Title: Efficient simulatability of continuous-variable circuits with large Wigner negativity - Laura GarcÃ-a-Ã•lvarez

Speakers:

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Abstract: Discriminating between quantum computing architectures that can provide quantum advantage from those that cannot is of crucial importance. From the fundamental point of view, establishing such a boundary is akin to pinpointing the resources for quantum advantage; from the technological point of view, it is essential for the design of non-trivial quantum computing architectures. Wigner negativity is known to be a necessary resource for computational advantage in several quantum-computing architectures, including those based on continuous variables (CVs). However, it is not a sufficient resource, and it is an open question under which conditions CV circuits displaying Wigner negativity offer the potential for quantum advantage. In this work, we identify vast families of circuits that display large Wigner negativity, and yet are classically efficiently simulatable, although they are not recognized as such by previously available theorems. These families of circuits employ bosonic codes based on either translational or rotational symmetries (e.g., Gottesman-Kitaev-Preskill or cat codes), and can include both Gaussian and non-Gaussian gates and measurements. Crucially, within these encodings, the computational basis states are described by intrinsically negative Wigner functions, even though they are stabilizer states if considered as codewords belonging to a finite-dimensional Hilbert space. We derive our results by establishing a link between the simulatability of high-dimensional discrete-variable quantum circuits and bosonic codes.

Efficient simulatability of continuous-variable circuits with large Wigner negativity

Physical Review Research 2, 043322 (2020)

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Outline

1. Introduction
2. Main results
3. Discrete-variable systems: qubits and qudits
4. DV systems: qubits into qudits
5. Efficient simulation of DV systems
6. Continuous-variable systems: qubits and qudits
7. CV systems: qubits into qudits
8. Efficient simulation of CV systems
9. Summary
10. Limitations and future work

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Introduction

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- Context: Resources for continuous-variable quantum computation
- Resource: Wigner negativity
- Wigner function: $$W_{\hat{\rho}}(q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}x\, e^{ipx} \left\langle q + \tfrac{x}{2} \middle| \hat{\rho} \middle| q - \tfrac{x}{2} \right\rangle$$
- Wigner logarithmic negativity: $$\mathcal{W}(\hat{\rho}) \equiv \log_2 \left(\int_{-\infty}^{\infty} \mathrm{d}q\, \mathrm{d}p \, |W_{\hat{\rho}}(q,p)| \right)$$

Also evolution and measurements: $\hat{\rho} \to \hat{O},\ \hat{\Pi}$

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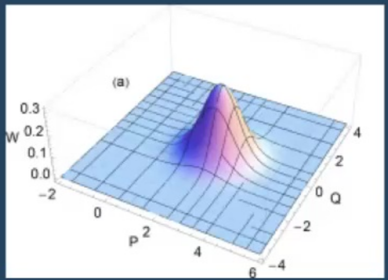
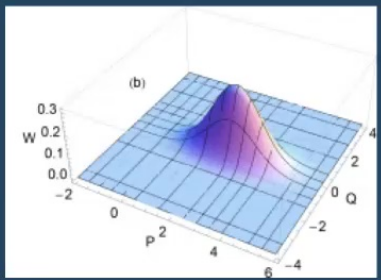
Introduction

- Wigner function: $W_{\hat{\rho}}(q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}x\, e^{ipx}\, \langle q+\frac{x}{2}|\hat{\rho}|q-\frac{x}{2}\rangle$

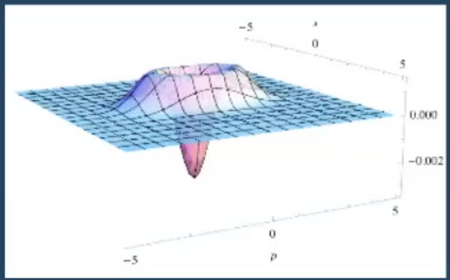
Coherent state $|\alpha\rangle$



Squeezed state $|\xi\rangle$



Single photon state $|1\rangle$



Introduction

- No-go theorem: if all Wigner functions of the quantum circuit (input state, gates, measurements) are positive, there is no quantum advantage

- Wigner negativity is a necessary resource to implement non-trivial quantum computations

