Title: A Mellin-Barnes Approach to Scattering in de Sitter Space

Speakers: Charlotte Sleight

Series: Quantum Fields and Strings

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Abstract: The last decade has seen significant progress in our understanding of scattering in anti-de Sitter (AdS) space. Through the AdS/CFT correspondence, we can reformulate scattering processes in AdS in terms of correlation functions in Conformal Field Theory (CFT), which are sharply defined by the requirements of Conformal Symmetry, Unitarity and a consistent Operator Product expansion. Accordingly, numerous highly effective techniques for the study of scattering in AdS have been developed. This has been driven largely by the Conformal Bootstrap programme, which aims to carve out the space of consistent CFTs (and, in turn, quantum gravities in AdS space) principally through the three basic consistency requirements above. In this talk I will describe some steps towards extending some of these techniques and results to boundary correlators in de Sitter (dS) space. Compared to AdS, we have little grasp of the properties required of consistent correlation functions in Euclidean CFTs dual to physics in dS. The boundaries at infinity in dS are space-like with no standard notion of locality and time, so the basic criteria that underpin the Conformal Bootstrap programme do not directly apply to the corresponding programme in dS, the so-called Cosmological bootstrap. I will show how boundary correlators in AdS and dS can be placed on a similar footing by introducing a Mellin-Barnes representation in momentum space, providing a framework that could facilitate bridging the gap between the Conformal and Cosmological bootstrap programmes. I will then discuss how the Mellin-Barnes representation itself can be a useful tool to study boundary correlators both in AdS and dS.

A Mellin-Barnes Approach to Scattering in de Sitter

Charlotte Sleight

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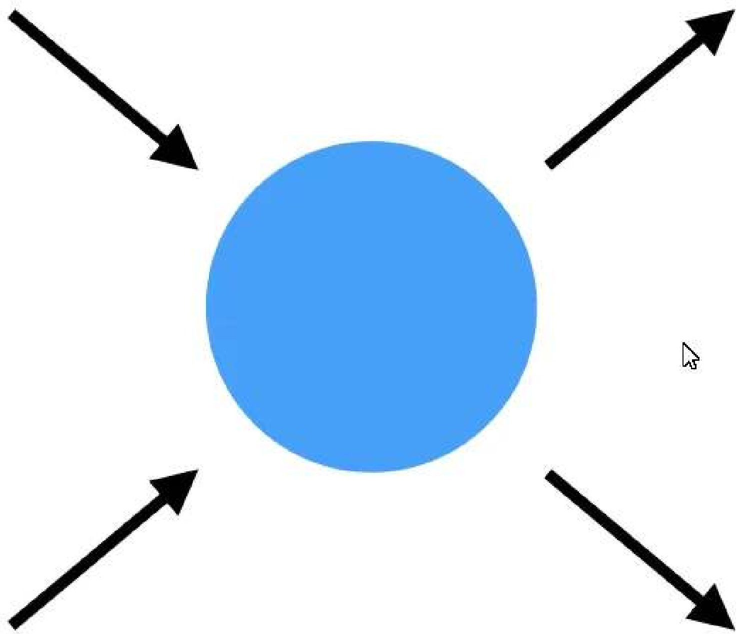
1906.12302 C.S.
1907.01143, 2007.09993 C.S. and M. Taronna
+ to appear.

Scattering Amplitudes



… are the bridge between theory and experiment.

Scattering Amplitudes

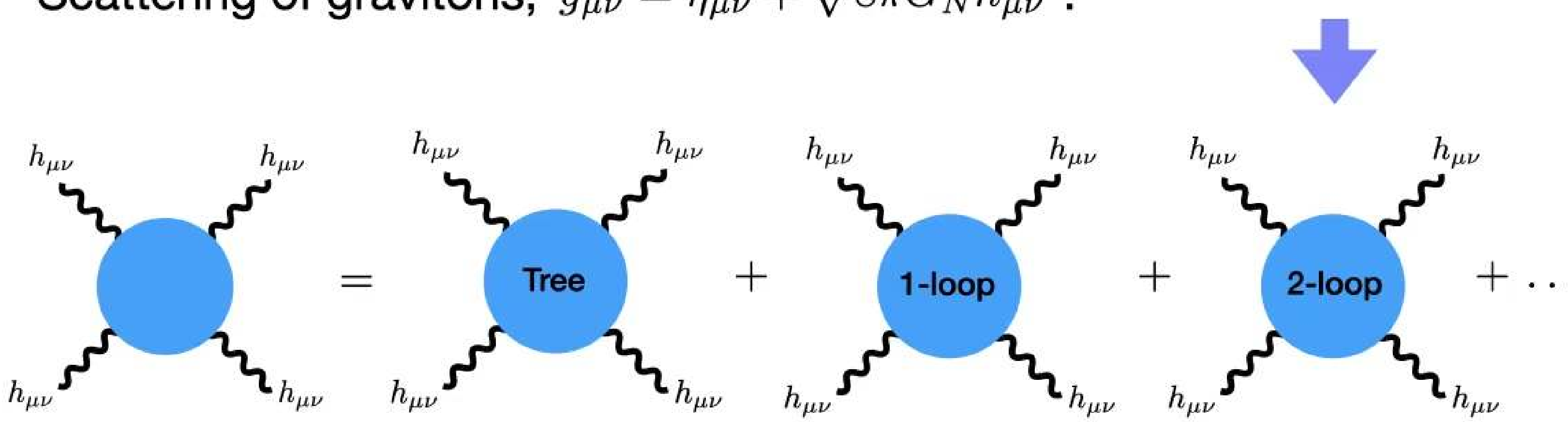
...provide a theoretical laboratory to test our theories.

General Relativity

$$\mathcal{L}_{EH}[g] = \frac{1}{16\pi G_N}\sqrt{-g}R$$

UV divergent!

Scattering of gravitons, $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G_N}h_{\mu\nu}$:



Scattering Amplitudes

Challenge: Quest for physics beyond the SM and GR

Access to theoretical and experimental laboratories is limited:

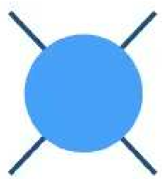
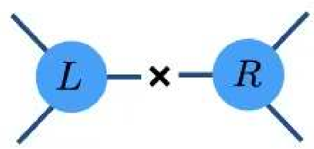
- Computing scattering amplitudes is **hard**.

- At high energies we **lack** experimental data.

Scattering Amplitudes: Bootstrap Approach

Challenge: Carve out space the of consistent theories

Collect theoretical data points by imposing basic physical criteria:

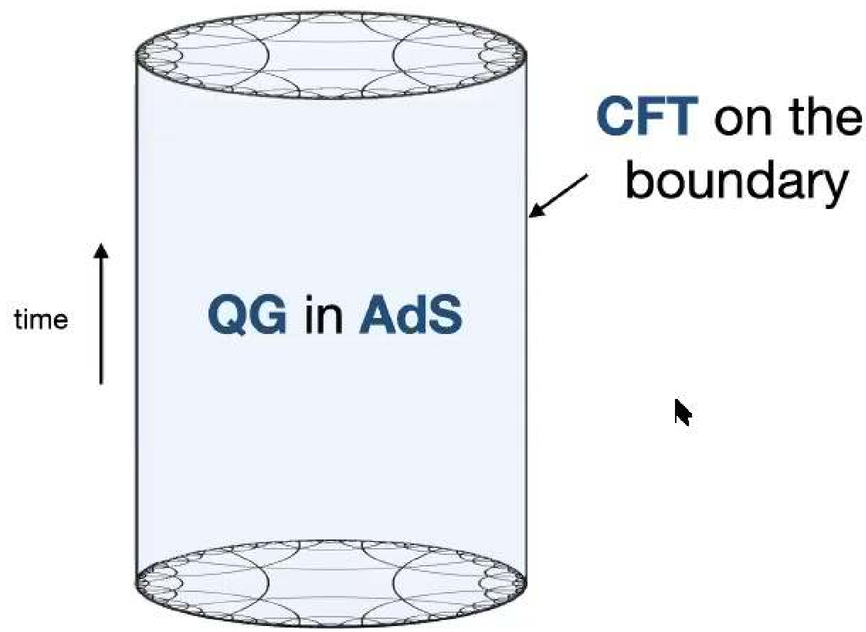
- Lorentz invariance
- Unitarity $SS^\dagger = 1$
- Locality  $\sim \frac{1}{s_{LR} - m^2}$ 

⋮

The AdS-CFT Correspondence

Maldacena 1997

Quantum Gravity in anti-de Sitter space	=	Conformal Field Theory on the boundary at infinity



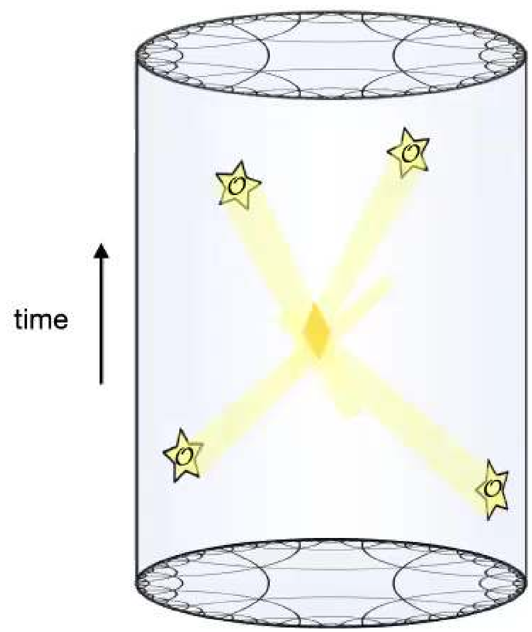
The AdS-CFT Correspondence

Maldacena 1997

Observables in
Quantum Gravity
in anti-de Sitter space

=

Correlation functions in the
Conformal Field Theory
on the boundary at infinity



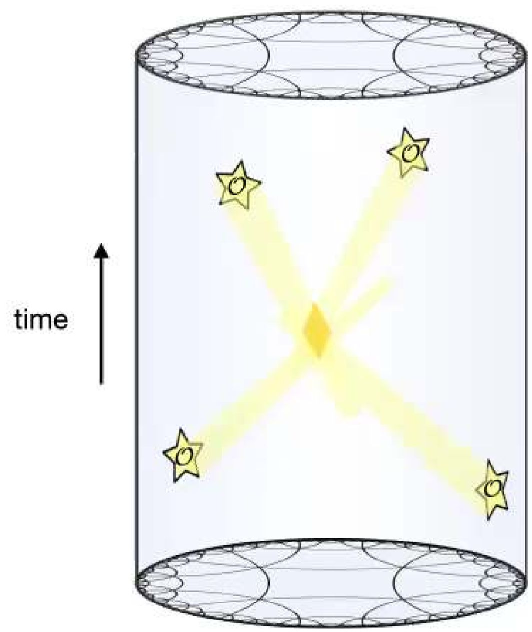
Scattering in AdS

The AdS-CFT Correspondence

Maldacena 1997

Observables in Quantum Gravity in anti-de Sitter space = **Correlation functions in the** Conformal Field Theory on the boundary at infinity

?!



