Title: Stiefel liquids: possible non-Lagrangian quantum criticality from intertwined orders

Speakers: Chong Wang
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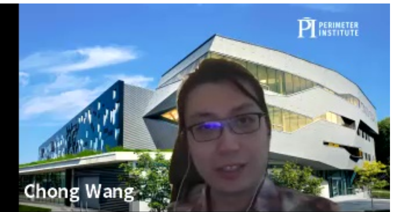
Abstract: We propose a new type of critical quantum liquids, dubbed Stiefel liquids, based on 2+1 dimensional Wess-Zumino-Witten models on target space SO(N)/SO(4). We show that the well known deconfined quantum critical point and U(1) Dirac spin liquid are unified as two special examples of Stiefel liquids, with N = 5 and N = 6, respectively. Furthermore, we conjecture that Stiefel liquids with N > 6 are non-Lagrangian, in the sense that the theories do not (at least not easily) admit any weakly-coupled UV completion. Such non-Lagrangian states are beyond the paradigm of parton gauge theory familiar in the study of exotic quantum liquids in condensed matter physics. The intrinsic absence of mean-field construction also makes it difficult to decide whether a non-Lagrangian state can emerge from a specific UV system (such as a lattice spin system). For this purpose we hypothesize that a quantum state is emergible from a lattice system if its quantum anomalies match with the constraints from the (generalized) Lieb-Schultz-Mattis theorems. Based on this hypothesis, we find that some of the non-Lagrangian Stiefel liquids can indeed be realized in frustrated quantum spin systems, for example, on triangular or Kagome lattice, through the intertwinement between non-coplanar magnetic orders and valence-bond-solid orders.



***Stiefel liquids*:**
possible non-Lagrangian quantum criticality from intertwined orders

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Quantum Matter Frontier Seminars
March 29, 2021

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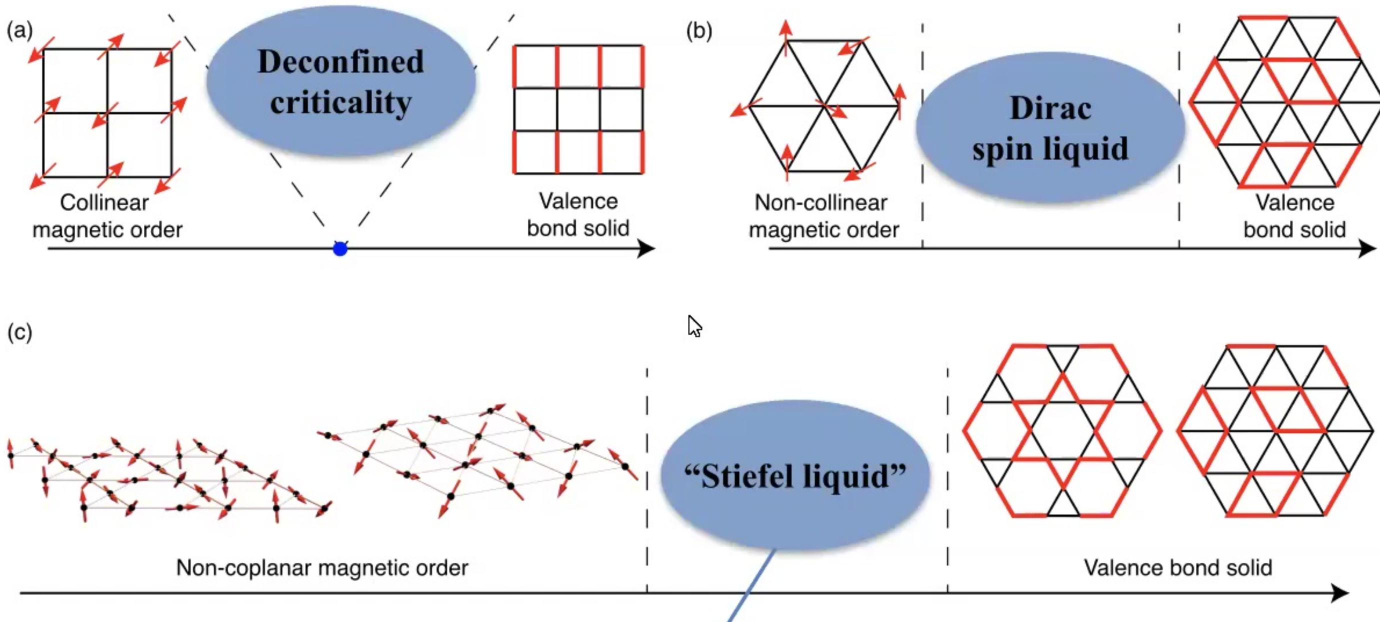
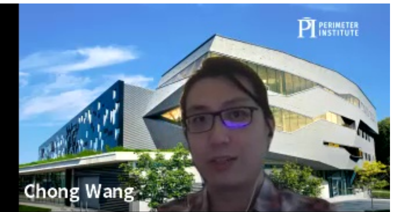
arXiv: 2101.07805

