Title: Black hole induced false vacuum decay from first principles

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Series: Particle Physics

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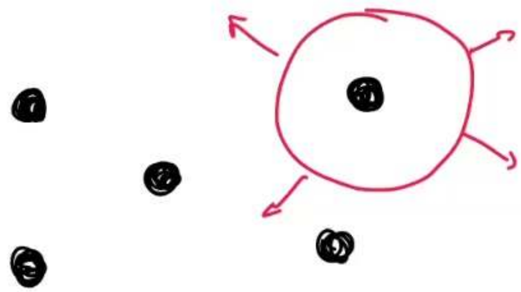
Abstract: It has been proposed that microscopic black holes can catalyze decay of metastable false vacuum. The calculations of the decay rate existing in the literature make use of the Euclidean time formalism developed for equilibrium configurations. This is not the case, however, for a realistic black hole formed by gravitational collapse and emitting Hawking radiation. I will review the motivations to study black hole catalysis of vacuum decay, propose a general method to calculate the decay rate, and illustrate it on a two-dimensional toy model.

Black hole catalysis of vacuum decay from first principles

Sergey Sibiryakov

w/ Andrey Shkerin



Question and history

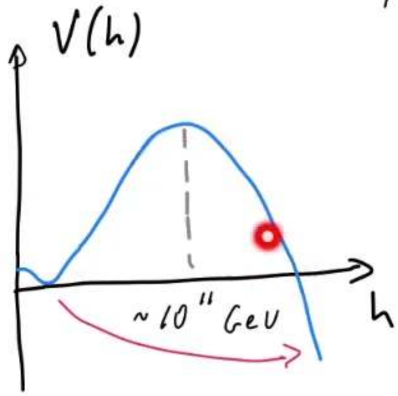
How does presence of a black hole affect the rate of false vacuum decay?

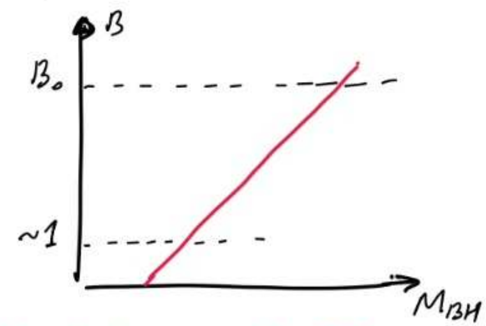
Hiscock (1987), Arnold (1990)
Berezin, Kuzmin, Tkachev (1988, 91)

Studied various thin shell solutions in Euclidean, interpreted as tunneling bubbles
Generally found catalysis, but not always clear if solutions make sense

Gregory, Moss, Withers (2014)
Burda, Gregory, Moss (2015)

$\Gamma \sim e^{-B}$ → in empty space $B_0 \sim 10^3$
$\Gamma \sim e^{-B}$ → with a black hole

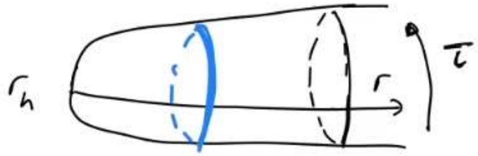




Not a single evaporating BH in our past!

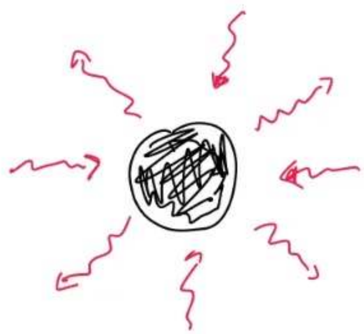
Caveat

Euclidean bounce = thermal equilibrium



$T_{BH} \propto \frac{1}{M_{BH}}$ grows to Planckian

at high temperature all transitions are unsuppressed

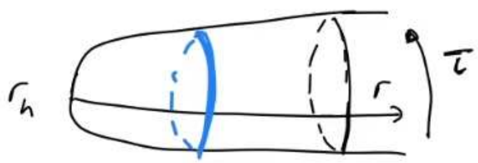


Hartle-Hawking vacuum

Caveat

$$P \sim e^{-E_{sph}/T}$$

Euclidean bounce = thermal equilibrium



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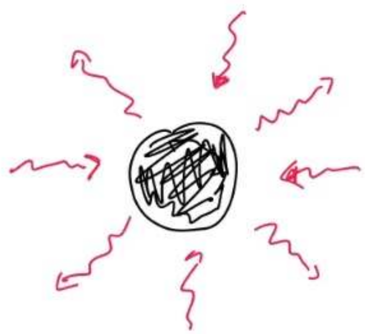


