Title: Primordial non-gaussianities from consistency relations: a proof of principle

Speakers: Angelo Esposito

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Abstract: In this talk I will discuss the application of the so-called â€œconsistency relationsâ€• for large scale structures to the study of primordial non-Gaussianities of the local f_NL type. I will first introduce the consistency relations themselves, commenting on some important aspects and underlying assumptions. I will then verify them (and their violation) using N-body simulations for the matter density in the Universe. This proves consistency relations to be a promising tool to apply in the forthcoming large scale structures surveys. I will conclude describing some work in progress aimed at finding a practical recipe to constrain f_NL from these relations.

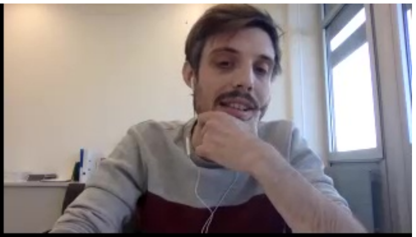
PRIMORDIAL NON-GAUSSIANITIES FROM CONSISTENCY RELATIONS: A PROOF OF PRINCIPLE

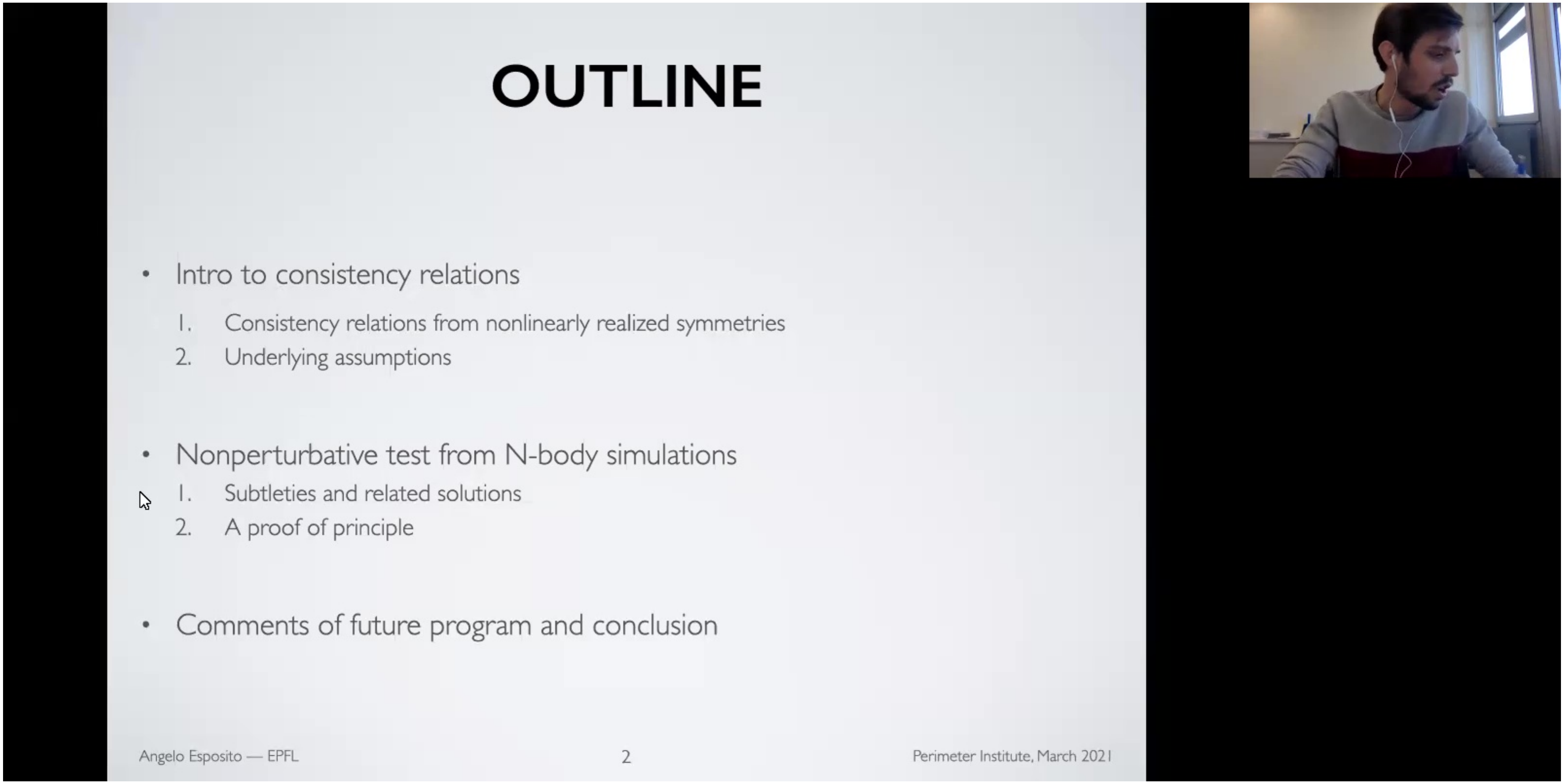
Angelo Esposito

École Polytechnique Fédérale de Lausanne (EPFL)

Perimeter Institute, March 2021

Talk mostly based on:
A.E., L. Hui, R. Scoccimarro — arXiv:1905.11423, PRD 2019
M. Abitbol, A.E., L. Hui, R. Scoccimarro — work in progress



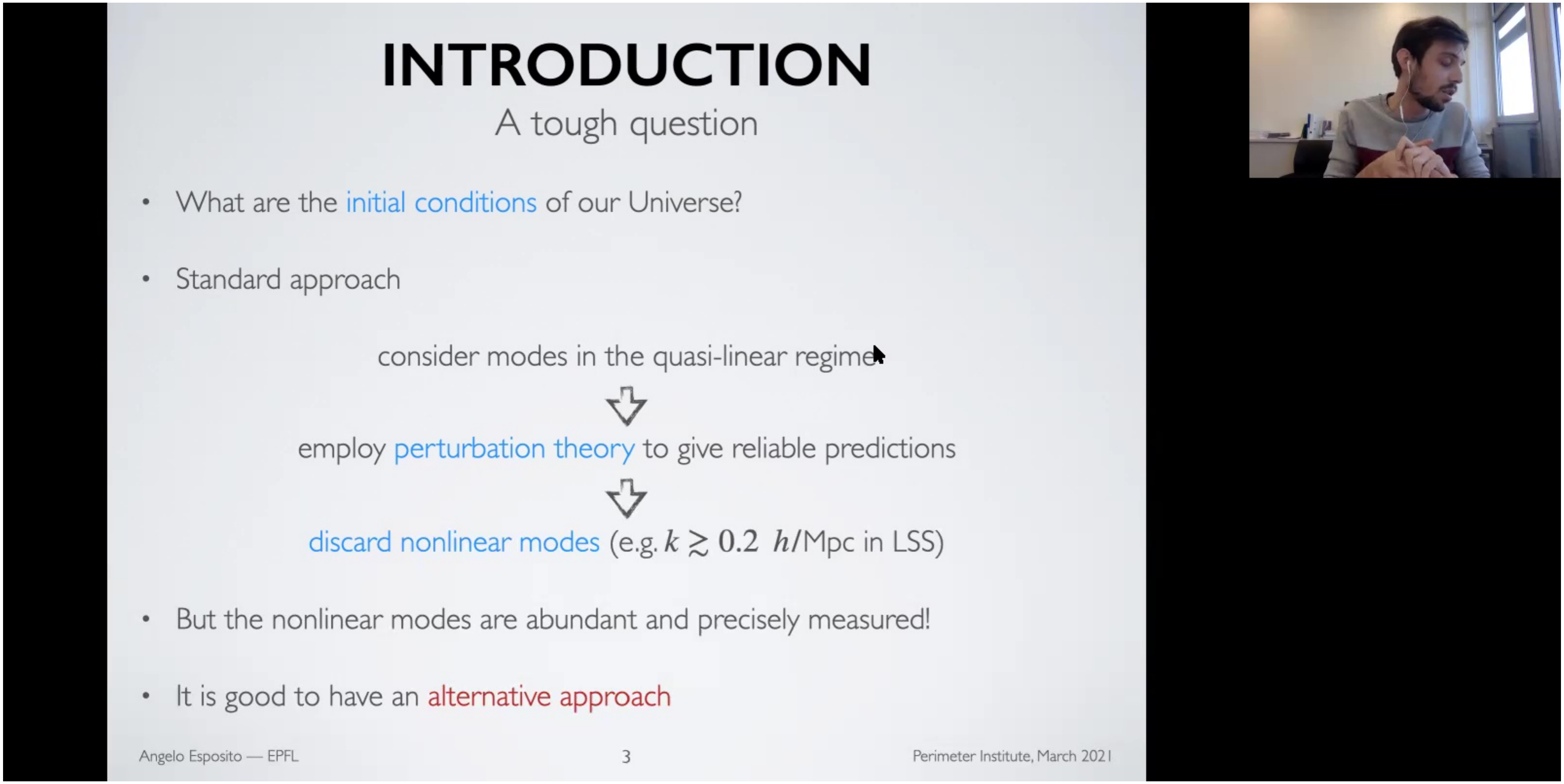


OUTLINE

- Intro to consistency relations
  1. Consistency relations from nonlinearly realized symmetries
  2. Underlying assumptions

- Nonperturbative test from N-body simulations
  1. Subtleties and related solutions
  2. A proof of principle

- Comments of future program and conclusion



INTRODUCTION

A tough question

- What are the initial conditions of our Universe?
- Standard approach

consider modes in the quasi-linear regime

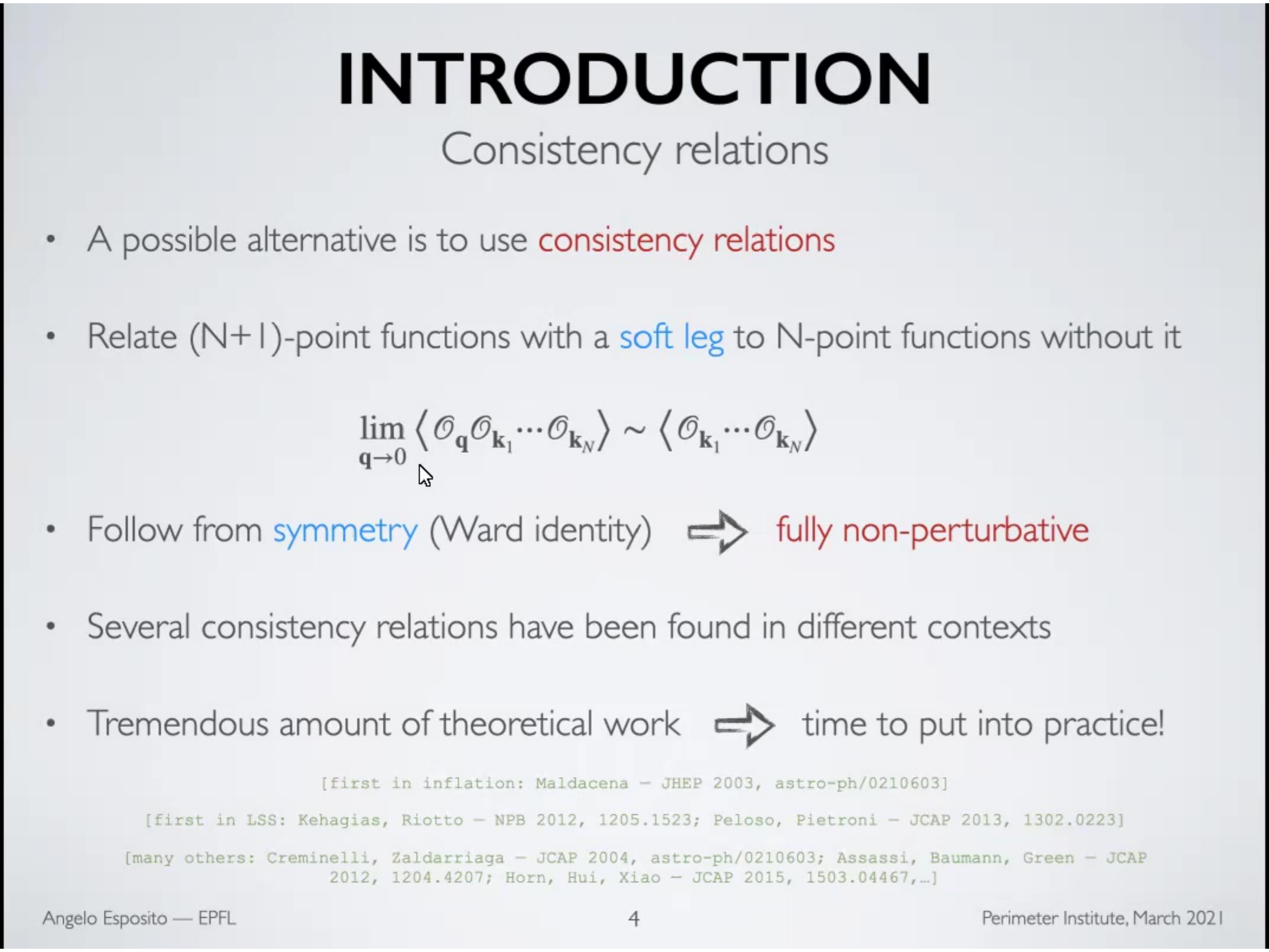
⇩

employ perturbation theory to give reliable predictions

⇩

discard nonlinear modes (e.g. $k \gtrsim 0.2\ h/\mathrm{Mpc}$ in LSS)

- But the nonlinear modes are abundant and precisely measured!
- It is good to have an alternative approach



INTRODUCTION

Consistency relations

- A possible alternative is to use consistency relations
- Relate (N+1)-point functions with a soft leg to N-point functions without it

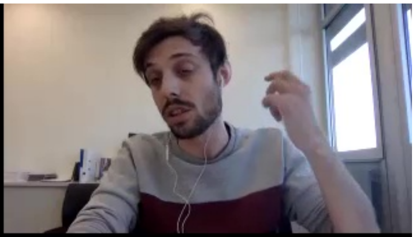
$$\lim_{\mathbf{q}\to 0}\langle \mathcal{O}_{\mathbf{q}}\mathcal{O}_{\mathbf{k}_1}\cdots\mathcal{O}_{\mathbf{k}_N}\rangle \sim \langle \mathcal{O}_{\mathbf{k}_1}\cdots\mathcal{O}_{\mathbf{k}_N}\rangle$$

- Follow from symmetry (Ward identity) ⇨ fully non-perturbative
- Several consistency relations have been found in different contexts
- Tremendous amount of theoretical work ⇨ time to put into practice!