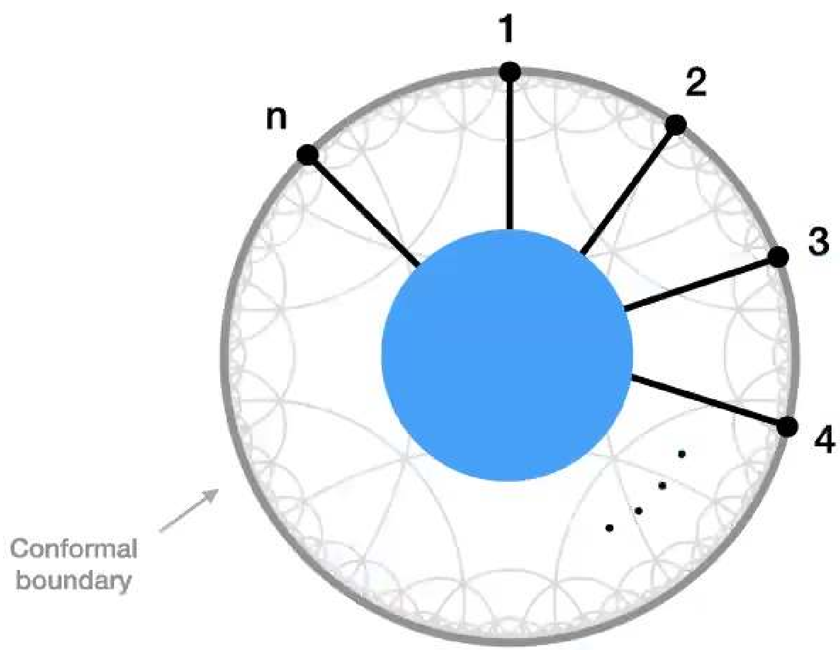
Scattering in AdS

Defined non-perturbatively by:

- Conformal symmetry
- Unitarity
- Consistent Operator Product Expansion

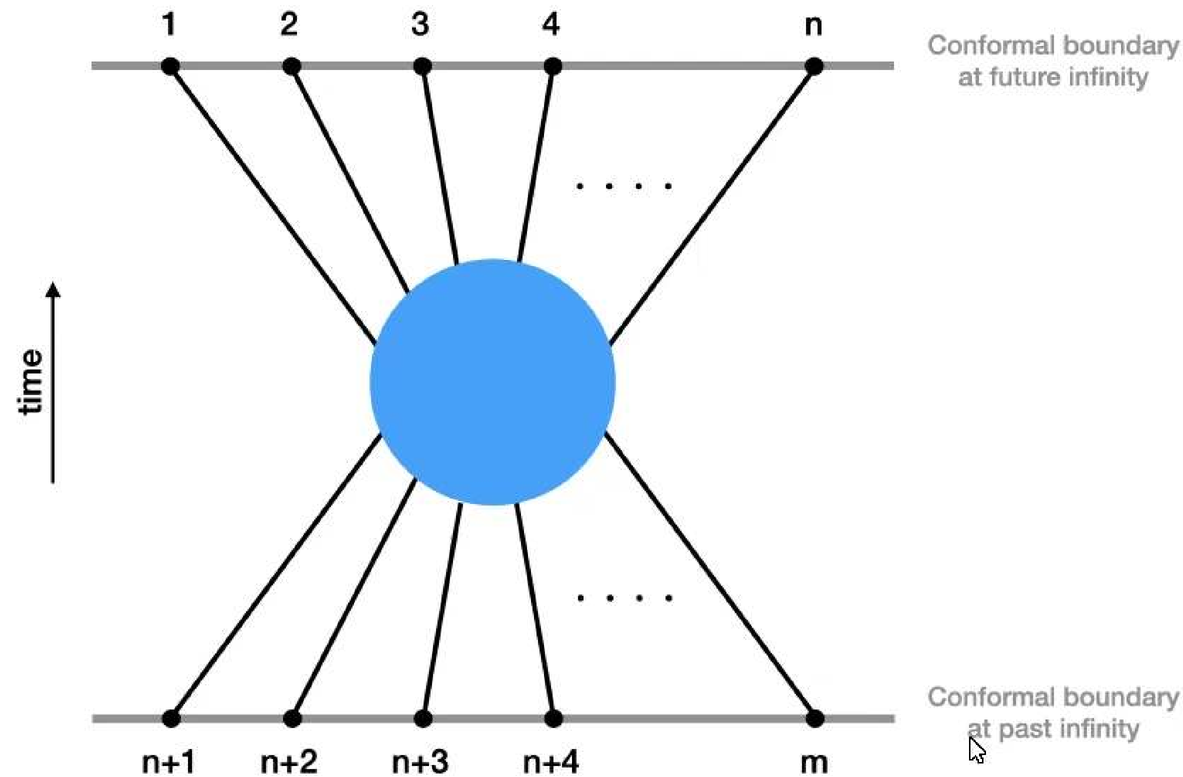
Scattering in anti-de Sitter

…in AdS we have a pretty good understanding.



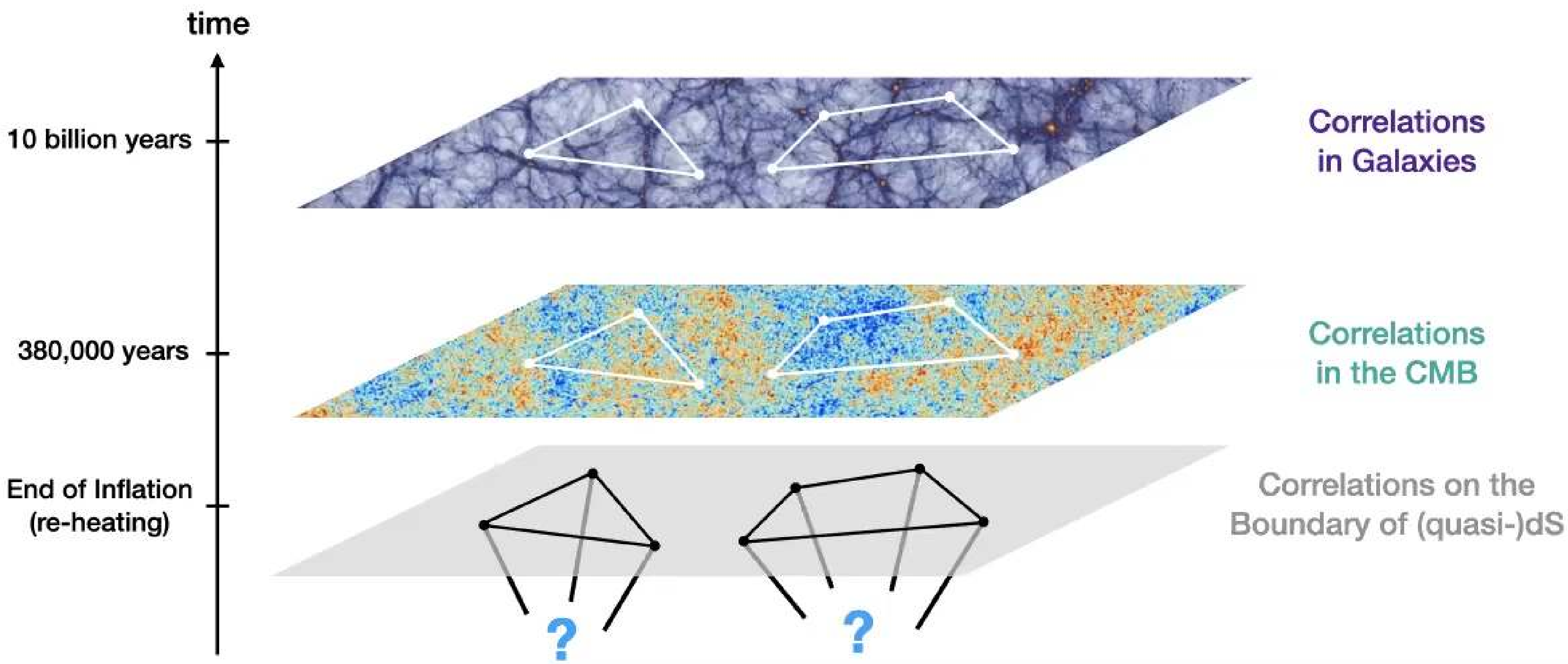
Can we adapt extend this understanding and techniques beyond the relative security of AdS space?

Scattering in de Sitter



Cosmological Collider Physics

Many groups, e.g.: Chen and Wang 2009, Baumann and Green 2011, Noumi, Yamaguchi and Yokoyama 2013, Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018, …

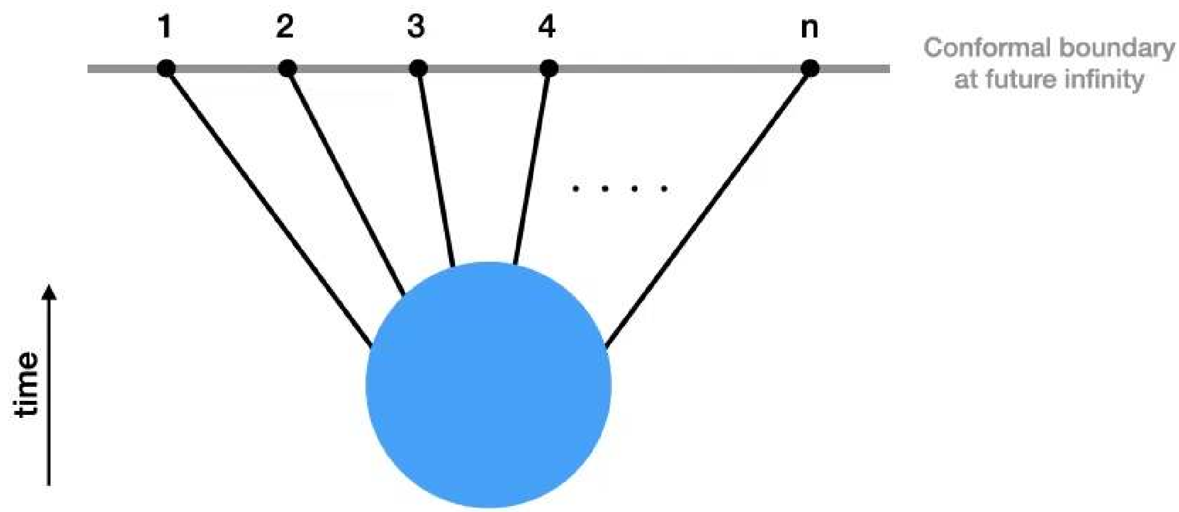


Task: Classify the imprints of new degrees of freedom

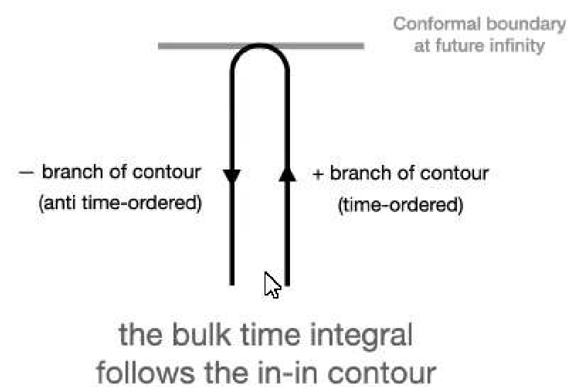
The Cosmological Bootstrap:

Ghosh, Kundu, Raju, Shukla, Trivedi 2014-; Arkani-Hamed, Maldacena 2015; Arkani-Hamed, Benincasa, Mcleod, Parisi, Postnikov, Vergu 2017-; Arkani-Hamed, Baumann, Duaso-Pueyo, Joyce Lee and Pimentel 2018-; **Sleight and Taronna 2019-**; Green and Pajer 2020; Pajer, David Stefanyszyn, Jakub Supeł 2020; Goodhew, Jazayeri, Pajer 2020; Céspedes, Davis, Melville 2020 …

Scattering in de Sitter



Computed within the in-in (Schwinger-Keldysh) formalism:



Outline

Part I: Can we place boundary correlators in (A)dS on a similar footing?

→ **Mellin-Barnes representation in momentum space**

cf. Mellin-Barnes representation of the Gauss Hypergeometric function

$$ {}_2F_1\left(a,b;c;z\right) = \frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\frac{\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(-s\right)}{\Gamma\left(c+s\right)}\left(-z\right)^s $$

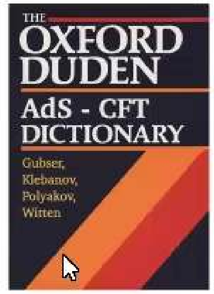
Part II: Applications.