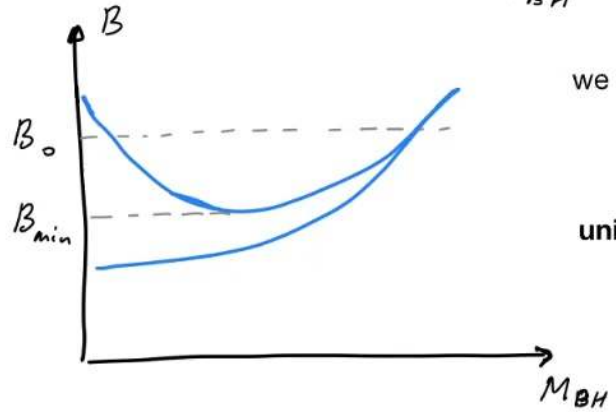
Hartle-Hawking vacuum

Realistic black hole

BH radiates, but nothing comes back

Unruh vacuum

size of hot region $\sim R_{BH} \sim \frac{1}{T_{BH}}$ $\Rightarrow$ bubble does not fit in



we want to know B_{min}

Gorbunov, Levkov, Panin (2017)

universe is big:

$N_{BH} \sim 10^{40}$

Other reasons

- non-perturbative non-local process

  $\Rightarrow$ teaches us smth about QFT in curved geommetry

- semiclassical description can potentially take into account gravity

  $\Rightarrow$ teaches as smith about semiclassical quantum gravity
  (cf. Coleman-De Lucia / Hawking-Moss instanton)

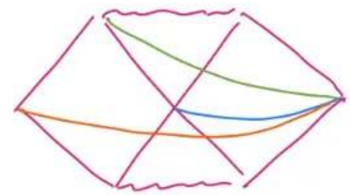
- the BH mass can change in the process *(increase / decrease?)*

  $\Rightarrow$ teaches us smth about BH entropy / information paradox

(Not so) technical puzzles

- ***where does bounce live?*** cannot be Euclidean time
- ***what quantization surface we shall choose for the field?***
  does tunneling depend on what's going on inside BH?

complete BH spacetime includes interior
and another asymptotic region...



- ***what are the conditions ensuring that bounce starts from Unruh vacuum?***

$\Longrightarrow$ **Address these questions in external BH metric**

N.B. change of BH mass is irrelevant as long as $R_{bubble} \ll t_{evap}$

Γ_{decay} can be bigger or smaller than t_{evap}^{-1}

Go to the basics

decay amplitude

near false vacuum

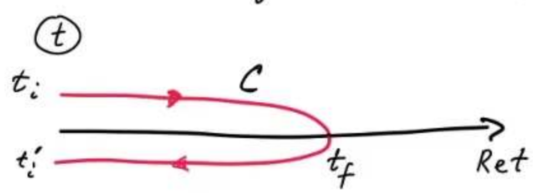
$$\langle f|i\rangle = \int D\varphi\, D\varphi_f\, D\varphi_i\, \langle f|\varphi_f, t_f\rangle\, e^{iS[\varphi]}\, \langle \varphi_i, t_i|\, i\rangle$$

near true vacuum

decay probability

$$\mathcal{P}_{decay} = \sum_f \langle i|\overbrace{f\rangle\langle f}^{\text{projector on true vacuum}}|i\rangle$$

$$= \int D\varphi_c\, D\varphi_i\, D\varphi_i'\, \langle i|\varphi_i', t_i'\rangle\, e^{iS[\varphi_c]}\, \langle \varphi_i t_i|i\rangle$$



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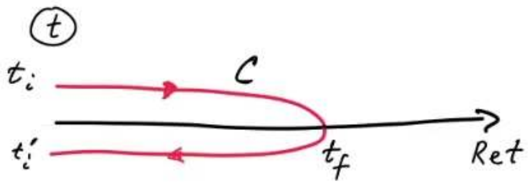
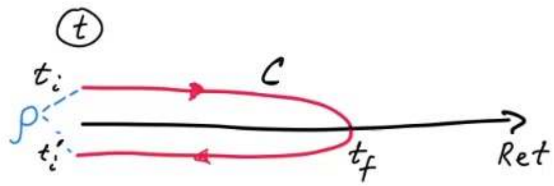