[first in inflation: Maldacena – JHEP 2003, astro-ph/0210603]

[first in LSS: Kehagias, Riotto – NPB 2012, 1205.1523; Peloso, Pietroni – JCAP 2013, 1302.0223]

[many others: Creminelli, Zaldarriaga – JCAP 2004, astro-ph/0210603; Assassi, Baumann, Green – JCAP 2012, 1204.4207; Horn, Hui, Xiao – JCAP 2015, 1503.04467,…]



CONSISTENCY RELATIONS

Linearly realized symmetries

- We are familiar with the consequences of linearly realized symmetries

- Example: spatial translations

depends linearly on the field

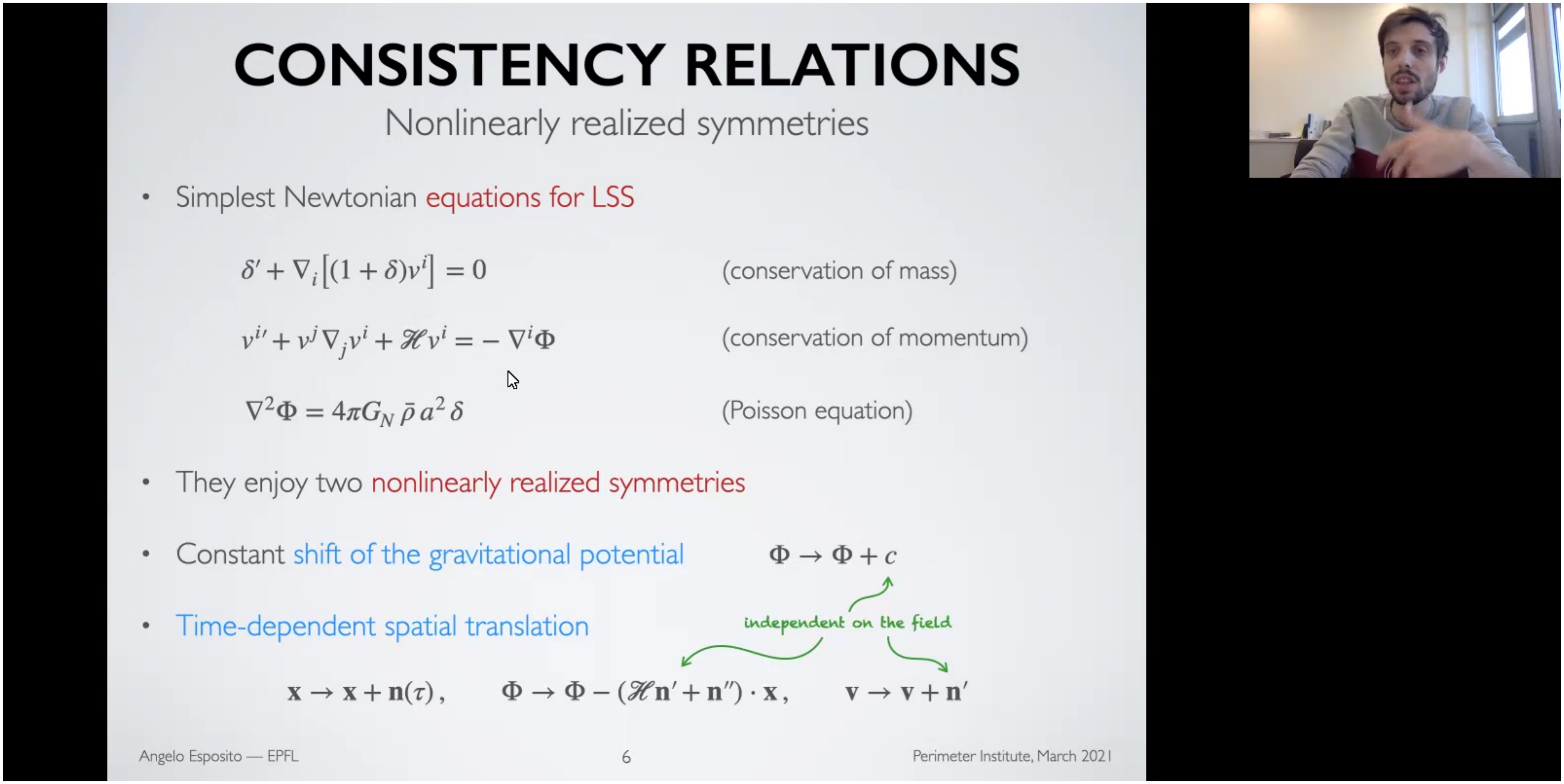
$$x^i \to x^i + \epsilon^i\,, \qquad \delta \to \delta + \Delta\delta \quad \text{with} \quad \Delta\delta = -\,\epsilon^i \nabla_i \delta$$

$$\Downarrow$$

$$\Delta\big\langle \delta(\mathbf{x}_1)\,\cdots\,\delta(\mathbf{x}_N)\big\rangle = -\,\epsilon_i \sum_{a=1}^{N} \nabla^i_a \big\langle \delta(\mathbf{x}_1)\,\cdots\,\delta(\mathbf{x}_N)\big\rangle = 0$$

- Assuming invariant initial conditions, the correlator is simply translationally invariant

- Neat, but a bit boring…



CONSISTENCY RELATIONS

Nonlinearly realized symmetries

- Simplest Newtonian equations for LSS

$$\delta' + \nabla_i\left[(1+\delta)v^i\right] = 0 \qquad \text{(conservation of mass)}$$

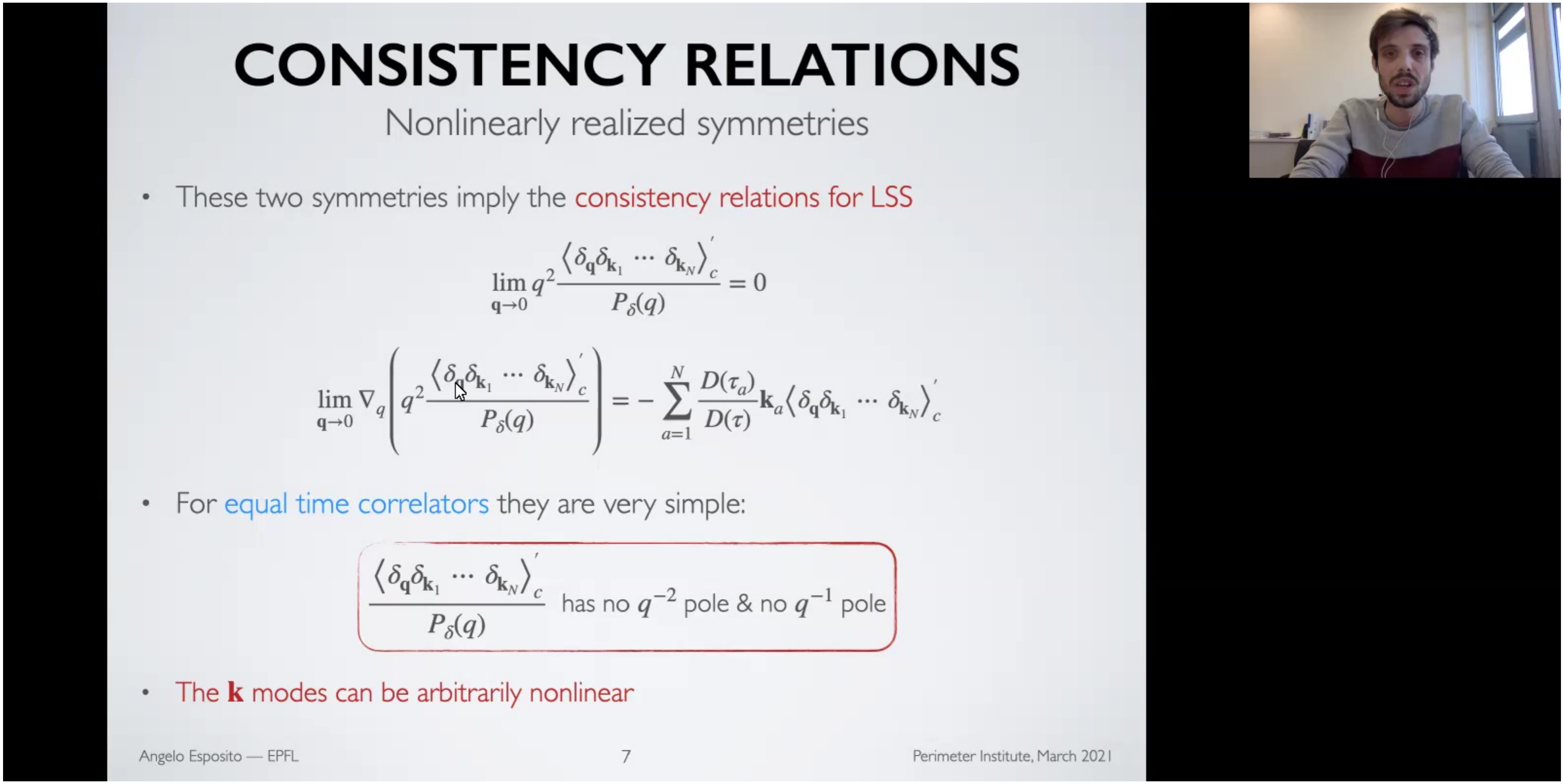
$$v^{i\prime} + v^j\nabla_j v^i + \mathscr{H} v^i = -\nabla^i\Phi \qquad \text{(conservation of momentum)}$$

$$\nabla^2\Phi = 4\pi G_N\,\bar\rho\,a^2\,\delta \qquad \text{(Poisson equation)}$$

- They enjoy two nonlinearly realized symmetries
- Constant shift of the gravitational potential $\Phi \to \Phi + c$
- Time-dependent spatial translation

independent on the field

$$\mathbf{x} \to \mathbf{x} + \mathbf{n}(\tau)\,, \qquad \Phi \to \Phi - (\mathscr{H}\mathbf{n}' + \mathbf{n}'')\cdot\mathbf{x}\,, \qquad \mathbf{v} \to \mathbf{v} + \mathbf{n}'$$



CONSISTENCY RELATIONS

Nonlinearly realized symmetries

- These two symmetries imply the consistency relations for LSS

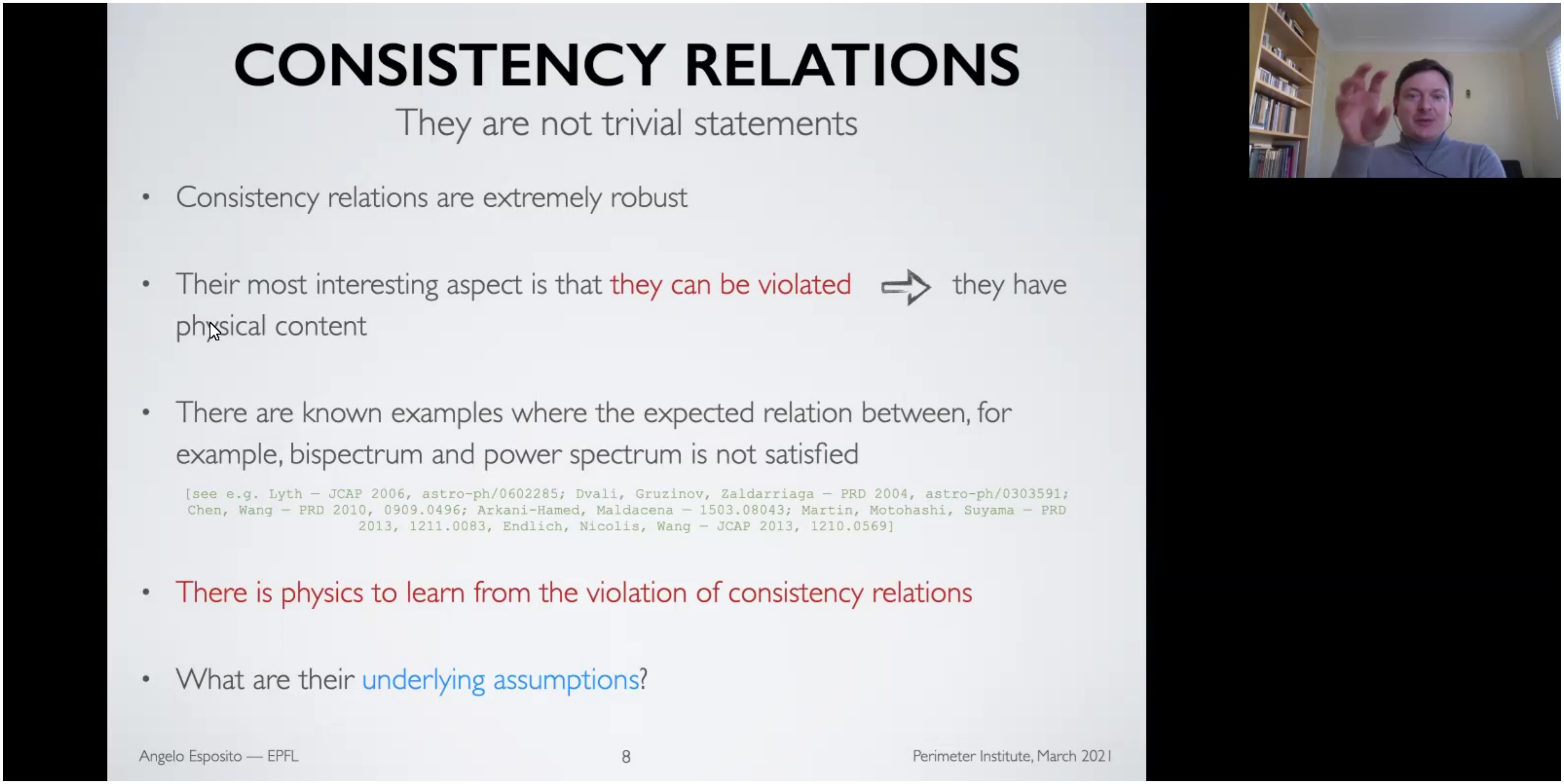
$$\lim_{\mathbf{q}\to 0} q^2 \frac{\langle \delta_{\mathbf{q}} \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N} \rangle_c'}{P_\delta(q)} = 0$$

$$\lim_{\mathbf{q}\to 0} \nabla_q \left(q^2 \frac{\langle \delta_{\mathbf{q}} \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N} \rangle_c'}{P_\delta(q)} \right) = - \sum_{a=1}^{N} \frac{D(\tau_a)}{D(\tau)} \mathbf{k}_a \langle \delta_{\mathbf{q}} \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N} \rangle_c'$$

- For equal time correlators they are very simple:

$$\boxed{\frac{\langle \delta_{\mathbf{q}} \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N} \rangle_c'}{P_\delta(q)} \text{ has no } q^{-2} \text{ pole \& no } q^{-1} \text{ pole}}$$