- **Contact diagrams**
- **Exchanges**
- **Constraints on interactions of massless spinning particles**

The AdS-CFT Dictionary

Maldacena 1997

Quantum **G**ravity
in **a**nti-**d**e **S**itter space



Conformal **F**ield **T**heory

boundary value

$$Z_{\mathrm{QG\ AdS}}\left[\varphi \to \bar{\varphi}\right]$$

source

$$Z_{\mathrm{CFT}}\left[\bar{\varphi}\right]$$

Elementary field φ

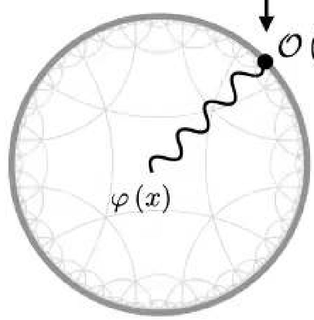
spin J, mass $m^2 R^2 = -\left(\Delta_+ \Delta_- + J\right)$

$\Delta_+ + \Delta_- = d \ , \ \Delta_+ \geq \Delta_-$

Local operator $\mathcal{O}$

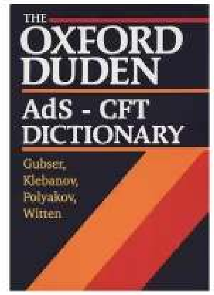
spin J, scaling dimension Δ_+

$$\int d^d\mathbf{x}\, \bar{\varphi}\left(\mathbf{x}\right) \mathcal{O}\left(\mathbf{x}\right)$$



The AdS-CFT Dictionary

Maldacena 1997



Quantum Gravity in anti-de Sitter space

boundary value ↓

$$Z_{\text{QG AdS}}\left[\varphi \rightarrow \bar{\varphi}\right]$$

Elementary field φ

spin J, mass $m^2R^2 = -(\Delta_+\Delta_- + J)$

$\Delta_+ + \Delta_- = d\ ,\ \Delta_+ \geq \Delta_-$

n-point scattering of particles φ_i

Conformal Field Theory

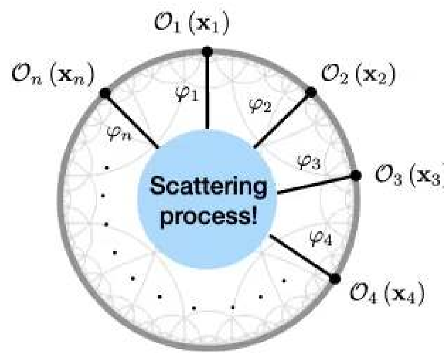
source ↓

$$Z_{\text{CFT}}\left[\bar{\varphi}\right]$$

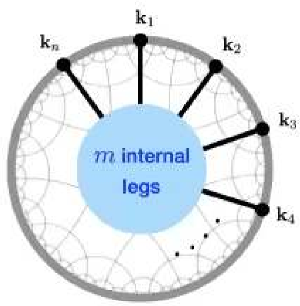
Local operator $\mathcal{O}$

spin J, scaling dimension Δ_+

n-point correlator of operators $\mathcal{O}_i$

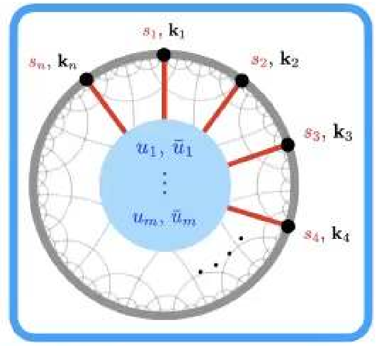


Mellin-Barnes Representation



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs internal legs



Mellin-Barnes representation

External leg, momentum $\mathbf{k}_i$ $\quad (|\mathbf{k}_i|)^{-2s_i+\Delta_i^+-\frac{d}{2}}$ $\quad$ **external Mellin variable**, s_i

Translation invariance: $\mathbf{k}_1 + \ldots + \mathbf{k}_n = 0$

$$(2\pi)^d\, \delta^{(d)}\left(\mathbf{k}_1 + \ldots + \mathbf{k}_n\right) = \int d^d\mathbf{x}\, e^{i\mathbf{x}\cdot(\mathbf{k}_1+\ldots+\mathbf{k}_n)}$$

Dilatation Ward identities: $s_1 + \ldots + s_n = \text{const.}$

$$2\pi i\, \delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$

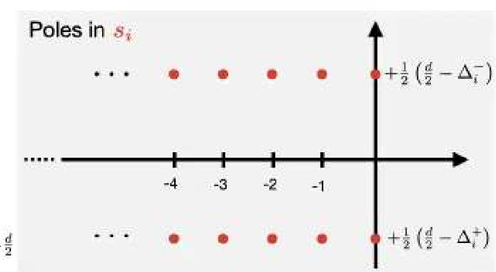
Poles from the Mellin-Barnes representation of the propagators for the external legs →

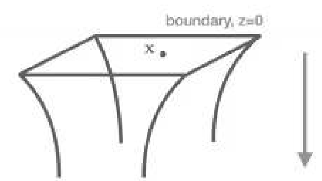
e.g. scalar field propagator:

$$= z^{\frac{d}{2}} \left(\frac{|\mathbf{k}_i|}{2}\right)^{\Delta_i^+-\frac{d}{2}} K_{\Delta_i^+-\frac{d}{2}}\left(z|\mathbf{k}_i|\right)$$

Modified Bessel function of the second kind

$$= \int_{-i\infty}^{+i\infty} \frac{ds_i}{2\pi i} z^{\frac{d}{2}-2s_i}\, \Gamma\left(s_i - \tfrac{1}{2}\left(\tfrac{d}{2} - \Delta_i^-\right)\right) \Gamma\left(s_i - \tfrac{1}{2}\left(\tfrac{d}{2} - \Delta_i^+\right)\right) \left(\frac{|\mathbf{k}_i|}{2}\right)^{-2s_i+\Delta_i^+-\frac{d}{2}}$$

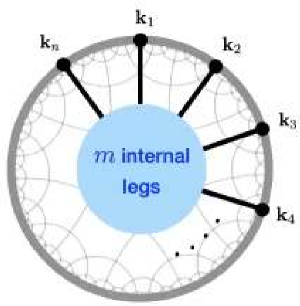




Euclidean AdS in Poincaré co-ordinates

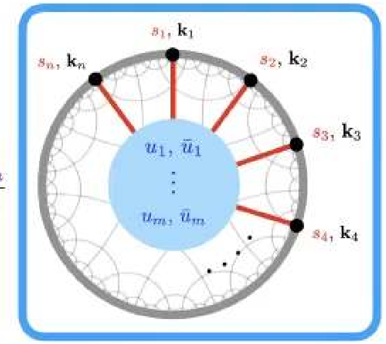
$$ds^2 = \left(\frac{R_{\text{AdS}}}{z}\right)^2 \left(dz^2 + d\mathbf{x}^2\right)$$

Mellin-Barnes Representation



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs — internal legs



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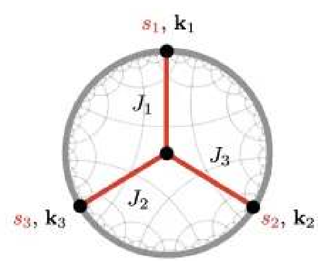
$$2\pi i\, \delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$

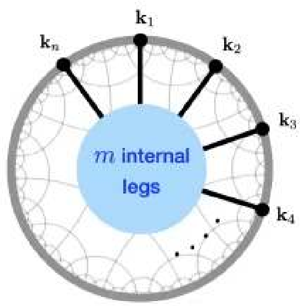
E.g. 3pt contact diagram, spins J1-J2-J3:

$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$$



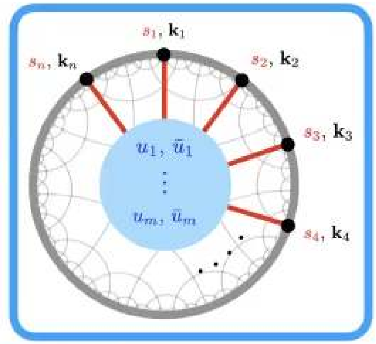
$$s_1 + s_2 + s_3 = \frac{d + 2\left(J_1 + J_2 + J_3\right)}{4}$$

Mellin-Barnes Representation



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs   internal legs



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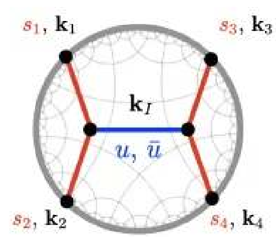
Internal leg, momentum $\mathbf{k}_I$ $(|\mathbf{k}_I|)^{-2(u+\bar{u})}$ Two **internal Mellin variables**, $u, \bar{u}$

E.g. 4pt spin J exchange:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_I,$$

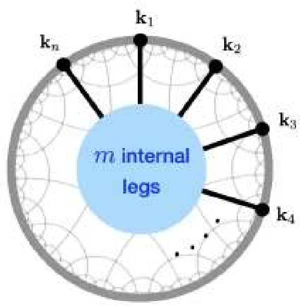
$$\mathbf{k}_3 + \mathbf{k}_4 = -\mathbf{k}_I$$



$$s_1 + s_2 + u = \frac{d + 2\left(J_1 + J_2 + J\right)}{4}$$

$$s_3 + s_4 + \bar{u} = \frac{d + 2\left(J + J_3 + J_4\right)}{4}$$

Mellin-Barnes Representation



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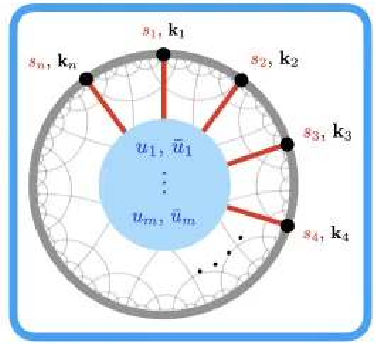
external legs   internal legs



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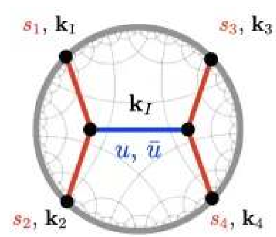


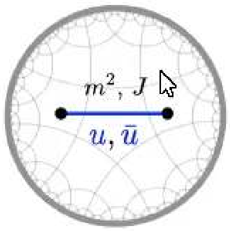
$$s_1 + s_2 + u = \frac{d + 2\left(J_1 + J_2 + J\right)}{4}$$

$$s_3 + s_4 + \bar{u} = \frac{d + 2\left(J + J_3 + J_4\right)}{4}$$

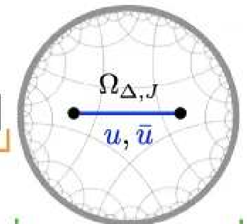
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In EAdS we have:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha\,\overset{\text{Dirichlet}}{\omega_D}\left(u,\bar{u}\right)+\beta\,\overset{\text{Neumann}}{\omega_N}\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



Harmonic function, $\left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0$

$\omega_{D/N}\left(u,\bar{u}\right)$ project onto Dirichlet/Neumann boundary conditions:

$m^2R^2=-\left(\Delta_+\Delta_-+J\right)$

$\Delta_++\Delta_-=d\ ,\ \Delta_+\geq\Delta_-$

$$\omega_{D/N}\left(u,\bar{u}\right)=\frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_\mp-\tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_\mp-\tfrac{d}{2}\right)\right)\right)$$

Recall the general solution to the wave equation near the boundary of EAdS, $z\to 0$:

$$\varphi\left(z,\mathbf{k}\right)=\alpha\,\underbrace{z^{\Delta_+}\left[\mathcal{O}_{\Delta_+}\left(\mathbf{k}\right)+O\left(z^2\right)\right]}_{\text{Dirichlet boundary condition, selected by } \omega_D(u,\bar{u})}+\beta\,\underbrace{z^{\Delta_-}\left[\mathcal{O}_{\Delta_-}\left(\mathbf{k}\right)+O\left(z^2\right)\right]}_{\text{Neumann boundary condition, selected by } \omega_N(u,\bar{u})}$$

$$ds^2=\left(\frac{R_{\text{AdS}}}{z}\right)^2\left(dz^2+d\mathbf{x}^2\right)$$

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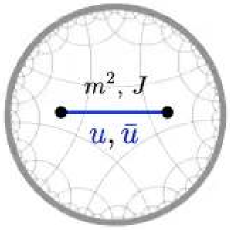
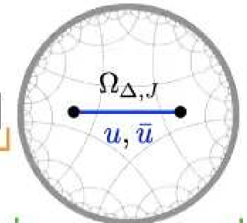


$$= \csc\left(\pi\left(u+\bar{u}\right)\right)\left[\alpha\,\omega_D\left(u,\bar{u}\right)+\beta\,\omega_N\left(u,\bar{u}\right)\right]$$