S. D. Bartlett et al, PRL 88, 097904 (2002)
A. Mari, J. Eisert, PRL 109, 230503 (2012)
V. Veitch et al, New J. Phys. 14, 113011 (2012)

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Introduction

- Classical algorithms for CV: exponential scaling with the Wigner logarithmic negativity

H. Pashayan, J. Wallman, S. Bartlett PRL 115, 070501 (2015)

- Quantum circuits: instances with negative Wigner elements that yield hardness of sampling

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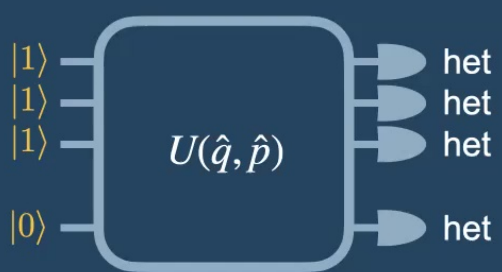
Introduction

- Quantum circuits: instances of negative Wigner elements that yield hardness of sampling

Boson sampling + heterodyne

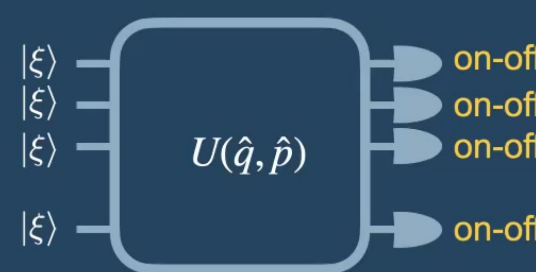


Chabaud et al, PRA 062307 (2017)
Chakhmakhchyan, PRA 032326 (2017)
Lund et al, PRA 022301 (2017)

CV Instantaneous Quantum Computing

Douce et al, PRL 118 070503 (2017)
Douce et al, PRA 99, 012344 (2019)

Gaussian Boson Sampling



Hamilton et al, PRL 119, 170501 (2017)

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Introduction

- Main motivation: What are the conditions for quantum advantage in CV?

- Is the Wigner negativity a sufficient condition?

  No.

- Immediate counter-examples: Qubits, qudits and logical gates in CV
  - **Bosonic codes:** Gottesman-Kitaev-Preskill (GKP) encoding and rotationally symmetric bosonic (RSB) codes

Introduction

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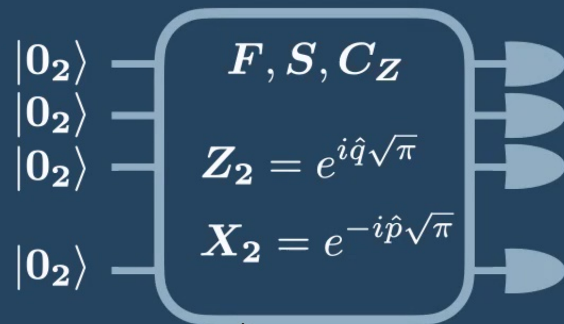
- Wigner negativity is not a sufficient condition
- Counter-example: GKP encoding

Encoded Pauli eigenstates: negative Wigner function



Encoded Clifford operations: positive Wigner function

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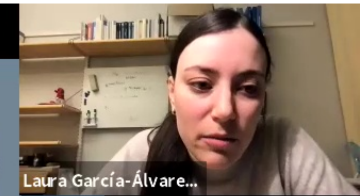
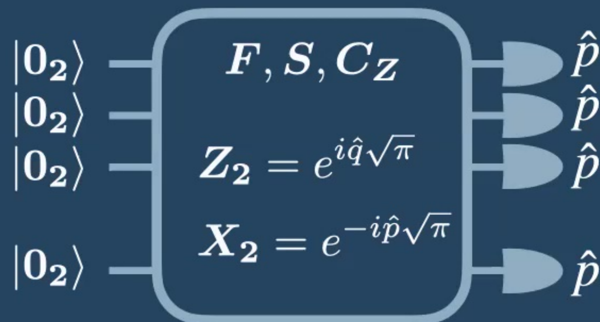


Introduction

- Wigner negativity is not a sufficient condition
- Counter-example: GKP encoding

