Title: A Generalized Hartle-Hawking State

Speakers: Stephon Alexander

Series: Quantum Gravity

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Abstract: The Hartle-Hawking-Vilenkin state is defined on mini-superspace and is semiclassically related to inflationary theory. However, the state suffers a few problems connected to the path integral of Euclidean or the Lorentzian measure of metrics. In this talk, we will explore a way around these issues by working in the (Ashtekar) connection representation, a real Kodama-Chern-Simons state. We find a generalized "Fourier Transform" that related the Chern-Simons-Kodama state to the Hartle-Hawking state beyond mini-superspace. We end with some discussion and open ended questions for cosmology and quantum gravity.

The Generalized Hartle-Hawking State

Stephon Alexander
Brown University
CCA, Flatiron Institute, NYC

Collaborators

- Gabe Herczeg and Joao Magueijo
- Ongoing discussions with Laurent Freidel

References

- Magueijo(PRD, 2020)
- S.A, Cortes, Liddle, Magueijo, Smolin (PRD 2019)
- Smolin, Freiidel (Class.QG, 2004)
- S.A, Herczeg, Magueijo, Class.QC ‘21)
- Vilenkin, (PRD 1986)
- Hartle, Hawking (PRD ‘83)

Quantum Cosmology

- The paradigm of cosmic inflation relies on a quantum origin of cosmic structure.
- The wavefunction of universe is a framework to realize a quantum early universe beyond perturbation theory.
- But can we go beyond mini-superspace?

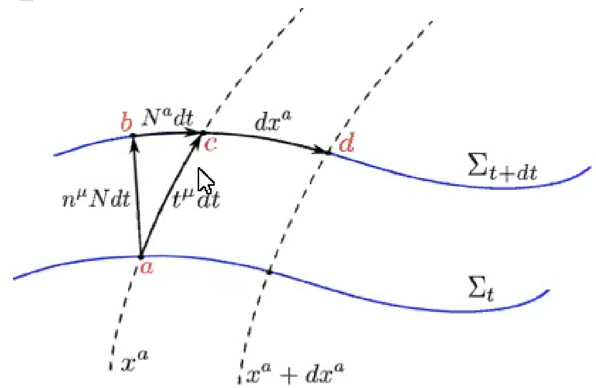
Classical Theory → Symmetry Reduction → Quantization

Classical Theory → Quantization → Symmetry Reduction

Review : Wavefunction of the Universe

$$S = \frac{m_p^2}{16\pi}\left[\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}(R-2\Lambda) + 2\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K\right] + S_m$$

$$S = \int \mathrm{d}^3x\mathrm{d}t\left[\dot{h}_{ij}\pi^{ij} + \dot{\Phi}\pi_\Phi - N\mathcal{H} - N^i\mathcal{H}_i\right]$$



$$\mathcal{H} = \frac{16\pi}{m_p^2}G_{ijkl}\pi^{ij}\pi^{kl} - \frac{m_p^2}{16\pi}\sqrt{h}\left({}^3R - 2\Lambda\right) + \mathcal{H}_m$$

$$\hat{\mathcal{H}}\Psi = \left[-\frac{16\pi}{m_p^2} G_{ijkl} \frac{\delta}{\delta h_{ij}} \frac{\delta}{\delta h_{kl}} - \frac{m_p^2}{16\pi}\sqrt{h}\left({}^3R - 2\Lambda\right) + \hat{\mathcal{H}}_m\right]\Psi = 0.$$

$$\Psi[\tilde{h}_{ij}, \tilde{\Phi}, \Sigma] = \sum_{\mathcal{M}} \int_{\mathcal{C}} \mathcal{D}g_{\mu\nu} \mathcal{D}\Phi e^{iS[g_{\mu\nu}, \Phi]}$$

$$\mathscr{H} = -\frac{G}{3\pi a} P_a{}^2 + \frac{1}{4\pi^2 a^3} P_\phi{}^2 - \frac{3\pi}{4G} a \left[1 - \frac{8\pi}{3} G a^2 V(\phi)\right]$$