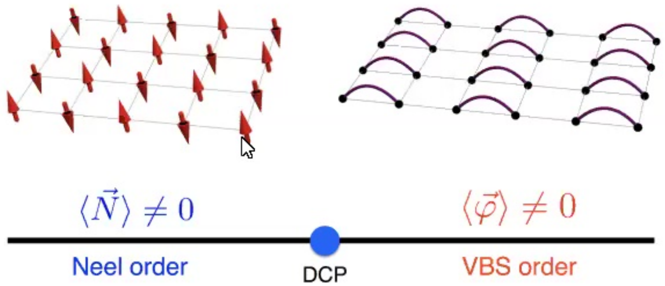
Exotic quantum criticality from intertwined orders



Possibly beyond the parton gauge theory paradigm



Deconfined quantum critical point



(Senthil, Vishwanath, Balents, Sachdev, Fisher, 2004)

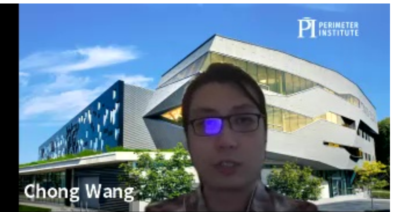
- A continuous* transition between very different symmetry-breaking phases

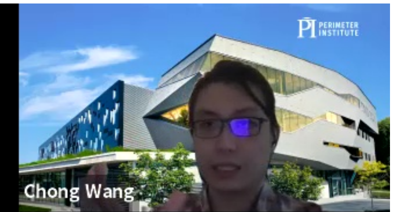
- Continuum field theory description: non-compact CP^1 in (2+1)d

$$\mathcal{L} = |D_b z_1|^2 + |D_b z_2|^2 - (|z_1|^2 + |z_2|^2)^2$$

- Continuum theory obtained from a lattice *parton* mean field

$$S^\mu \sim \frac{1}{2} z_\alpha^\dagger \sigma^\mu_{\alpha\beta} z_\beta$$



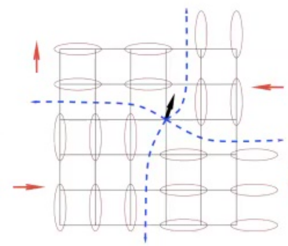


Sigma model formulation

$$S = \int d^3x \frac{1}{2g}(\nabla\hat{n})^2 + S_{\text{WZW}}[\hat{n}]$$

(Tanaka, Hu, 05; Senthil, Fisher, 06)

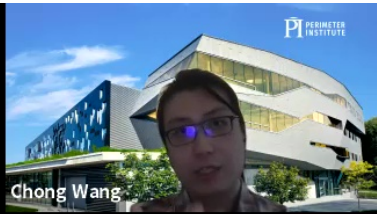
$$\hat{n} = (\underbrace{N_1, N_2, N_3}_{\text{Neel}}, \underbrace{\Phi_1, \Phi_2}_{\text{VBS}})$$