- The $\mathbf{k}$ modes can be arbitrarily nonlinear



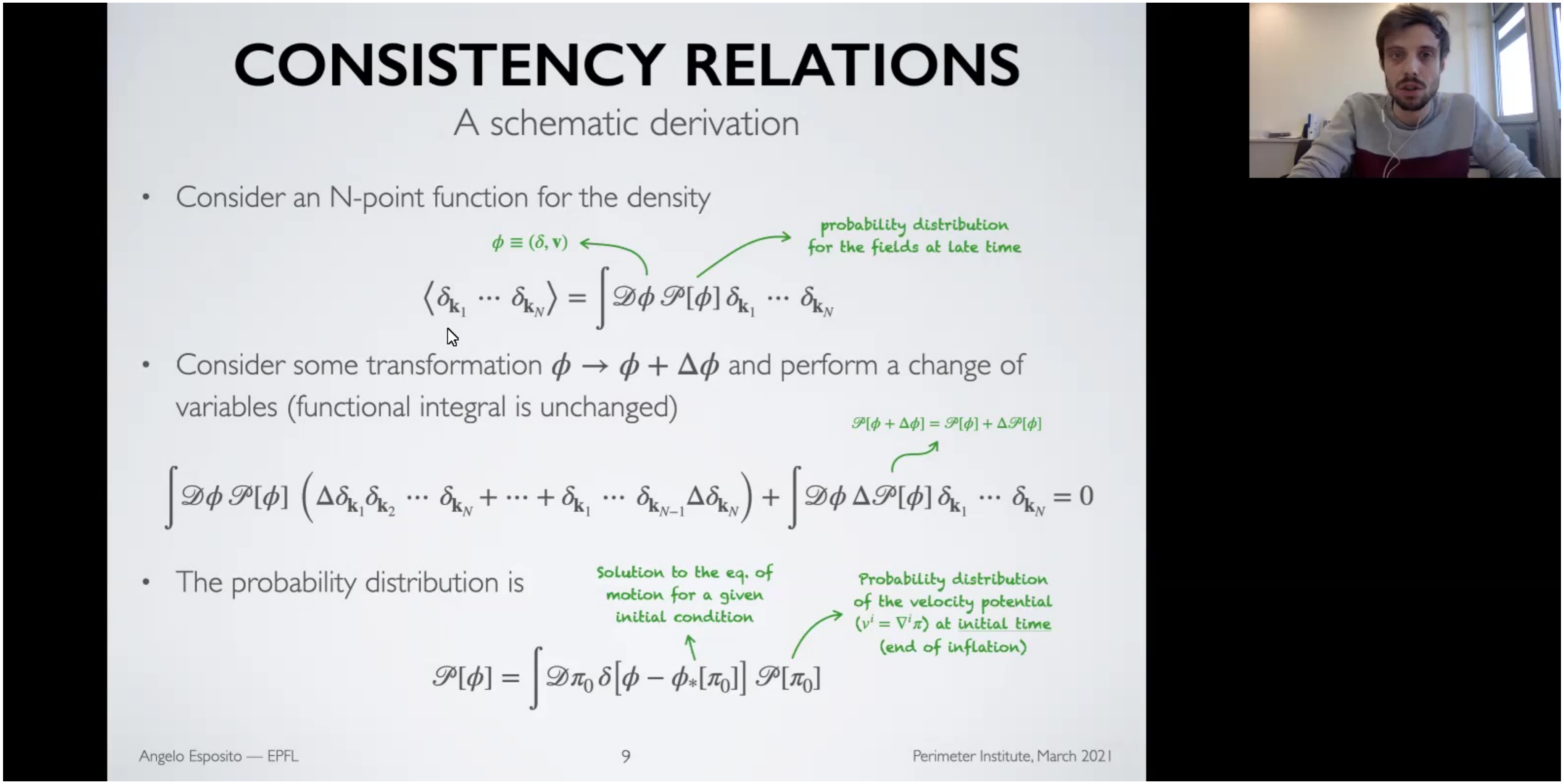
CONSISTENCY RELATIONS

They are not trivial statements

- Consistency relations are extremely robust
- Their most interesting aspect is that they can be violated ⇒ they have physical content
- There are known examples where the expected relation between, for example, bispectrum and power spectrum is not satisfied

[see e.g. Lyth – JCAP 2006, astro-ph/0602285; Dvali, Gruzinov, Zaldarriaga – PRD 2004, astro-ph/0303591; Chen, Wang – PRD 2010, 0909.0496; Arkani-Hamed, Maldacena – 1503.08043; Martin, Motohashi, Suyama – PRD 2013, 1211.0083, Endlich, Nicolis, Wang – JCAP 2013, 1210.0569]

- There is physics to learn from the violation of consistency relations
- What are their underlying assumptions?



CONSISTENCY RELATIONS

A schematic derivation

- Consider an N-point function for the density

$\phi \equiv (\delta, \mathbf{v})$

probability distribution for the fields at late time

$$\langle \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N} \rangle = \int \mathscr{D}\phi \, \mathscr{P}[\phi] \, \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N}$$

- Consider some transformation $\phi \to \phi + \Delta\phi$ and perform a change of variables (functional integral is unchanged)

$\mathscr{P}[\phi + \Delta\phi] = \mathscr{P}[\phi] + \Delta\mathscr{P}[\phi]$

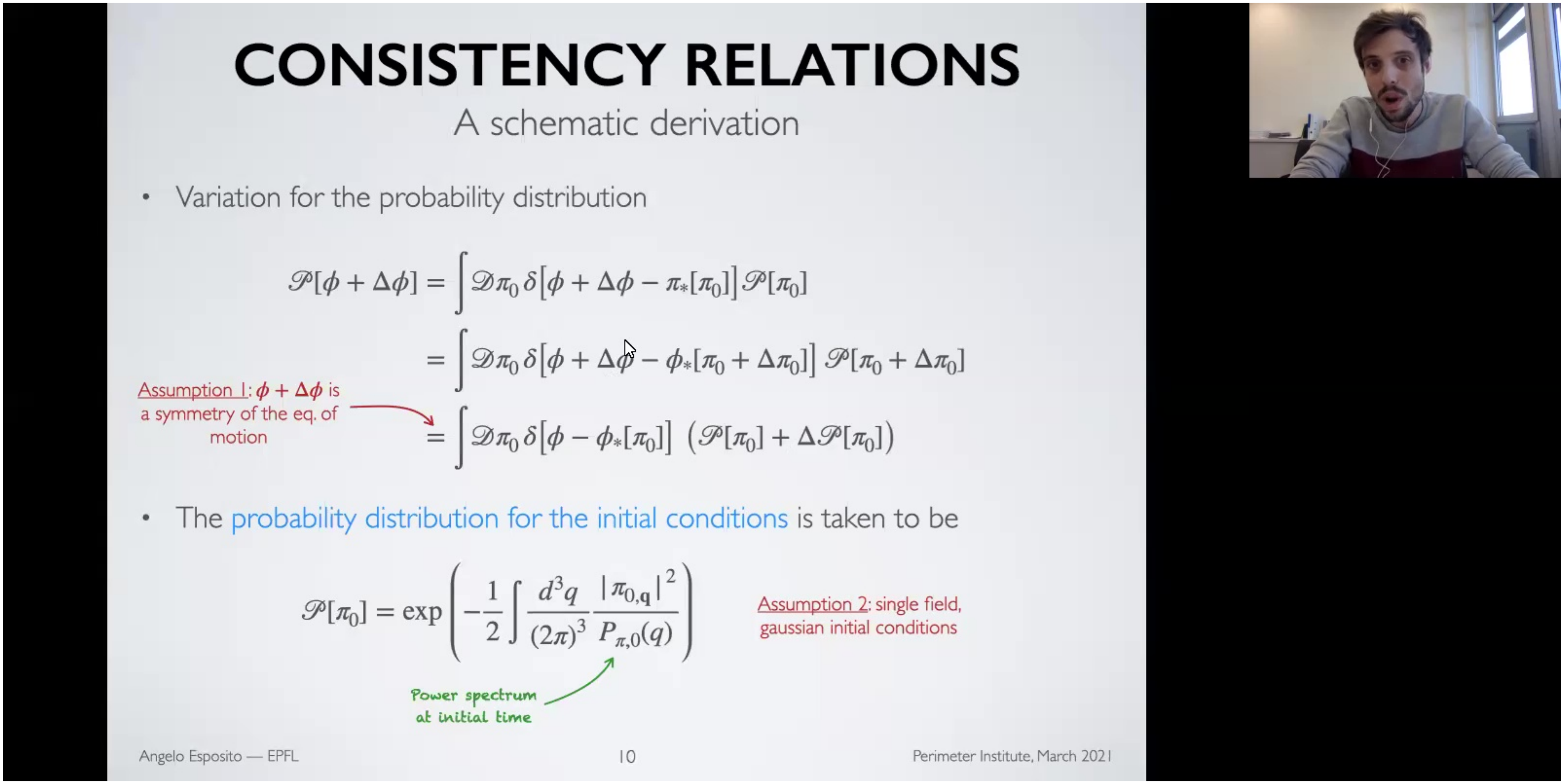
$$\int \mathscr{D}\phi \, \mathscr{P}[\phi] \left(\Delta\delta_{\mathbf{k}_1}\delta_{\mathbf{k}_2} \cdots \delta_{\mathbf{k}_N} + \cdots + \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_{N-1}} \Delta\delta_{\mathbf{k}_N} \right) + \int \mathscr{D}\phi \, \Delta\mathscr{P}[\phi] \, \delta_{\mathbf{k}_1} \cdots \delta_{\mathbf{k}_N} = 0$$

- The probability distribution is

Solution to the eq. of motion for a given initial condition

Probability distribution of the velocity potential ($v^i = \nabla^i \pi$) at initial time (end of inflation)

$$\mathscr{P}[\phi] = \int \mathscr{D}\pi_0 \, \delta\left[\phi - \phi_*[\pi_0]\right] \mathscr{P}[\pi_0]$$



CONSISTENCY RELATIONS

A schematic derivation

- Variation for the probability distribution

$$\mathscr{P}[\phi + \Delta\phi] = \int \mathscr{D}\pi_0\, \delta\big[\phi + \Delta\phi - \pi_*[\pi_0]\big]\, \mathscr{P}[\pi_0]$$

$$= \int \mathscr{D}\pi_0\, \delta\big[\phi + \Delta\phi - \phi_*[\pi_0 + \Delta\pi_0]\big]\, \mathscr{P}[\pi_0 + \Delta\pi_0]$$

Assumption 1: $\phi + \Delta\phi$ is a symmetry of the eq. of motion

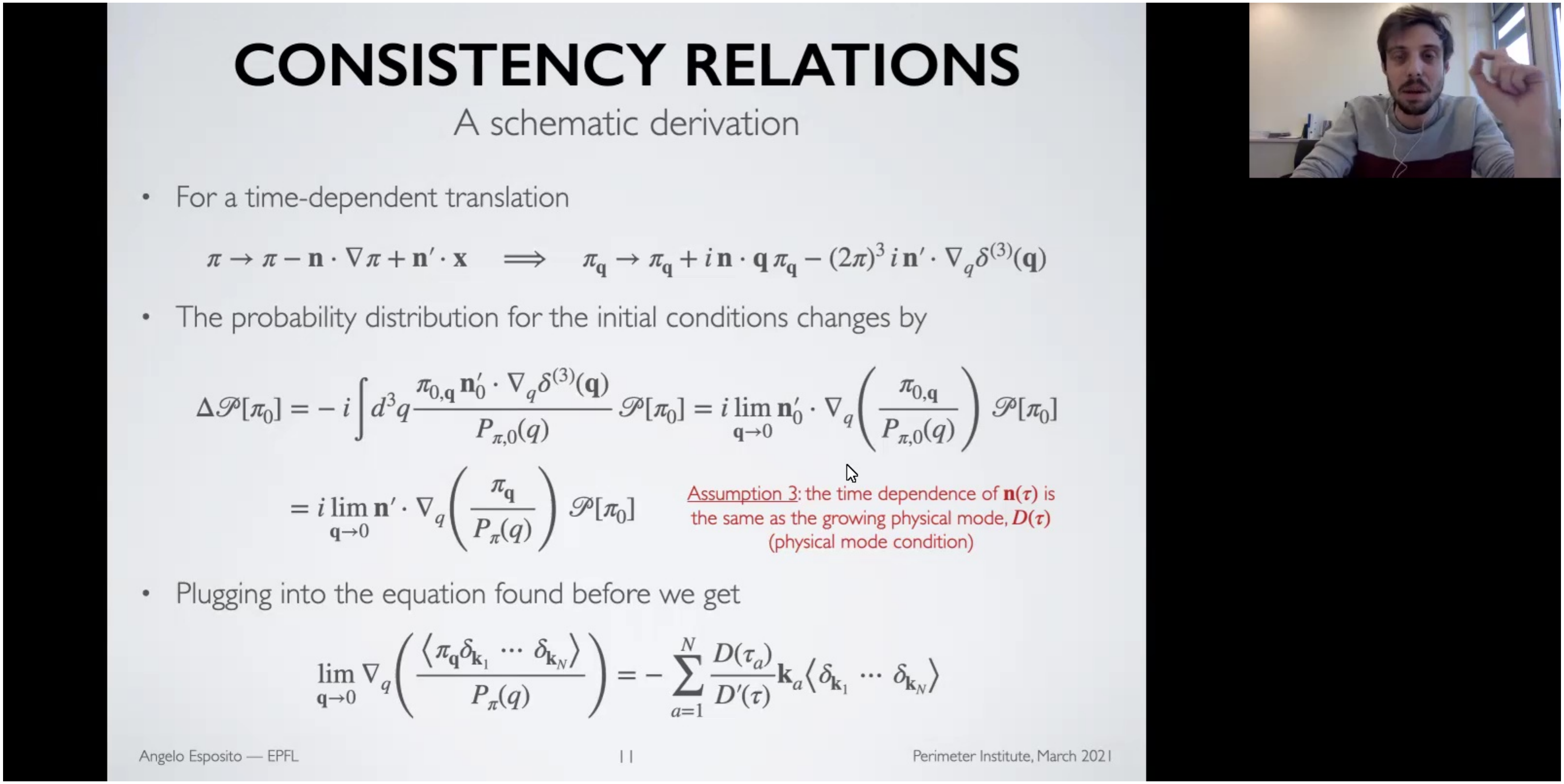
$$= \int \mathscr{D}\pi_0\, \delta\big[\phi - \phi_*[\pi_0]\big]\, \big(\mathscr{P}[\pi_0] + \Delta\mathscr{P}[\pi_0]\big)$$

- The probability distribution for the initial conditions is taken to be

$$\mathscr{P}[\pi_0] = \exp\left(-\frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{|\pi_{0,\mathbf{q}}|^2}{P_{\pi,0}(q)}\right)$$

Assumption 2: single field, gaussian initial conditions

Power spectrum at initial time



CONSISTENCY RELATIONS

A schematic derivation

- For a time-dependent translation

$$\pi \to \pi - \mathbf{n}\cdot\nabla\pi + \mathbf{n}'\cdot\mathbf{x} \quad \Longrightarrow \quad \pi_{\mathbf{q}} \to \pi_{\mathbf{q}} + i\,\mathbf{n}\cdot\mathbf{q}\,\pi_{\mathbf{q}} - (2\pi)^3\, i\,\mathbf{n}'\cdot\nabla_q\delta^{(3)}(\mathbf{q})$$