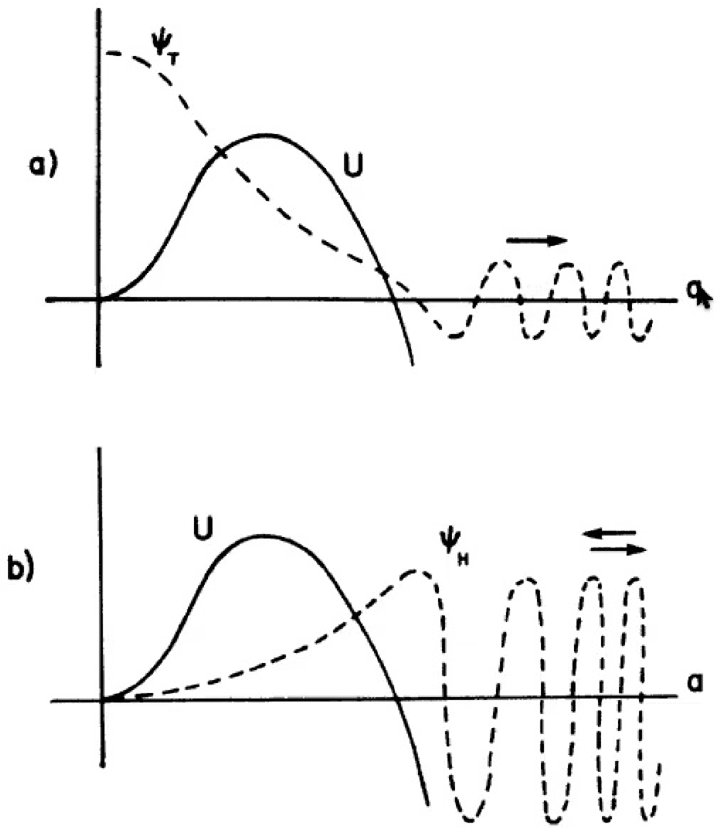
Minisuperspace WDW equation

$$\left[\frac{d^2}{da^2} - \frac{1}{a}\frac{d}{da} - U(a)\right]\psi = 0$$

$$U(a) = 4\left(\frac{3V_c}{l_P^2}\right)^2 a^2\left(k - \frac{\Lambda}{3}a^2\right)$$

$$\left[\frac{\partial^2}{\partial z^2} + z\right]\psi = 0 \qquad z = -\left(\frac{9V_c}{\Lambda l_P^2}\right)^{2/3}\left(k - \frac{\Lambda a^2}{3}\right)$$

Wavefunction and Boundary Conditions



$$\psi_{\pm}^{(1)}(a) = \exp\left[\pm i \int_{H^{-1}}^{a} p(a')da' \mp \frac{i\pi}{4}\right]$$

and the underbarrier ($a < H^{-1}$) solutions are

$$\psi_{\pm}^{(2)}(a) = \exp\left[\pm \int_{a}^{H^{-1}} |p(a')|\, da'\right],$$

where $p(a) \equiv [-U(a)]^{1/2}$

$$\hat{\mathcal{H}}\Psi = \left[-\frac{16\pi}{m_p^2}G_{ijkl}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}} - \frac{m_p^2}{16\pi}\sqrt{h}\left({}^3R - 2\Lambda\right) + \hat{\mathcal{H}}_m\right]\Psi = 0.$$

$$\Psi[\tilde{h}_{ij}, \tilde{\Phi}, \Sigma] = \sum_{\mathcal{M}} \int_{\mathcal{C}} \mathcal{D}g_{\mu\nu}\mathcal{D}\Phi e^{iS[g_{\mu\nu},\Phi]}$$

$$\mathscr{H} = -\frac{G}{3\pi a}P_a{}^2 + \frac{1}{4\pi^2 a^3}P_\phi{}^2 - \frac{3\pi}{4G}a\left[1 - \frac{8\pi}{3}Ga^2V(\phi)\right]$$

The Kodama-Chern-Simons State

$$R_I{}^J = \mathrm{d}\omega_I{}^J + \omega_I{}^K \wedge \omega_K{}^J \qquad A^i_\alpha(x) \equiv -\tfrac{1}{2}\epsilon^{ij}{}_k \omega_{\alpha j}{}^k - \mathrm{i}\omega_{\alpha 0}{}^i$$

$$\hat{\mathcal{H}}\Psi(A) = \ell^4_{\mathrm{Pl}} \epsilon_{ijl}\epsilon_{\alpha\beta\gamma} \frac{\delta}{\delta A_{\alpha i}} \frac{\delta}{\delta A_{\beta j}} \hat{\mathcal{S}}^{\gamma l}\Psi(A) = 0$$

$$\hat{\mathcal{G}}_i\Psi(A) = -\ell^2_{\mathrm{Pl}} D_\alpha \frac{\delta}{\delta A_{\alpha i}}\Psi(A) = 0,$$

