Title: Speck of Chaos

Speakers: Lea Santos

Series: Perimeter Institute Quantum Discussions

Date: March 24, 2021 - 4:00 PM

URL: http://pirsa.org/21030034

Abstract: It has been shown that, despite being local, a perturbation applied to a single site of the one-dimensional XXZ model is enough to bring this interacting integrable spin-1/2 system to the chaotic regime. In this talk, we show that this is not unique to the XXZ model, but happens also to the spin-1/2 Ising model in a transverse field and to the spin-1 Lai-Sutherland chain. The larger the system is, the smaller the amplitude of the local perturbation for the onset of chaos. We focus on two indicators of chaos, the correlation hole, which is a dynamical tool, and the distribution of off-diagonal elements of local observables, which is used in the eigenstate thermalization hypothesis. Both methods avoid spectrum unfolding and can detect chaos even when the eigenvalues are not separated by symmetry sectors.

Speck of Chaos

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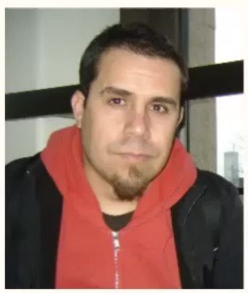
Jonathan Torres-Herrera



Francisco Pérez-Bernal

Lea F. Santos, Yeshiva University

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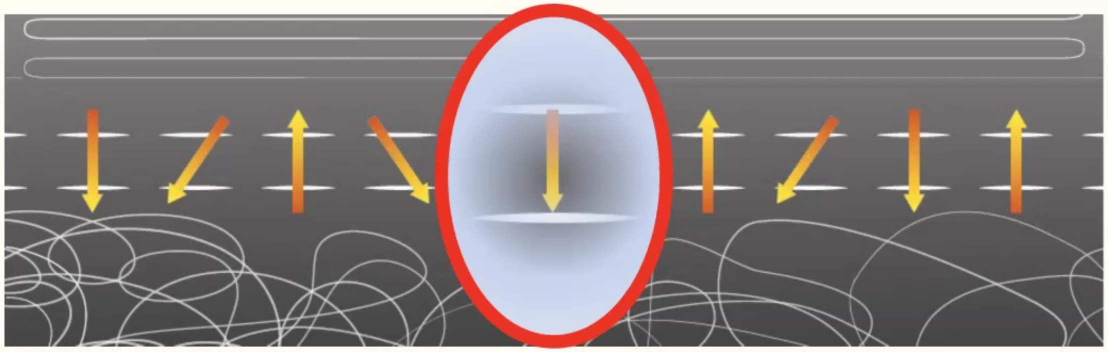
Plan



Local perturbation induces quantum chaos in systems with many interacting particles

➢ Indicators of Quantum Chaos:

- Level statistics
- Eigenstates
- Thermalization
- Correlation hole (ramp)
- OTOC

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

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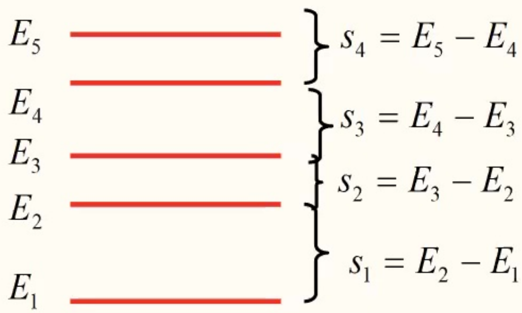
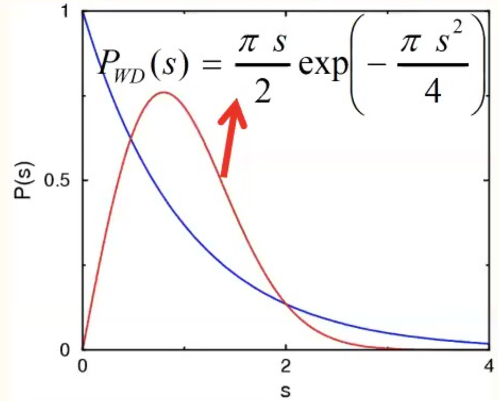
Level Statistics

Full random matrices from the GOE: real and symmetric
(Wigner in the 50's to describe statistically the spectra of heavy nuclei)

Level spacing distribution



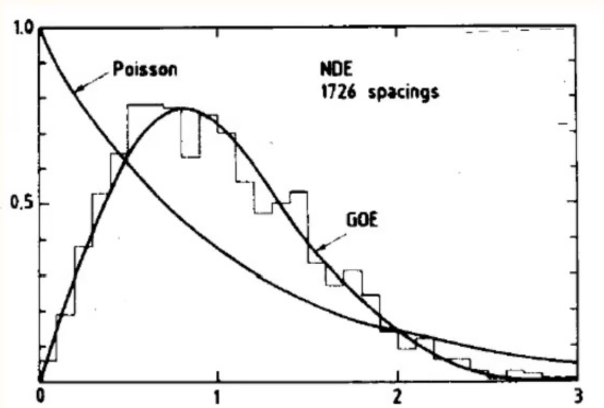
Wigner-Dyson distribution
(time reversal symmetry)



Eigenvalues are correlated
Level repulsion
Rigid spectrum

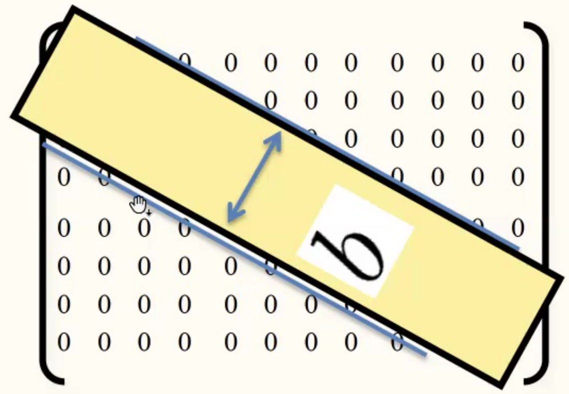
Level Spacing Distribution and Random Matrices

Nearest neighbor spacing distribution for the "**Nuclear Data Ensemble**" comprising 1726 spacings s = S/D with D the mean level spacing and S the actual spacing.



Bohigas, Haq and Pandey (1983)
Nuclear Data for Science and Technology

- Wigner band random matrix



-- Sparse banded random matrix
Fyodorov, Casati, Izrailev, Prosen

-- Power-law banded random matrix
Seligman, Kravtsov, Mirlin

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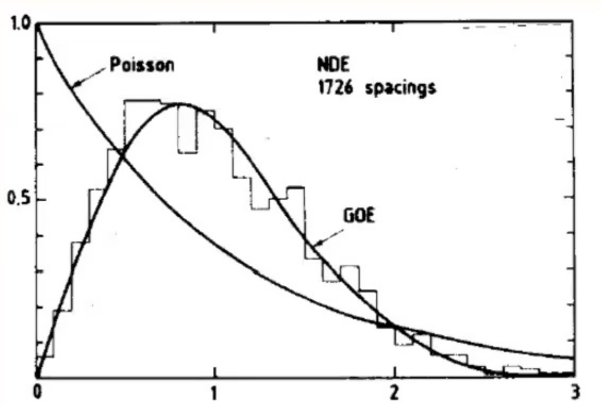
Level Spacing Distribution and TBRE

Nearest neighbor spacing distribution for the "**Nuclear Data Ensemble"** comprising 1726 spacings s = S/D with D the mean level spacing and S the actual spacing.



Bohigas, Haq and Pandey (1983)
Nuclear Data for Science and Technology

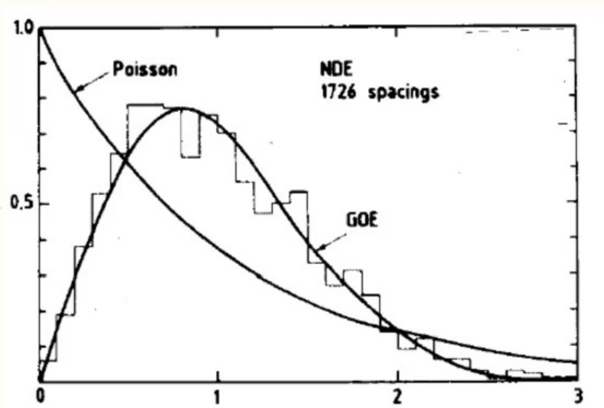
- Wigner band random matrix
- Sparse banded random matrix
- Power-law banded random matrix
- Two-body random ensemble (TBRE) or SYK model [Embedded ensembles]

French, Wong, Flores, Bohigas, Brody, Mello, Guhr, Weidenmüller, Izrailev, Flambaum, Kota, Zelevinsky, Horoi, Volya, Alhassid, Prosen, Seligman,…

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k\leq l, p\leq q} \langle pq|\, V \,|kl\rangle\, a_p^\dagger a_q^\dagger a_l a_k,$$

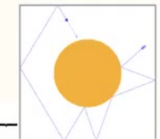
Level Spacing Distribution and Chaos

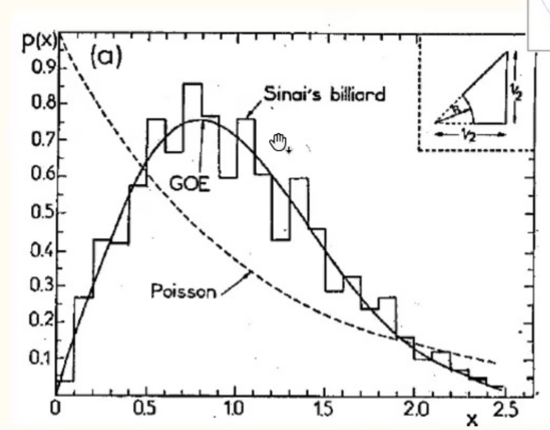
Nearest neighbor spacing distribution for the **"Nuclear Data Ensemble"** comprising 1726 spacings s = S/D with D the mean level spacing and S the actual spacing.



Bohigas, Haq and Pandey (1983)
Nuclear Data for Science and Technology

The nearest neighbor spacing distribution versus s for the ***quantum*** **Sinai billiard**. The histogram comprises about 1000 consecutive eigenvalues.