- The Wess-Zumino-Witten (WZW) model is well defined since

$$\pi_4(S^4) = \mathbb{Z} \qquad \pi_{n<4}(S^4) = 0$$

- Physics of WZW: intertwinement between different orders (Levin, Senthil, 04)
- DQCP: a strong-coupling fixed point* with emergent $SO(5)$ symmetry (Nahum, Chalker, Serna, Ortuno, Somoza, 15; CW, Nahum, Metlitski, Xu, Senthil, 17)

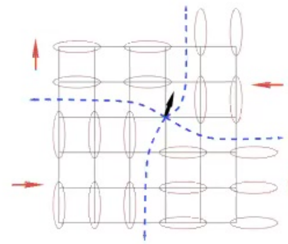


Anomaly

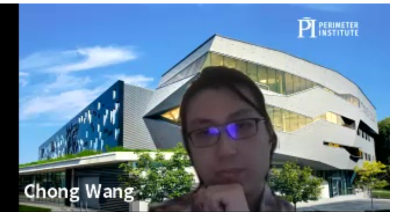
- $SO(5)$ has anomaly: if gauged, the response function lives in $(3+1)d$ bulk

$$\pi \int_{X_4} w_4^{SO(5)}$$

- Physical meaning: think about subgroup $SO(2) \times SO(3) \subset SO(5)$
A 2π flux in $SO(2)$ traps a half-integer spin in $SO(3)$
(analogous to fractional Hall conductance)



- Strongly constraints UV symmetry implementation: e.g. $SO(2)$ & $SO(3)$ cannot both be on-site — in DQCP, on-site $SO(3)$, lattice $C_4 \subset SO(2)$



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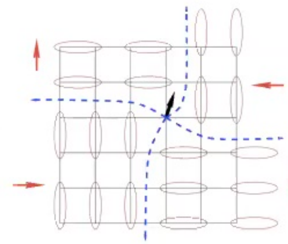

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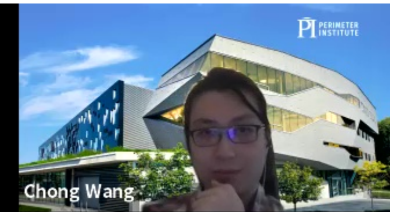
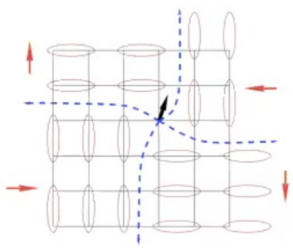


Anomaly and Lieb-Schultz-Mattis

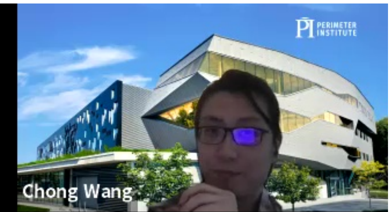
- Mutual anomaly between on-site $SO(3)$ and lattice C_4
- Manifestation of (generalized) Lieb-Schultz-Mattis Thm: ground state cannot be trivial if a half-integer spin sits on a C_4 rotation center



Cheng, Zaletel, Barkeshli, Vishwanath, Bonderson, 15;
Thorngren, Else, 16;
Po, Watanabe, Jian, Zaletel, 17;
And many others…

- Similar anomalies & LSM conditions involving translation, reflection, time-reversal — strongly constraint possible phases & phase transitions

Sigma model on Stiefel manifold



Instead of $S^4 = \dfrac{SO(5)}{SO(4)}$, let's try $\dfrac{SO(N)}{SO(4)}$ $(N \geq 5)$