$$\hat{\mathcal{V}}_\alpha\Psi(A) = -\ell^2_{\mathrm{Pl}} \frac{\delta}{\delta A^i_\beta} F^i_{\alpha\beta}\Psi(A) = 0,$$

$$\frac{\delta}{\delta A^i_a(\vec{x})} Y_{CS} = -B^a_i(\vec{x})$$

$$\hat{\mathcal{S}}^{\gamma l} \equiv \hat{B}^{\gamma l} - \frac{2\pi}{k}\frac{\delta}{\delta A_{\gamma l}}$$

$$k \equiv \frac{6\pi}{\ell^2_{\mathrm{Pl}}\Lambda},$$

$$\Psi_{\mathrm{CS}} = \mathcal{N}\exp\left(\frac{k}{4\pi}\int_{\mathcal{M}_3} Y_{\mathrm{CS}}\right)$$

$$Y_{\mathrm{CS}} \equiv \frac{1}{2}\mathrm{tr}\left(A\wedge \mathrm{d}A + \frac{2}{3}A\wedge A\wedge A\right) = \mathrm{d}^3x\,\epsilon^{\alpha\beta\gamma}\left(A^i_\alpha\partial_\beta A_{\gamma i} + \tfrac{1}{3}\epsilon_{ijk}A^i_\alpha A^j_\beta A^k_\gamma\right)$$

$$\hat{\mathcal{H}}\Psi = \left[-\frac{16\pi}{m_p^2} G_{ijkl} \frac{\delta}{\delta h_{ij}} \frac{\delta}{\delta h_{kl}} - \frac{m_p^2}{16\pi} \sqrt{h} \left({}^3R - 2\Lambda \right) + \hat{\mathcal{H}}_m \right] \Psi = 0.$$

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$$K_a^i \to \Im A_a^i$$
$$\Gamma_a^i \to \Re A_a^i$$

$$\left[\Im A_a^i(\vec{x}), E_j^b(\vec{y})\right] = i l_P^2 \delta_a^b \delta_j^i \delta(\vec{x} - \vec{y})$$

$$H = \epsilon^{ij}{}_k E_i^a E_j^b \left(\Re F_{ab}^k + \frac{\Lambda}{3} \epsilon_{abc} E^{ck} \right).$$

$$\left(\Re B^{kc} - \frac{i l_P^2 \Lambda}{3} \frac{\delta}{\delta \Im A_c^k(\vec{x})} \right) \psi = 0$$

$$\frac{\delta}{\delta \Im A_a^i(\vec{x})} \Im Y_{CS} = -\Re B_i^a(\vec{x})$$

$$\psi_{CS}(A) = \mathcal{N} \exp\left(\frac{3i}{l_P^2 \Lambda} \Im Y_{CS} \right)$$

The Kodama-Chern-Simons State

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Minisuperspace WDW equation

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$$\left[\hat{b}, \hat{a^2}\right] = \frac{i l_P^2}{3V_c}$$

$$\hat{\mathcal{H}}\psi = \left(\frac{i\Lambda l_P^2}{9V_c}\frac{d}{db} + k + b^2\right)\psi = 0.$$

$$\hat{b} = \frac{i l_P^2}{3V_c}\frac{d}{d(a^2)}$$

Its most general solution has the form:

$$\psi_{CS} = \mathcal{N}\exp\left[i\left(\frac{9V_c}{\Lambda l_P^2}\left(\frac{b^3}{3} + kb\right) + \phi_0\right)\right]$$

The Generalized Fourier Transform Reduces to:

$$\psi_{a^2}(a^2) = \frac{3V_c}{l_P^2} \int \frac{db}{\sqrt{2\pi}} e^{-i\frac{3V_c}{l_P^2}a^2 b} \psi_b(b)$$

Which is an integral rep of Airy Function

$$\phi(z) = \frac{1}{2\pi} \int e^{i\left(\frac{t^3}{3}+zt\right)} dt$$

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- We Propose a "Fourier Transformed" Kodama-Chern-State

$$\psi_E = \prod_{\vec{x},a,i} \int \frac{d[\Im(A_a^i(\vec{x}))]}{\sqrt{2\pi l_P^2}} e^{-\frac{i}{l_P^2} E_i^a(\vec{x}) \Im A_a^i(\vec{x})} \psi_A$$

$$\left[\Im A_a^i(\vec{x}), E_j^b(\vec{y})\right] = i l_P^2 \delta_a^b \delta_j^i \delta(\vec{x} - \vec{y})$$