Classical chaos – Level Statistics
Quantum chaos = signatures of chaos

Correspondence established – few degrees of freedom

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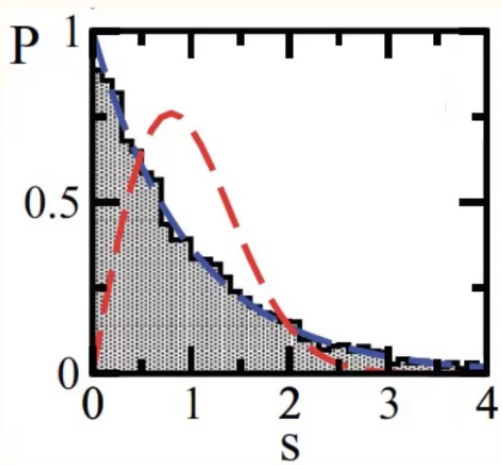
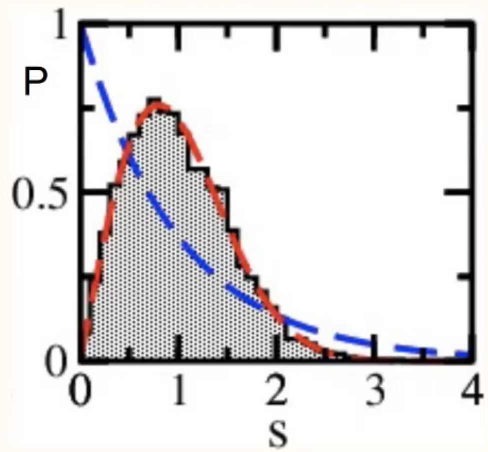
Poisson vs Wigner-Dyson

Integrable model



Chaotic model



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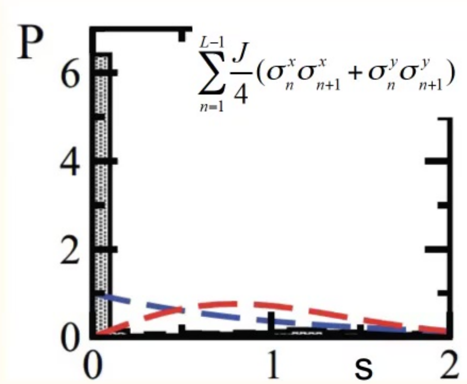
Poisson distribution

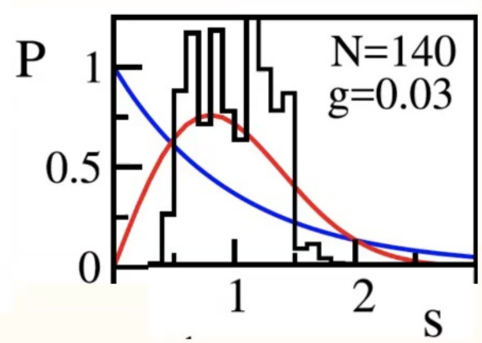
Integrable models:

- Degeneracies
- Picket-fence spectrum



PRE**88**, 032913 (2013)

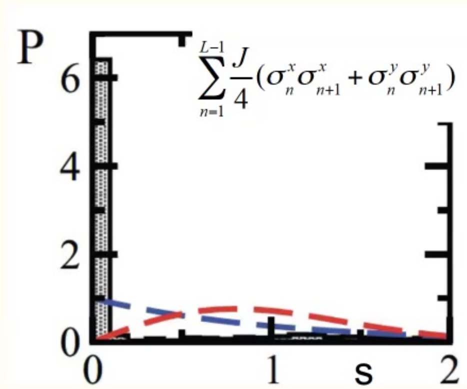
NJP**20**, 113039 (2018)



Poisson distribution

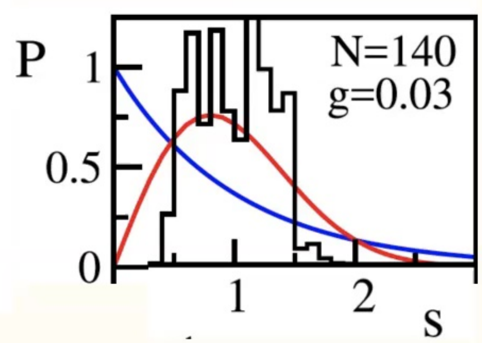
Integrable models:

- ➢ Degeneracies
- ➢ Picket-fence spectrum



PRE**88,** 032913 (2013)

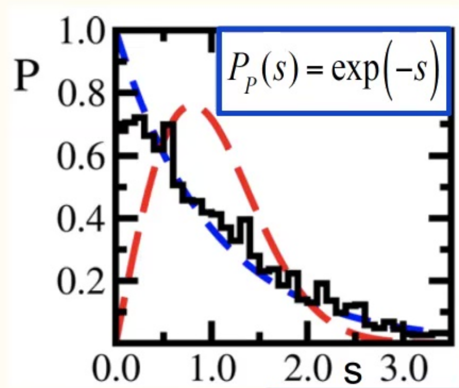
NJP**20**, 113039 (2018)



➢ **Chaotic** models but mixed symmetries:

$$H_{NN}+H_{NNN}=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)+$$

$$+\lambda\sum_{n=1}^{L-2}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+2}^z+\sigma_n^x\sigma_{n+2}^x+\sigma_n^y\sigma_{n+2}^y)$$



AJP**80**, 246 (2012)

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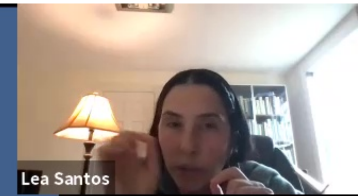
Wigner-Dyson Distribution and Integrability



- We can construct integrable Hamiltonians with WD distribution

Relaño, Dukelsky, Gómez, Retamosa
PRE **70**, 026208 (2004)

- Finite-size effect: Localization length is larger than the system size

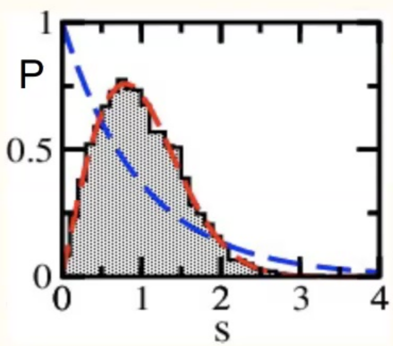
1D Anderson model

$$H = \sum_{n=1}^{L} \epsilon_n c_n^\dagger c_n - J \sum_{n=1}^{L-1} (c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n)$$

1D Aubry-André model

$$H = \sum_{j=1}^{L} h \cos[(\sqrt{5}-1)\pi j + \phi] c_j^\dagger c_j - J \sum_{j=1}^{L-1} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$

Tight-binding models



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PRE**100**, 022142 (2019)

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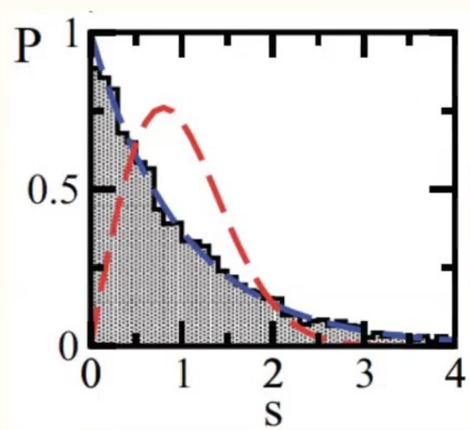
1D Spin-1/2 Systems

Integrable system:

XXZ model

$$H = \sum_{n=1}^{L-1} \frac{J}{4} \left(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right)$$

POISSON DISTRIBUTION



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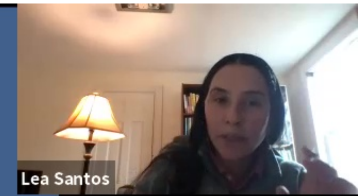
1D Spin-1/2 Systems



Integrable system:

XXZ model

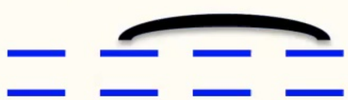
$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

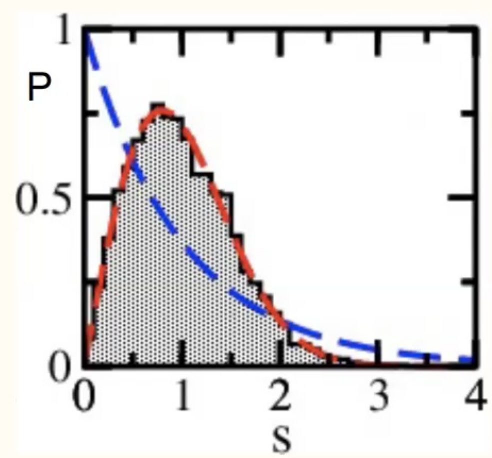
Interaction between next-nearest neighbors

WIGNER-DYSON DISTRIBUTION

$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) +$$
$$+ \lambda \sum_{n=1}^{L-2} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+2}^z + \sigma_n^x \sigma_{n+2}^x + \sigma_n^y \sigma_{n+2}^y)$$

NNN model





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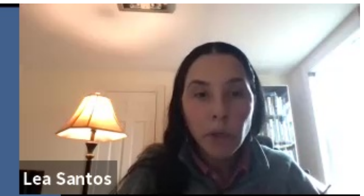
1D Spin-1/2 Systems



Integrable system:

XXZ model

$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

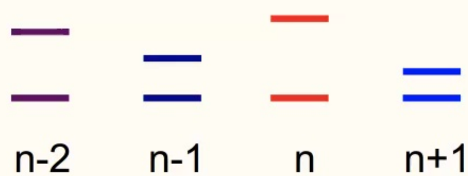
WIGNER-DYSON DISTRIBUTION

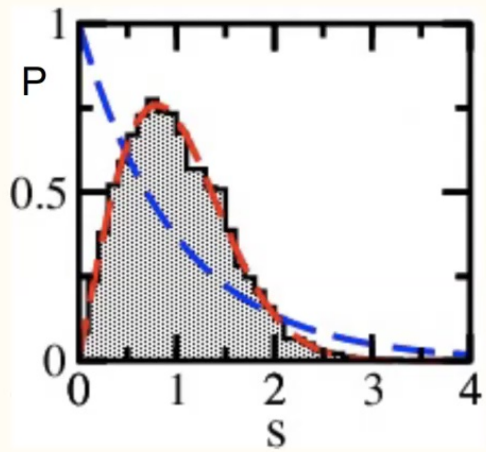
Many-body localization

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z)$$

Random numbers

$h_n \in [-h, h]$





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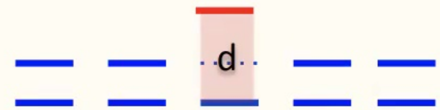
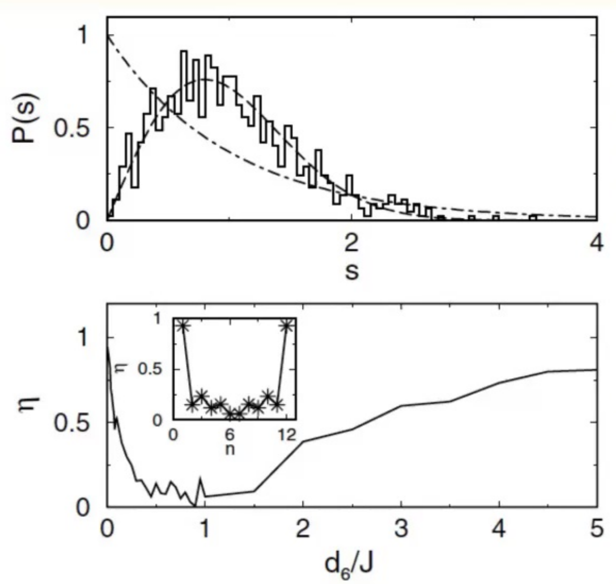
Single-Defect Model

$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1})$$



LFS,
JPA **37**, 4723 (2004)

(local perturbation)



INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **37** (2004) 4723–4729 PII: S0305-4470(04)74620-X

Integrability of a disordered Heisenberg spin-1/2 chain

L F Santos

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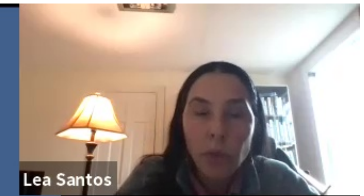
Single-Defect vs Second Neighbors



$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1})$$

vs

$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma^z_n\sigma^z_{n+1} + \sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1}) + \lambda\sum_{n=1}^{L-2}\frac{J}{4}(\Delta\sigma^z_n\sigma^z_{n+2} + \sigma^x_n\sigma^x_{n+2} + \sigma^y_n\sigma^y_{n+2})$$

Perturbed Hamiltonian | Unperturbed Hamiltonian | Perturbation

$$H = H_0 + \lambda V$$

Show very similar properties when: $\Delta = 0.48$

$$d = 0.9$$

$$\lambda = 0.44$$

Torres & LFS
PRE **89**, 062110 (2014)