Order parameter: n_{ij}

$N \times (N-4)$ matrix with orthonormal columns

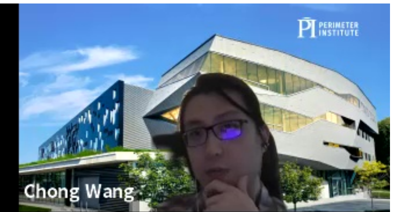
$$S^{(N,k)}[n] = \frac{1}{2g} \int d^{2+1}x \mathrm{Tr}(\partial_\mu n^T \partial^\mu n) + k \cdot S^{(N)}_{\mathrm{WZW}}$$

WZW well defined since

$$\pi_4 \left(\frac{SO(N)}{SO(4)} \right) = \mathbb{Z}$$

(and lower homotopy groups trivial)

Some intuitions for WZW



Take:

Unit vector in S^4, can wind in 4d

$$n_{ij} = \begin{pmatrix} I_{N-5} & 0 \\ & n'_1 \\ & n'_2 \\ 0 & n'_3 \\ & n'_4 \\ & n'_5 \end{pmatrix}$$

$$S^{(N)}_{\mathrm{WZW}}[n] = \frac{2\pi}{\Omega_4} \int_0^1 du \int d^3x \sum_{i,i'=1}^{N-4} \det\left(\tilde{n}_{(ii')}\right)$$

$$\tilde{n}_{(ii')} = (n, \partial_x n_i, \partial_y n_i, \partial_t n_{i'}, \partial_u n_{i'})$$

Sigma model on Stiefel manifold

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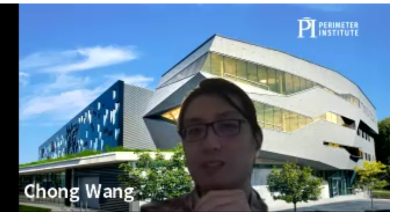


Symmetries

$$n \to Ln, L \in SO(N)$$

$$n \to nR, R \in SO(N-4)$$

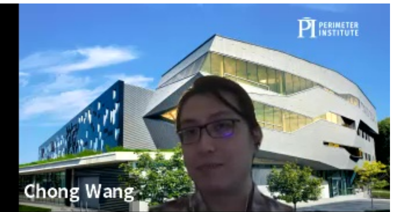
"Charge conjugation": reverse orientation in both $SO(N)$ & $SO(N-4)$

Time-reversal: reverse orientation in either $SO(N)$ or $SO(N-4)$

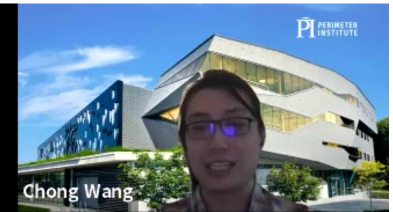
Total symmetry*:

$$(SO(N) \times SO(N-4)) \rtimes (\mathbb{Z}_2^{\mathcal{C}} \times \mathbb{Z}_2^{\mathcal{T}})$$

*: neglected a $\mathbb{Z}_2$ quotient for even N



$N = 6$: $U(1)$ Dirac spin liquid



The $SO(6)/SO(4)$ WZW model ($k = 1$) is dual* to $N_f = 4$ QED_3

$$\mathcal{L} = \sum_{i=1}^{4} \bar{\psi}_i i \not{D}_a \psi_i + \frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu}$$

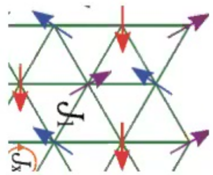
Flavour symmetry: $SU(4) \sim SO(6)$

Conservation of gauge flux $j^\mu = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$: $U(1) \sim SO(2)$

The $SO(6)/SO(4)$ order parameter: monopoles in QED_3

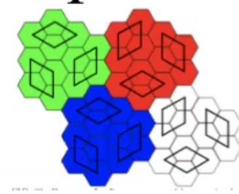
*: see our paper for derivation (similar logic as in Senthil, Fisher 06)

Dirac spin liquid



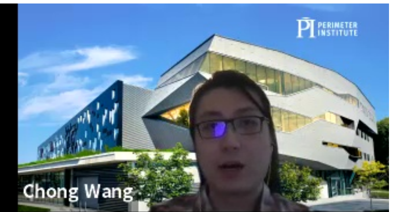
120 degree order

vs.