- The probability distribution for the initial conditions changes by

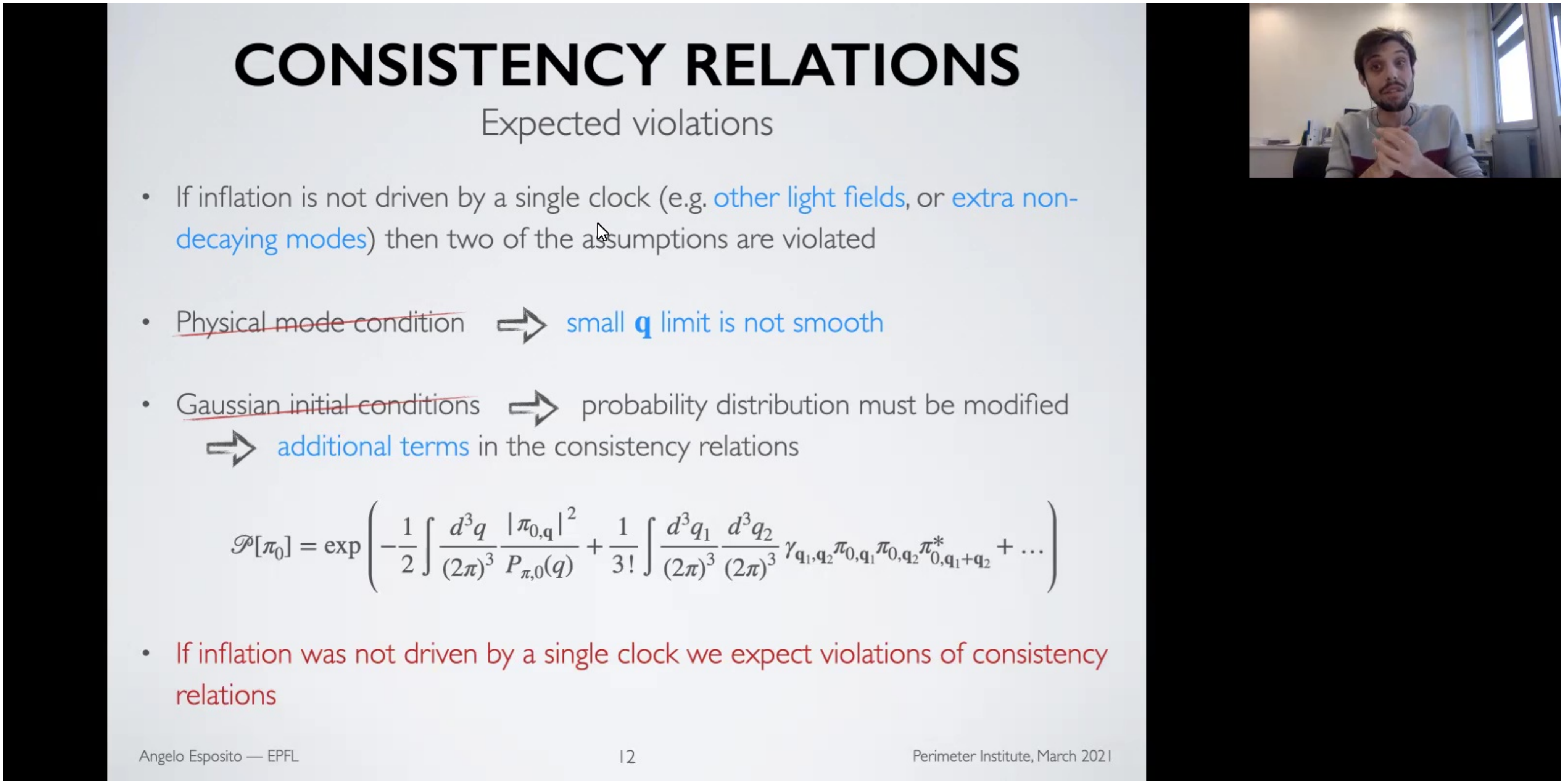
$$\Delta\mathscr{P}[\pi_0] = -\,i\int d^3q\,\frac{\pi_{0,\mathbf{q}}\,\mathbf{n}_0'\cdot\nabla_q\delta^{(3)}(\mathbf{q})}{P_{\pi,0}(q)}\,\mathscr{P}[\pi_0] = i\lim_{\mathbf{q}\to 0}\mathbf{n}_0'\cdot\nabla_q\left(\frac{\pi_{0,\mathbf{q}}}{P_{\pi,0}(q)}\right)\mathscr{P}[\pi_0]$$

$$= i\lim_{\mathbf{q}\to 0}\mathbf{n}'\cdot\nabla_q\left(\frac{\pi_{\mathbf{q}}}{P_\pi(q)}\right)\mathscr{P}[\pi_0]$$

Assumption 3: the time dependence of $\mathbf{n}(\tau)$ is the same as the growing physical mode, $D(\tau)$ (physical mode condition)

- Plugging into the equation found before we get

$$\lim_{\mathbf{q}\to 0}\nabla_q\left(\frac{\langle\pi_{\mathbf{q}}\delta_{\mathbf{k}_1}\cdots\delta_{\mathbf{k}_N}\rangle}{P_\pi(q)}\right) = -\sum_{a=1}^{N}\frac{D(\tau_a)}{D'(\tau)}\mathbf{k}_a\langle\delta_{\mathbf{k}_1}\cdots\delta_{\mathbf{k}_N}\rangle$$



CONSISTENCY RELATIONS

Expected violations

- If inflation is not driven by a single clock (e.g. other light fields, or extra non-decaying modes) then two of the assumptions are violated

- ~~Physical mode condition~~ ⇨ small $\mathbf{q}$ limit is not smooth

- ~~Gaussian initial conditions~~ ⇨ probability distribution must be modified
  ⇨ additional terms in the consistency relations

$$\mathscr{P}[\pi_0] = \exp\left(-\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}\frac{|\pi_{0,\mathbf{q}}|^2}{P_{\pi,0}(q)} + \frac{1}{3!}\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\gamma_{\mathbf{q}_1,\mathbf{q}_2}\pi_{0,\mathbf{q}_1}\pi_{0,\mathbf{q}_2}\pi^*_{0,\mathbf{q}_1+\mathbf{q}_2} + \dots\right)$$

- If inflation was not driven by a single clock we expect violations of consistency relations

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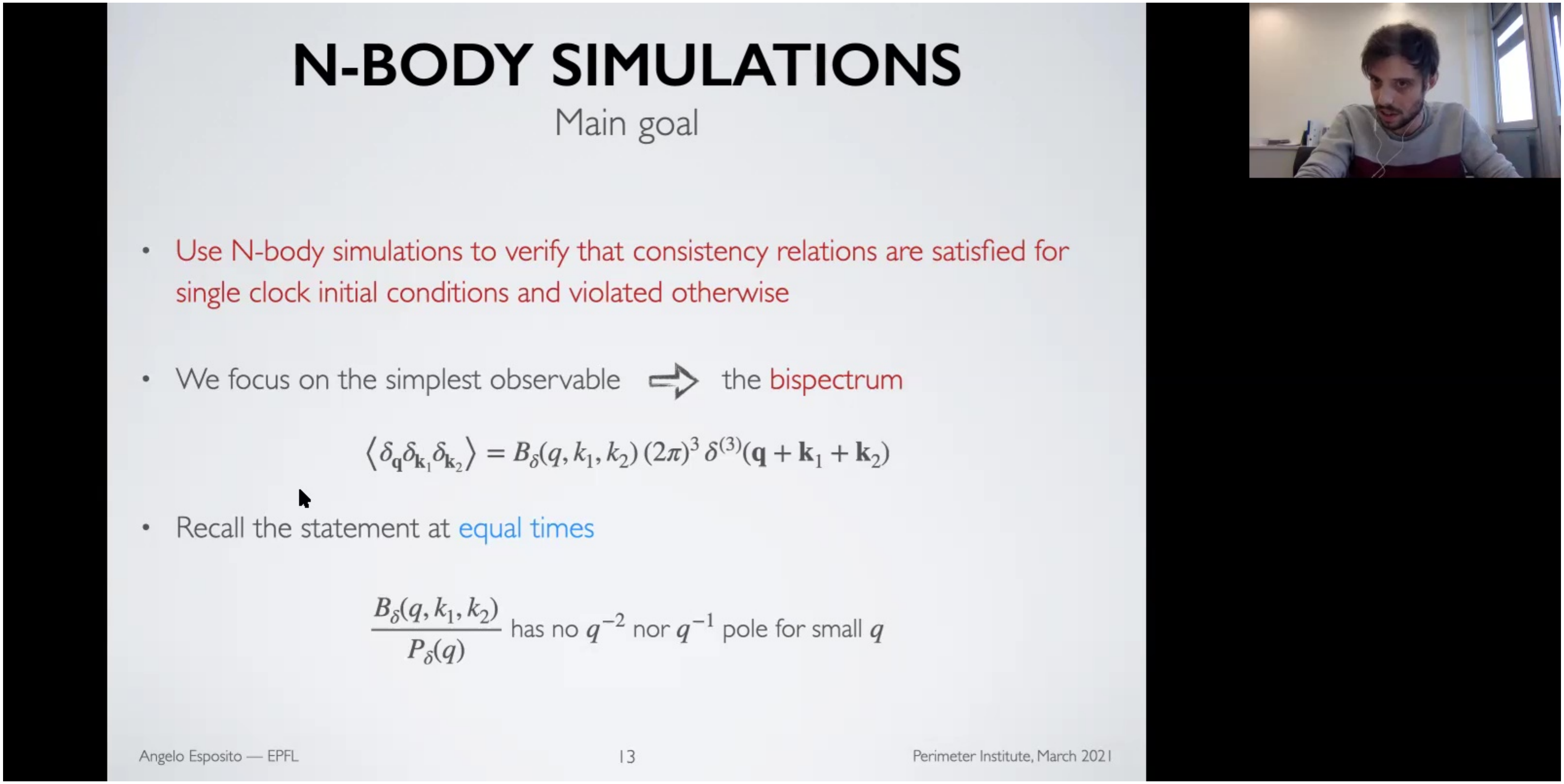


N-BODY SIMULATIONS

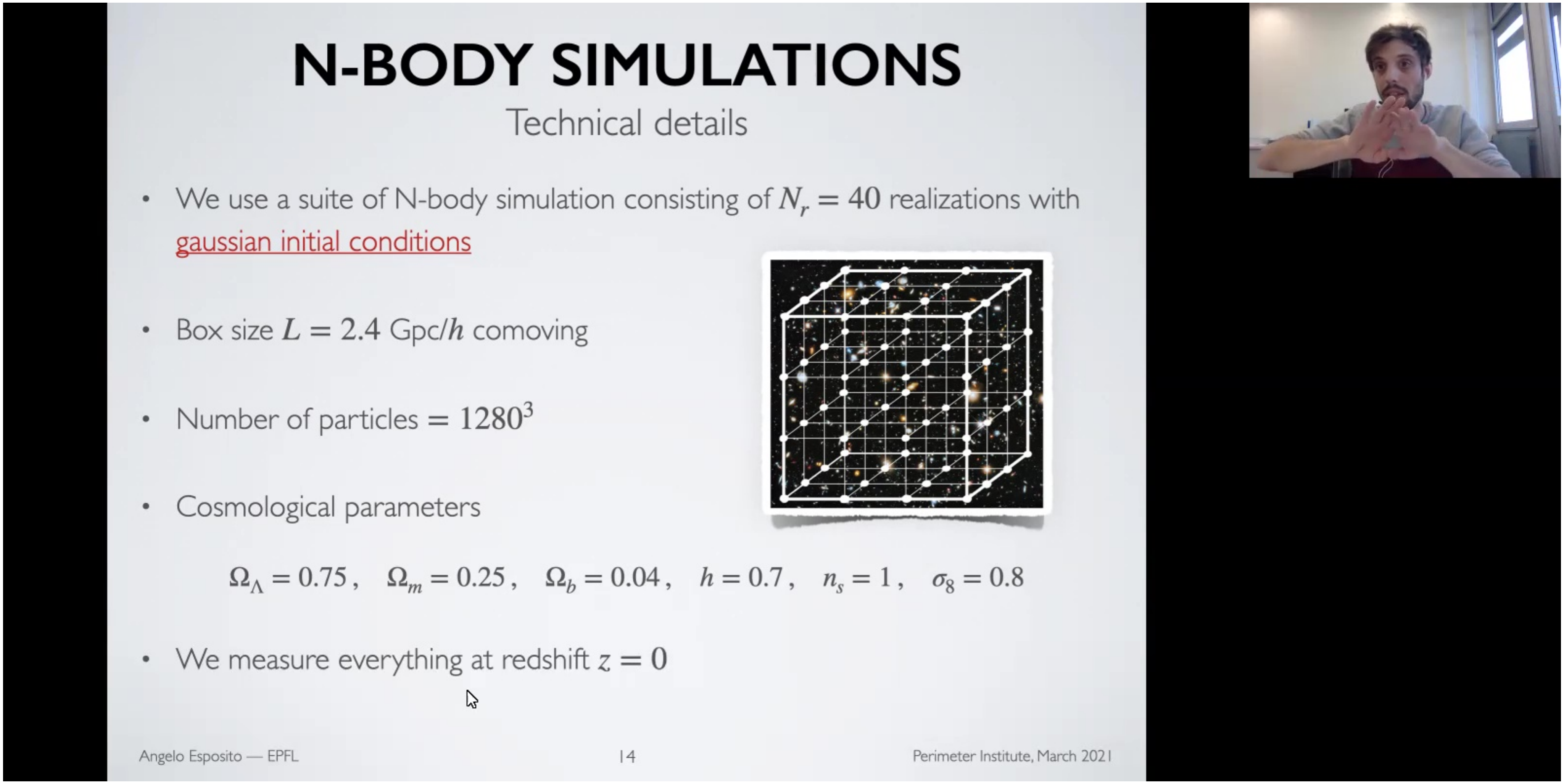
Main goal

- Use N-body simulations to verify that consistency relations are satisfied for single clock initial conditions and violated otherwise
- We focus on the simplest observable ⇨ the bispectrum

$$\left\langle \delta_{\mathbf{q}}\delta_{\mathbf{k}_1}\delta_{\mathbf{k}_2}\right\rangle = B_\delta(q,k_1,k_2)\,(2\pi)^3\,\delta^{(3)}(\mathbf{q}+\mathbf{k}_1+\mathbf{k}_2)$$

- Recall the statement at equal times

$$\frac{B_\delta(q,k_1,k_2)}{P_\delta(q)} \text{ has no } q^{-2} \text{ nor } q^{-1} \text{ pole for small } q$$