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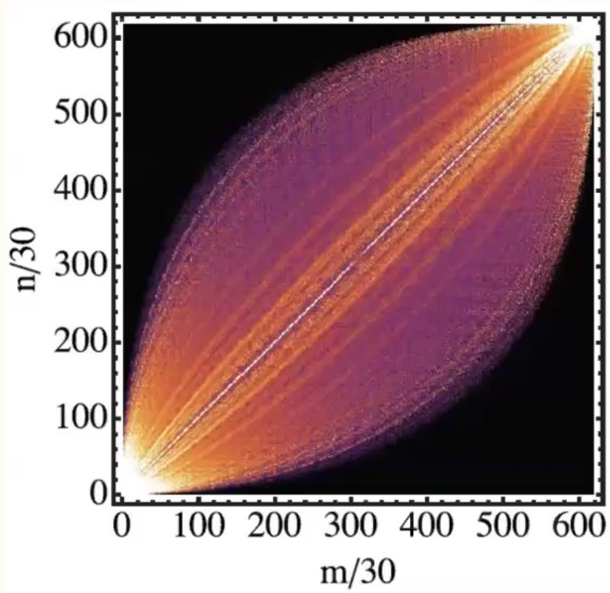
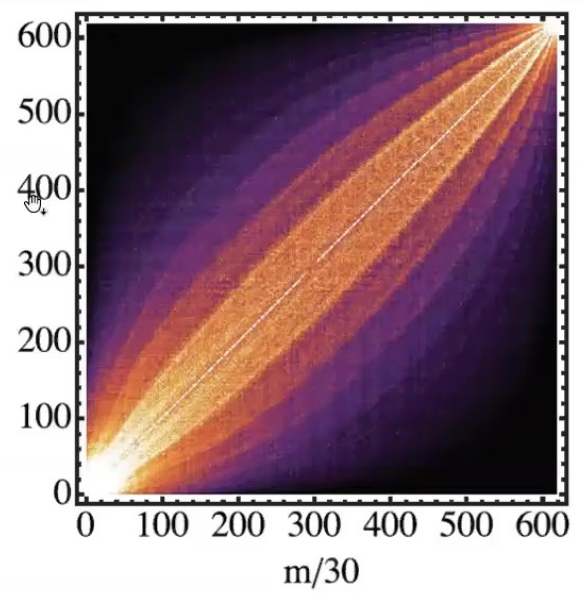
Hamiltonian Matrix

BASIS for both models: XXZ part

Single-defect model



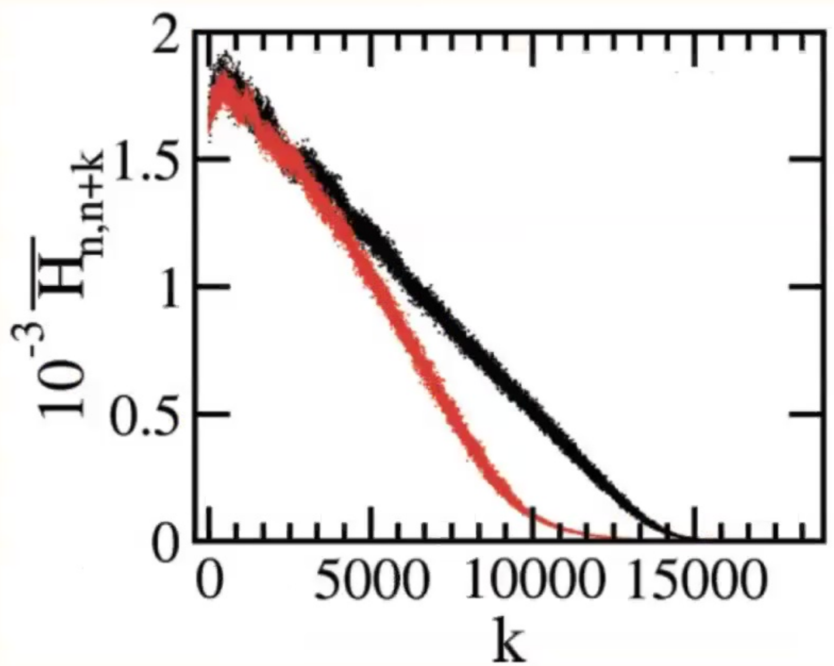
2^{nd}-neighbors couplings



Torres & LFS
PRE **89**, 062110 (2014)

Strength of Off-Diagonal Elements

BASIS for both models: XXZ part
Matrices ordered by the diagonal values



RED: single-defect

BLACK: 2^{nd}-neighbors

L=18
6 excitations

$d = 0.9$
$\lambda = 0.44$

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PRE **89**, 062110 (2014)

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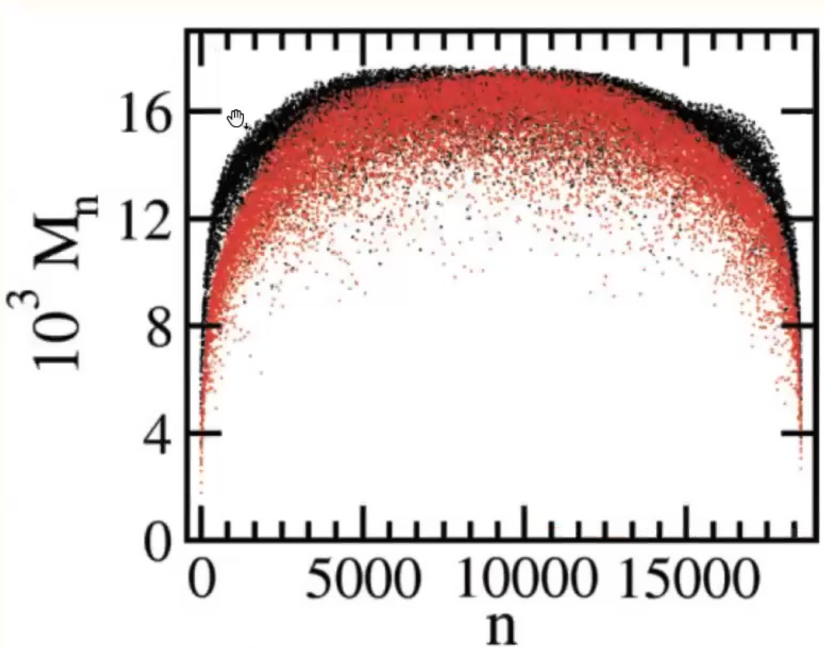
Connectivity

BASIS for both models: XXZ part
Matrices ordered by the diagonal values



RED: single-defect

BLACK: 2nd-neighbors

$d = 0.9$

$\lambda = 0.44$

L=18
6 excitations

Torres & LFS
PRE **89**, 062110 (2014)

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Coupling Effectiveness

$$\begin{pmatrix} H_{nn} & \cdots & \cdots & H_{nm} \\ \cdots & H & \cdots & \cdots \\ \cdots & \cdots & H & \cdots \\ \cdots & \cdots & \cdots & H_{mm} \end{pmatrix}$$

Effective coupling:

$$|H_{nm}| > |H_{nn} - H_{mm}|$$

BASIS for both models: XXZ part
Matrices ordered by the diagonal values

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PRE **89**, 062110 (2014)

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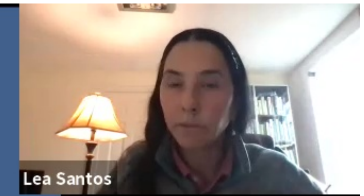
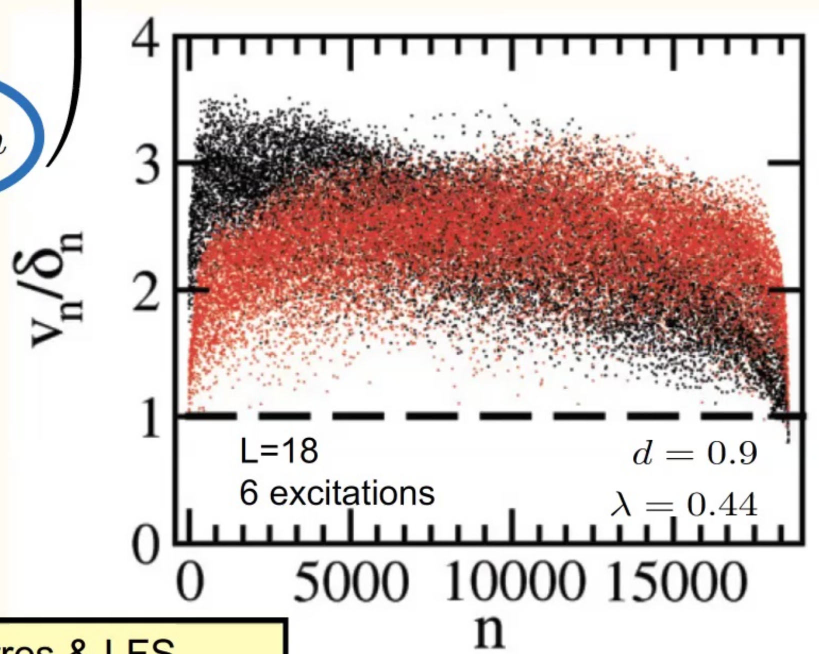
Coupling Effectiveness



$$\begin{pmatrix} H_{nn} & \dots & \dots & H_{nm} \\ \dots & H & \dots & \dots \\ \dots & \dots & H & \dots \\ \dots & \dots & \dots & H_{mm} \end{pmatrix}$$

Effective coupling:

$$|H_{nm}| > |H_{nn} - H_{mm}|$$



RED: single-defect

BLACK: 2^{nd}-neighbors

BASIS for both models: XXZ part
Matrices ordered by the diagonal values

Torres & LFS
PRE **89**, 062110 (2014)

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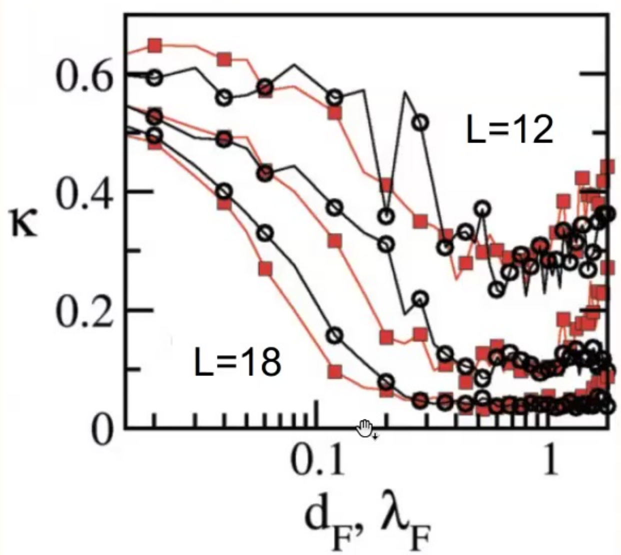
Level Statistics

$$\kappa \equiv \frac{\sum_i [\mathcal{P}(s_i) - \mathcal{P}_{\mathrm{WD}}(s_i)]}{\sum_i \mathcal{P}_{\mathrm{WD}}(s_i)}$$

P(r): ratio of consecutive level spacings



RED single-defect

BLACK 2^{nd}-neighbors

Torres & LFS
PRE **89**, 062110 (2014)