Dirichlet — Neumann

contact terms — boundary conditions

Harmonic function, $\left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0$

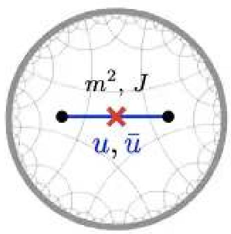
$\omega_{D/N}\left(u,\bar{u}\right)$ project onto Dirichlet/Neumann boundary conditions:

$m^2R^2=-\left(\Delta_+\Delta_-+J\right)$

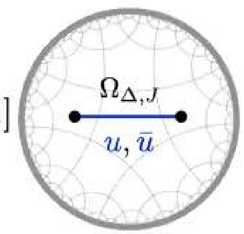
$\Delta_++\Delta_-=d\ ,\ \Delta_+\geq\Delta_-$

$$\omega_{D/N}\left(u,\bar{u}\right)=\frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_{\mp}-\tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_{\mp}-\tfrac{d}{2}\right)\right)\right)$$

On shell, the factor $\csc\left(\pi\left(u+\bar{u}\right)\right)$ gets cancelled:



$$=\left[\alpha\,\omega_D\left(u,\bar{u}\right)+\beta\,\omega_N\left(u,\bar{u}\right)\right]$$

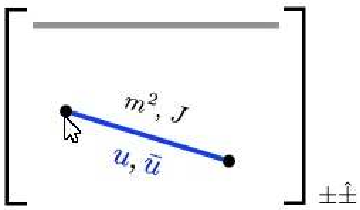


i.e. $\csc\left(\pi\left(u+\bar{u}\right)\right)$ is generated by the source term in the propagator equation.

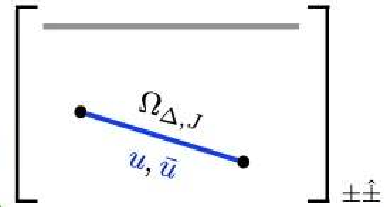
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In dS, for the $\pm\hat{\pm}$ branch of the in-in contour, we have:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

$m^2 R_{\rm dS}^2 = \left(\Delta_+\Delta_- + J\right),\ \Delta_+ + \Delta_- = d$

Recall the general solution to the wave equation near the boundary of dS, $\eta \to 0$

$$\varphi\left(\eta,\mathbf{k}\right) = \alpha_{\pm\hat{\pm}} \underbrace{\left(-\eta\right)^{\Delta_+}\left[\mathcal{O}_{\Delta_+}\left(\mathbf{k}\right) + O\left(\eta^2\right)\right]}_{\text{Selected by } \omega_D\left(u,\bar{u}\right)} + \beta_{\pm\hat{\pm}} \underbrace{\left(-\eta\right)^{\Delta_-}\left[\mathcal{O}_{\Delta_-}\left(\mathbf{k}\right) + O\left(\eta^2\right)\right]}_{\text{Selected by } \omega_N\left(u,\bar{u}\right)}$$

$$ds^2 = \left(\frac{R_{\rm dS}}{\eta}\right)^2\left(-d\eta^2 + d\mathbf{x}^2\right)$$

For the Bunch Davies (Euclidean) vacuum we have:

$$\alpha_{\pm\pm} = \beta_{\mp\mp} = \csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[-\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp} = -\beta_{\mp\pm} = \pm\csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right]$$

Bridging the Gap between EAdS and dS

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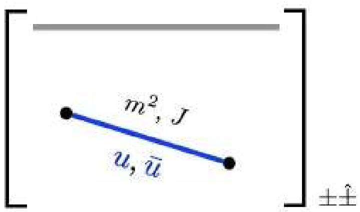
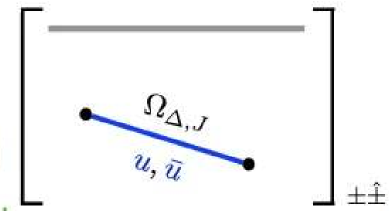


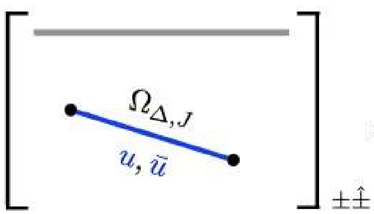
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



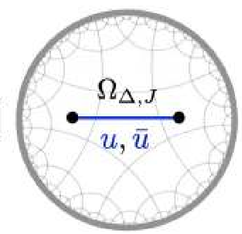
Harmonic function, $\left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0$

dS and EAdS Harmonic functions differ by a simple phase:

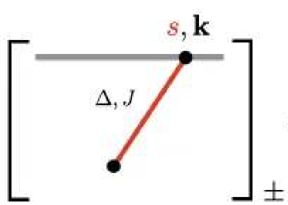
$m^2R_{\text{dS}}^2=\left(\Delta_+\Delta_-+J\right)$



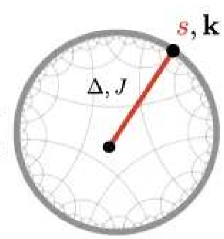
$$= \exp\left[\mp\left(u+\tfrac{1}{2}\left(\Delta_+-\tfrac{d}{2}\right)\right)\pi i\right]\exp\left[\hat{\mp}\left(\bar{u}+\tfrac{1}{2}\left(\Delta_--\tfrac{d}{2}\right)\right)\pi i\right]$$



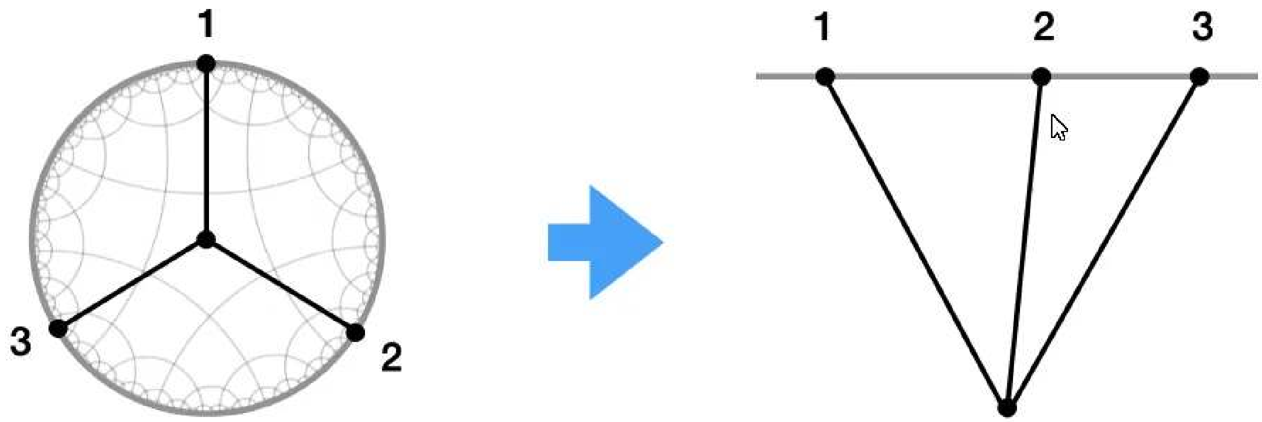
Also the bulk-boundary propagators:



$$= \exp\left[\mp\left(s+\tfrac{1}{2}\left(\Delta-\tfrac{d}{2}\right)\right)\pi i\right]$$

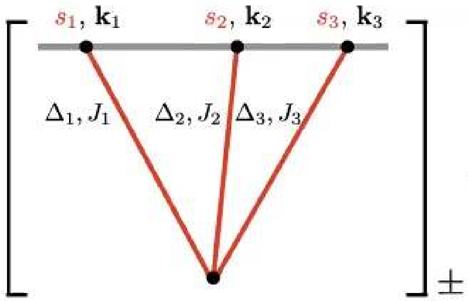


3pt Contact

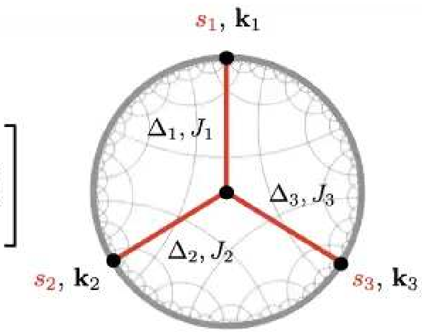


3pt Contact

Contact amplitudes in dS can be obtained directly from their EAdS counterparts:



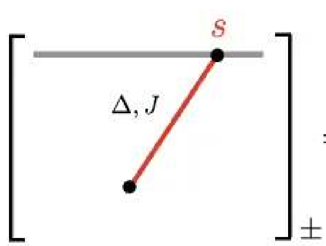
$$= \pm i \exp\left[\mp \pi i \sum_{j=1}^{3}\left(s_j + \tfrac{1}{2}\left(\Delta_j - \tfrac{d}{2}\right)\right)\right]$$



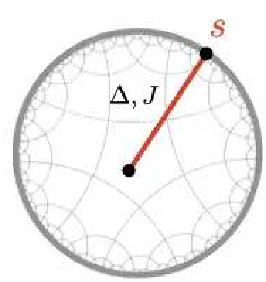
Overall phase is constant, as required by the Dilatation Ward identity, since:

$$s_1 + s_2 + s_3 = \frac{d + 2\left(J_1 + J_2 + J_3\right)}{4}$$

Above we simply used that:

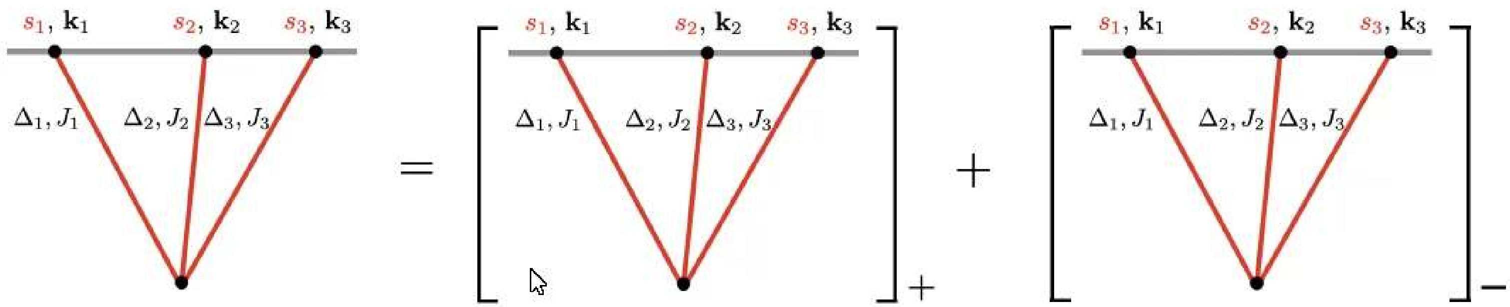


$$= \exp\left[\mp\left(s + \tfrac{1}{2}\left(\Delta - \tfrac{d}{2}\right)\right)\pi i\right]$$

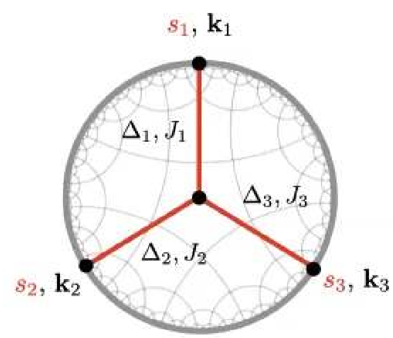


3pt Contact

The full de Sitter 3pt function is the sum from each branch of the in-in contour:

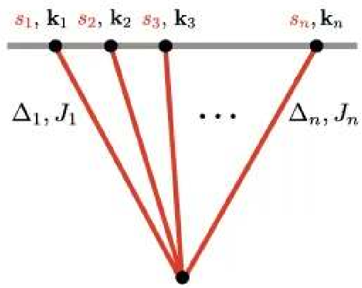


$$= \sin\left[\left(-d + \sum_{i=1}^{3} (\Delta_i + J_i)\right)\frac{\pi}{2}\right]$$

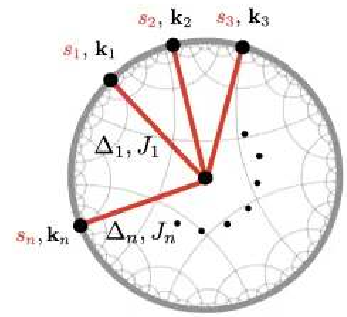


n ~~3~~pt Contact

The full de Sitter n ~~3~~pt function is the sum from each branch of the in-in contour:



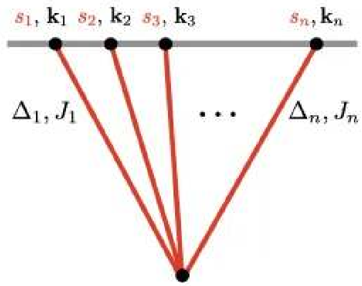
$$= \sin\left[\left(-d + \sum_{i=1}^{n} (\Delta_i + J_i)\right)\frac{\pi}{2}\right]$$



→ de Sitter contact diagrams can vanish when the sine factor has a zero!