$|i\rangle\langle i|$

for mixed states

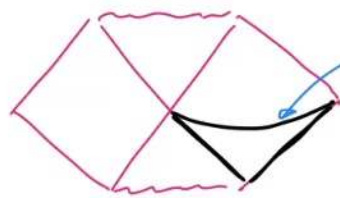


density matrix

$$\mathcal{P}_{decay} = \int D\varphi_c \, D\varphi_i \, D\varphi_i' \, e^{iS[\varphi_c]} \underbrace{\langle \varphi_i \, t_i | \rho | \varphi_i' \, t_i' \rangle}_{\text{sets the boundary conditions at the ends of the contour}}$$

bounce is the saddle-point,
lives on the in-in contour *C*

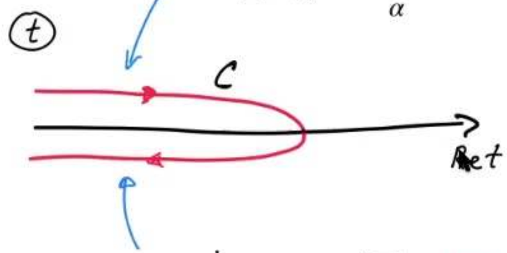
We can stay outside BH: just treat H-H and Unruh vacua as mixed states



quantization surface

Boundary conditions

$$\varphi^{\mathrm{up}}\Big|_{t\to-\infty} = \sum_{\alpha}\left(c_{\alpha}^{\mathrm{up}} f_{\alpha}(x) e^{-i\omega_{\alpha}t} + \bar{c}_{\alpha}^{\mathrm{up}} f_{\alpha}^{*}(x) e^{i\omega t}\right)$$



$$\varphi^{\mathrm{low}}\Big|_{t\to-\infty} = \sum_{\alpha}\left(c_{\alpha}^{\mathrm{low}} f_{\alpha}(x) e^{-i\omega_{\alpha}t} + \bar{c}_{\alpha}^{\mathrm{low}} f_{\alpha}^{*}(x) e^{i\omega t}\right)$$

for empty vacuum: $c_{\alpha}^{up} = \bar{c}_{\alpha}^{low} = 0$ $\quad (*)$

for thermal bath: $c_{\alpha}^{up} = c_{\alpha}^{low} e^{-\frac{\omega_\alpha}{T}}, \quad \bar{c}_{\alpha}^{low} = \bar{c}_{\alpha}^{up} e^{-\frac{\omega_\alpha}{T}}$ $\quad (**)$

for Unruh vacuum: $\begin{cases} \text{cond. } (*) \text{ for infalling modes} \\ \text{cond. } (**) \text{ for outgoing modes} \end{cases}$

A twist

Consider the generating functional for time-ordered Green's function in the in-in formalism

$$G_T = \langle T\left(\varphi(t)\,\varphi(0)\right)\rangle_\rho$$

$$Z[J] = \int D\varphi_c \, D\varphi_i \, D\varphi_i' \, e^{iS[\varphi_c] + iJ\cdot\varphi_c} \underbrace{\langle \varphi_i\, t_i \,|\, \rho \,|\, \varphi_i', t_i' \rangle}_{\text{same BCs as for the bounce!}}$$

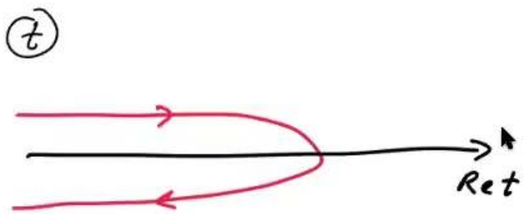
$\Longrightarrow$ BCs are automatically implemented if we take the right Green's function

$$\Box \varphi_b - m^2 \varphi_b - \frac{\partial V_{int}}{\partial \varphi} = 0 \quad \Longrightarrow \quad \underbrace{\varphi_b = -i\, G_T * \frac{\partial V_{int}}{\partial \varphi}}_{} \qquad (***)$$

can be easier to solve if V_{int} is non-zero only in a small region $\ll \frac{1}{m}$