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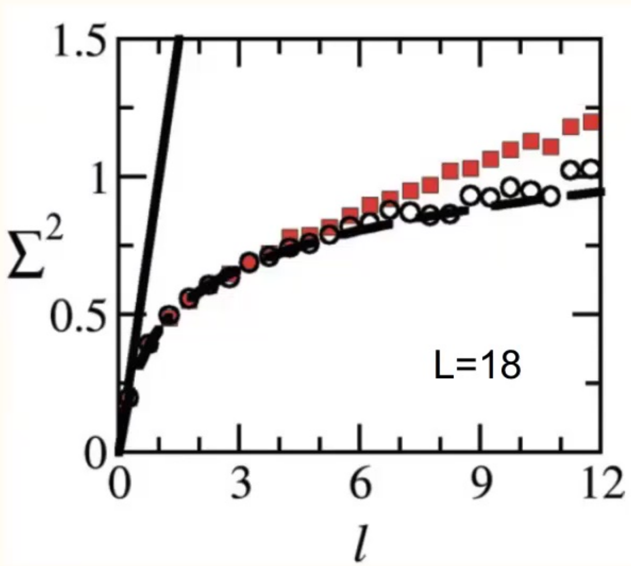
Level Statistics: Level Number Variance

$$\Sigma^2(l) \equiv \langle N(l,g)^2 \rangle - \langle N(l,g) \rangle^2$$



RED
single-defect

BLACK
2nd-neighbors

$d = 0.9$
$\lambda = 0.44$

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PRE **89**, 062110 (2014)

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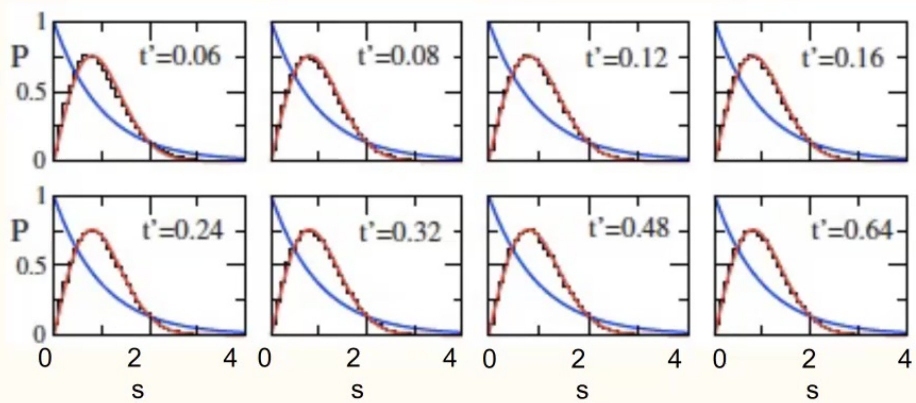
Long-Range Correlations

$$H=\sum_{n=1}^{L}\left[t\left(b_n^{+}b_n-\frac{1}{2}\right)\left(b_{n+1}^{+}b_{n+1}-\frac{1}{2}\right)-t\left(b_n^{+}b_{n+1}+h.c.\right)\right]+\sum_{n=1}^{L}\left[t'\left(b_n^{+}b_n-\frac{1}{2}\right)\left(b_{n+2}^{+}b_{n+2}-\frac{1}{2}\right)-t'\left(b_n^{+}b_{n+2}+h.c.\right)\right]$$

Hardcore bosons

- Level spacing distribution
- Ratio consecutive level spacings

detect SHORT-range correlations

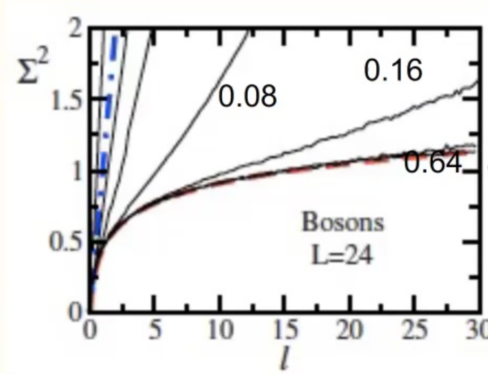


- Level Number Variance
- Spectral rigidity

detect long-range correlations also

$$\Sigma^2(l) \equiv \langle [N(l,\epsilon)]^2\rangle - \langle N(l,\epsilon)\rangle^2,$$

LFS & Rigol
PRE **80** 036206 (2010)



THERMALIZATION in the SINGLE-DEFECT MODEL

Lea F. Santos, Yeshiva University

Torres & LFS
PRE **89**, 062110 (2014)

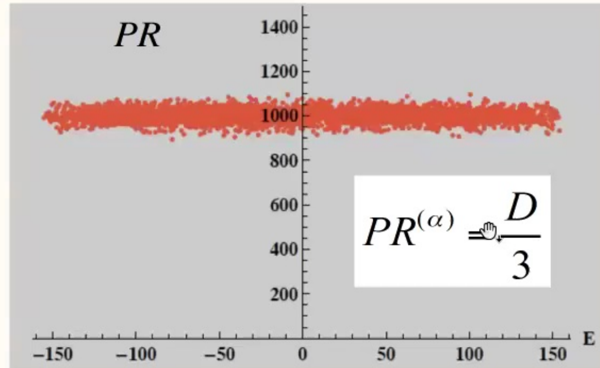
Perimeter Institute, Canada 2021

Eigenstates

Eigenstates of full random matrices are random vectors

$$|\alpha\rangle = \sum_{n=1}^{D} C_n^{\alpha} |n\rangle$$

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{n=1}^{D} |C_n^{(\alpha)}|^4}$$



$$|\alpha\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ \cdots \end{pmatrix}$$

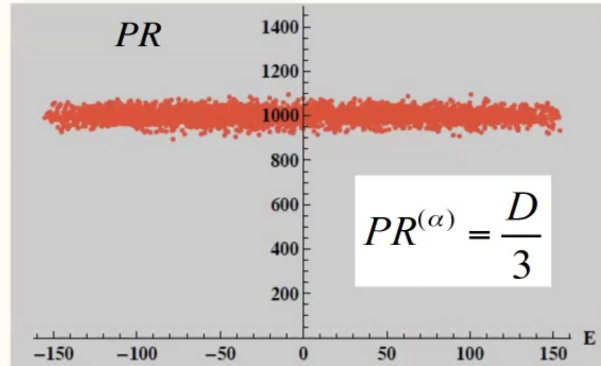
Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

Eigenstates

LFS, Borgonovi, Izrailev
PRL **108**, 094102 (2012)
PRE **85**, 036209 (2012)

Eigenstates of full random matrices are random vectors

$$|\alpha\rangle = \sum_{n=1}^{D} C_n^{\alpha} |n\rangle \qquad PR^{(\alpha)} \equiv \frac{1}{\sum_{n=1}^{D} |C_n^{(\alpha)}|^4}$$

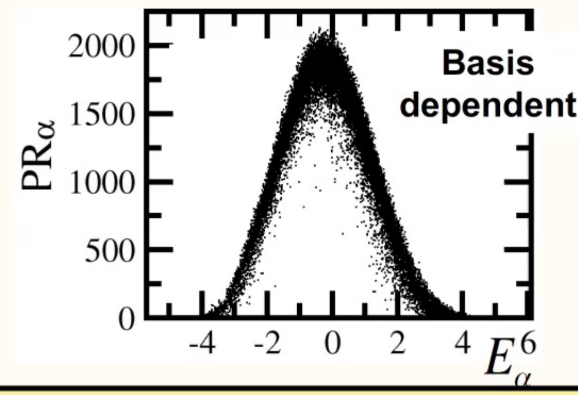


(mean-field basis)

$$H_{NN} + H_{NNN} =$$

$$\sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) +$$

$$+\lambda \sum_{n=1}^{L-2} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+2}^z + \sigma_n^x \sigma_{n+2}^x + \sigma_n^y \sigma_{n+2}^y)$$



Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

Structure of Initial State

Perturbed Hamiltonian | Unperturbed Hamiltonian | Perturbation

$$H = H_0 + \lambda V$$

Energy Eigenbasis $|\alpha\rangle$

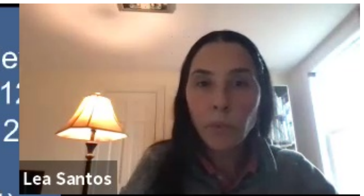
$$|\Psi(0)\rangle = \sum_{\alpha} C_{\alpha}^{ini} |\alpha\rangle$$

$$|\Psi(0)\rangle = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ \dots \end{pmatrix}$$

LFS, Borgonovi, Izrailev
PRL **108**, 094102 (2012)
PRE **85**, 036209 (2012)

Structure of Initial State

LFS, Borgonovi, Izraile
PRL **108**, 094102 (201
PRE **85**, 036209 (2012
Torres, Vyas, LFS
NJP**16**, 063010 (2014)



Perturbed Hamiltonian — Unperturbed Hamiltonian — Perturbation

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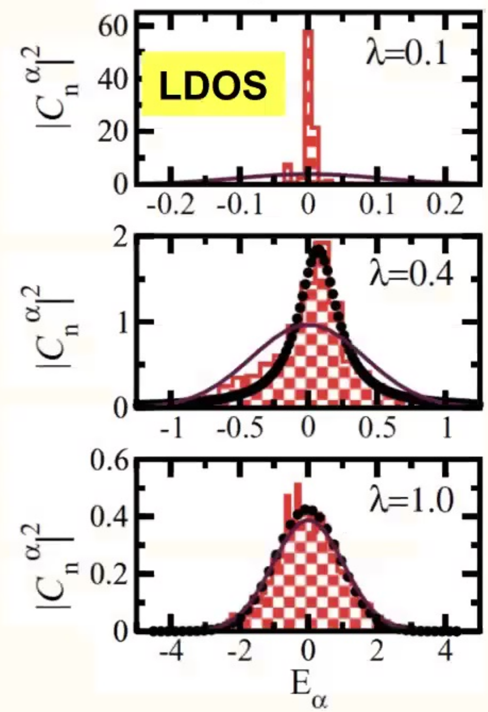
$$\rho_{ini}(E) = \sum_\alpha |C_\alpha^{ini}|^2 \delta(E - E_\alpha)$$



Chaotic Hamiltonian/
Chaotic Eigenstates

Middle of the Spectrum

$$E_{ini} = \langle \Psi(0) | H | \Psi(0) \rangle \sim 0$$

Strong Perturbation

$$|\Psi(0)\rangle = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ \dots \end{pmatrix}$$

Uncorrelated components

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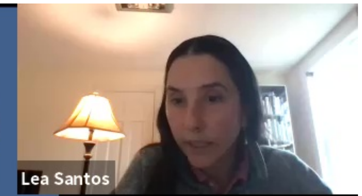
Perimeter Institute, Canada 2021

Eigenstates

LFS, Borgonovi, Izrailev
PRL **108**, 094102 (2012)
PRE **85**, 036209 (2012)



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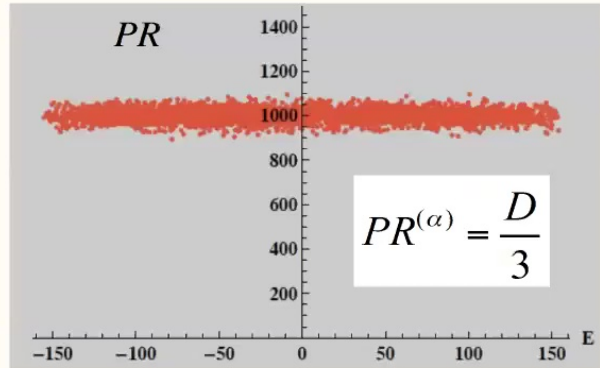

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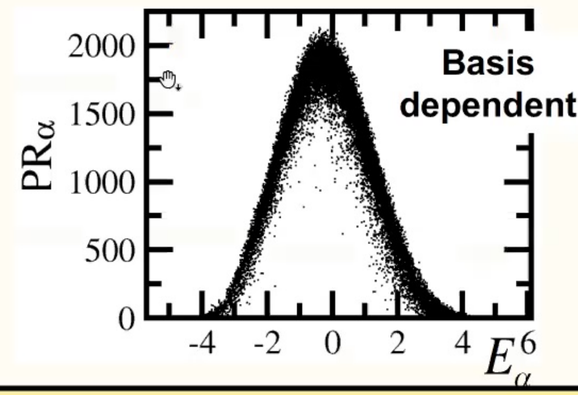