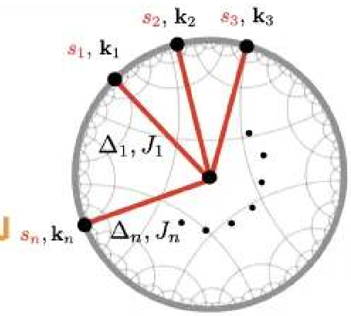
~~3~~ n pt Contact

The full de Sitter ~~3~~ n pt function is the sum from each branch of the in-in contour:



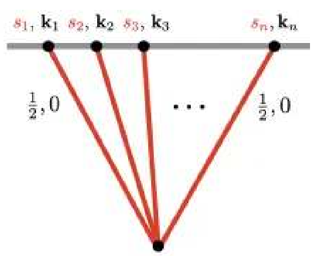
$$= \sin\left[\left(-d + \sum_{i=1}^{n} (\Delta_i + J_i)\right)\frac{\pi}{2}\right]$$

selected by unitarity

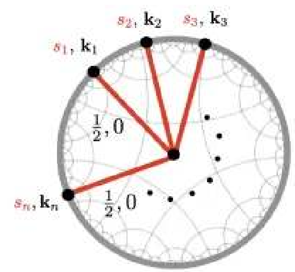


→ de Sitter contact diagrams can vanish when the sine factor has a zero!

E.g. conformally coupled scalars for d=3 $(\Delta_i = 1,\ J_i = 0)$:



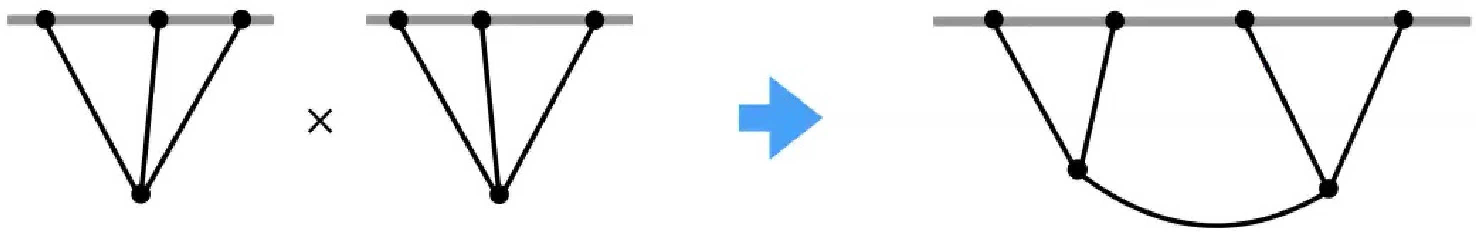
$$= \sin\left[\left(\frac{n-3}{2}\right)\pi\right]$$



$= 0$ for n odd!

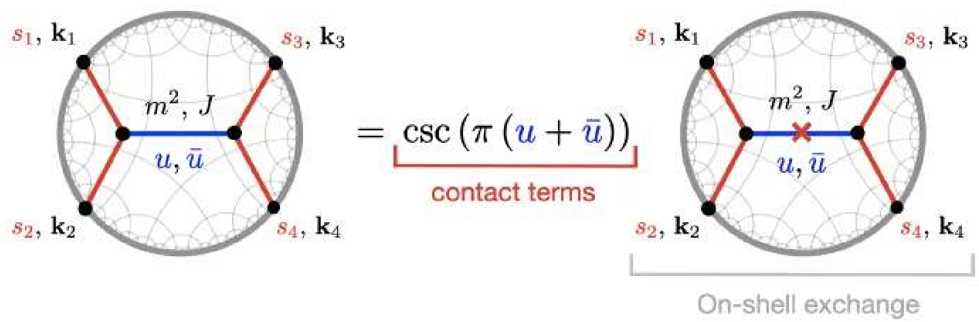
Shown to follow from unitarity by Goodhew, Jazayeri & Pajer - [Cosmological Optical Theorem, 2020]

Exchanges



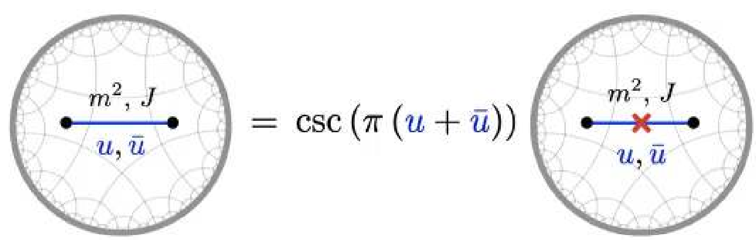
Exchanges in EAdS

Exchanges are straightforwardly reconstructed from their on-shell part:



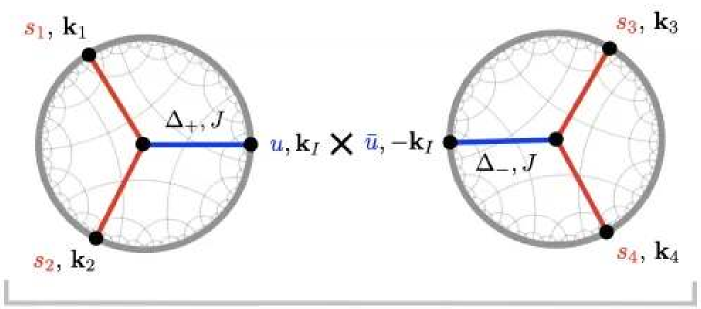
Constrained by:
- Factorisation
- Conformal Symmetry
- Boundary Conditions

Simply follows from:



Exchanges in EAdS

Factorisation and Conformal Symmetry:



"Conformal Partial Wave",
single valued Eigenfunction of Conformal Casimirs

Mack, Dobrev, Petkova, Petrova, Todorov, 1974-7

$$m^2 R^2_{\text{AdS}} = -(\Delta_+\Delta_- + J)$$

Exchanges in EAdS

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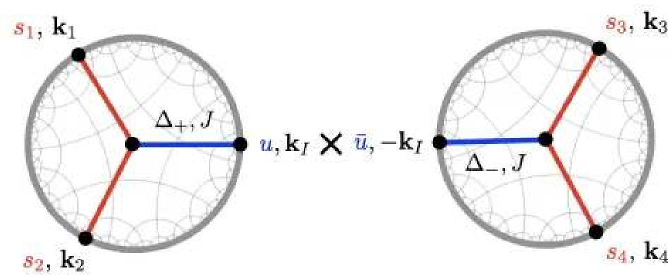

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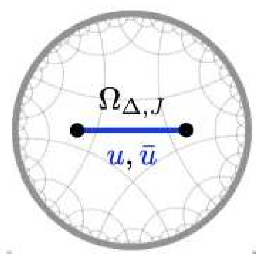
attach/remove external legs



Harmonic function, $(\nabla^2 - m^2)\,\Omega_{\Delta,J} = 0$

e.g. Leonhardt, Manvelyan, Rühl 2003;
Costa, Gonçalves, Penedones 2014

Exchanges in EAdS

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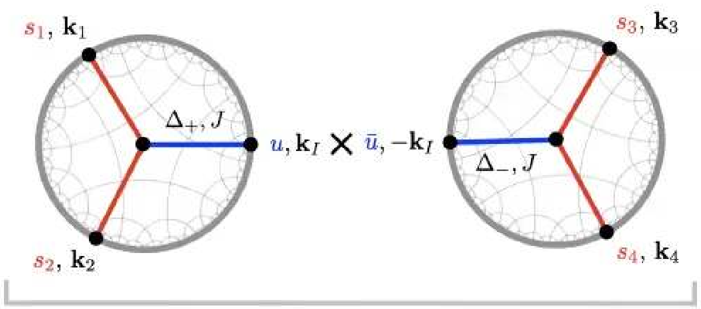

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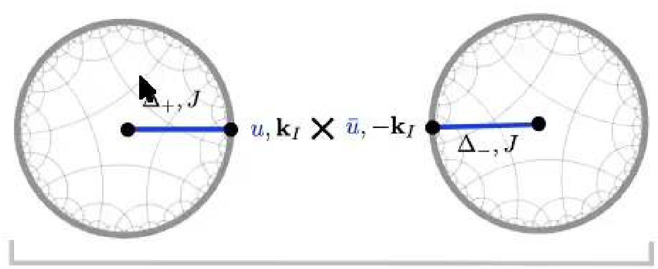


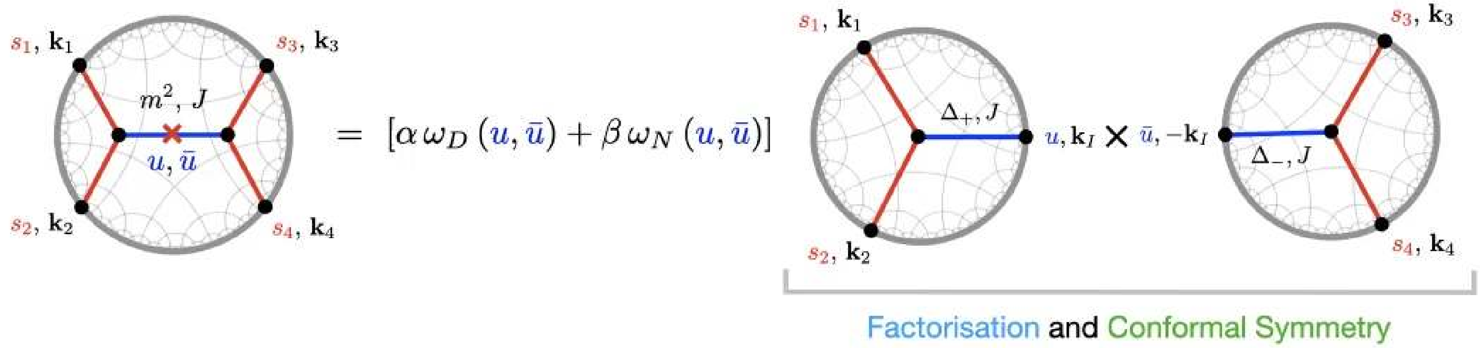
Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

This duality is made manifest by the "split representation" of $\Omega_{\Delta,J}$