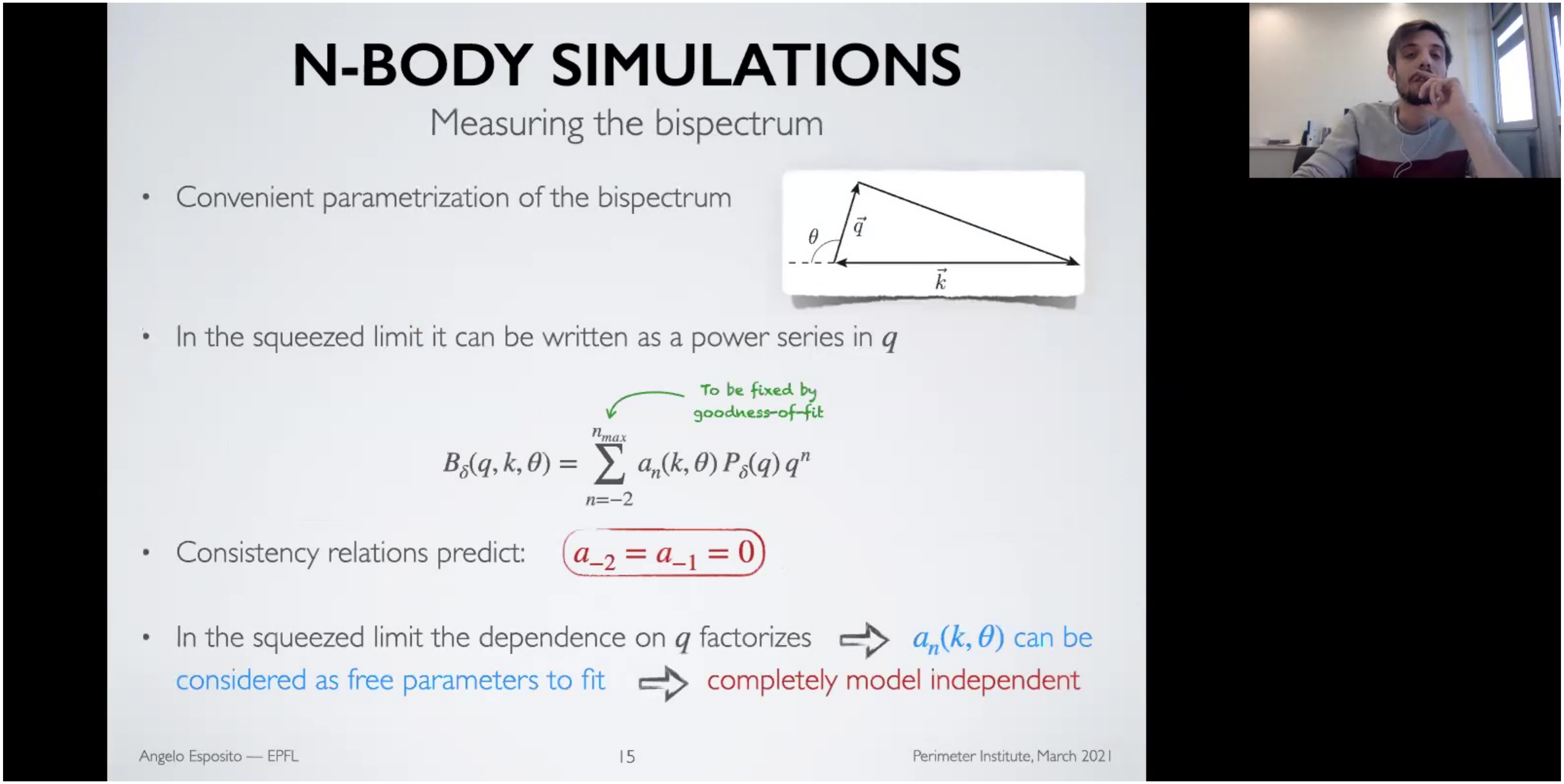
N-BODY SIMULATIONS

Technical details

- We use a suite of N-body simulation consisting of $N_r = 40$ realizations with gaussian initial conditions
- Box size $L = 2.4$ Gpc/h comoving
- Number of particles $= 1280^3$
- Cosmological parameters

$$\Omega_\Lambda = 0.75\,, \quad \Omega_m = 0.25\,, \quad \Omega_b = 0.04\,, \quad h = 0.7\,, \quad n_s = 1\,, \quad \sigma_8 = 0.8$$

- We measure everything at redshift $z = 0$



N-BODY SIMULATIONS

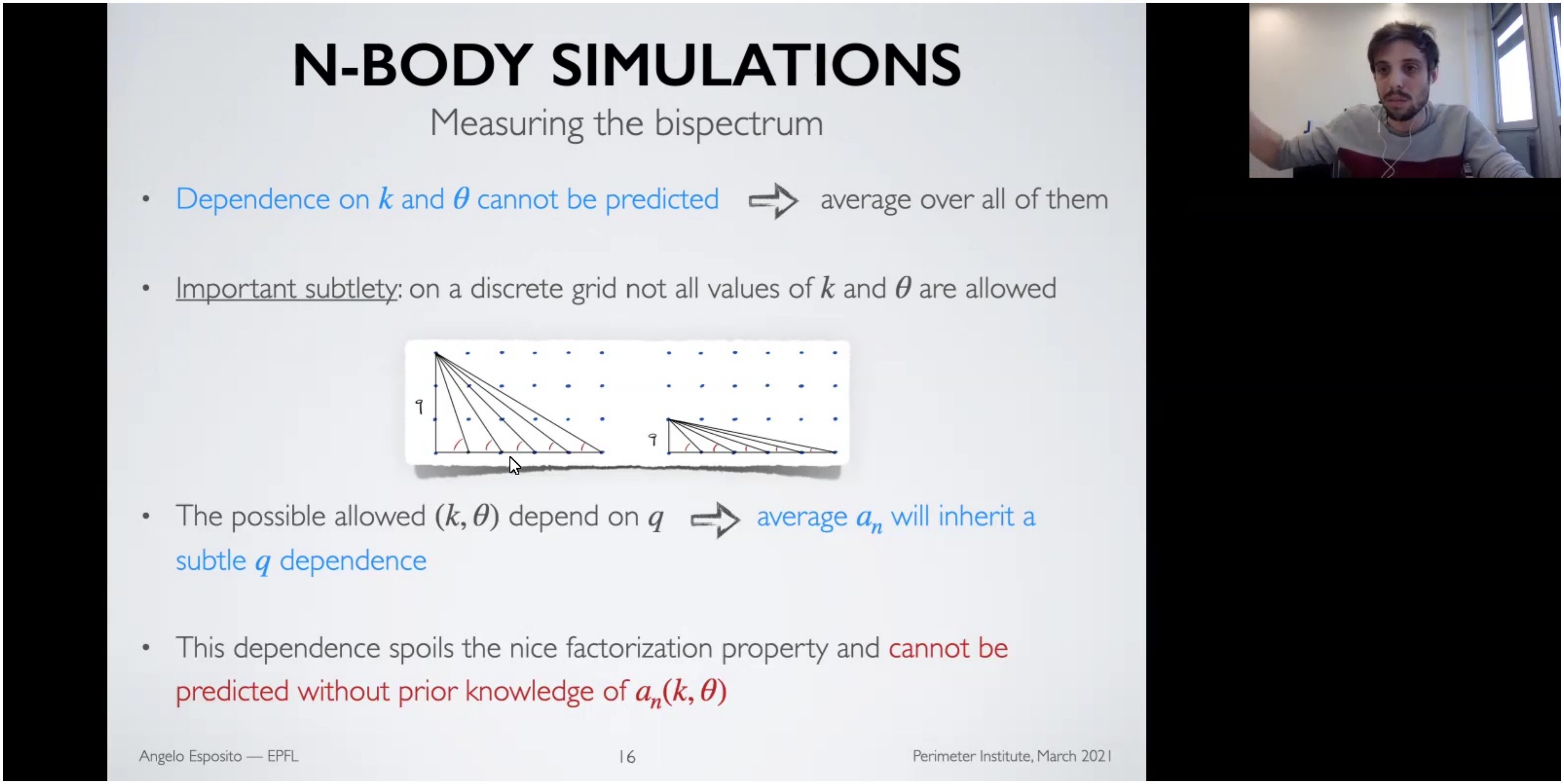
Measuring the bispectrum

- Convenient parametrization of the bispectrum
- In the squeezed limit it can be written as a power series in q

To be fixed by goodness-of-fit

$$B_\delta(q,k,\theta) = \sum_{n=-2}^{n_{max}} a_n(k,\theta)\, P_\delta(q)\, q^n$$

- Consistency relations predict: $\boxed{a_{-2} = a_{-1} = 0}$
- In the squeezed limit the dependence on q factorizes ⇨ $a_n(k,\theta)$ can be considered as free parameters to fit ⇨ completely model independent



N-BODY SIMULATIONS

Measuring the bispectrum

- Dependence on k and θ cannot be predicted ⇨ average over all of them
- Important subtlety: on a discrete grid not all values of k and θ are allowed
- The possible allowed (k, θ) depend on q ⇨ average a_n will inherit a subtle q dependence
- This dependence spoils the nice factorization property and cannot be predicted without prior knowledge of $a_n(k, \theta)$

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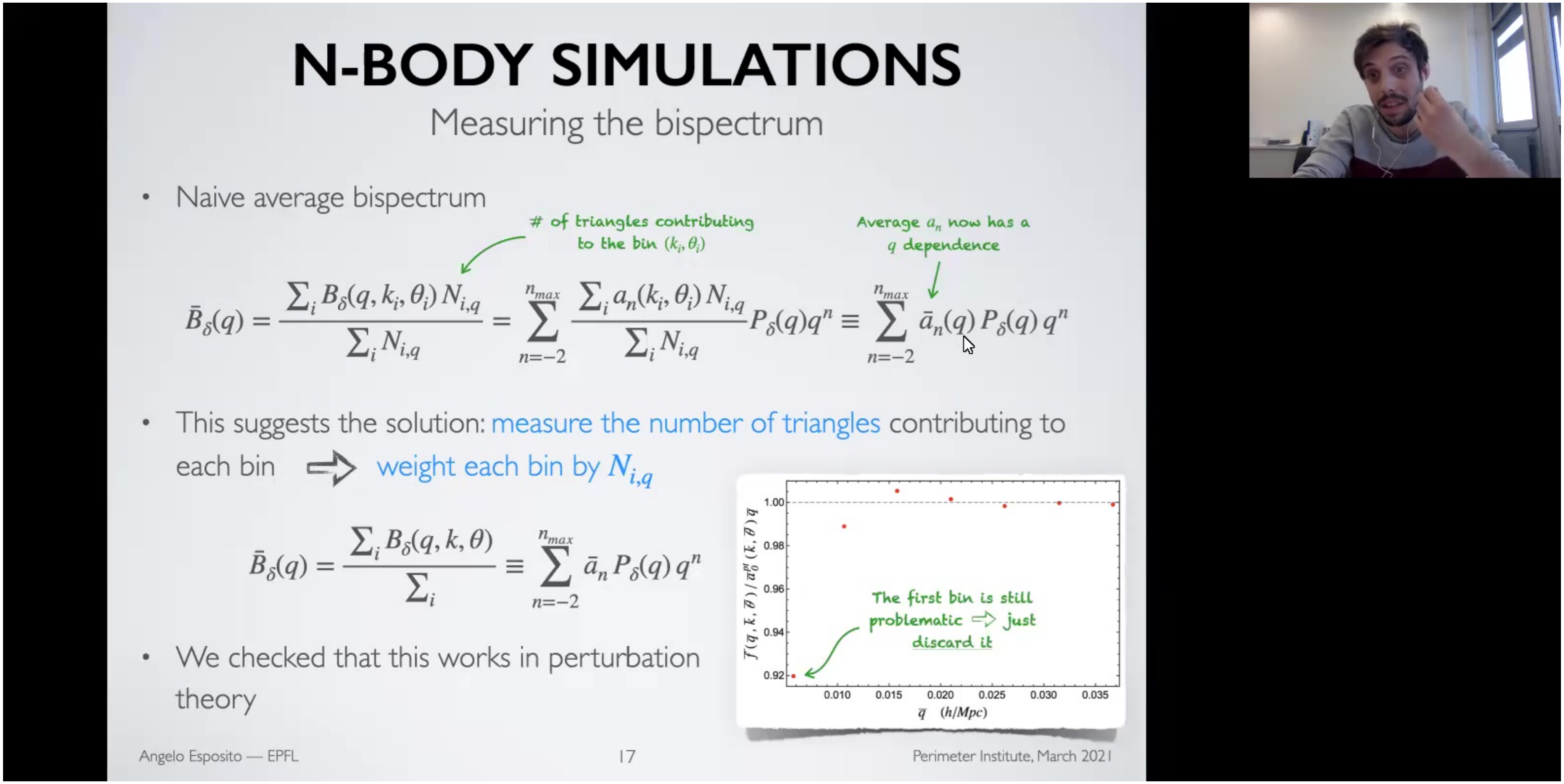


N-BODY SIMULATIONS

Measuring the bispectrum

- Naive average bispectrum

\# of triangles contributing to the bin (k_i, θ_i)

Average a_n now has a q dependence

$$\bar{B}_\delta(q) = \frac{\sum_i B_\delta(q, k_i, \theta_i)\, N_{i,q}}{\sum_i N_{i,q}} = \sum_{n=-2}^{n_{max}} \frac{\sum_i a_n(k_i, \theta_i)\, N_{i,q}}{\sum_i N_{i,q}} P_\delta(q) q^n \equiv \sum_{n=-2}^{n_{max}} \bar{a}_n(q)\, P_\delta(q)\, q^n$$

- This suggests the solution: measure the number of triangles contributing to each bin ⇨ weight each bin by $N_{i,q}$

$$\bar{B}_\delta(q) = \frac{\sum_i B_\delta(q, k, \theta)}{\sum_i} \equiv \sum_{n=-2}^{n_{max}} \bar{a}_n\, P_\delta(q)\, q^n$$

- We checked that this works in perturbation theory

The first bin is still problematic ⇨ just discard it

N-BODY SIMULATIONS

Measuring the bispectrum

- For simplicity we also bin in soft momentum

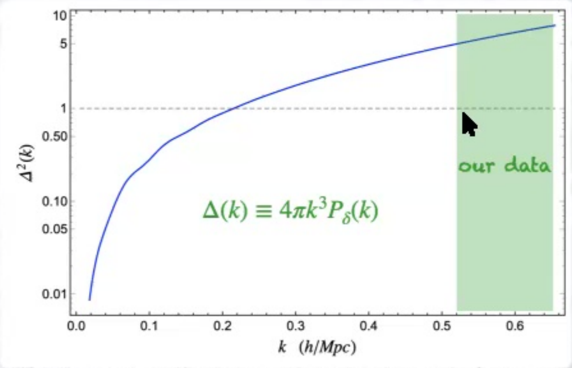
$$\bar{B}_\delta(\bar{q}) = \sum_{q \in \bar{q} \pm \frac{\Delta k}{2}} B_\delta(q) = \sum_{n=-2}^{n_{max}} \bar{a}_n \sum_{q \in \bar{q} \pm \frac{\Delta k}{2}} P_\delta(q)\, q^n \equiv \sum_{n=-2}^{n_{max}} \bar{a}_n \bar{M}_n(\bar{q})$$