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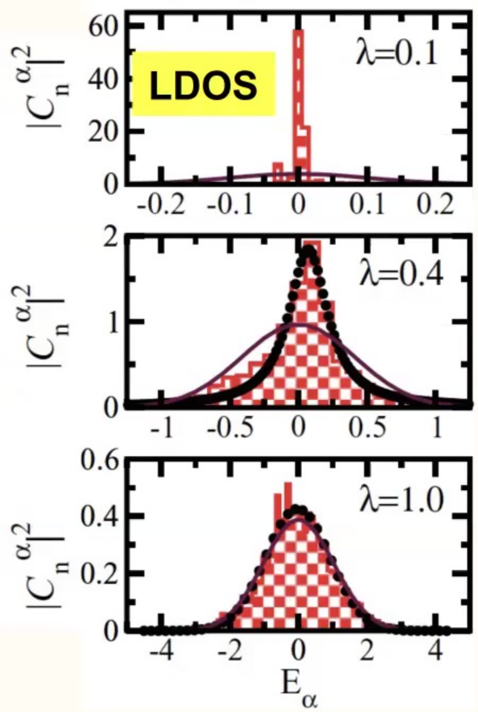

Chaotic Hamiltonian/
Chaotic Eigenstates

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Uncorrelated components

Structure of Initial State

LFS, Borgonovi, Izraile[…]
PRL **108**, 094102 (201[…]
PRE **85**, 036209 (2012[…]
Torres, Vyas, LFS
NJP**16**, 063010 (2014)

Perturbed Hamiltonian | Unperturbed Hamiltonian | Perturbation

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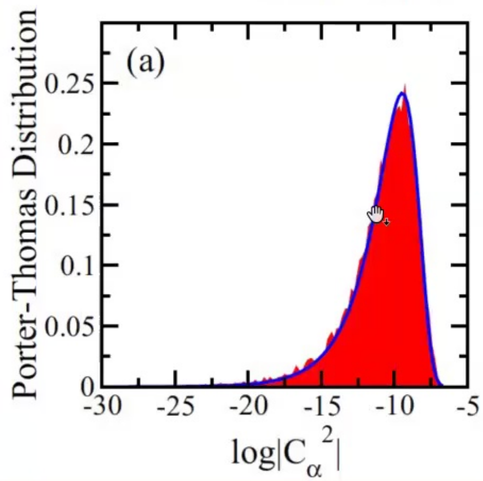
$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$\rho_{ini}(E) = \sum_\alpha \left|C_\alpha^{ini}\right|^2 \delta(E - E_\alpha)$$

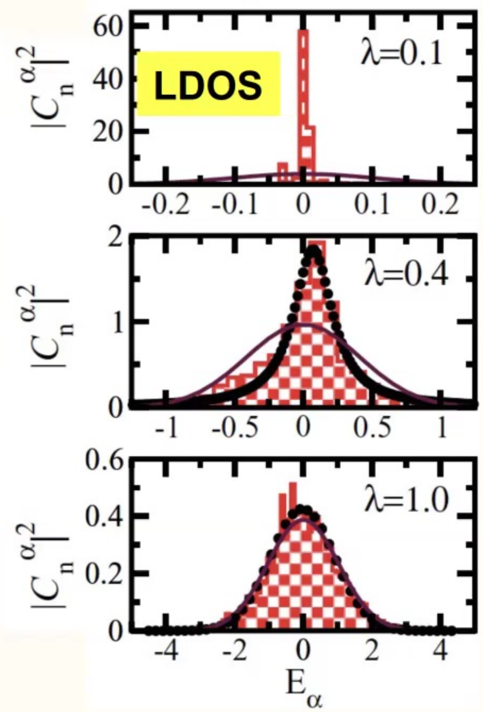
Porter-Thomas distribution

$$PT\left(|C_\alpha^{(0)}|^2\right) = \left(\frac{D}{2\pi |C_\alpha^{(0)}|^2}\right)^{1/2} \exp\left(-\frac{D}{2}|C_\alpha^{(0)}|^2\right)$$



$$|\Psi(0)\rangle = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ \dots \end{pmatrix}$$

Uncorrelated components



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Thermalization

$$\langle O(t)\rangle = \langle \Psi(t) | O | \Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle \qquad O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics,
L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol,
Adv. Phys. **65**, 239 (2016)

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Thermalization

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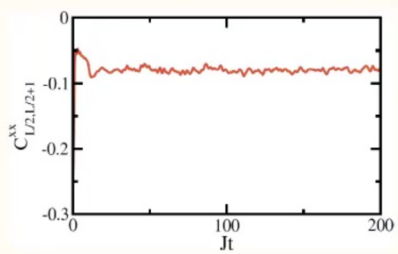
$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

Equilibration:



Size of the fluctuations
PRE **88**, 032913 (2013)

Components are small and **uncorrelated**
Lack of degeneracies: eigenvalues are **correlated**
Off-diagonal elements of local observables are small

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Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

Infinite time average Thermodynamic average

$$\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{N_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha|<\Delta E}} O_{\alpha\alpha}$$

$$\langle\alpha|O|\alpha\rangle$$

depends on the initial conditions depends only on the energy

ETH: the expectation values $O_{\alpha\alpha}$ of few-body observables do not fluctuate for eigenstates close in energy

Chaos guarantees thermalization, ETH

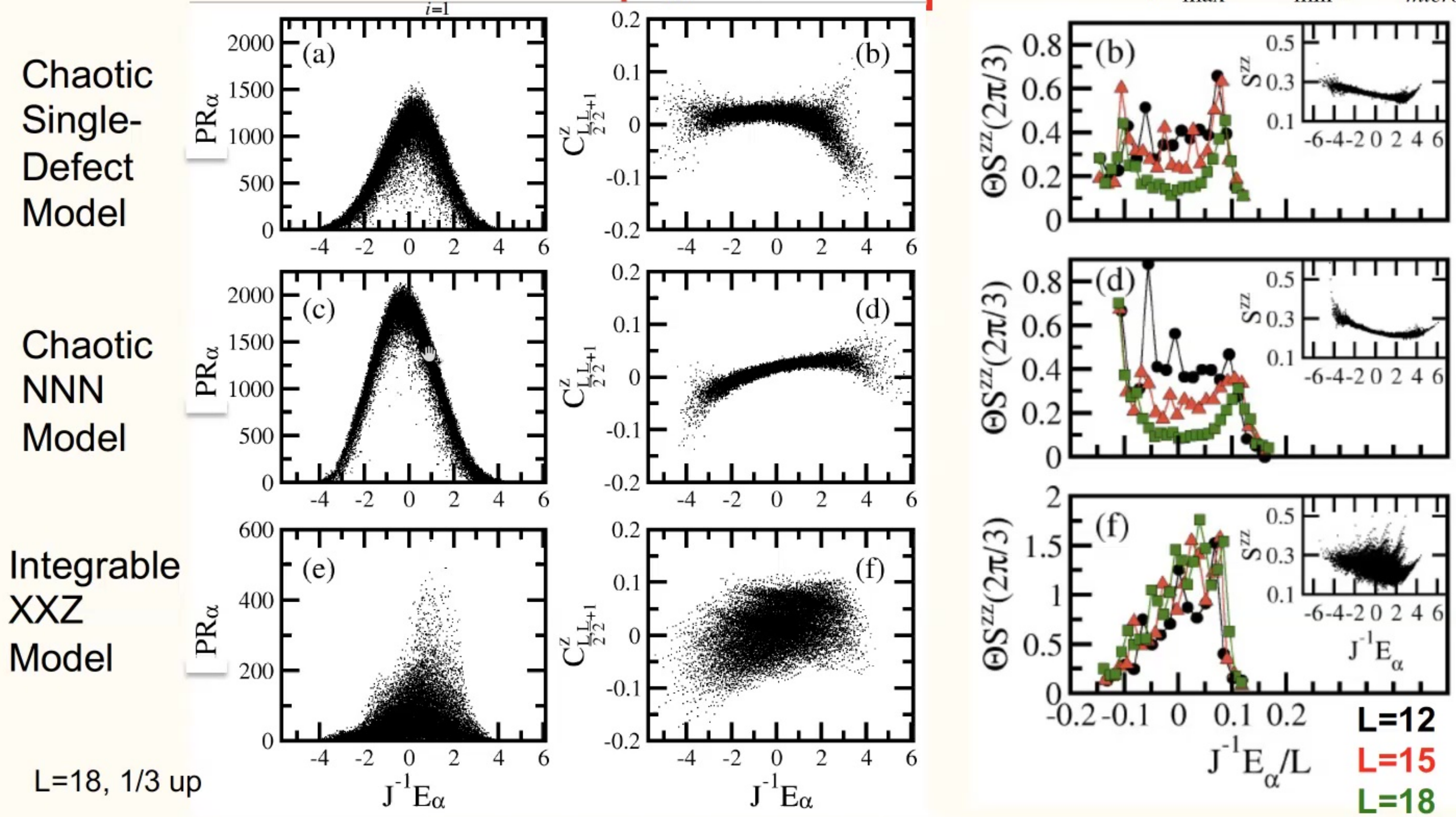
Chaotic states

Peres Lattice: Diagonal ETH

$PR^{(\alpha)} = 1/\sum_{i=1}^{D} |c_i^{(\alpha)}|^4$

$O_{\alpha\alpha} = \langle \alpha | O | \alpha \rangle$

$\Theta O = (O_{max} - O_{min})/O_{micro}$

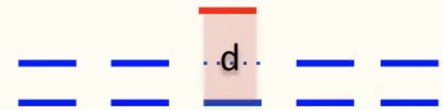


PRE **89**, 062110 (2014)

Phys. Scr. **T165**, 014018 (2015)

Speck of Chaos

$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}\left(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1}\right)$$



LFS,
JPA **37**, 4723 (2004)

Chaos: Level statistics, chaotic eigenstates, diagonal and off-diagonal elements of O

Chaos is the mechanism for **thermalization**

Chaos is the condition for the validity of ETH

Torres & LFS
PRE **89**, 062110 (2014)

Ballistic quantum transport

Brenes, Mascarenhas, Rigol & Goold
PRB **98**, 235128 (2018)

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

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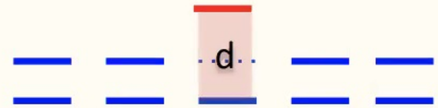
Speck of Chaos

$$H_{XXZ} = d_{L/2} S^z_{L/2} + J \sum_{n=1}^{L-1} \left(S^x_n S^x_{n+1} + S^y_n S^y_{n+1} + \Delta S^z_n S^z_{n+1} \right)$$



LFS,
JPA **37**, 4723 (2004)

$$H_{ZZ} = d_{L/2} S^z_{L/2} + Jh_x \sum_{n=1}^{L-1} S^x_n - J \sum_{n=1}^{L-1} S^z_n S^z_{n+1}$$

Ising model in a transverse field
Spin-1/2

$$H_{S1} = d_{L/2} S^z_{L/2} + J \sum_{n=1}^{L-1} \left(S^x_n S^x_{n+1} + S^y_n S^y_{n+1} + S^z_n S^z_{n+1} \right)$$
$$+ J \sum_{n=1}^{L-1} \left((S^x_n S^x_{n+1})^2 + (S^y_n S^y_{n+1})^2 + (S^z_n S^z_{n+1})^2 \right)$$