Summary I

We need to solve classical field eqs. on a contour in complex time plane



with BC's on the mode decomposition

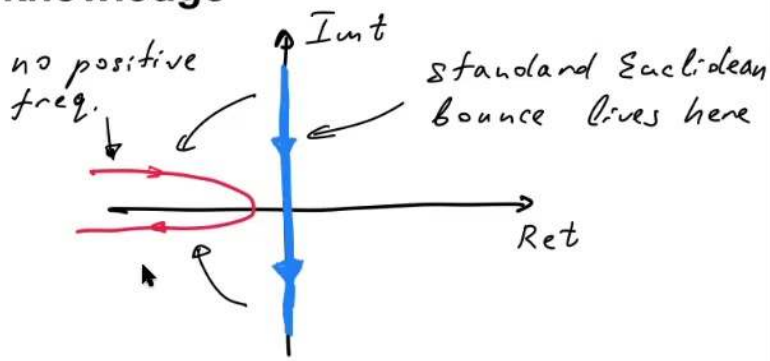
or

the integral eq. $(***)$

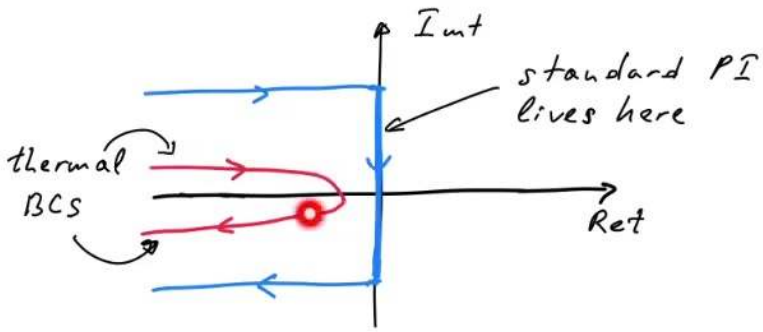
$$\Gamma \sim e^{-B}, \quad B = -i\, S[\varphi_{bounce}]$$

Contact with prior knowledge

vacuum bounce



periodic instantons



N.B. Can also treat formation of sphaleron (over-barrier jumps) at high temperature

In general, no useful Euclidean picture

$\Rightarrow$ hard numerical problem

seems the case for Unruh vacuum

$\Rightarrow$ **go to (1+1) dimensions!**

“A man grows stale if he works all the time on insoluble problems, and a trip to the beautiful world of one dimension will refresh his imagination better than a dose of LSD.”

Freeman Dyson

The toy model

dilaton black hole

$$ds^2 = \Omega(x)\left(-dt^2 + dx^2\right)$$

$$\Omega(x) = \frac{1}{1+e^{-2\lambda x}}$$

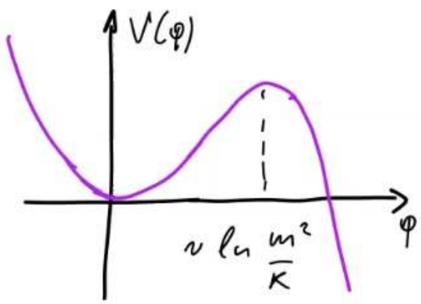
temperature

$$T_{BH} = \frac{\lambda}{2\pi}$$

inverted Liouville potential with a mass

$$S = \frac{1}{\alpha^2}\int d^2x \sqrt{-g}\left(-\frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right)$$

$$V(\varphi) = \frac{m^2\varphi^2}{2} - 2\kappa\left(e^{\varphi}-1\right)$$



assume : $\boxed{\ln \frac{m}{\sqrt{\kappa}} \gg 1}$

N.B. We do not add non-minimal coupling to gravity

$$\Box \varphi - m^2 \Omega \varphi + 2\kappa \Omega e^{\varphi} = 0$$

- at short distances $|\Delta x| \ll \frac{1}{m}$ and if $(\ln \Omega)'' = 0 \implies$ exactly solvable

$$\varphi = \ln \left[\frac{4 f_1'(u) f_2'(v)}{\left(1 + \kappa f_1(u) f_2(v)\right)^2} \right]$$

$u = t - x$
$v = t + x$

when $\kappa f_1 \cdot f_2 \gg 1 \implies$ becomes solution of massless free eq.