√12x√12 VBS

Jian, Thomson, Rasmussen, Bi, Xu, 17; Song, CW, Vishwanath, He, 18

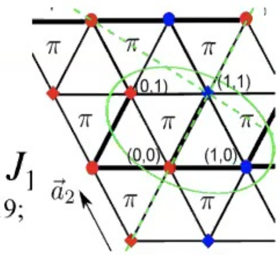


- $U(1)$ Dirac spin liquid from fermonic parton construction

(Rantner, Wen; Hermele, et al, Hasting…)

$$\mathbf{S}_i = \frac{1}{2} f^\dagger_{i,\alpha} \sigma_{\alpha\beta} f_{i,\beta}$$

$$H_{MF} = -\sum_{ij} f_i^\dagger t_{ij} f_j$$



Picture from Lu, 2016

- Numerical realizations: triangular $J_1 - J_2$ & Kagome J_1

Ferrari, Becca 19; Hu, Zhu, Eggert, He 19; He, Zaletel, Oshikawa, Pollman 17

- Candidate materials (?): $NaYbO_2$

Ding, Tsirlin et al, 19

- Physical meaning of $n \in SO(6)/SO(4)$ (monopoles): coplanar magnet and VBS order parameters

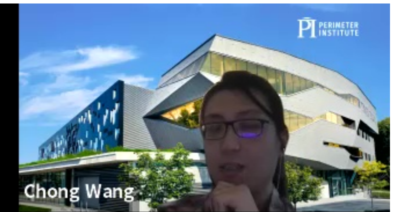
Stiefel Liquid

So far: $(2+1)d$ WZW models on target manifold $SO(N)/SO(4)$

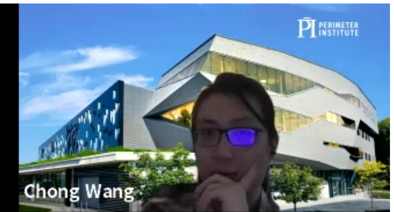
$N = 5$: Deconfined criticality

$N = 6$: $U(1)$ Dirac spin liquid

Generalize to any $N \geq 5$
postulate $\exists$ strong-coupling fixed points: "Stiefel liquids"



Anomaly of Stiefel liquids



The anomaly is characterized by an SPT in (3+1)d
with a topological response
to a probe gauge field of the symmetry

$$(SO(N) \times SO(N-4)) \rtimes (\mathbb{Z}_2^{\mathcal{C}} \times \mathbb{Z}_2^{\mathcal{T}})$$

The result*:

$$ik\pi \int_{X_4} \left\{ w_4^{O(N)} + w_4^{O(N-4)} + \left[w_2^{O(N-4)} + \left(w_1^{O(N-4)} \right)^2 \right] \left(w_2^{O(N)} + w_2^{O(N-4)} \right) + \left(w_1^{O(N-4)} \right)^4 \right\}$$

with constraint

$$w_1^{O(N)} + w_1^{O(N-4)} + w_1^{TM} = 0 \pmod 2$$

*: calculated indirectly using a "cascade structure"

Gauge theory description?