Lai-Sutherland model
Spin-1

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

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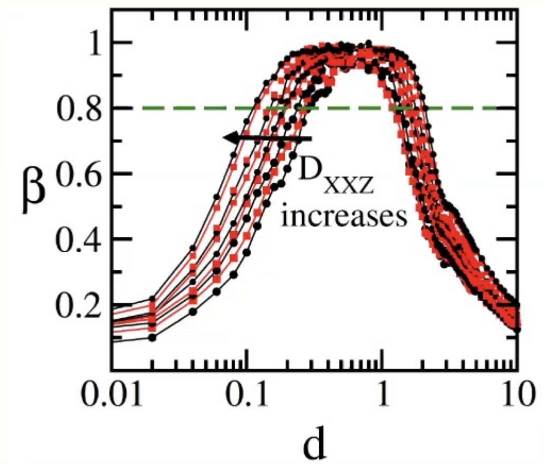
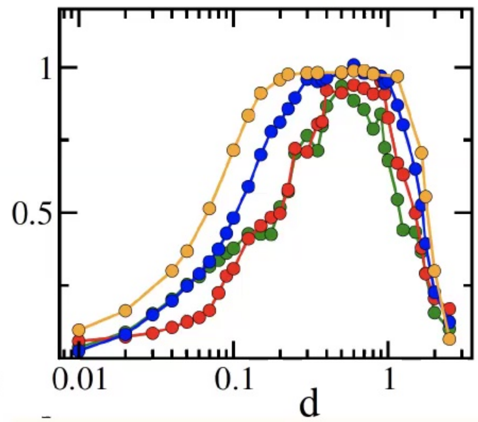
Speck of Chaos

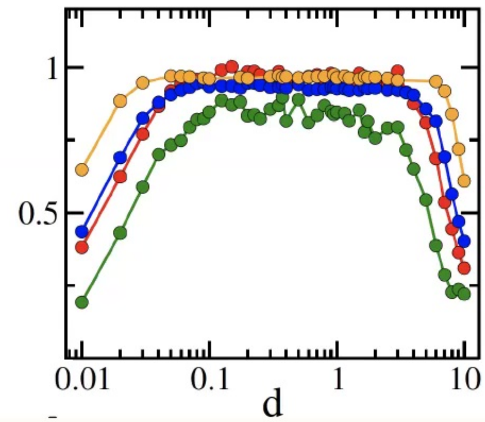
XXZ + defect
Spin-1/2

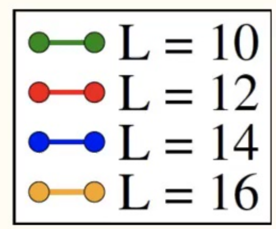


Ising + defect
in a transverse field
Spin-1/2



Lai-Sutherland + defect
Spin-1





$$P(s) = (\beta + 1) b s^{\beta} \exp(-b s^{\beta+1})$$

$\beta \sim 1$ **chaos**

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

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Off-diagonal elements

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

Beugeling, Moessner, Haque
PRE **91**, 012144 (2015)
(Gaussian distribution)

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

$$R(\omega) = \frac{\overline{\left|\langle \alpha | O | \beta \rangle\right|^2}}{\overline{\left|\langle \alpha | O | \beta \rangle\right|}^2}$$

$R(\omega) = \pi / 2$

means Gaussian distribution

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Defect models
Off-diagonal elements

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

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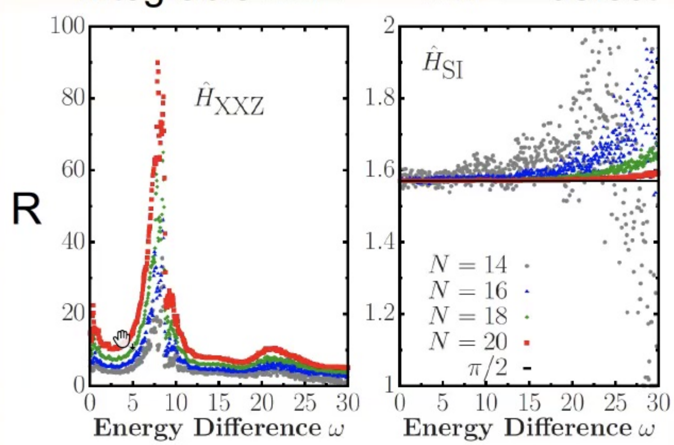
Beugeling, Moessner, Haque
PRE **91**, 012144 (2015)

Integrable XXZ XXZ + defect



Brenen, Goold, Rigol
PRB **102**, 075127 (2020)

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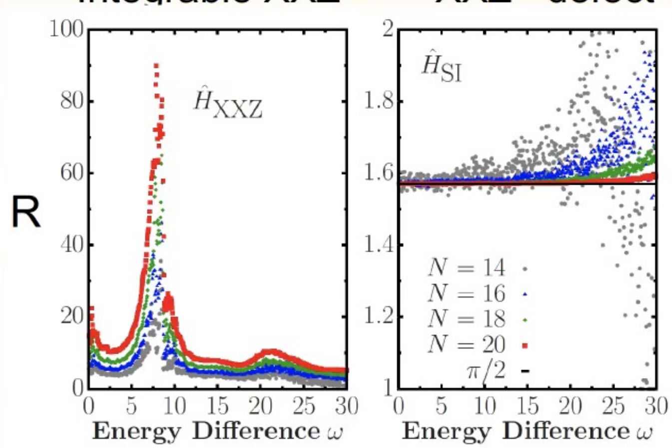
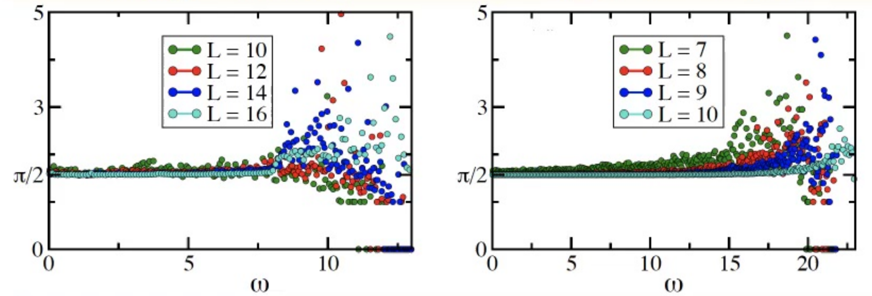
Integrable XXZ | XXZ +defect



Ising + defect | Spin 1 + defect



Brenen, Goold, Rigol
PRB **102**, 075127 (2020)

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

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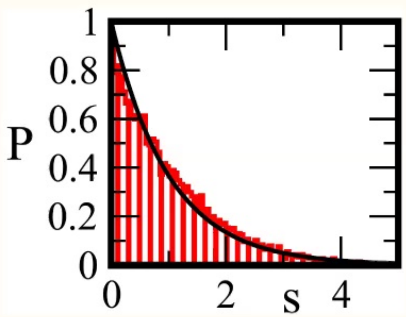
Off-diagonal elements and symmetries

$$R(\omega) = \overline{\left|\langle \alpha | S^z_{L/2} | \beta \rangle\right|^2} / \overline{\left|\langle \alpha | S^z_{L/2} | \beta \rangle\right|}^2$$

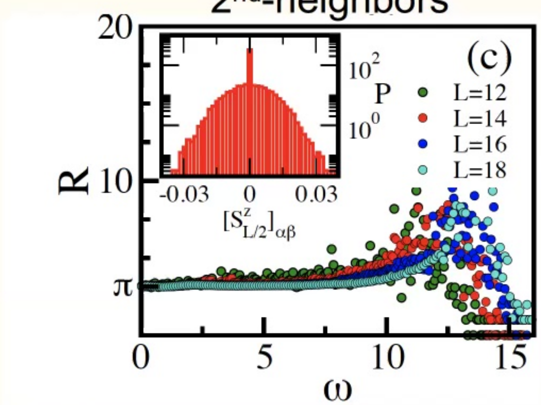
No need for unfolding
Detect chaos despite symmetries



Eigenvalues **NOT** separated by symmetry sectors

$$H_{NN} + H_{NNN} = \sum_{n=1}^{L-1} \frac{J}{4}(\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) + $$
$$+\lambda \sum_{n=1}^{L-2} \frac{J}{4}(\Delta \sigma_n^z \sigma_{n+2}^z + \sigma_n^x \sigma_{n+2}^x + \sigma_n^y \sigma_{n+2}^y)$$

2nd-neighbors



Speck of Chaos
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LFS, Bernal, Torres

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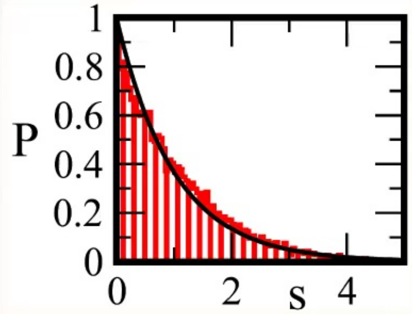

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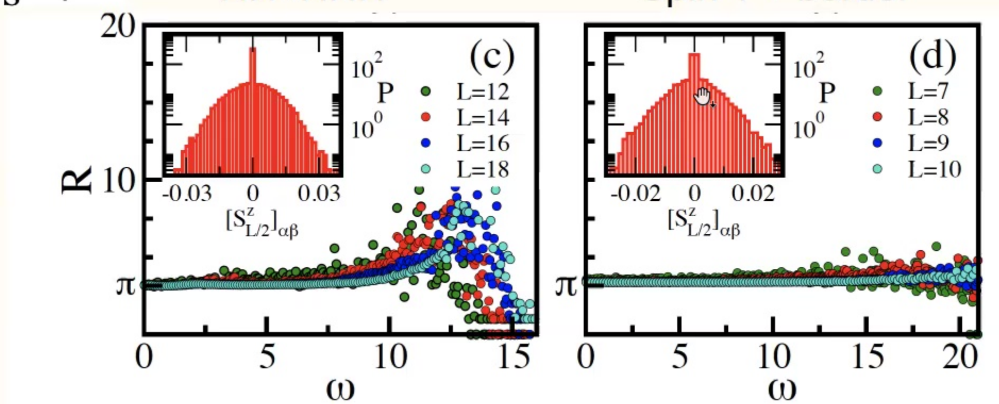

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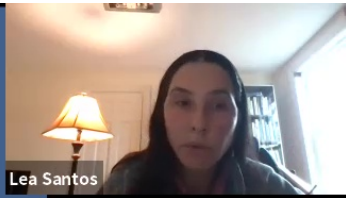

Manifestations of Chaos in the Dynamics

Level statistics is a good approach when we have access to the spectrum: Nuclear Physics

How about experiments with cold atoms and ion traps?

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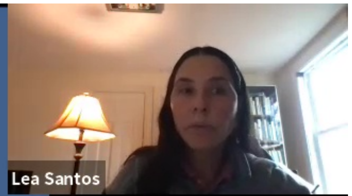
Dynamics: OTOC



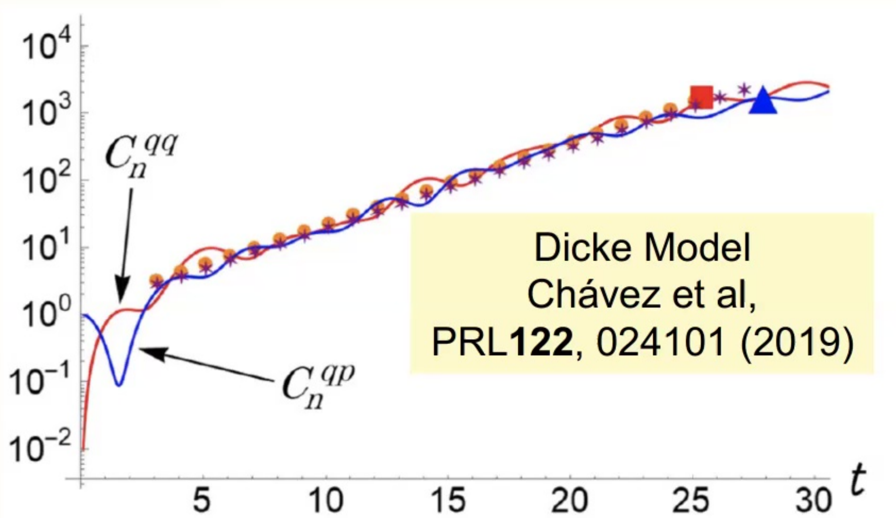
Loschmidt Echo $L(t) = \left| \left\langle \Psi(0) \,|\, e^{iH_2 t} e^{-iH_1 t} \,|\, \Psi(0) \right\rangle \right|^2$

Out-of-time-ordered four-point correlator

$$C(t) = -\left\langle [W(t), V(0)]^2 \right\rangle$$

$$C(t) = -\left\langle W^+(t) V^+(0) W(t) V(0) \right\rangle$$

Kicked Rotor
Rozenbaum, Ganeshan, Galitski,
PRL**118**, 086801(2017)



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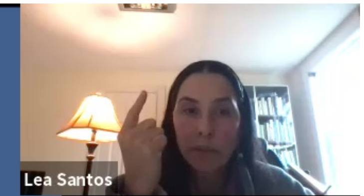
OTOC detects instability



Is the OTOC a detector of chaos?