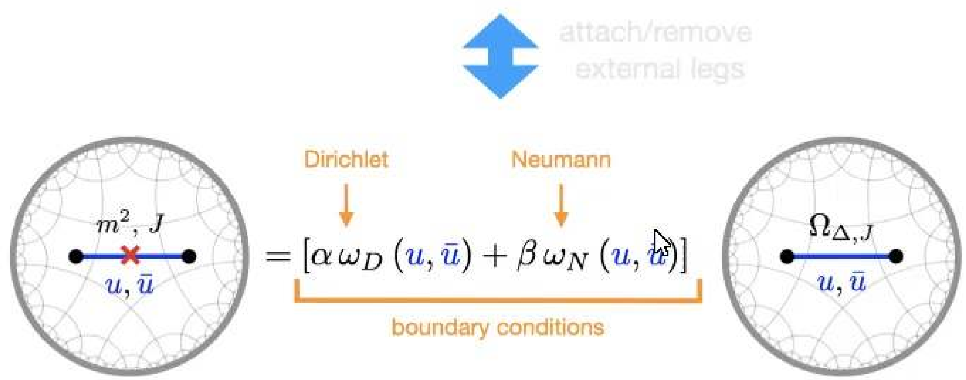
e.g. Leonhardt, Manvelyan, Rühl 2003;
Costa, Gonçalves, Penedones 2014

Exchanges in EAdS

Factorisation, Conformal Symmetry and boundary conditions:



$m^2 R^2_{\mathrm{AdS}} = -(\Delta_+\Delta_- + J)$



Exchanges in EAdS

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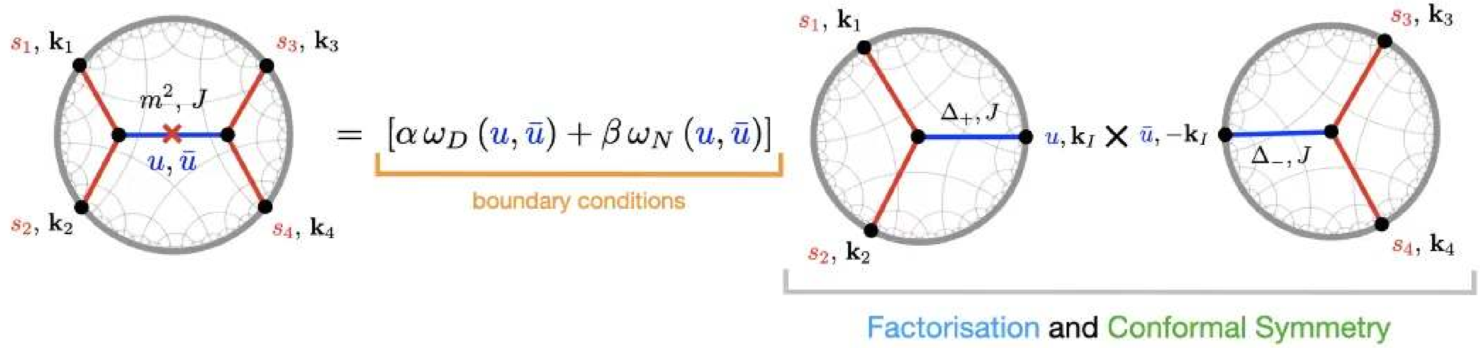
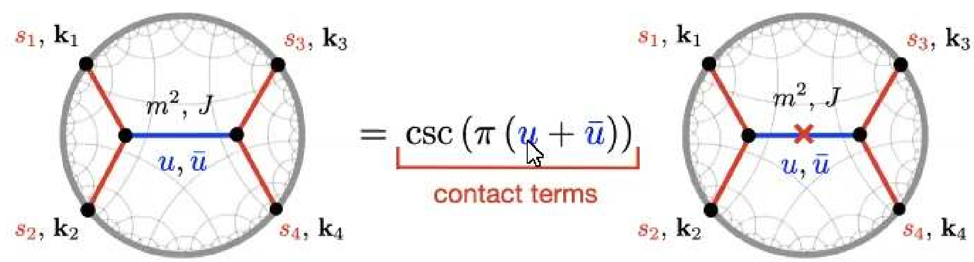
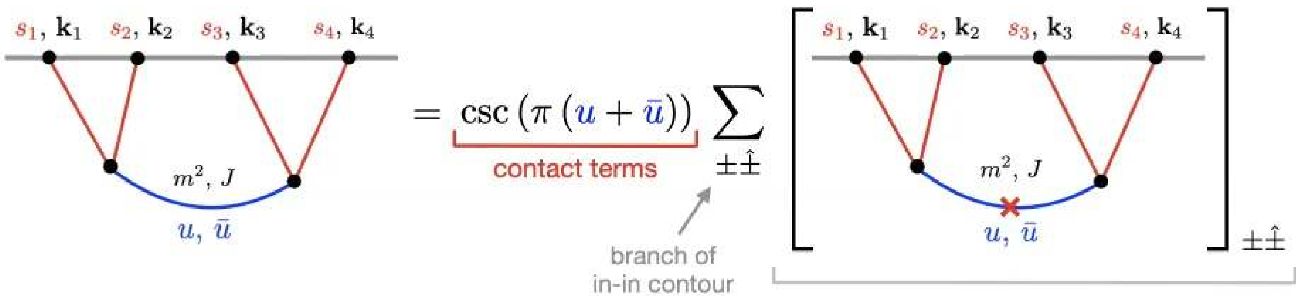


The full exchange is reconstructed via:

$m^2 R^2_{\text{AdS}} = -\left(\Delta_+\Delta_- + J\right)$



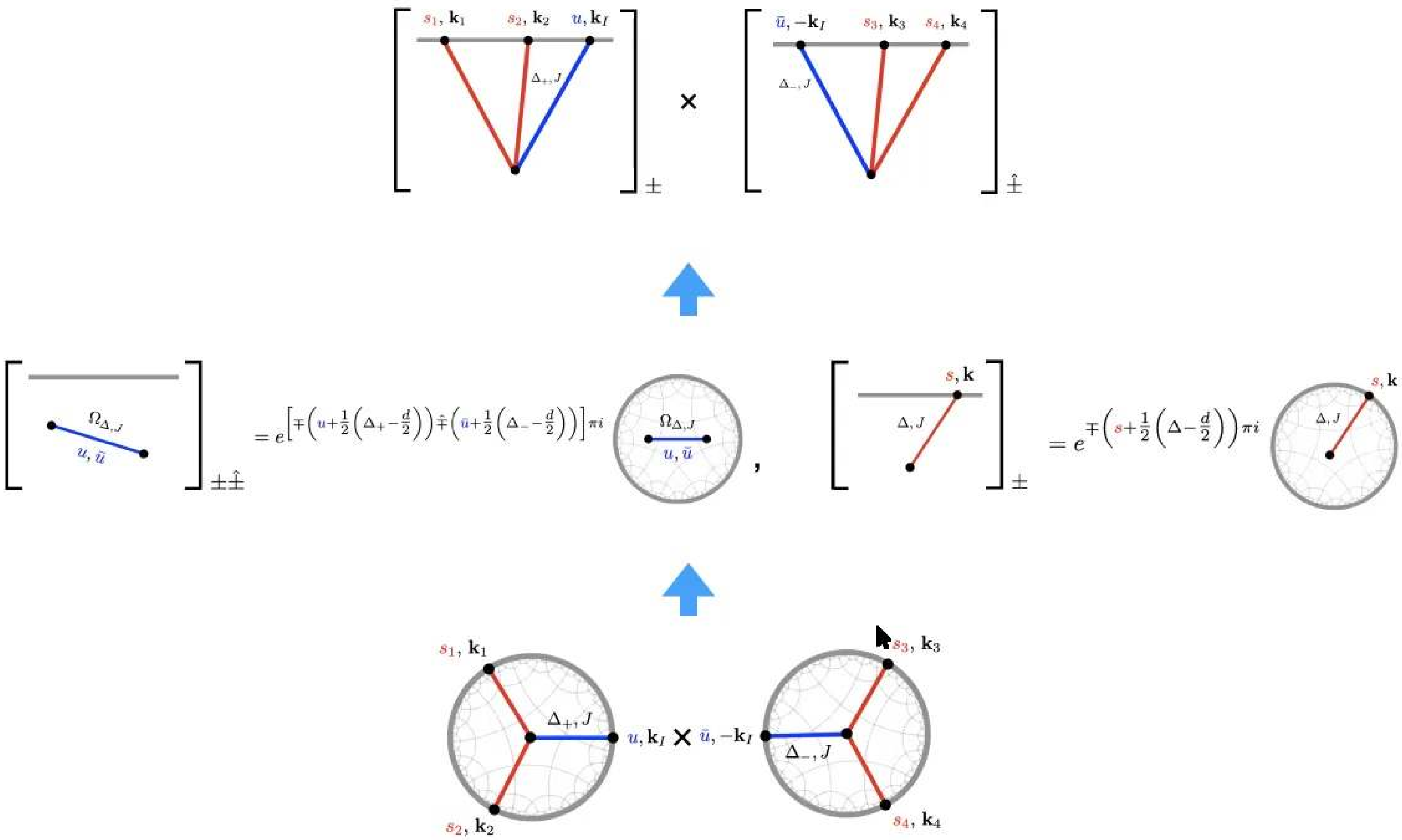
Exchanges in dS



Constrained by:
- Factorisation
- Conformal Symmetry
- Boundary Conditions

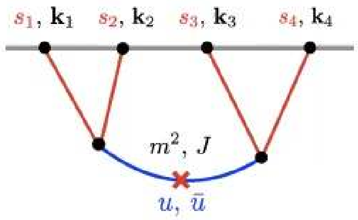
Exchanges in dS

Factorisation and Conformal Symmetry:

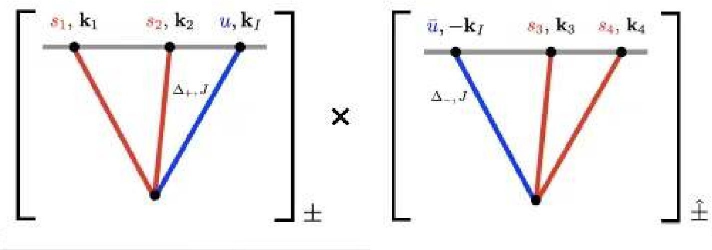


Exchanges in dS

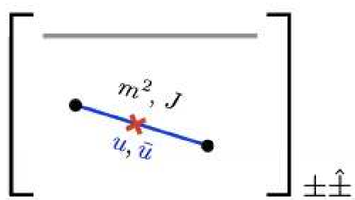
Factorisation, Conformal Symmetry and boundary conditions:



$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u, \bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u, \bar{u}\right)\right]$$



attach/remove external legs



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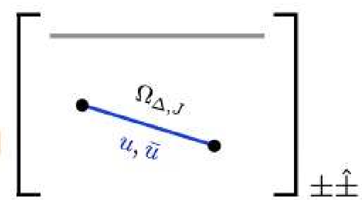
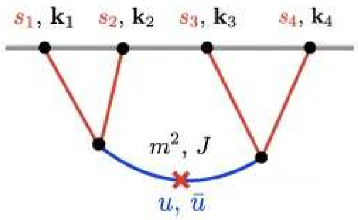


For the Bunch Davies (Euclidean) vacuum:

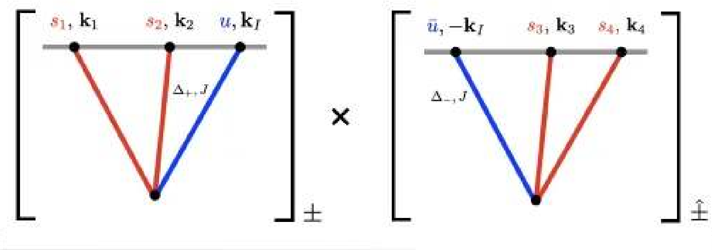
$$\alpha_{\pm\pm} = \beta_{\mp\mp} = \csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right) \exp\left[-\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp} = -\beta_{\mp\pm} = \pm\csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right) \exp\left[\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right]$$

Exchanges in dS

Factorisation, Conformal Symmetry and boundary conditions:



$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\,(u,\bar{u}) + \beta_{\pm\hat{\pm}}\, \omega_N\,(u,\bar{u})\right]$$