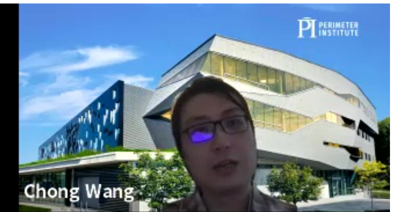
- $N = 5 \sim$ many gauge theories, e.g. $SU(2)$ QCD$_3$ with $N_f = 2$ Dirac fermions, flavour symmetry $Sp(2)/\mathbb{Z}_2 = SO(5)$

- $N = 6 \sim U(1)$ QED$_3$ with $N_f = 4$ Dirac fermions, flavour symmetry $SU(4)/\mathbb{Z}_2 = SO(6)$

- The extra $SO(2) = SO(N-4)$ for $N = 6$: the conservation of $U(1)$ gauge flux

$$j_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda$$

- For $N > 7$, we run out of imagination, so conjecture: no (renormalizable) Lagrangian description



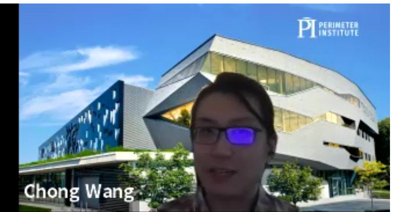
No mean field construction!

- To study a phase in condensed matter, we usually start from a mean field theory

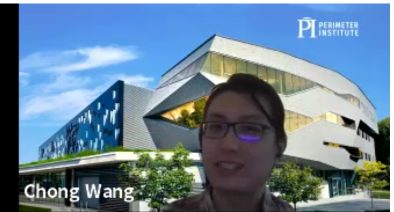
$$J\sum \mathbf{S}_i \cdot \mathbf{S}_j \to J\sum \langle \mathbf{S}_i \rangle \cdot \mathbf{S}_j \qquad \text{or} \qquad \begin{aligned} \mathbf{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ H_{MF} &= \sum_{ij} t_{ij} f_i^\dagger f_j \end{aligned}$$

- Then introduce **weak** fluctuations and let RG take us to the ultimate low energy phase (strongly coupled if fluctuation RG relevant)

- This is so fundamental that some consider
  "understanding a phase ≈ having a mean field"
  but doesn't work for non-Lagrangian theories ($N > 6$ Stiefel liquids)



Emergibility



A Basic question for any quantum phase of matter:

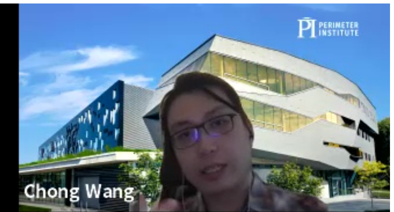
Given a UV system (say a lattice of spins), can the phase emerge (even in principle) as the low energy states of some local Hamiltonians?

Traditional approach:
construct a mean-field on the lattice system,
if can be done, then the phase is likely emergible

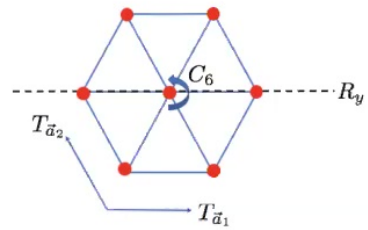
Doesn't work for $N > 6$ Stiefel liquids

So what can we do? — Anomaly matching

Anomaly on lattice



We illustrate with the triangular lattice
with one $SO(3)$ spin-1/2 moment (Kramers doublet) on each site



Consider gauging the $SO(3)$ spin rotation, time-reversal (gauge field t),
lattice translations (x and y), inversion (c) and reflection (r)

Anomaly from generalized LSM:

$$S_{\rm LSM} = i\pi \int_{X^4} (w_2^{SO(3)} + t^2)[xy + c^2 + r(x+c)]$$

with constraint: $t + r = w_1^{TM} \pmod 2$

Cheng, Zaletel, Barkeshli, Vishwanath, Bonderson, 2015;
Thorngren, Else, 2016;
And many others…

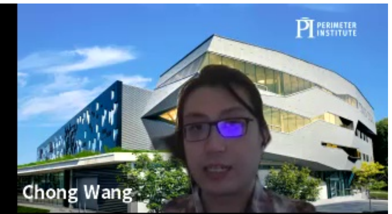
Lattice realization

The microscopic symmetry G_{UV} acts in the IR theory as a subgroup of the IR symmetry G_{IR}