WDW equation

Had we chosen the metric representation versus connection

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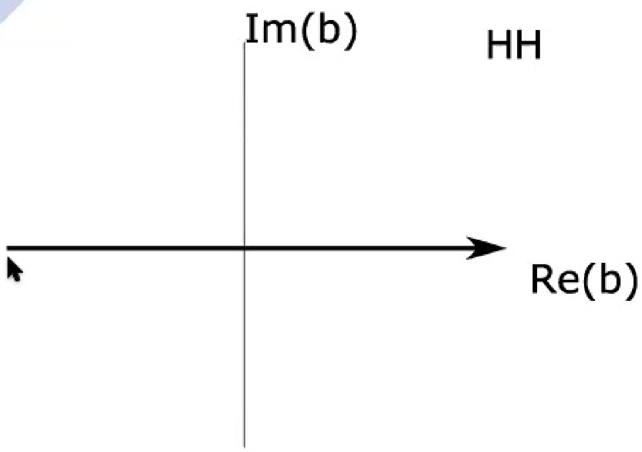
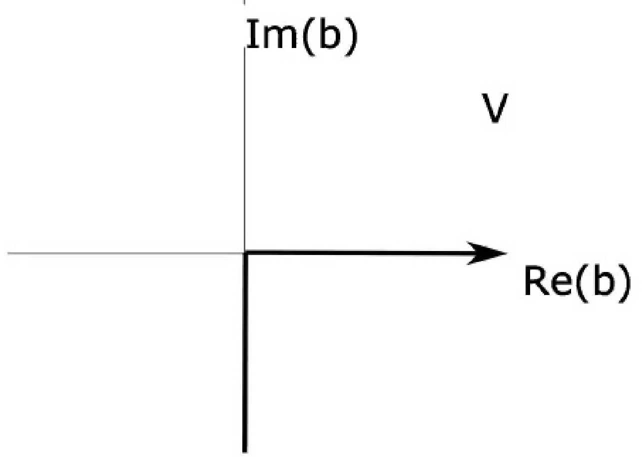


Hartle-Hawking corresponds
To real axis contour



Vilenkin State corresponds to
Negative imaginary to positive real contour

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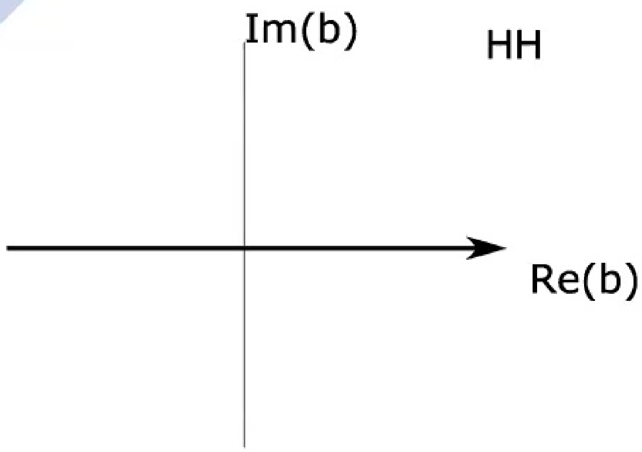

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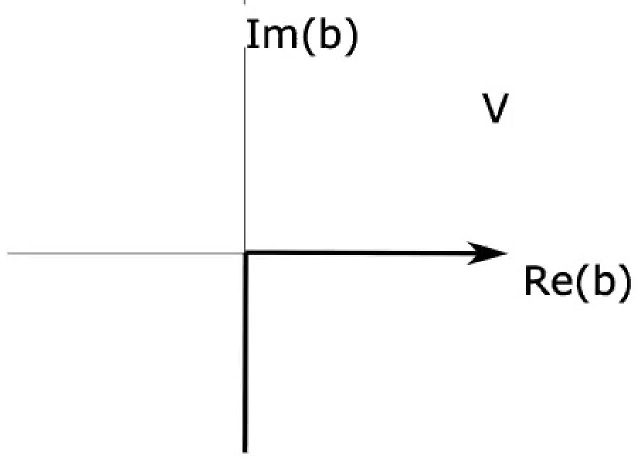

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But we can go beyond mini-superspace and consider anisotropic spacetimes

$$ds^2 = -dt^2 + a_i^2(t)dx_i^2.$$

$$E_i^a = \frac{1}{2}\delta_i^a \epsilon_{ijk}\mathrm{sgn}(a_i)a_j a_k$$
$$A_i^a = i\delta_i^a b_i$$