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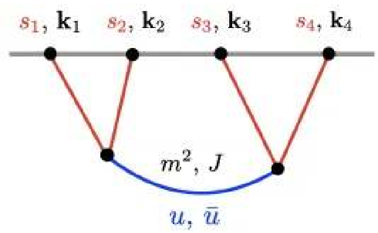
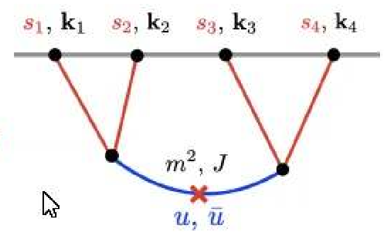


$$= \csc\left(\pi\left(u+\bar{u}\right)\right)$$

contact terms



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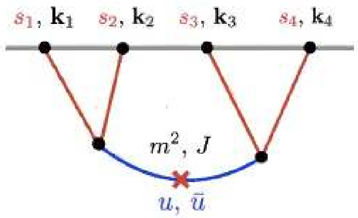
Exchanges in (A)dS

Factorisation, Conformal Symmetry and boundary conditions:



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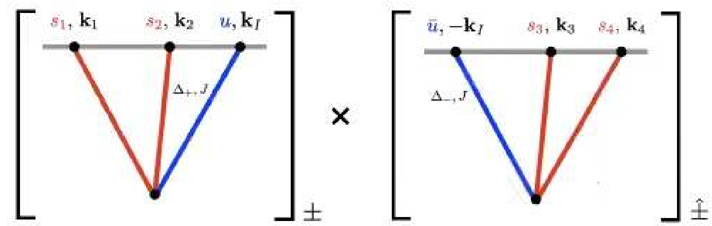
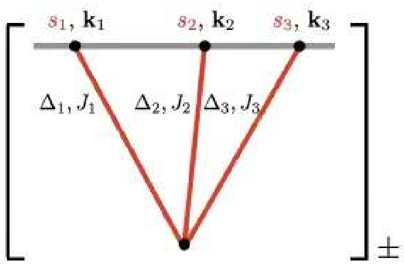
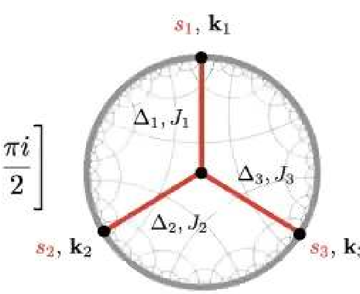


Factorisation + Conformal Symmetry

The bridge to the EAdS exchanges is via:



$$= \pm i \exp\left[\mp\left(-d + \sum_{k=1}^{3}(\Delta_k + J_k)\right)\frac{\pi i}{2}\right]$$

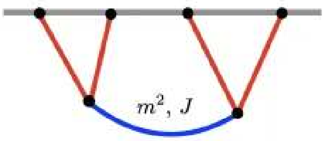


For the Bunch Davies (Euclidean) vacuum:

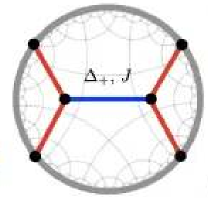
$$\alpha_{\pm\pm} = \beta_{\mp\mp} = \csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[-\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp} = -\beta_{\mp\pm} = \pm\csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right]$$

Exchanges in (A)dS

dS exchange in the Bunch-Davies vacuum is a linear combination of AdS exchanges:



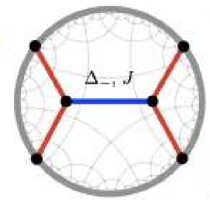
$$= \sin\left(\frac{\pi}{2}\left(-d+\Delta_+ + J + \sum_{k=1}^{2}(\Delta_k+J_k)\right)\right)\sin\left(\frac{\pi}{2}\left(-d+\Delta_+ + J + \sum_{k=3}^{4}(\Delta_k+J_k)\right)\right)$$



Dirichlet $\omega_D(u,\bar{u})$

Change in 3pt function (OPE) coefficient, selected by unitarity

$$+ \sin\left(\frac{\pi}{2}\left(-d+\Delta_- + J + \sum_{k=1}^{2}(\Delta_k+J_k)\right)\right)\sin\left(\frac{\pi}{2}\left(-d+\Delta_- + J + \sum_{k=3}^{4}(\Delta_k+J_k)\right)\right)$$



Neumann $\omega_N(u,\bar{u})$

This identity can be used to directly import techniques and results from AdS to dS!

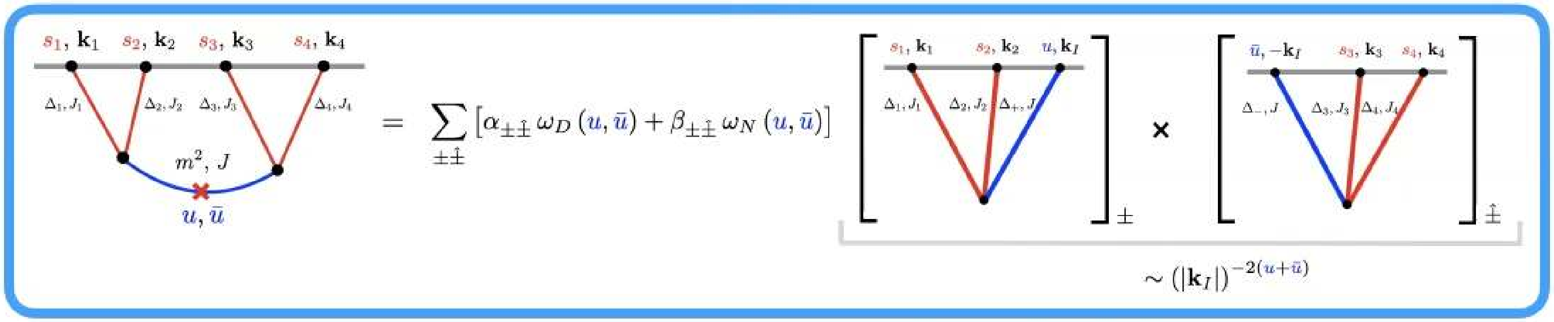
Some small steps in 2007.09993 [hep-th]:

- AdS exchanges are basic solutions to the crossing equation (associativity of operator algebra)

→ **dS exchanges are also solutions to crossing**. Their decomposition into conformal blocks (in all channels) is inherited from those of AdS exchanges (which are known)

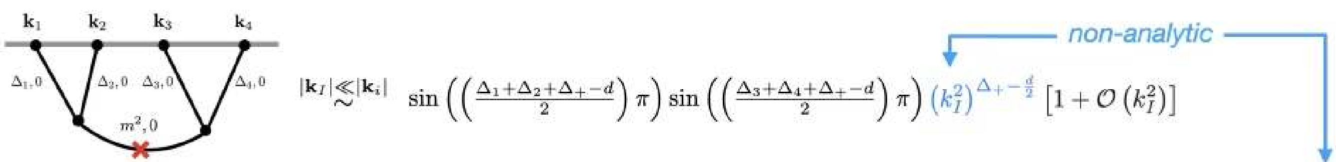
- **Mellin amplitudes for dS correlators**. For AdS, Mellin amplitudes have been an instrumental tool owing to striking parallels with scattering amplitudes — Mack 2009, Penedones 2010

Exchanges in dS



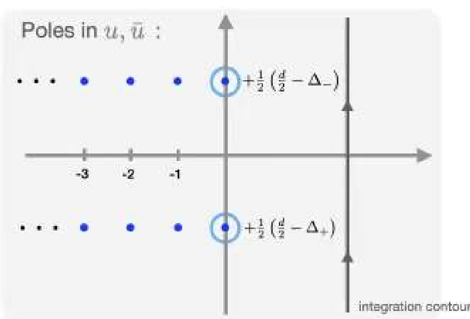
The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)

The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If all the fields are **scalars**:



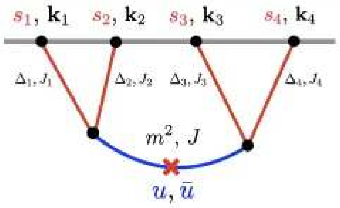
$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right)(k_I^2)^{\Delta_+-\frac{d}{2}}\left[1+\mathcal{O}(k_I^2)\right]$$

$$+\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right)(k_I^2)^{\Delta_--\frac{d}{2}}\left[1+\mathcal{O}(k_I^2)\right]+\dots$$

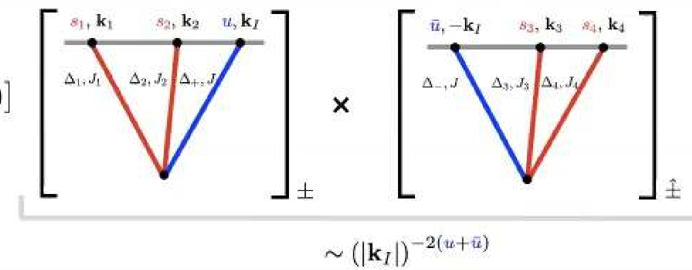


External conformally coupled/massless scalars:
Arkani-Hamed and Maldacena 2015;
Arkani-Hamed, Baumann, Lee and Pimentel 2018

Exchanges in dS



$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D(u,\bar{u}) + \beta_{\pm\hat{\pm}}\, \omega_N(u,\bar{u})\right]$$



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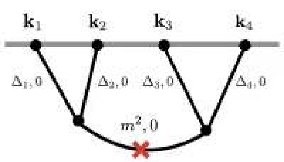
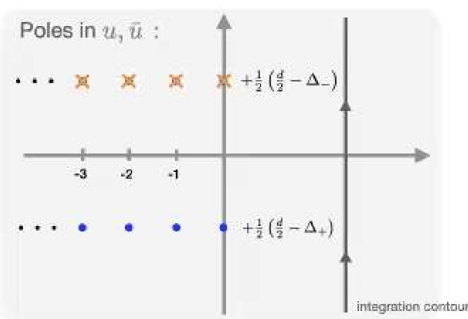


non-analytic

$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right) (k_I^2)^{\Delta_+-\frac{d}{2}} \left[1+\mathcal{O}(k_I^2)\right]$$

$$+ \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right) (k_I^2)^{\Delta_--\frac{d}{2}} \left[1+\mathcal{O}(k_I^2)\right] + \ldots$$



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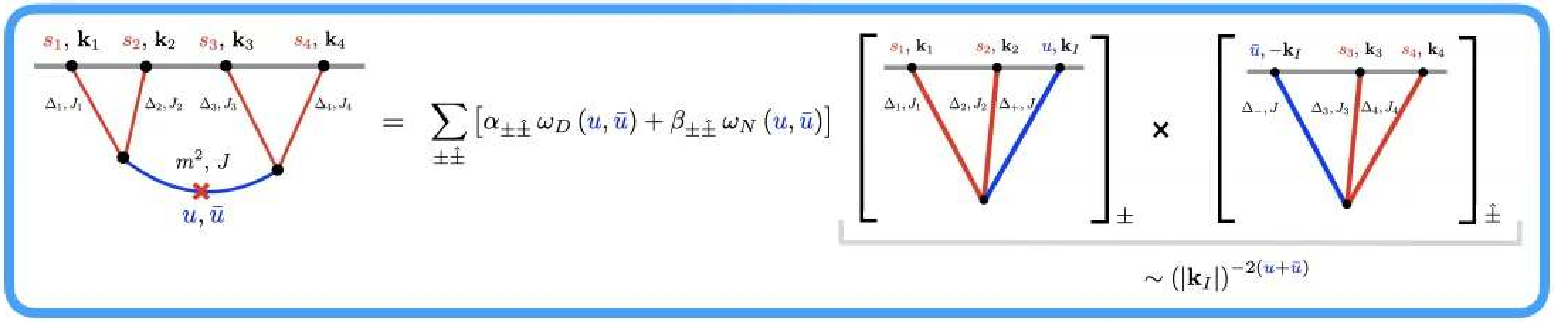

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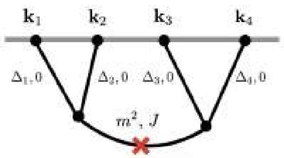
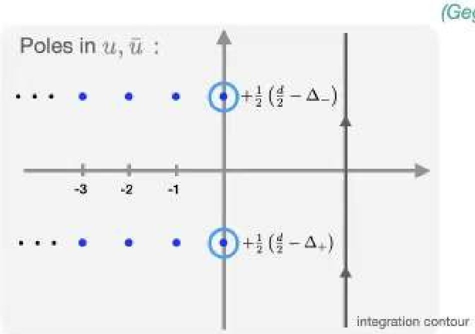


$$\overset{|\mathbf{k}_I|\ll|\mathbf{k}_i|}{\sim} C_J^{(\frac{d-2}{2})}(\cos\theta)\left[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right)(k_I^2)^{\Delta_+-\frac{d}{2}} + \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right)(k_I^2)^{\Delta_--\frac{d}{2}}\right] + \dots$$

non-analytic

angular dependence, (Gegenbauer polynomial)



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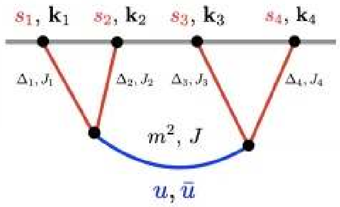
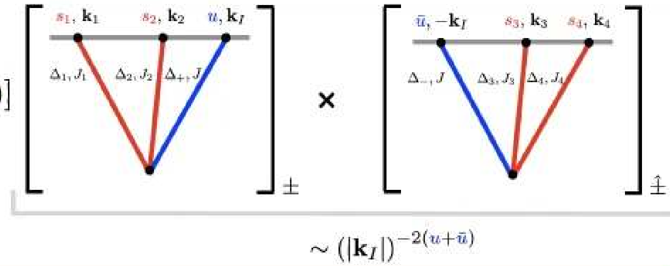
Exchanges in dS



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$



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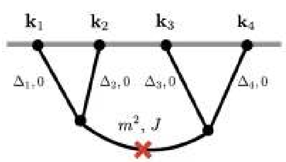

$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} C_J^{\left(\frac{d-2}{2}\right)}(\cos\theta) \Bigg[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right) (k_I^2)^{\Delta_+-\frac{d}{2}}$$

$$+ \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right) (k_I^2)^{\Delta_--\frac{d}{2}} \Bigg]$$

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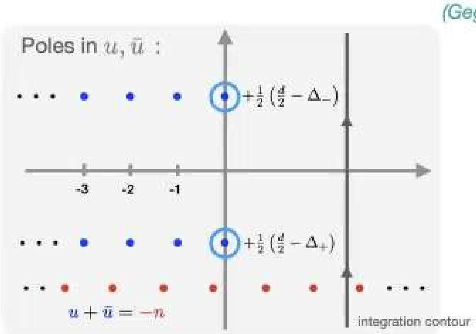
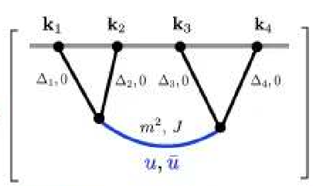


$$+ \sum_{n=0}^{\infty} \mathrm{Res}_{u+\bar{u}=-n} \left[\quad \right] \qquad + \dots$$



analytic $\propto (k_I^2)^n$

$\sim H^2/m^2$, EFT expansion

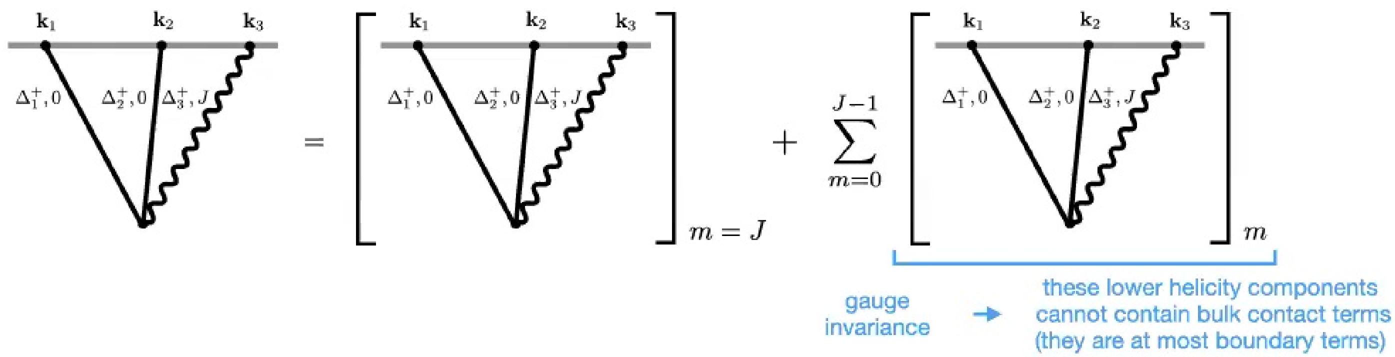
External conformally coupled/massless scalars: Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018

Constraints on Massless Particles

Toy model: Cubic coupling of a massless spin-J field to scalars.

$\Delta_3^+ = d - 2 + J$

Decomposition into helicities m = 0, 1, …, J :



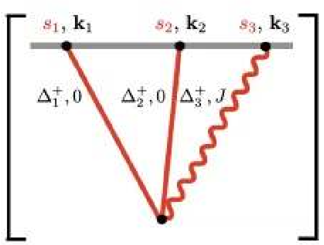
Constraints on Massless Particles

Toy model: Cubic coupling of a massless spin-J field to scalars.

$\Delta_3^+ = d - 2 + J$

Recall that bulk contact singularities are encoded in Dirac delta functions in the external Mellin variables.

Gauge invariance requires that for the lower helicity components m < J we *must have*:



This factor cancels the bulk contact singularity encoded by the Dirac delta function

$$\propto \left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right) \delta\left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)$$

$$\lim_{\eta_0 \to 0} \int_{-\infty}^{\eta_0} d\eta \, \partial_\eta \left(\eta^{\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3}\right) = \lim_{\eta_0 \to 0} \left(\eta_0^{\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3}\right)$$

This is a boundary term.

Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✔

Scalars of unequal mass ✘

(Consistent with Berends, Burgers and van Dam 1986)

Constraints on Massless Particles

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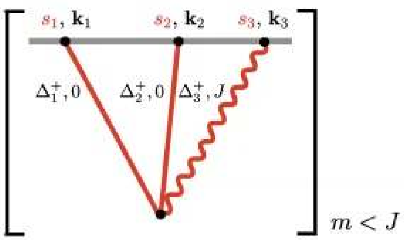

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Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✔

A non-trivial Ward-Takahashi identity is generated by the finite number of poles that satisfy:

$$\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3 = 0$$

which are: $s_1 = \pm\frac{1}{2}\left(\Delta - \frac{d}{2}\right) - n_1,\quad s_2 = \mp\frac{1}{2}\left(\Delta - \frac{d}{2}\right) - n_2,\quad s_3 = \frac{1}{2}\left(\Delta_3^+ - \frac{d}{2}\right) - n_3,\quad n_i \in \mathbb{N}$

with $(n_1 + n_2 + n_3) = (J - 1 - m)$

Constraints on Massless Particles

Toy model: Cubic coupling of a massless spin-J field to scalars.

$\Delta_3^+ = d - 2 + J$

Recall that bulk contact singularities are encoded in Dirac delta functions in the external Mellin variables.

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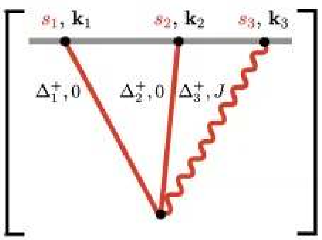

$m < J$

This factor cancels the bulk contact singularity encoded by the Dirac delta function

$$\propto \left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right) \delta\left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)$$

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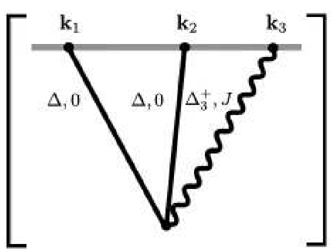
Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✓

A non-trivial Ward-Takahashi identity is generated by the finite number of poles that satisfy:

$$\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3 = 0$$

For example:



$m = J - 1$

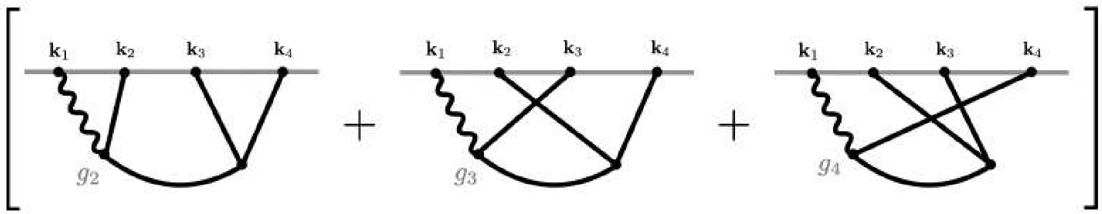
$$\sim \quad (\xi \cdot \mathbf{k}_{12})^{J-1} \langle \mathcal{O}_\Delta(\mathbf{k}_1) \mathcal{O}_\Delta(-\mathbf{k}_1) \rangle - (\xi \cdot \mathbf{k}_{12})^{J-1} \langle \mathcal{O}_\Delta(\mathbf{k}_2) \mathcal{O}_\Delta(-\mathbf{k}_2) \rangle$$

where $\xi \cdot \xi = 0, \quad \xi \cdot \mathbf{k}_3 = 0$

Constraints on Massless Particles

Toy model: Cubic coupling of a massless spin-J field to scalars.

$\Delta_3^+ = d - 2 + J$

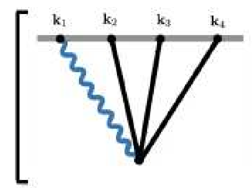


~ [Ward-Takahashi identity] + [Bulk contact singularities $E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4| \to 0$]

Comes from the on-shell exchange, inherited from the gauge invariant 3pt functions

Violates gauge invariance. *Can they be compensated by:*

Some singularities in E_T cannot be cancelled! (by local quartic vertices)



These singularities must therefore cancel by themselves → constrains g_i

The helicity-(J-1) component this gives the constraint:

$$\left[\sum_{i=2}^{4} g_i \, (\xi \cdot \mathbf{k}_i)^{J-1}\right] (E_T)^{\#} = 0$$

cf. Weinberg soft theorem in flat space!

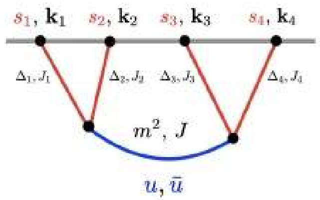
$J = 1: \; g_2 + g_3 + g_4 = 0,$ *charge conservation*

$J = 2: \; g_2 = g_3 = g_4,$ *equivalence principle*

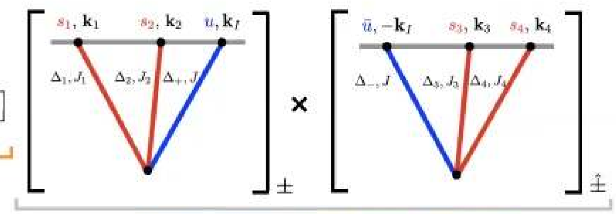
See also Baumann et al. May 2020

$J > 2: \; g_2 = g_3 = g_4 = 0,$ *no consistent coupling (in local theories)*

Summary



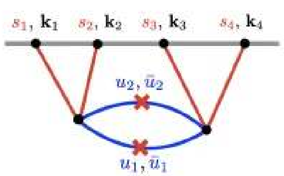
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{EFT}} \underbrace{\sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



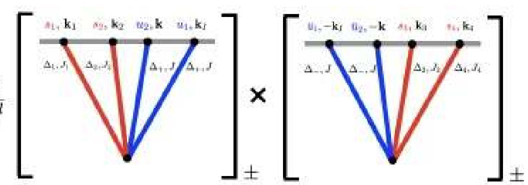
Factorisation + Conformal Symmetry

Plenty of diverse directions for the future!

- Higher points and Loops. Nice parallel with generalised unitarity methods/Cutkosky rules:



$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u_1,\bar{u}_1\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u_1,\bar{u}_1\right)\right] \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u_2,\bar{u}_2\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u_2,\bar{u}_2\right)\right] \int \frac{d^d\mathbf{k}}{(2\pi)^d}$$



- Bootstrap of Euclidean CFTs dual to dS physics?

- Celestial Amplitudes?

⋮