Different lattice realizations of the field theory correspond to different ways of embedding G_{UV} into G_{IR}

A "lattice realization" means a homomorphism
$$\varphi : G_{UV} \to G_{IR}$$

Anomaly matching requires
$$w[G_{UV}] = \varphi^* w[G_{IR}]$$



So basically…

Find some embeddings $\varphi : G_{UV} \to G_{IR}$

So that

$$w[G_{UV}] = \varphi^* w[G_{IR}]$$

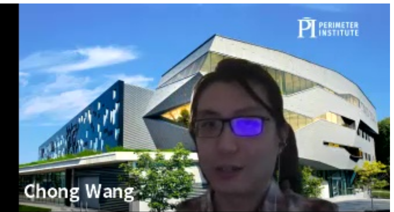
where $w[G_{IR}]$ is

$$ik\pi \int_{X_4} \left\{ w_4^{O(N)} + w_4^{O(N-4)} + \left[w_2^{O(N-4)} + \left(w_1^{O(N-4)} \right)^2 \right] \left(w_2^{O(N)} + w_2^{O(N-4)} \right) + \left(w_1^{O(N-4)} \right)^4 \right\}$$

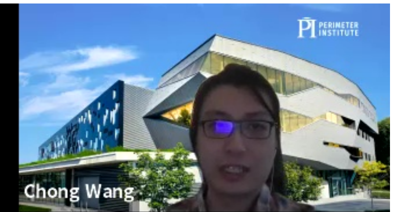
and $w[G_{UV}]$ is

$$S_{\mathrm{LSM}} = i\pi \int_{X^4} (w_2^{SO(3)} + t^2)[xy + c^2 + r(x+c)]$$

This requirement is quite nontrivial
since both anomalies are somewhat complicated



An example (N=7 on triangular)



$$SO(3): n \to \begin{pmatrix} SO^s(3) & 0 \\ 0 & I_{4\times4} \end{pmatrix} n$$

$$C_6 : n \to \begin{pmatrix} I_{3\times3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} n \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\mathscr{T}: \quad n \to \begin{pmatrix} -I_{3\times3} & 0 \\ 0 & I_{4\times4} \end{pmatrix} n, \quad i \to -i.$$

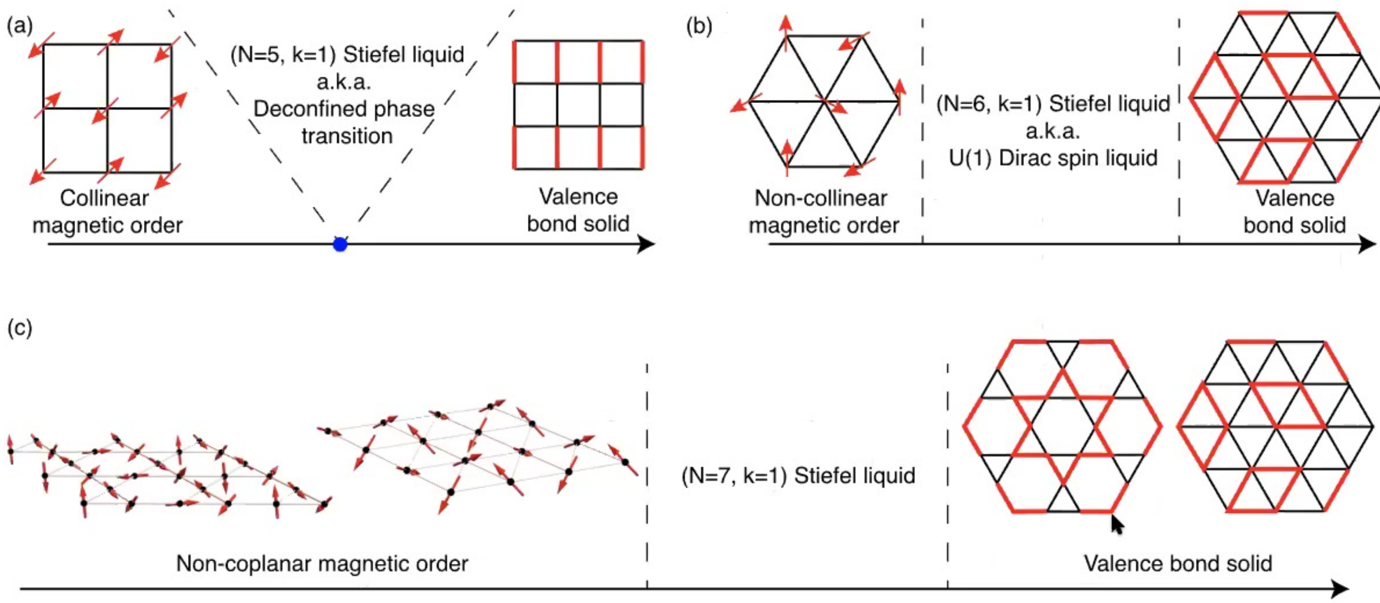
$$\mathcal{R}_y : n \to \begin{pmatrix} I_{3\times3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} n \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