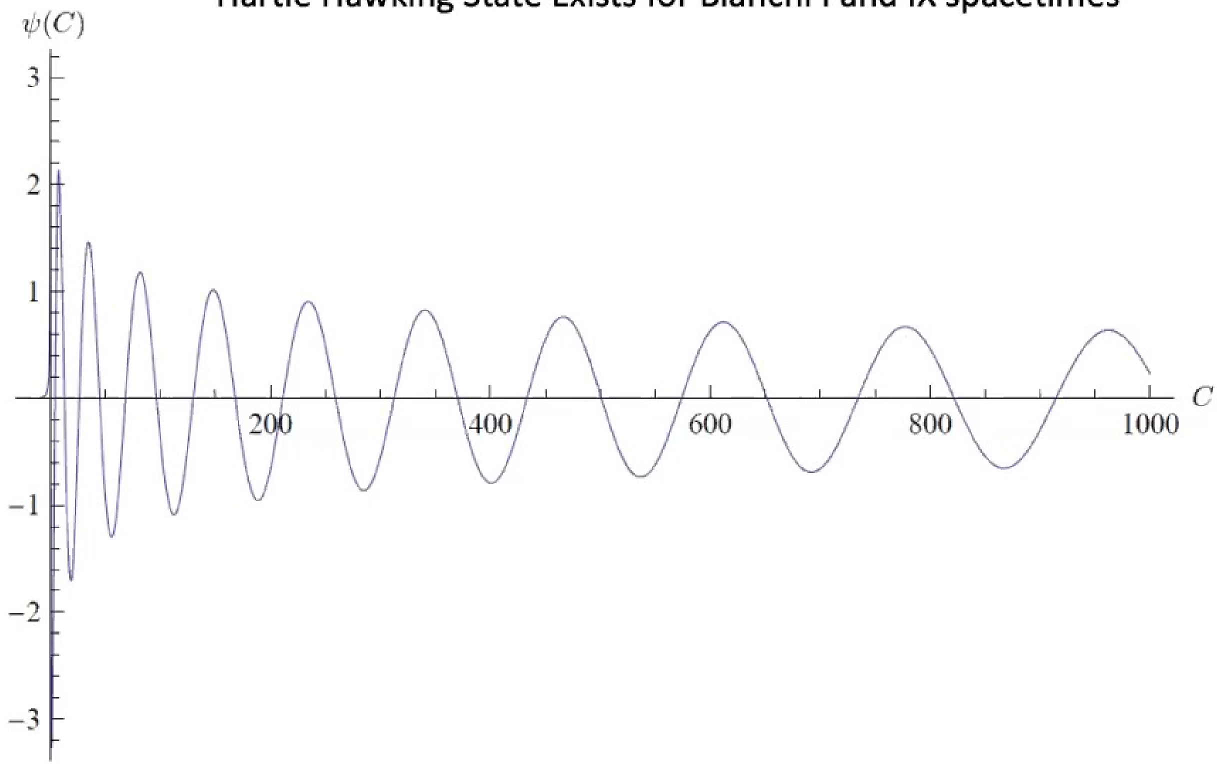
$$[b_i, p_j] = i\delta_{ij}\frac{l_P^2}{V_c},$$

$$p_1 = \mathrm{sign}(a_1)a_2 a_3$$
$$p_2 = \mathrm{sign}(a_2)a_1 a_3$$
$$p_3 = \mathrm{sign}(a_3)a_1 a_2.$$

$$\psi(a_1, a_2, a_3) = \left(\frac{V_c}{l_P^2}\right)^{3/2} \int \frac{db_1 db_2 db_3}{(2\pi)^{3/2}} \times$$
$$\times e^{-i\frac{V_c}{l_P^2}(b_1 p_1 + b_2 p_2 + b_3 p_3)} \psi_{CS}$$

$$\psi = \mathcal{N}\frac{\Lambda}{3l_P}\sqrt{\frac{V_c}{2\pi}} \int_{-\infty}^{\infty} \frac{dx}{|x|} e^{-i(x + C/x)}$$

Hartle Hawking State Exists for Bianchi I and IX spacetimes



Observation

- In mini-superspace the abelian part of CS state dominates over non-abelian
- In anisotropic case, the non-abelian parts dominate
- Does the non-abelian CS state encode a quantum version of the BKL conjecture?
- Work in progress with Laurent Friedel

$$Y_{CS} = -\int A^1 A^2 A^3$$

$$\Im(Y_{CS}) = -V_c(-b_1 b_2 b_3 + b_1 c_2 c_3 + b_2 c_1 c_3 + b_3 c_1 c_2)$$

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