- also at $\Delta x \ll \frac{1}{m}$ the Green's function becomes massless

$$G_T = \ln g_1(u) + \ln g_2(v) + \text{const}$$

$$g_2(v) = \begin{cases} m v & \textbf{for modes in vacuum (Unruh)} \\ \operatorname{sh} \frac{\lambda v}{2} & \textbf{for thermal modes (H-H)} \end{cases}$$

we can make $f_{1,2}$ to match these

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crucial for bounce action

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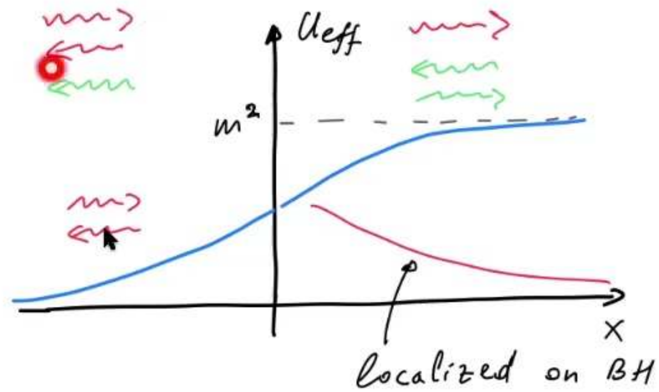
On modes and constants

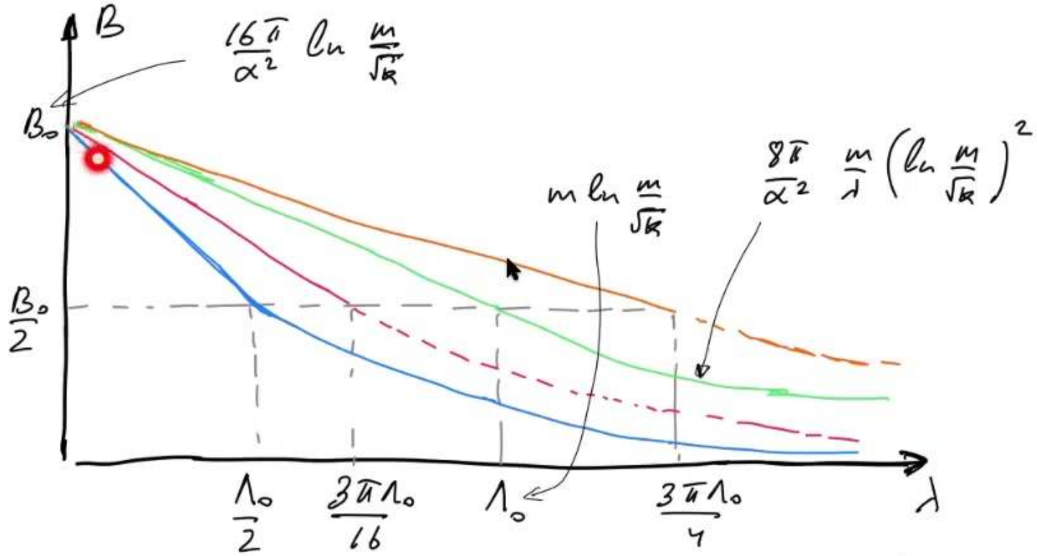
$$-f'' + m^2 \Omega f = \omega^2 f$$



const gets contributions from soft modes with $\omega \sim m \ll \lambda$

$$const_{HH} = \begin{cases} \dfrac{\lambda}{2\pi m} \\[2ex] \dfrac{\lambda}{4\pi m} \end{cases} \qquad const_{u} = \begin{cases} \dfrac{4\lambda}{3\pi^2 m} & \textbf{near horizon} \\[2ex] \dfrac{\lambda}{3\pi^2 m} & \textbf{far from BH} \end{cases}$$

Results



alternative estimate at large λ: $P_{decay} \sim e^{-\frac{\varphi_{max}^2}{2\langle\delta\varphi^2\rangle}}$

$\langle\delta\varphi^2\rangle \sim \alpha^2 \cdot \text{const} \sim \# \alpha^2 \frac{\lambda}{m}$