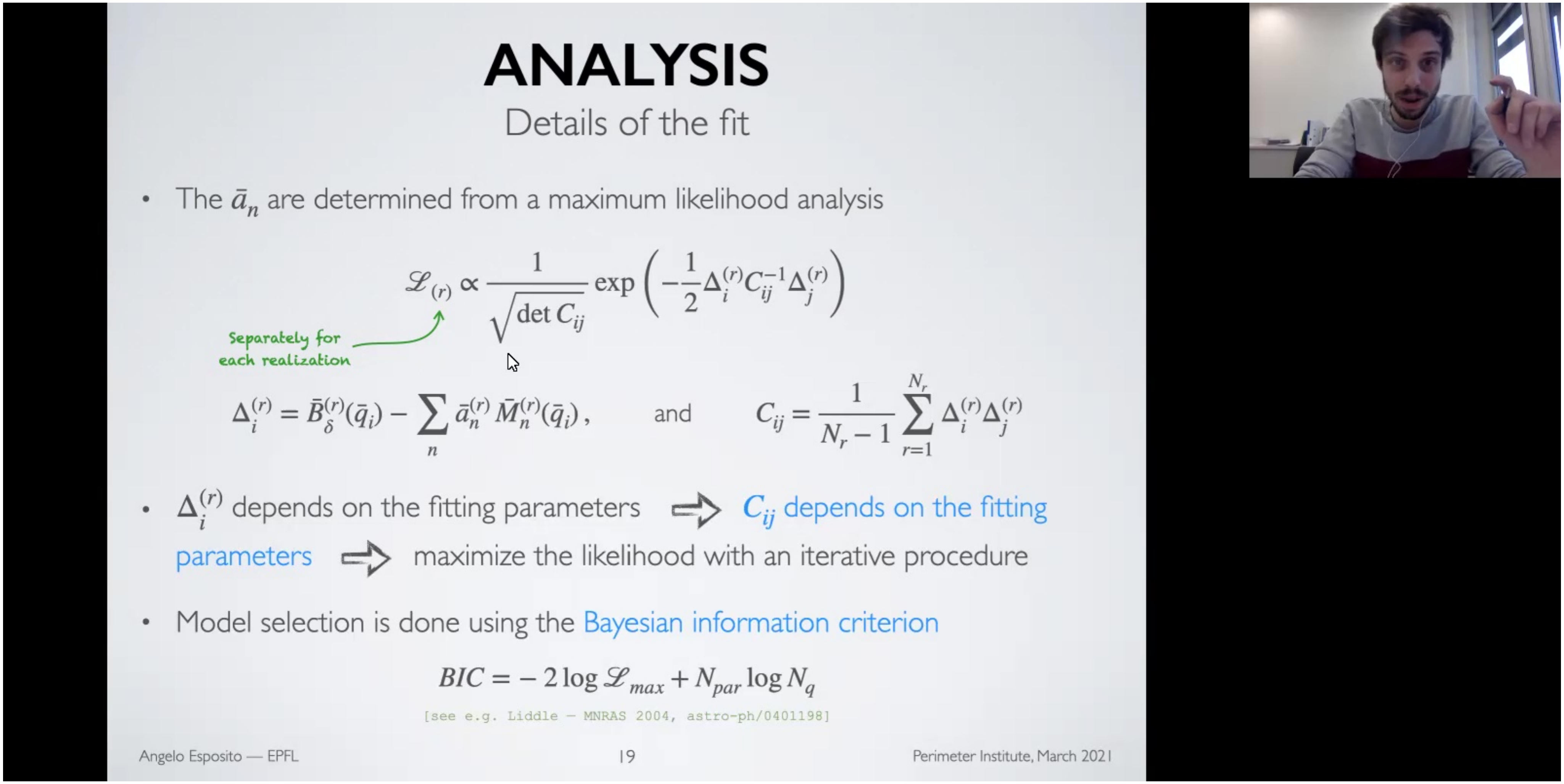
- To make our analysis completely model independent we also measure $\bar{M}_n(\bar{q})$ directly from data

- The range of hard modes we consider is

$$k \in [0.52, 0.65]\ \ h/\mathrm{Mpc}$$

- Deep into the nonlinear regime





ANALYSIS

Details of the fit

- The $\bar{a}_n$ are determined from a maximum likelihood analysis

$$\mathscr{L}_{(r)} \propto \frac{1}{\sqrt{\det C_{ij}}} \exp\left(-\frac{1}{2}\Delta_i^{(r)} C_{ij}^{-1} \Delta_j^{(r)}\right)$$

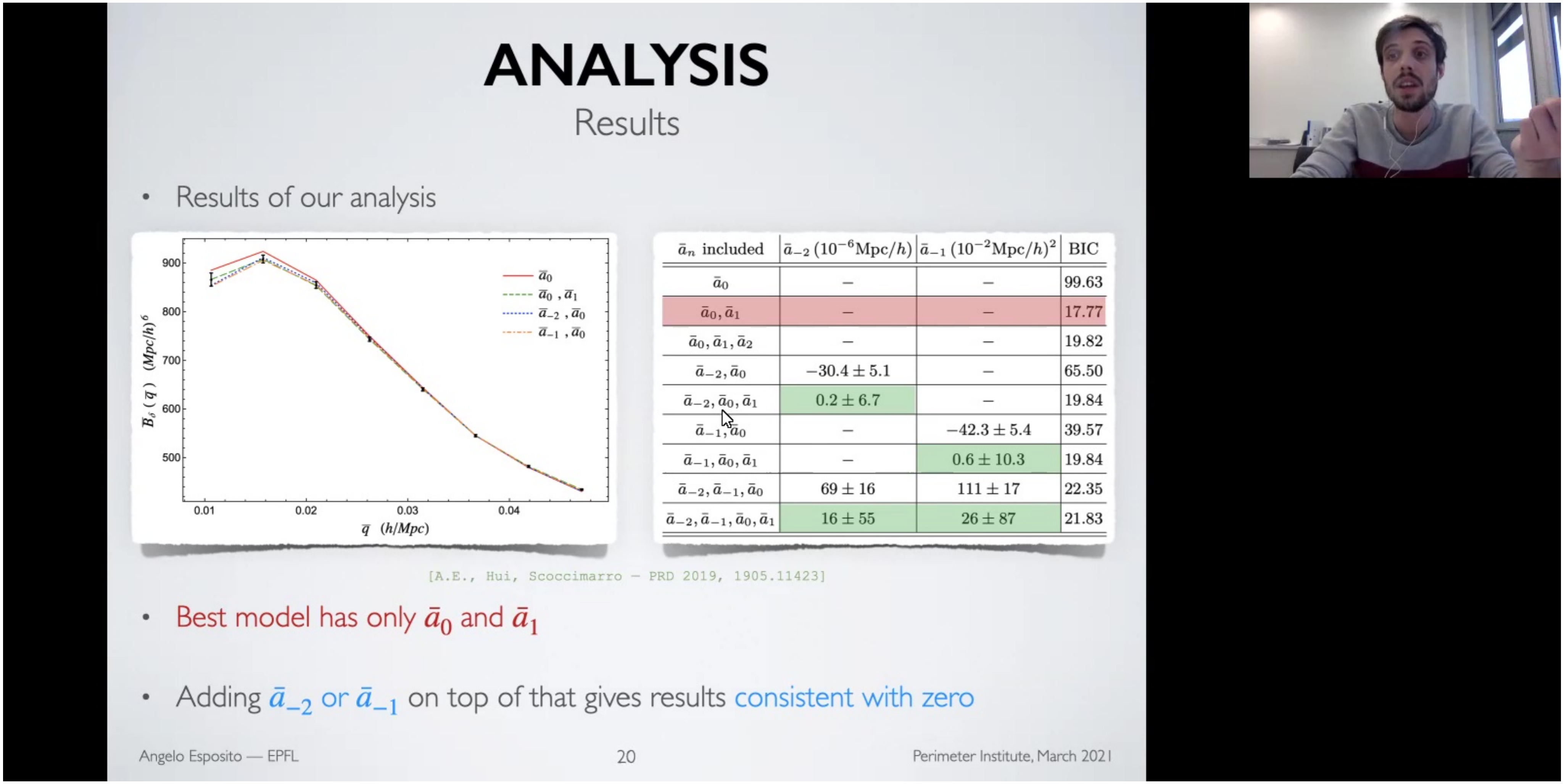
Separately for each realization

$$\Delta_i^{(r)} = \bar{B}_\delta^{(r)}(\bar{q}_i) - \sum_n \bar{a}_n^{(r)} \bar{M}_n^{(r)}(\bar{q}_i) , \qquad \text{and} \qquad C_{ij} = \frac{1}{N_r - 1} \sum_{r=1}^{N_r} \Delta_i^{(r)} \Delta_j^{(r)}$$

- $\Delta_i^{(r)}$ depends on the fitting parameters ⇨ C_{ij} depends on the fitting parameters ⇨ maximize the likelihood with an iterative procedure
- Model selection is done using the Bayesian information criterion

$$BIC = -2 \log \mathscr{L}_{max} + N_{par} \log N_q$$

`[see e.g. Liddle – MNRAS 2004, astro-ph/0401198]`



ANALYSIS

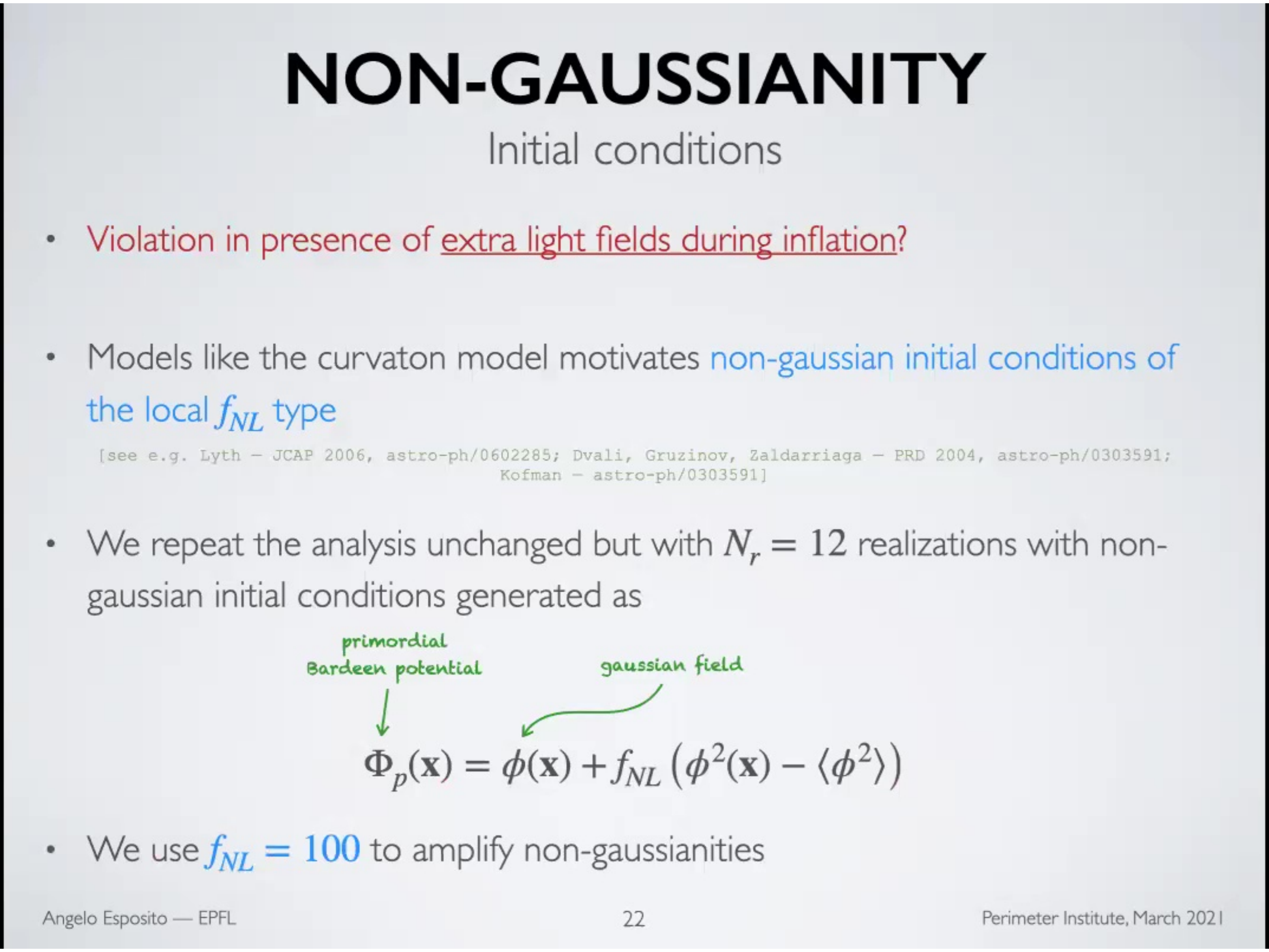
Results

- Results of our analysis

$\bar a_n$ included	$\bar a_{-2}\,(10^{-6}\mathrm{Mpc}/h)$	$\bar a_{-1}\,(10^{-2}\mathrm{Mpc}/h)^2$	BIC
$\bar a_0$	–	–	99.63
$\bar a_0, \bar a_1$	–	–	17.77
$\bar a_0, \bar a_1, \bar a_2$	–	–	19.82
$\bar a_{-2}, \bar a_0$	-30.4 ± 5.1	–	65.50
$\bar a_{-2}, \bar a_0, \bar a_1$	0.2 ± 6.7	–	19.84
$\bar a_{-1}, \bar a_0$	–	-42.3 ± 5.4	39.57
$\bar a_{-1}, \bar a_0, \bar a_1$	–	0.6 ± 10.3	19.84
$\bar a_{-2}, \bar a_{-1}, \bar a_0$	69 ± 16	111 ± 17	22.35
$\bar a_{-2}, \bar a_{-1}, \bar a_0, \bar a_1$	16 ± 55	26 ± 87	21.83

[A.E., Hui, Scoccimarro – PRD 2019, 1905.11423]

- Best model has only $\bar a_0$ and $\bar a_1$
- Adding $\bar a_{-2}$ or $\bar a_{-1}$ on top of that gives results consistent with zero



NON-GAUSSIANITY

Initial conditions

- Violation in presence of extra light fields during inflation?
- Models like the curvaton model motivates non-gaussian initial conditions of the local f_{NL} type

[see e.g. Lyth – JCAP 2006, astro-ph/0602285; Dvali, Gruzinov, Zaldarriaga – PRD 2004, astro-ph/0303591; Kofman – astro-ph/0303591]

- We repeat the analysis unchanged but with $N_r = 12$ realizations with non-gaussian initial conditions generated as

primordial Bardeen potential

gaussian field

$$\Phi_p(\mathbf{x}) = \phi(\mathbf{x}) + f_{NL}\left(\phi^2(\mathbf{x}) - \langle\phi^2\rangle\right)$$