(is the exponential growth of the OTOC at short times an indicator of chaos?)

In classical systems:
Positive Lyapunov exponent does not necessarily imply chaos.

Example: **inverted simple pendulum**.
Its upright position corresponds to a stationary point that is unstable.
It has a positive LE, as any genuine chaotic system, but it is completely integrable.
The pendulum does not exhibit chaotic behaviors, such as nonperiodicity and mixing.

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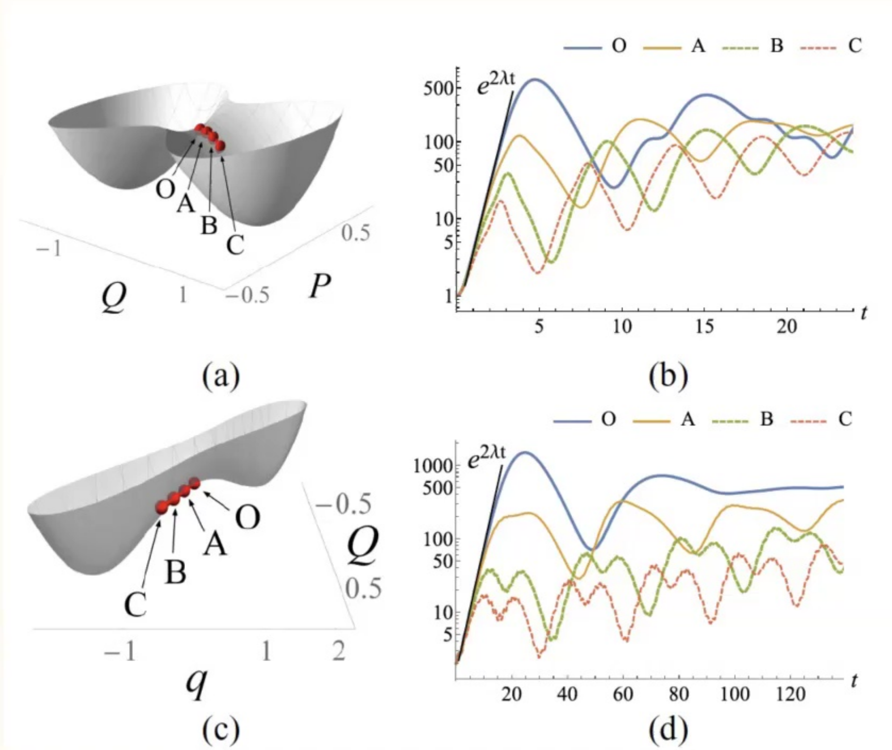
OTOC detects instability

Lipkin-Meshkov-Glick model

Dicke model in the regular regime



PRB 98, 134303 (2018)
PRL123,160401 (2019)
PRL124, 140602 (2020)
JHEP 68, (2020)

Pilatowsky et al
PRE 101, 010202(R) (2020)

Long-Time Dynamics: Correlation Hole

Manifestations of spectral correlations in the dynamics?

At long times, when the dynamics resolve the discreteness of the spectrum:
correlation hole (ramp)

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

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Survival Probability

Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

Nonlocal quantity
Autocorrelation function

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha^{ini} e^{-iE_\alpha t} |\alpha\rangle$$

$$SP(t) = \left| \sum_\alpha |C_\alpha^{ini}|^2 e^{-iE_\alpha t} \right|^2$$

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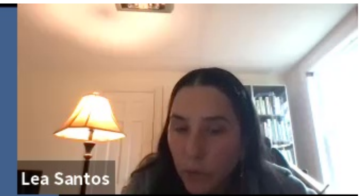
Survival Probability vs Spectral Form Factor



Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$SP(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha} t} \right|^2$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t} \rangle$$

Spectral form factor: $$SFT(t) = \langle \sum_{\alpha,\beta} e^{-i(E_{\alpha}-E_{\beta})t} \rangle$$

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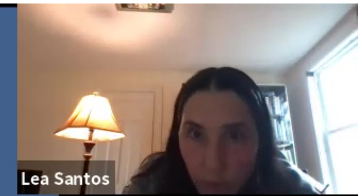

Survival Probability & LDOS

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

$$\rho_{ini}(E) = \sum_\alpha \left|C_\alpha^{ini}\right|^2 \delta(E - E_\alpha)$$

Energy distribution of the initial state

LDOS

Strength function

- Middle of the spectrum
- Strong perturbation

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Survival Probability & LDOS

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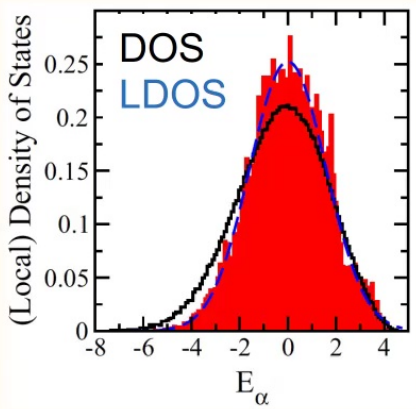

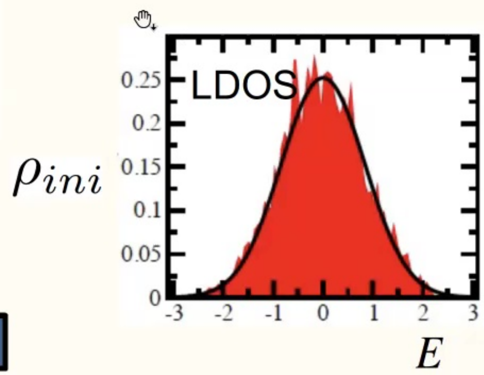

- Middle of the spectrum
- Strong perturbation

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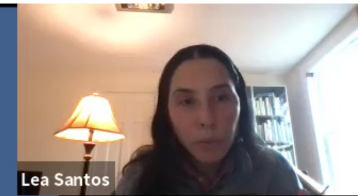
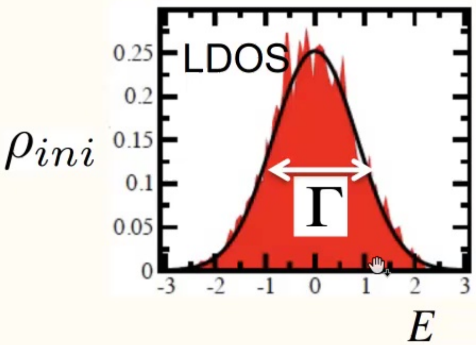
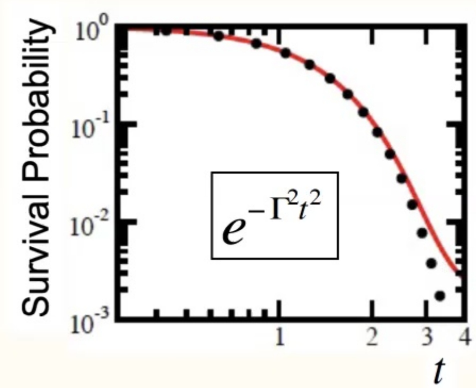
Initial Fast Decay





Γ Width of the distribution

$1/\Gamma$



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Torres-Herrera & LFS
PRA, NJP (2014)

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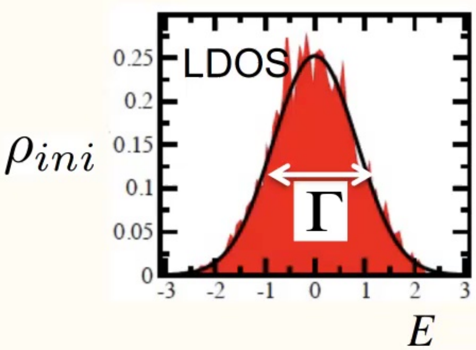
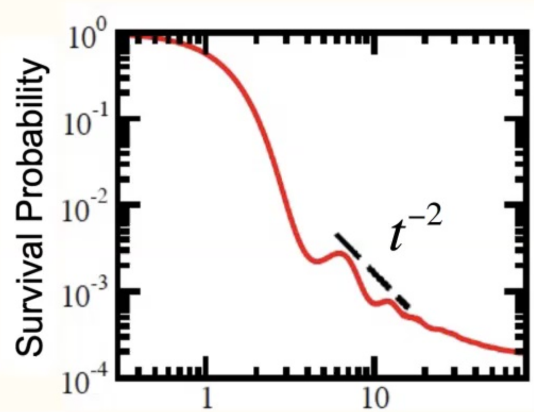
Power-law Decay



Γ Width of the distribution

$$\int_{E_{low}}^{E_{up}} \rho(E) e^{-iEt} dE$$

$$\frac{e^{-\Gamma^2 t^2}}{4N^2}\left|\left[erf\left(\frac{E_0 - E_{low} + i\Gamma^2 t}{\sqrt{2}\Gamma}\right) - erf\left(\frac{E_0 - E_{up} + i\Gamma^2 t}{\sqrt{2}\Gamma}\right)\right]\right|^2$$



Távora, Torres, LFS
PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

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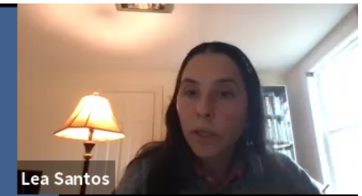
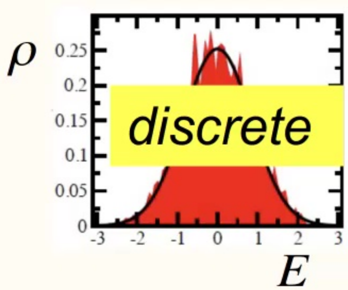
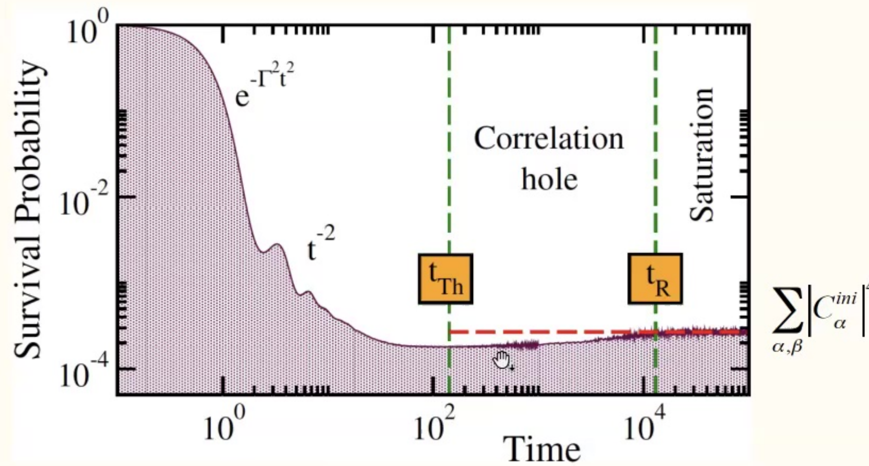
Correlation Hole



$$\langle SP(t)\rangle = \langle \sum_{\alpha\neq\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle + \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^4\rangle$$