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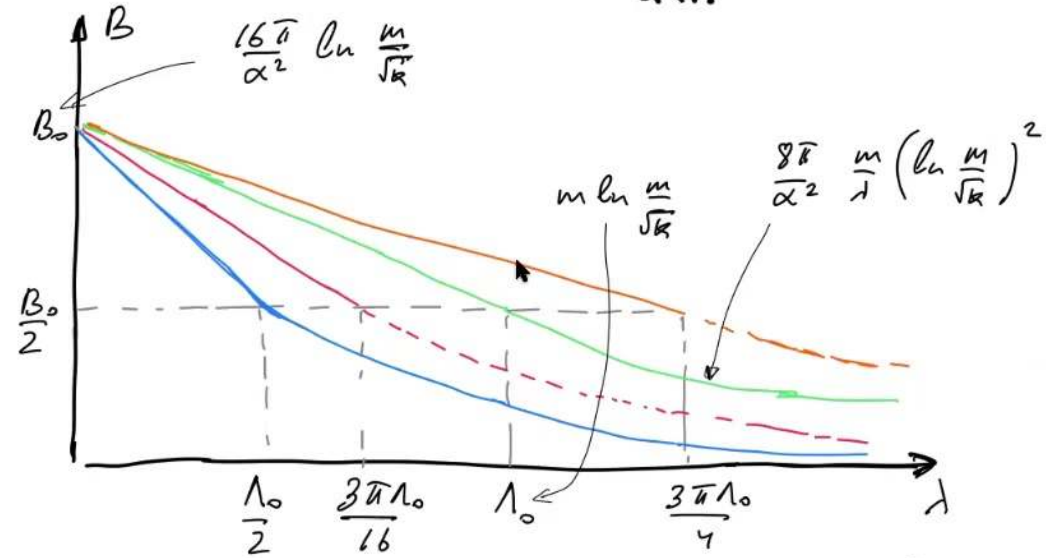
$\alpha \ll 1$



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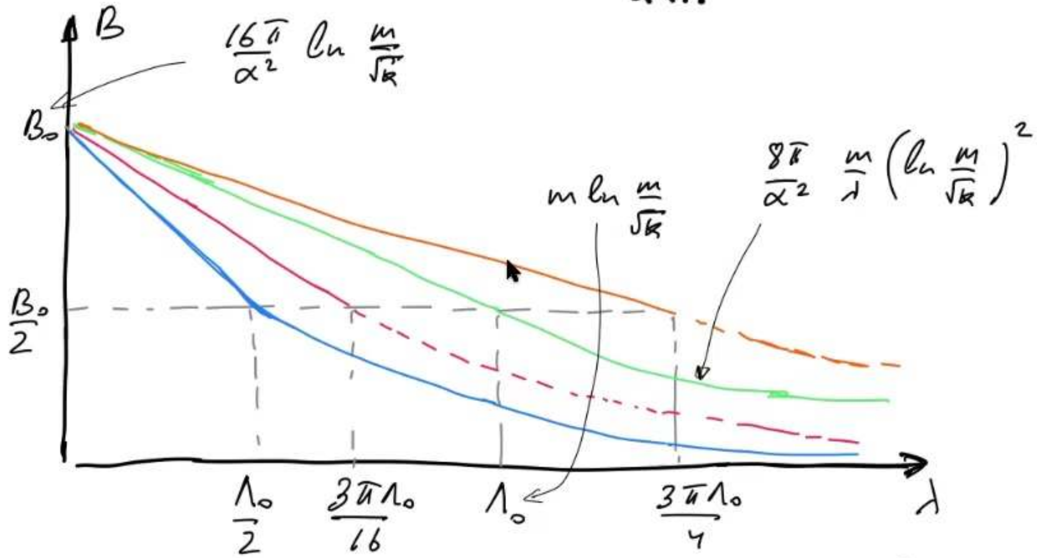

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Summary I I

* The toy model allowed us to explicitly see the difference for decay of different states
* Unruh decay is more suppressed than H-H
* Still, its suppression goes to zero at high T_{BH}
* Does the model capture all relevan physics?

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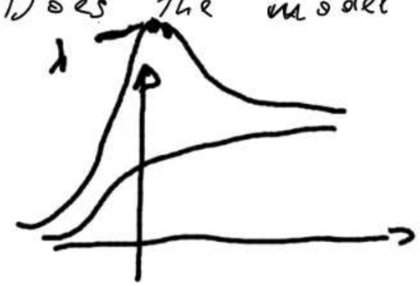

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