- We use $f_{NL} = 100$ to amplify non-gaussianities

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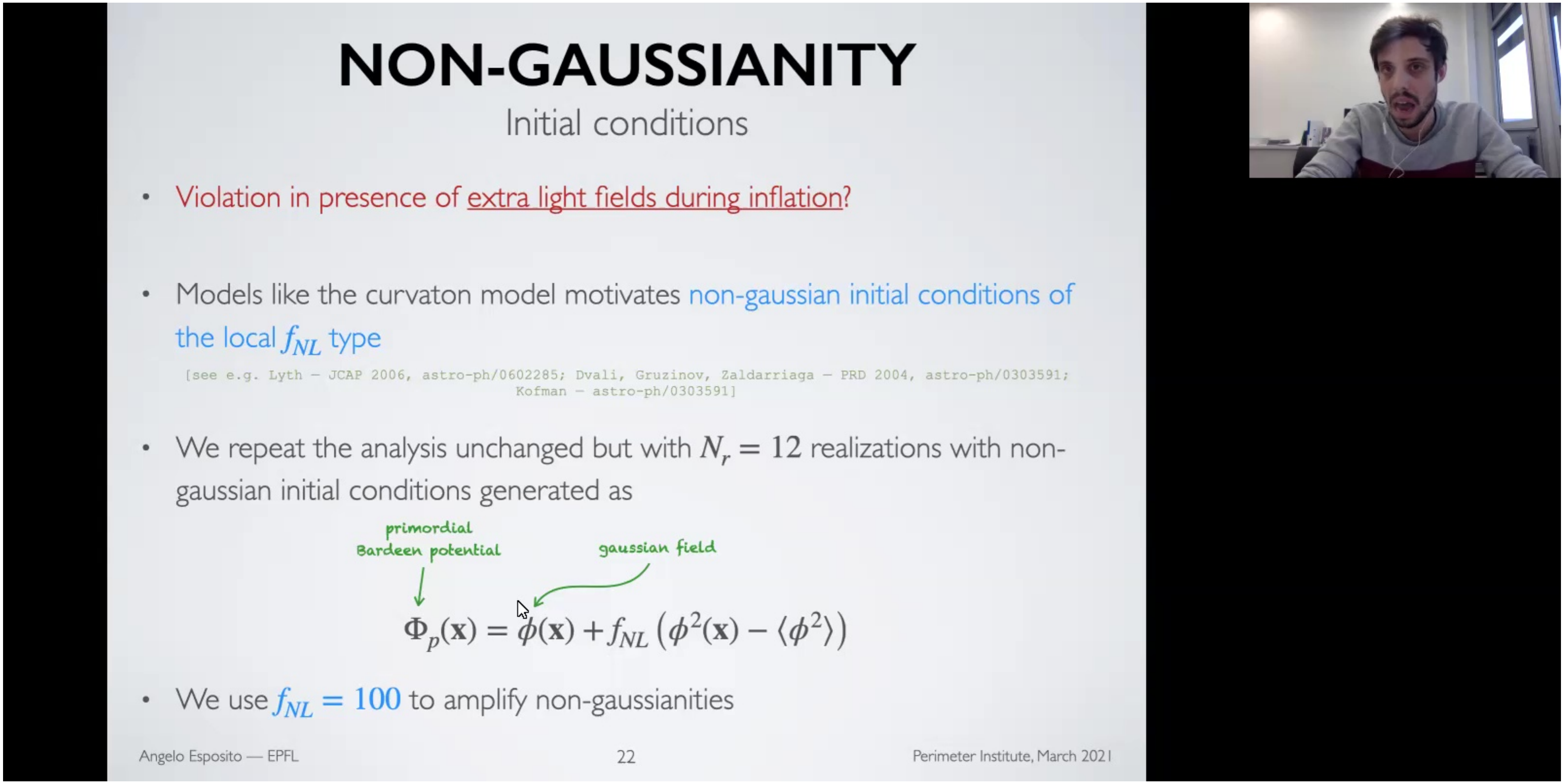

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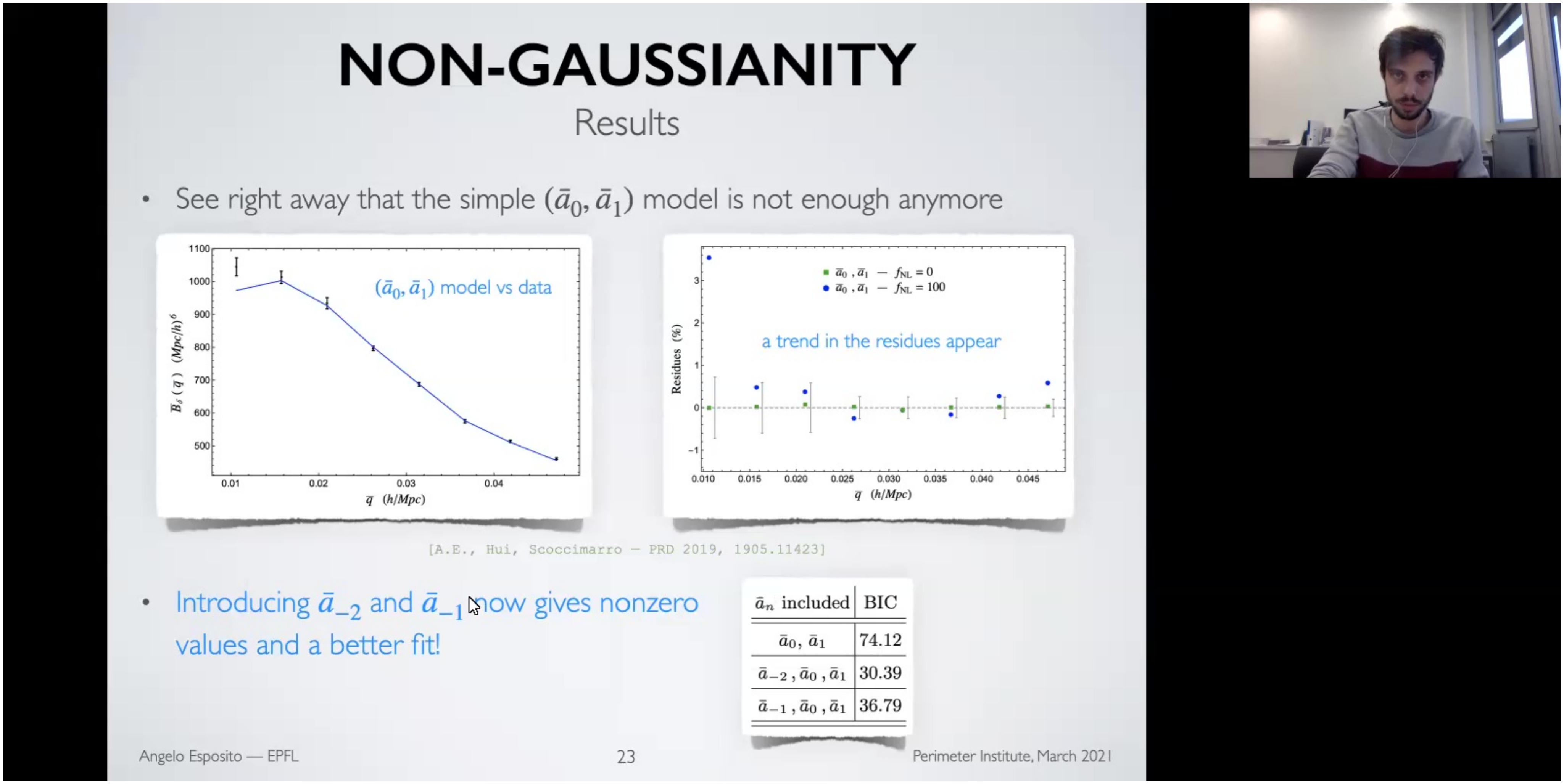
- We use $f_{NL} = 100$ to amplify non-gaussianities



NON-GAUSSIANITY

Results

- See right away that the simple $(\bar{a}_0, \bar{a}_1)$ model is not enough anymore

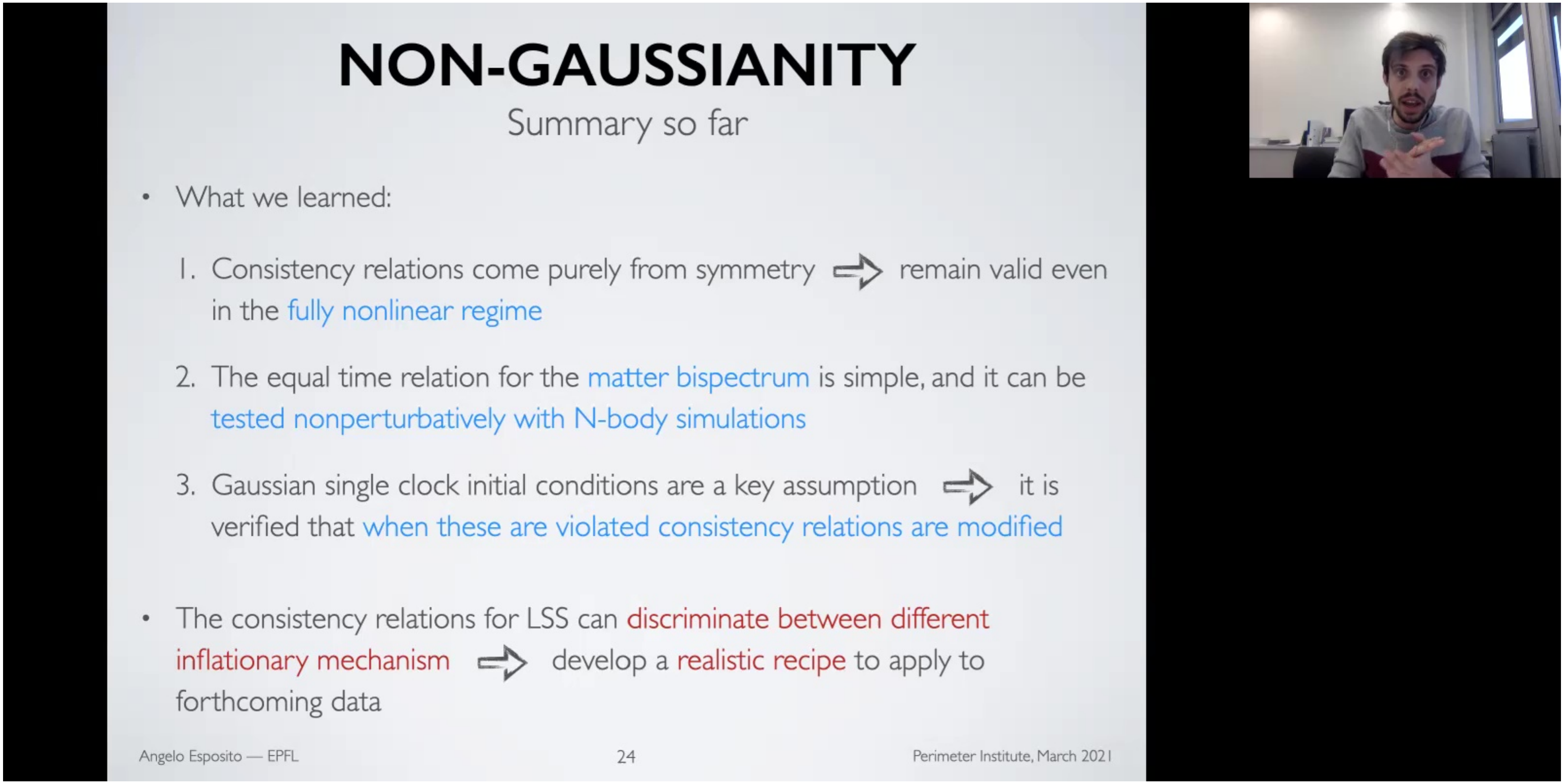
[A.E., Hui, Scoccimarro — PRD 2019, 1905.11423]

- Introducing $\bar{a}_{-2}$ and $\bar{a}_{-1}$ now gives nonzero values and a better fit!

$\bar{a}_n$ included	BIC
$\bar{a}_0,\ \bar{a}_1$	74.12
$\bar{a}_{-2}\,,\bar{a}_0\,,\bar{a}_1$	30.39
$\bar{a}_{-1}\,,\bar{a}_0\,,\bar{a}_1$	36.79

Angelo Esposito — EPFL

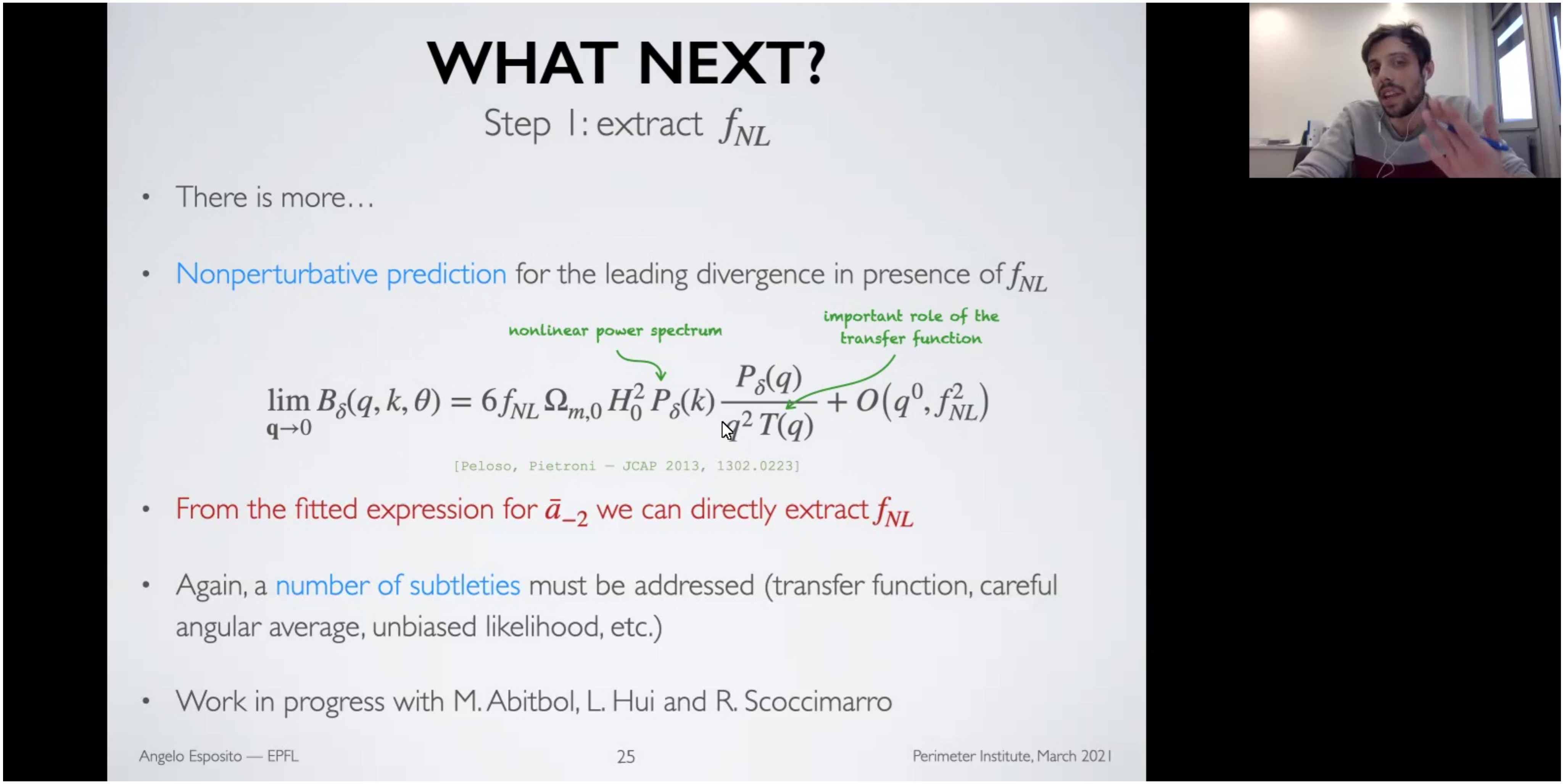
Perimeter Institute, March 2021



NON-GAUSSIANITY

Summary so far

- What we learned:
  1. Consistency relations come purely from symmetry ⇨ remain valid even in the fully nonlinear regime
  2. The equal time relation for the matter bispectrum is simple, and it can be tested nonperturbatively with N-body simulations
  3. Gaussian single clock initial conditions are a key assumption ⇨ it is verified that when these are violated consistency relations are modified
- The consistency relations for LSS can discriminate between different inflationary mechanism ⇨ develop a realistic recipe to apply to forthcoming data



WHAT NEXT?