Fourier transform of the 2-level cluster function:

2-level form factor b_2

$$b_2(t) = \begin{cases} 1 - 2t + t\ln(1+2t), & t \le 1 \\ t\ln\left(\frac{2t+1}{2t-1}\right) - 1, & t > 1 \end{cases}$$

Schiulaz, Torres & LFS, PRB **99**, 174313 (2019)

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Correlation hole = ramp

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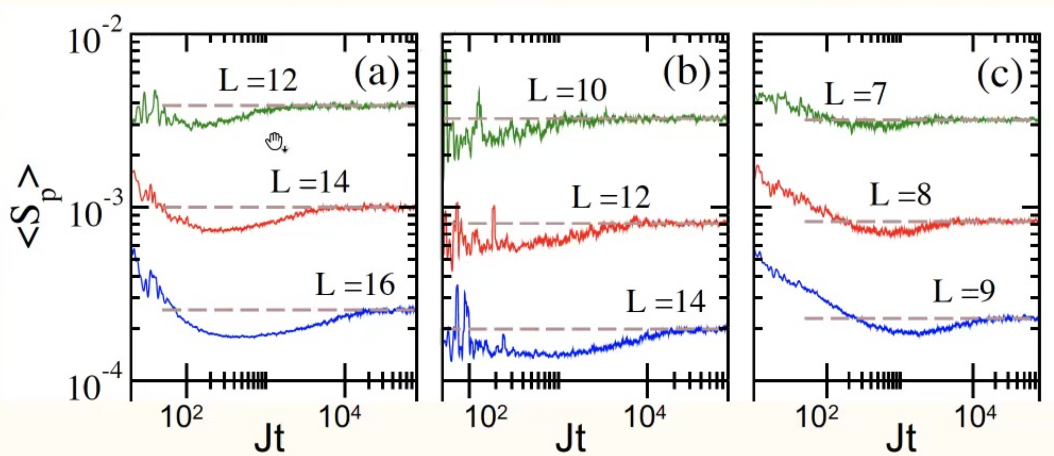
Correlation Hole: Single-Defect Models

No need for unfolding
Detect chaos despite symmetries

P(r): ratio of consecutive level spacings

XXZ + def   Ising + def   Spin 1 + def



Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

$|\Psi(0)\rangle = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$

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Self-Averaging

Disordered Systems

A quantity O is self-averaging when its relative variance (the ratio between its variance and the square of its mean) goes to zero as the system size increases

$$\mathcal{R}_O(t) = \frac{\sigma_O^2(t)}{\langle O(t) \rangle^2} = \frac{\langle O^2(t) \rangle - \langle O(t) \rangle^2}{\langle O(t) \rangle^2}$$

By increasing the system size, one can **reduce** the number of samples used in

- experiments
- statistical analysis.

If the system exhibits self-averaging, its physical properties are independent of the specific realization.

arXiv:2102.02824 (2021)
PRE 102, 062126 (2020)
PRB 102, 094310 (2020)
PRB 101, 174312 (2020)

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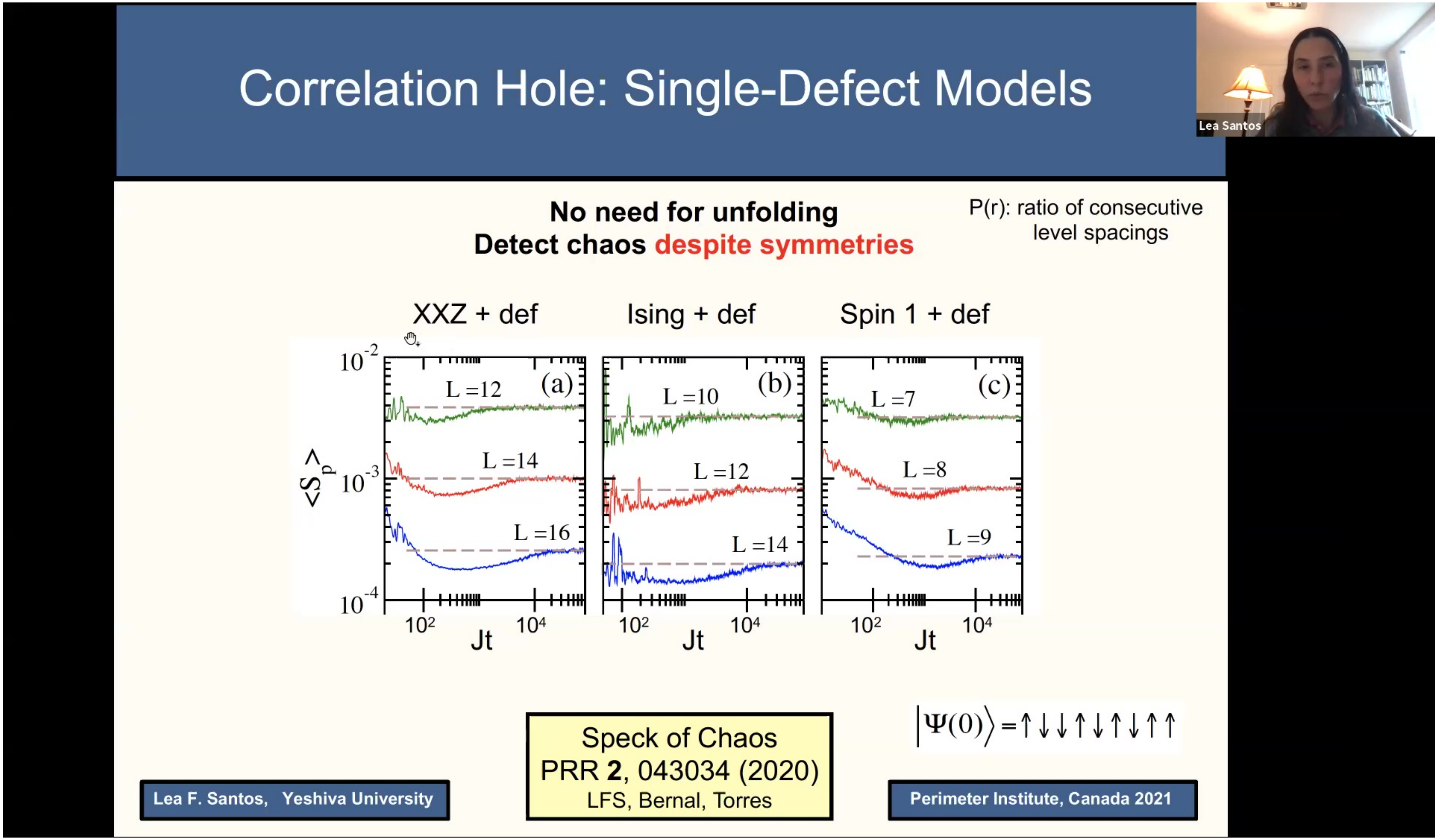
Correlation Hole: Single-Defect Models
Lea Santos
No need for unfolding
Detect chaos despite symmetries
P(r): ratio of consecutive level spacings
XXZ + def
Ising + def
Spin 1 + def
<S_p>
10^-2
10^-3
10^-4
L =12
L =14
L =16
(a)
L =10
L =12
L =14
(b)
L =7
L =8
L =9
(c)
10^2
10^4
Jt
|Ψ(0)⟩ = ↑↓↓↑↓↑↓↑↑
Speck of Chaos
PRR 2, 043034 (2020)
LFS, Bernal, Torres
Lea F. Santos, Yeshiva University
Perimeter Institute, Canada 2021

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arXiv:2102.02824 (2021)
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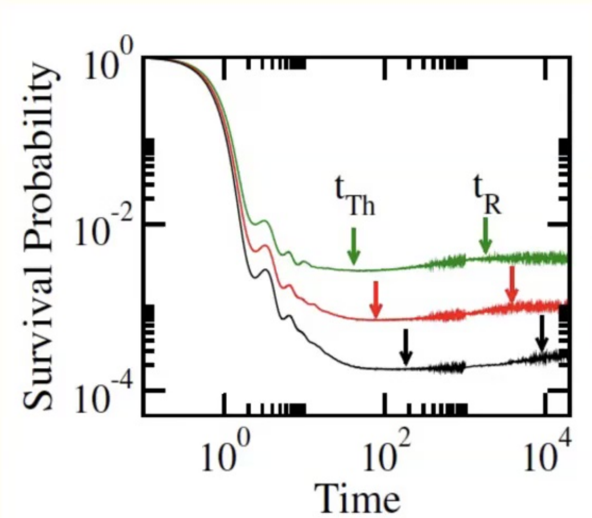
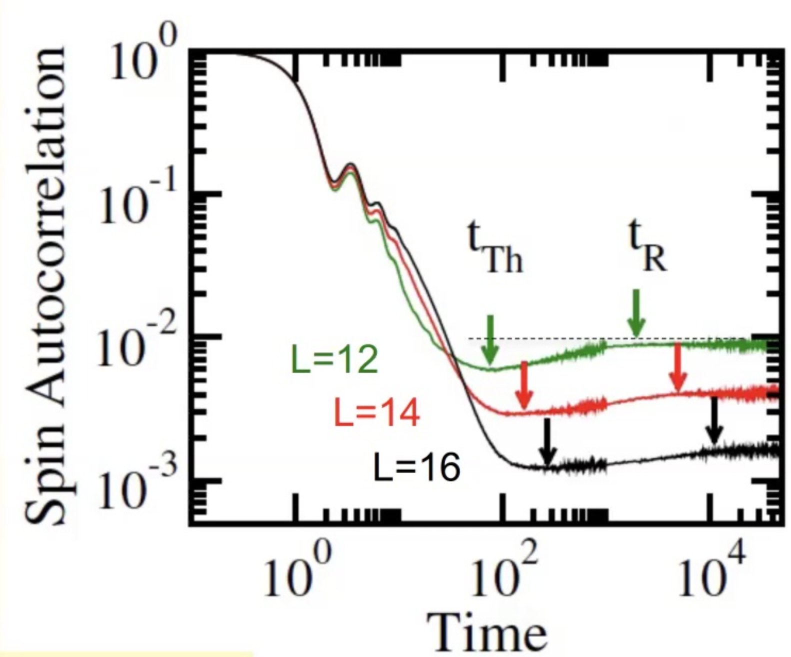
PRB 102, 094310 (2020)
PRB 101, 174312 (2020)

Correlation Hole and Experiments

Torres, García-García, LFS
PRB **97**, 060303 (R) (2018)

$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$





$$H = \sum_{n=1}^{L}\frac{\varepsilon_n}{2}\sigma_n^z + \sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z + \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)$$

Schiulaz, Torres & LFS,
PRB **99**, 174313 (2019)

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Conclusions

- Local perturbation in quantum many-body systems can lead to chaos.
- Defect + XXZ, Ising in transv., spin-1…
- Quantum transport, dynamics

JPA **37**, 4723 (2004)
PRE **89**, 062110 (2014)
Phys. Scr. **T165**, 014018 (2015)

- Correlation hole + off-diagonal elements: indicators of chaos
  no unfolding, no separation by symmetries

- Correlation hole + off-diagonal elements to identify symmetries
  to search for integrable models

- **Correlation hole**: a **dynamical** indicator of chaos (experiments – dynamics)

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres