \sigma^z_R$$

- Free spin-1/2 degree of freedom per unit cell on the boundary
- Transforms projectively (according to $\pi^{x,z}$)under subsystem symmetry
- Hence cannot be gapped without breaking subsystem symmetry

Strong SSPT phases: the 2D cluster state

(You, Devakul, FJB, Sondhi)



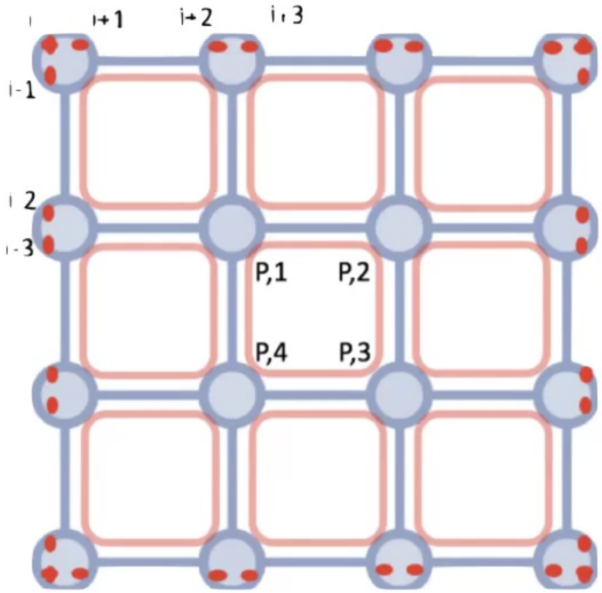


- Distinct from stacks of 1D SPTs ("Chaining" of commutators between group elements along the edge cannot be undone by adding stacks of 1D systems)

  (See also Devakul, Williamson, You)

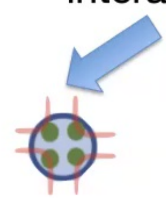
Subsystem SPT's and corners

(You, Devakul, FJB, Neupert; You, FJB, Hughes)



Each site: spin 2

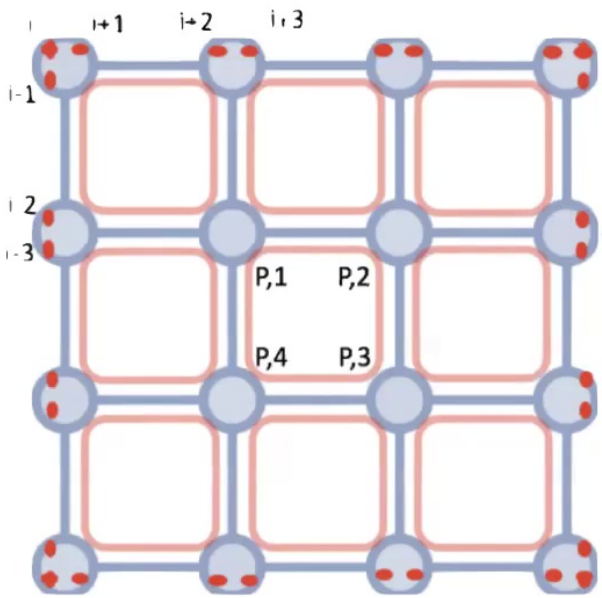
(Bond with strong plaquette interactions)



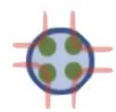
Symmetry: U(1) = spin rotation about the z-axis; global time reversal

Subsystem SPT's and corners

(You, Devakul, FJB, Neupert; You, Hughes, FJB)



Each site on the edge: left-over spin 1



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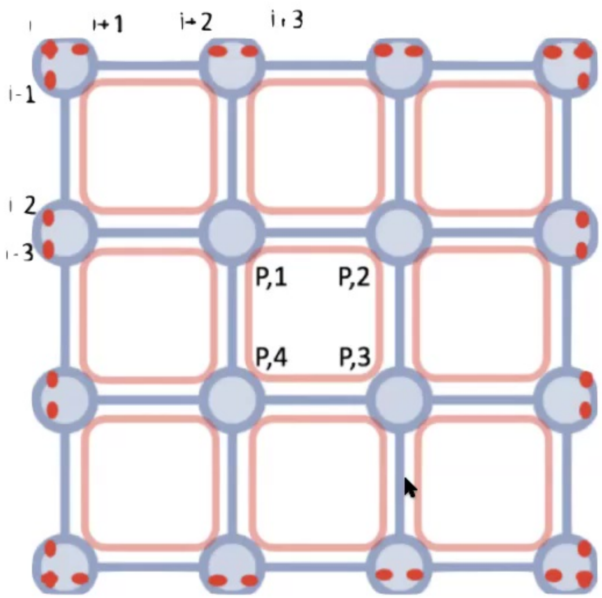

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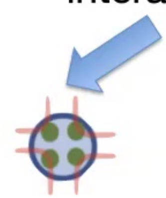

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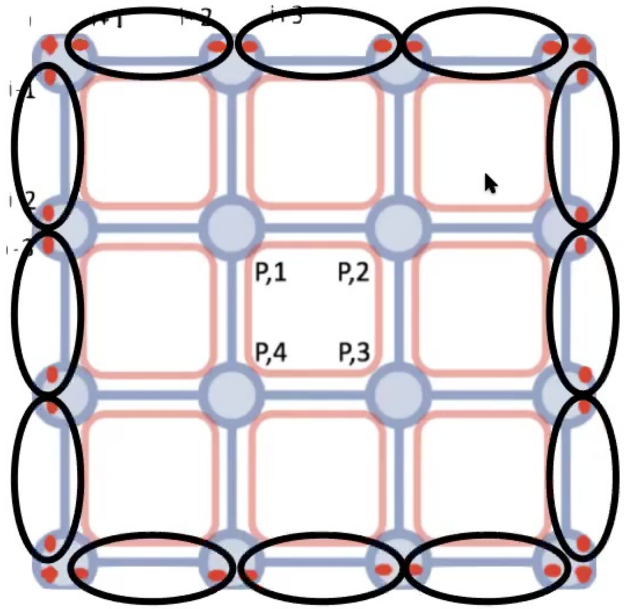

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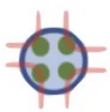

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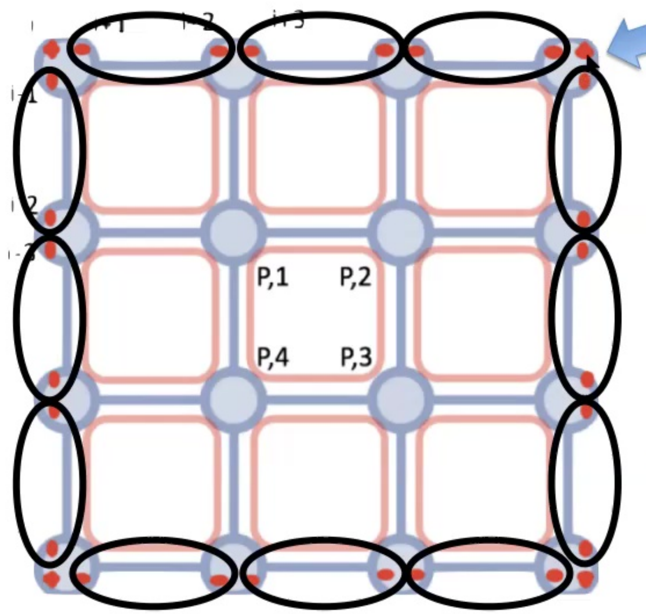
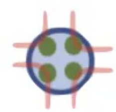
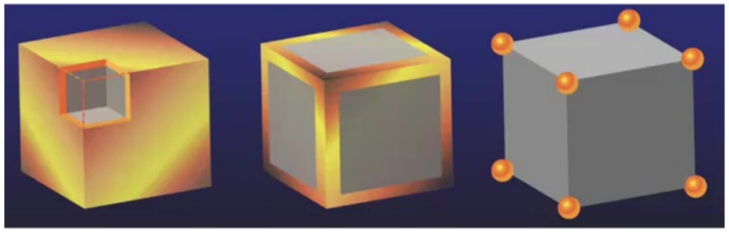
(You, Devakul, FJB, Neupert; You, Hughes, FJB)



Each site on a corner: left-over spin 1/2



Connections back to band topology



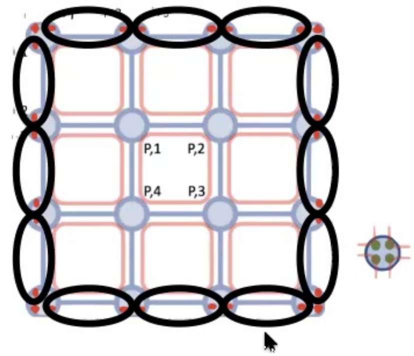
“Higher-order” topological insulators

(Benalcazar, Bernevig, Hughes)

- When quadrupole moment of a filled band is quantized, there can be additional topological invariants of the band structure (Symmetry: global U(1) *with dipole moment conservation*, time reversal.)
- Can lead to protected gapless modes at corners (2D, 3D) and hinges (3D)
- Requires some crystal symmetry (reflection, rotation)

Connections back to band topology

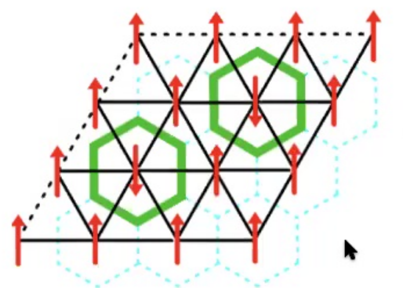
- If we break subsystem symmetry, but preserve some lattice symmetries, keep protected corner modes
- Subsystem symmetry: strong form of dipole moment conservation
- U(1) subsystem-SPT: HOTI band insulator; Cluster state: interacting HOTI



SSPT's in 3D: The twist

Levin, Gu '12

- 2D models with Ising symmetry:



2D models with Ising symmetry: The twist

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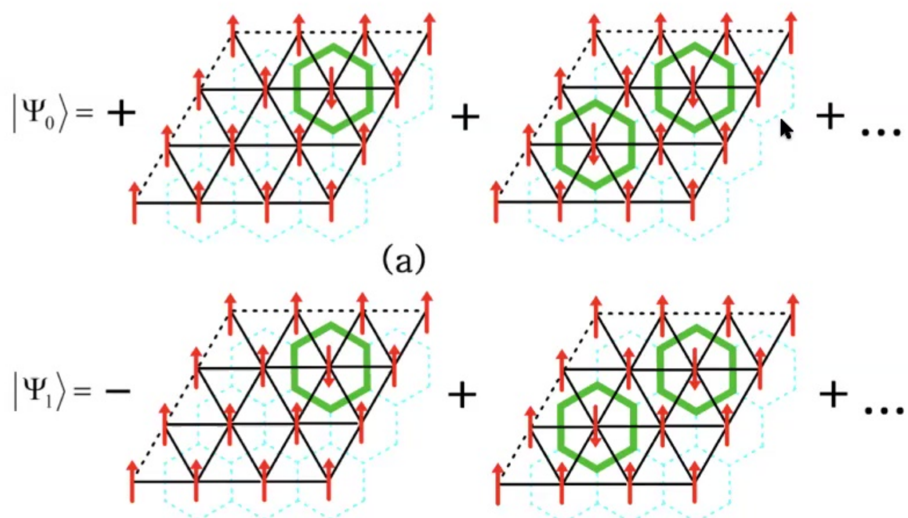


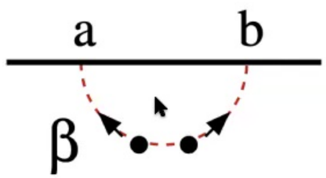
Domain wall loop soup (untwisted)

Domain wall loop soup with - signs (twisted)

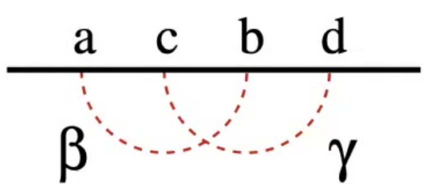
SPT's in 2D: Braiding statistics and gapless edges

Levin, Gu '12

- What's the difference? Bring domain walls to the boundary…

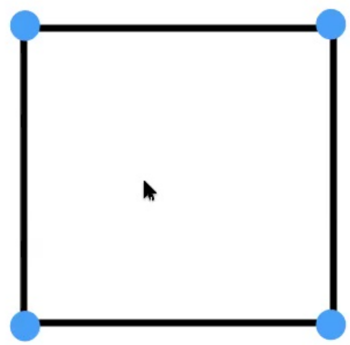


Untwisted: net phase +1



Twisted: Crossing leads to a net phase of -1. But how can this be, if the first pair was already gone?

3D Ising models with subsystem symmetry

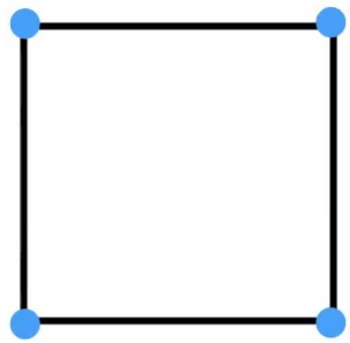


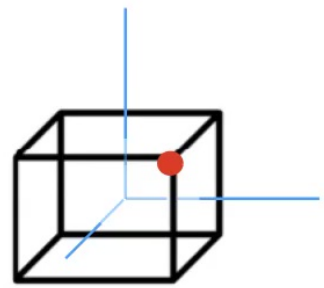
- Plaquette Ising model

$$H = -h \sum_j \sigma_j^x - J \sum_p \sigma_{p,1}^z \sigma_{p,2}^z \sigma_{p,3}^z \sigma_{p,4}^z$$

- Conserves $\prod_j \sigma_j^x$ in each plane of cubic lattice

3D Ising models with subsystem symmetry





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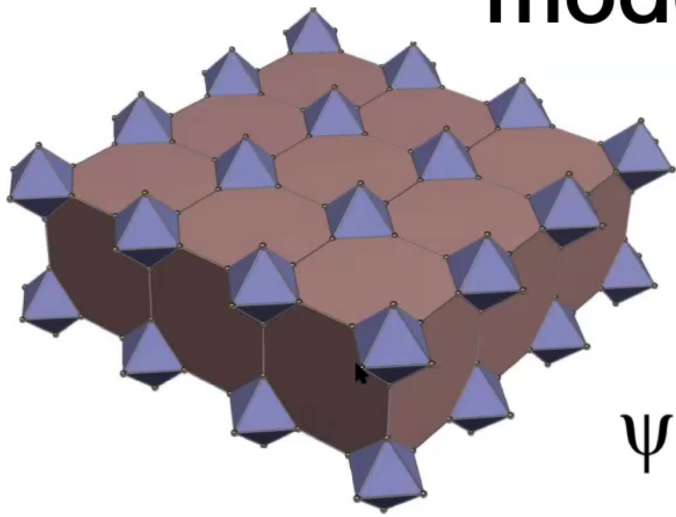
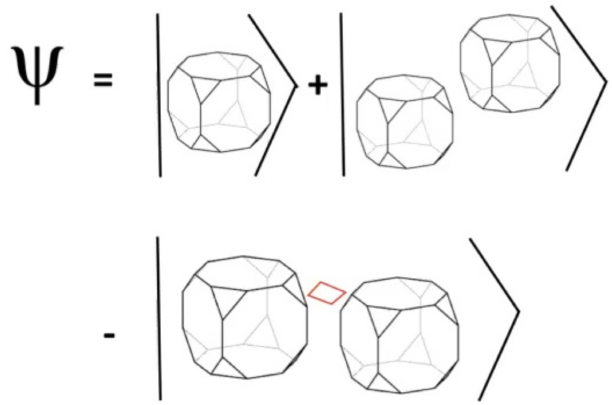
- Domain “walls” = Domain frames

Twisted 3D subsystem Ising model

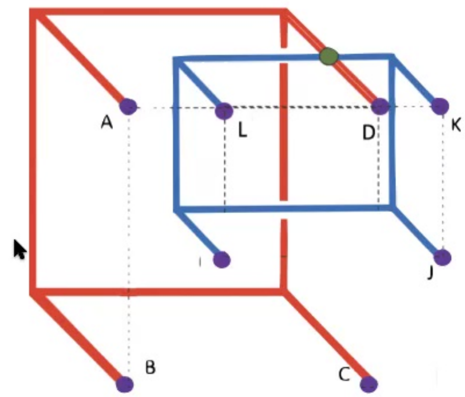




- As in 2D, modify H to change relative signs between some domain frame configurations

Twisted 3D subsystem Ising model: statistics and gapless boundaries

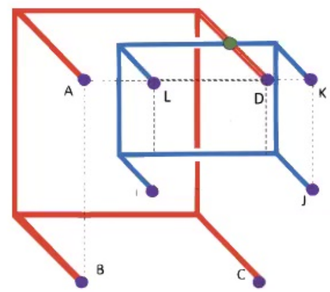
- Intersecting domain frames acquire a phase after twisting
- Boundary must be gapless if symmetry is preserved!

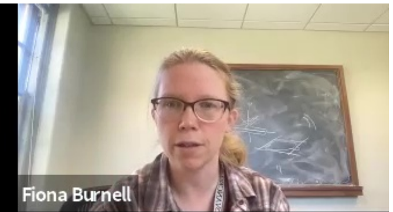


Weak vs strong planar SSPT

- This is weak: 2 edges together can be trivialized; the boundary (but not the bulk) is a stack of 2D things.
- Picture: lineon braiding non-trivial only in the same layer
- Strong versions also possible, with some subtleties

Devakul; Shirley; Wang





Summary

- Interacting symmetry-protected phases: anomalous boundaries
  - 1D, 2D : gapless. 3D: seemingly richer set of possibilities
- Subsystem symmetry on d-dimensional subsystems: boundary is similar to “stack” of d-dimensional SPT’s, but is a distinct phase of matter
- Unknowns (3+1 D): chirality and un-gappable boundaries?