Step 1: extract f_{NL}

- There is more…
- Nonperturbative prediction for the leading divergence in presence of f_{NL}

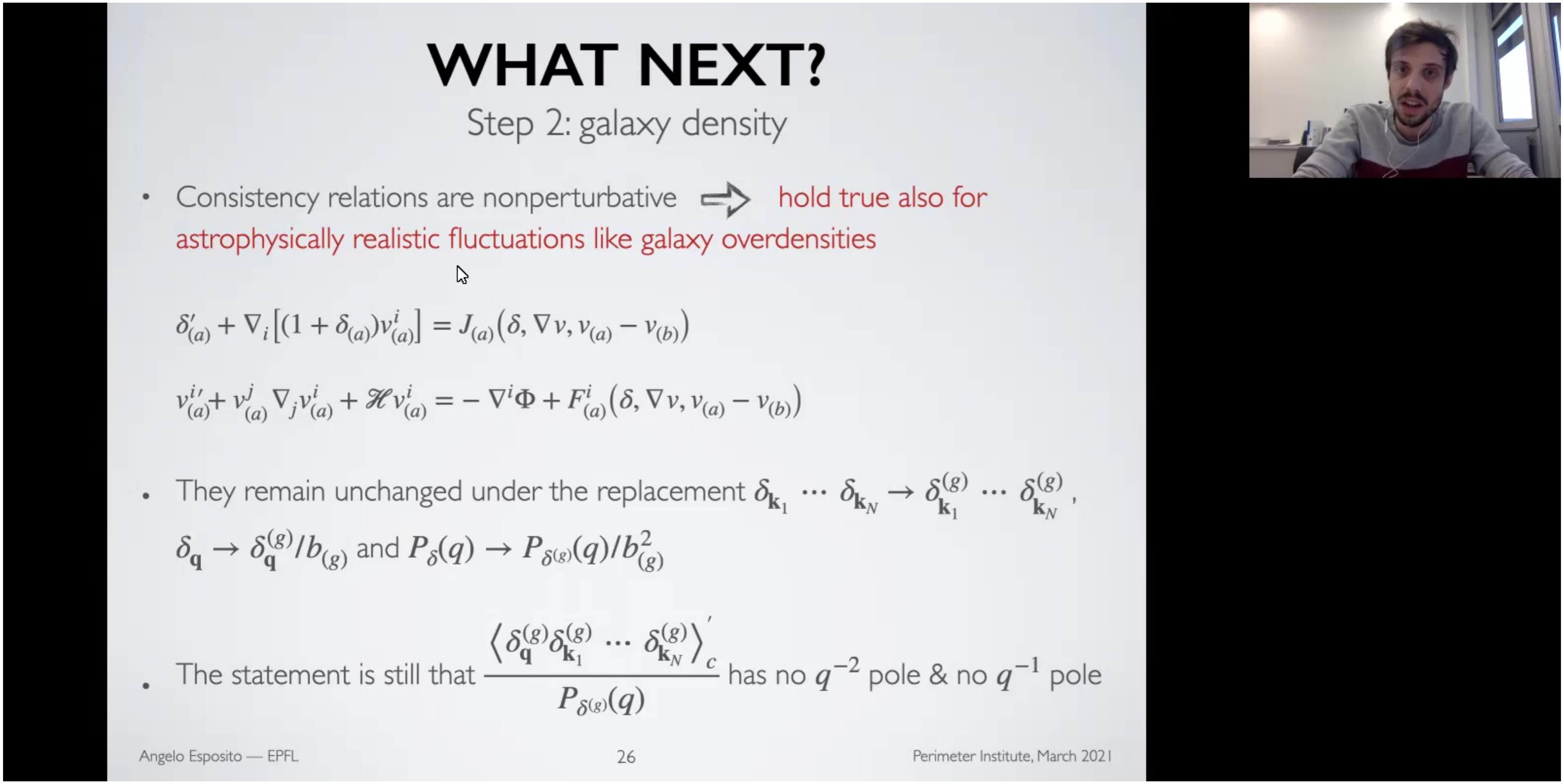
nonlinear power spectrum

important role of the transfer function

$$\lim_{\mathbf{q}\to 0} B_\delta(q,k,\theta) = 6 f_{NL}\,\Omega_{m,0}\, H_0^2\, P_\delta(k)\, \frac{P_\delta(q)}{q^2\, T(q)} + O\big(q^0, f_{NL}^2\big)$$

[Peloso, Pietroni – JCAP 2013, 1302.0223]

- From the fitted expression for $\bar{a}_{-2}$ we can directly extract f_{NL}
- Again, a number of subtleties must be addressed (transfer function, careful angular average, unbiased likelihood, etc.)
- Work in progress with M. Abitbol, L. Hui and R. Scoccimarro



WHAT NEXT?

Step 2: galaxy density

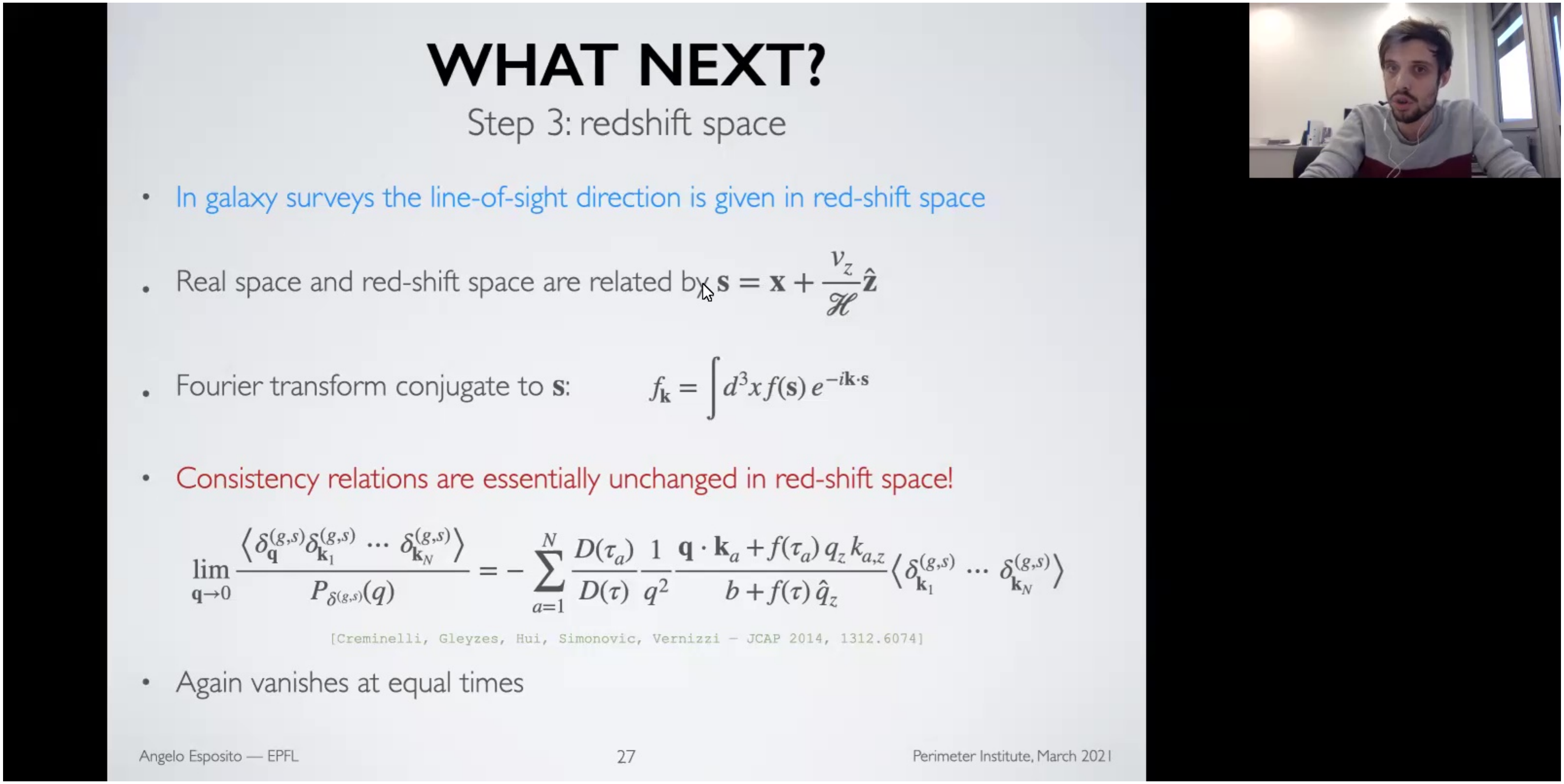
- Consistency relations are nonperturbative ⇨ hold true also for astrophysically realistic fluctuations like galaxy overdensities

$$\delta'_{(a)} + \nabla_i\left[(1+\delta_{(a)})v^i_{(a)}\right] = J_{(a)}\left(\delta, \nabla v, v_{(a)} - v_{(b)}\right)$$

$$v^{i\prime}_{(a)} + v^j_{(a)}\nabla_j v^i_{(a)} + \mathscr{H} v^i_{(a)} = -\nabla^i\Phi + F^i_{(a)}\left(\delta, \nabla v, v_{(a)} - v_{(b)}\right)$$

- They remain unchanged under the replacement $\delta_{\mathbf{k}_1}\cdots\delta_{\mathbf{k}_N} \to \delta^{(g)}_{\mathbf{k}_1}\cdots\delta^{(g)}_{\mathbf{k}_N}$, $\delta_{\mathbf{q}} \to \delta^{(g)}_{\mathbf{q}}/b_{(g)}$ and $P_\delta(q) \to P_{\delta^{(g)}}(q)/b^2_{(g)}$

- The statement is still that $\dfrac{\left\langle \delta^{(g)}_{\mathbf{q}}\delta^{(g)}_{\mathbf{k}_1}\cdots\delta^{(g)}_{\mathbf{k}_N}\right\rangle'_c}{P_{\delta^{(g)}}(q)}$ has no q^{-2} pole & no q^{-1} pole



WHAT NEXT?

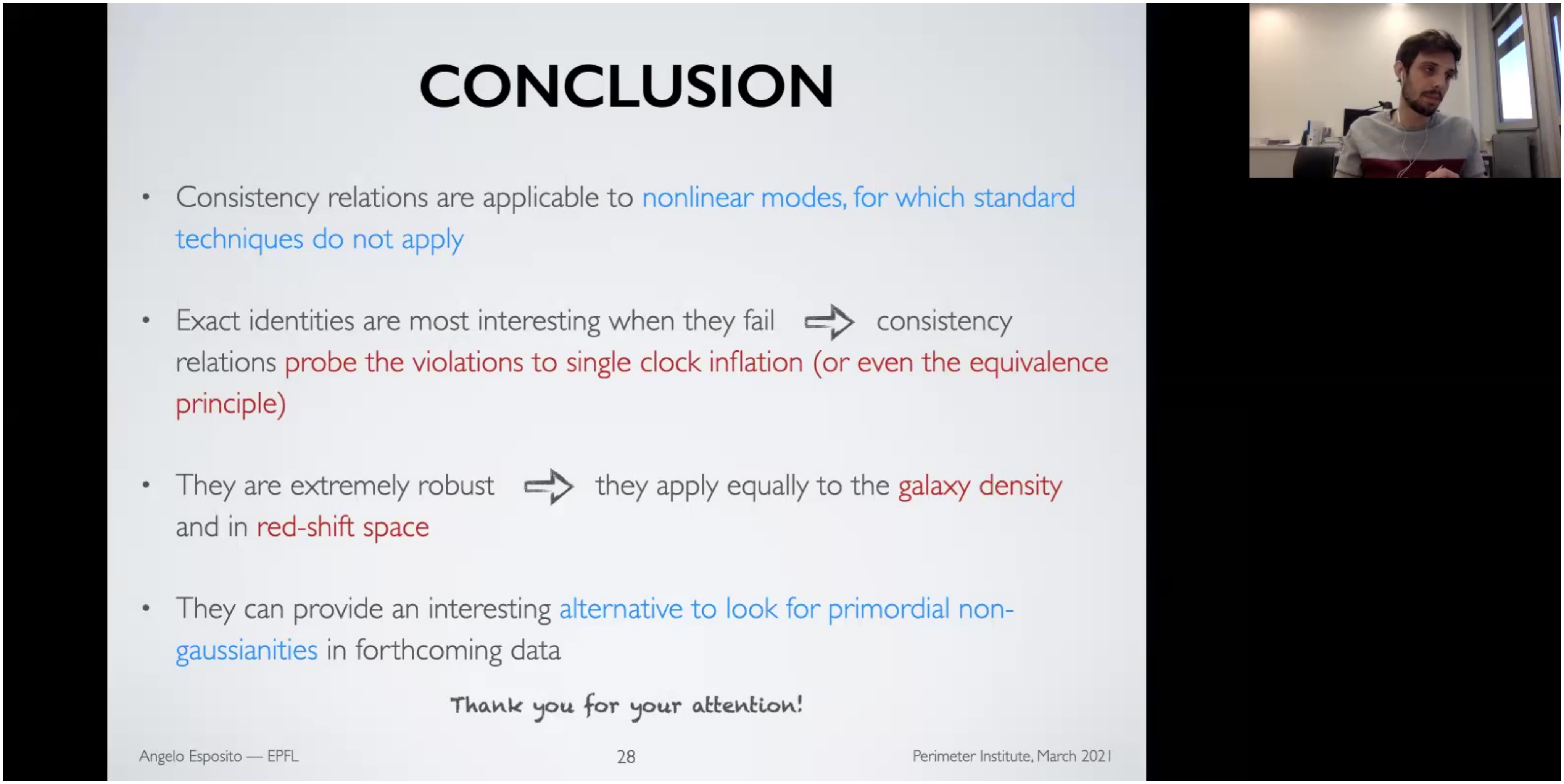
Step 3: redshift space

- In galaxy surveys the line-of-sight direction is given in red-shift space
- Real space and red-shift space are related by $\mathbf{s} = \mathbf{x} + \frac{v_z}{\mathscr{H}}\hat{\mathbf{z}}$
- Fourier transform conjugate to $\mathbf{s}$: $f_{\mathbf{k}} = \int d^3x\, f(\mathbf{s})\, e^{-i\mathbf{k}\cdot\mathbf{s}}$
- Consistency relations are essentially unchanged in red-shift space!

$$\lim_{\mathbf{q}\to 0} \frac{\langle \delta_{\mathbf{q}}^{(g,s)} \delta_{\mathbf{k}_1}^{(g,s)} \cdots \delta_{\mathbf{k}_N}^{(g,s)} \rangle}{P_{\delta^{(g,s)}}(q)} = - \sum_{a=1}^{N} \frac{D(\tau_a)}{D(\tau)} \frac{1}{q^2} \frac{\mathbf{q}\cdot\mathbf{k}_a + f(\tau_a)\, q_z\, k_{a,z}}{b + f(\tau)\,\hat{q}_z} \langle \delta_{\mathbf{k}_1}^{(g,s)} \cdots \delta_{\mathbf{k}_N}^{(g,s)} \rangle$$

[Creminelli, Gleyzes, Hui, Simonovic, Vernizzi – JCAP 2014, 1312.6074]

- Again vanishes at equal times



CONCLUSION

- Consistency relations are applicable to nonlinear modes, for which standard techniques do not apply
- Exact identities are most interesting when they fail ⇨ consistency relations probe the violations to single clock inflation (or even the equivalence principle)
- They are extremely robust ⇨ they apply equally to the galaxy density and in red-shift space
- They can provide an interesting alternative to look for primordial non-gaussianities in forthcoming data

Thank you for your attention!