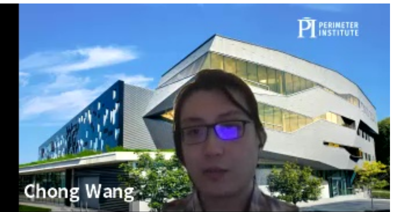
$$T_{\boldsymbol{a}_1} : n \to \begin{pmatrix} I_{3\times3} & 0 & 0 \\ 0 & \exp\left(i\frac{2\pi}{3}\sigma_y^{4,5}\right) & 0 \\ 0 & 0 & \exp\left(-i\frac{2\pi}{3}\sigma_y^{6,7}\right) \end{pmatrix} n \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{\boldsymbol{a}_2} : n \to \begin{pmatrix} I_{3\times3} & 0 & 0 \\ 0 & \exp\left(i\frac{2\pi}{3}\sigma_y^{4,5}\right) & 0 \\ 0 & 0 & \exp\left(-i\frac{2\pi}{3}\sigma_y^{6,7}\right) \end{pmatrix} n \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

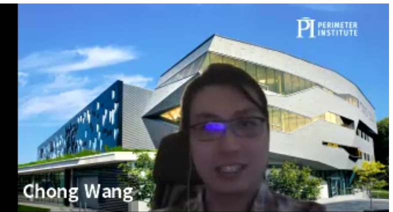
$$T_{-\boldsymbol{a}_1-\boldsymbol{a}_2} : n \to \begin{pmatrix} I_{3\times3} & 0 & 0 \\ 0 & \exp\left(i\frac{2\pi}{3}\sigma_y^{4,5}\right) & 0 \\ 0 & 0 & \exp\left(-i\frac{2\pi}{3}\sigma_y^{6,7}\right) \end{pmatrix} n \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Interpretation





Summary



- Stiefel liquids: possibly non-Lagrangian quantum criticalities, generalizing deconfined criticality and Dirac spin liquids

- The simplest such state (*N*=7) can emerge from the intertwinement between non-coplanar magnetic order and VBS

- In the absence of mean-field construction, anomaly-matching becomes a handy tool to establish emergibility

Thanks!

The meaning of "constructing" a phase on lattice

- Classic:

$$J \sum \mathbf{S}_i \cdot \mathbf{S}_j \to J \sum \langle \mathbf{S}_i \rangle \cdot \mathbf{S}_j$$

- Modern:

$$\mathbf{S} = \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta$$

$$H_{MF} = \sum_{ij} t_{ij} f_i^\dagger f_j$$

- Postmodern?

$$\varphi : G_{UV} \to G_{IR}$$

$$w[G_{UV}] = \varphi^* w[G_{IR}]$$