Encoded Pauli eigenstates: negative Wigner function



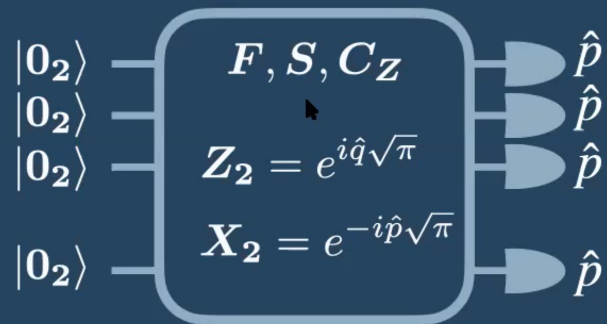
Pauli measurements: positive Wigner function

Encoded Clifford operations: positive Wigner function

Introduction

- Wigner negativity is not a sufficient condition
- Counter-example: GKP encoding

Encoded Pauli eigenstates: negative Wigner function



Pauli measurements: positive Wigner function

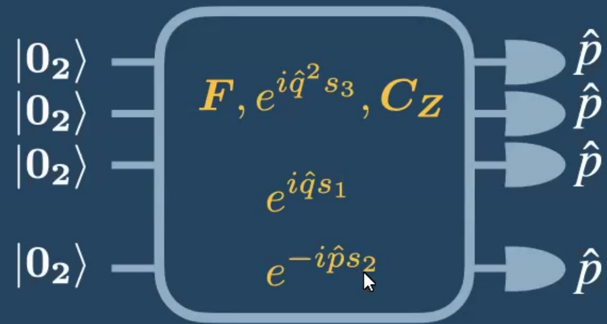
Encoded Clifford operations: positive Wigner function

- This circuit can be simulated efficiently: Gottesman-Knill theorem

Introduction

- Research question: Can we simulate this circuit efficiently?

Encoded Pauli eigenstates: negative Wigner function



Pauli measurements: positive Wigner function

General Gaussian operations: positive Wigner function

- General Gaussian operations are not logical qubit gates: Gottesman-Knill theorem ✗

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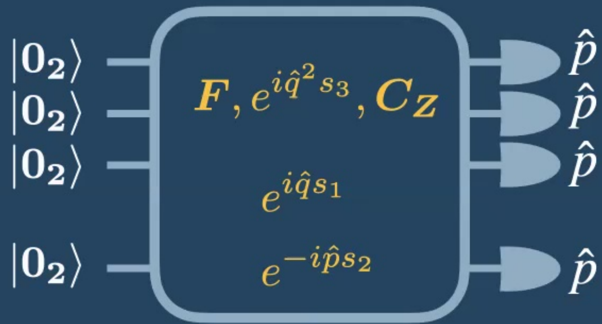
Introduction

- Research question: Can we simulate this circuit efficiently?

Encoded Pauli eigenstates: negative Wigner function



Pauli measurements: positive Wigner function

General Gaussian operations: positive Wigner function

- General Gaussian operations are not logical qubit gates: Gottesman-Knill theorem
- Wigner function is negative: Mari-Eisert theorem

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6. Continuous-variable systems: qubits and qudits
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Main results

- Basic idea:

  (i) Encode **qubits into qudits** → extended set of logical gates in CV

  (ii) Identify families of circuits as **Clifford circuits**

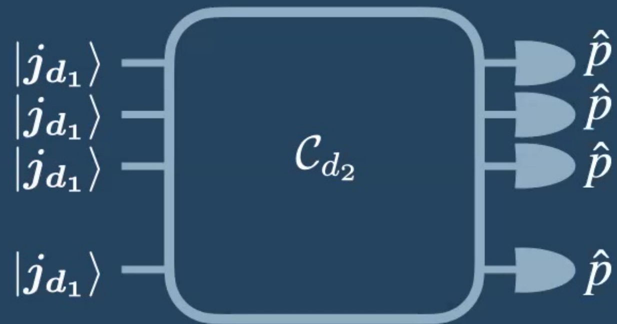
  (iii) Use **existing no-go theorems** of DV quantum computation

Main results

- Results: Translation-symmetric codes

Encoded Pauli eigenstates:
GKP qudit dimension d_1

$j = 0, \ldots, d_1 - 1$



Pauli measurements:
Homodyne
measurements (HM)

GKP Clifford group dimension d_2

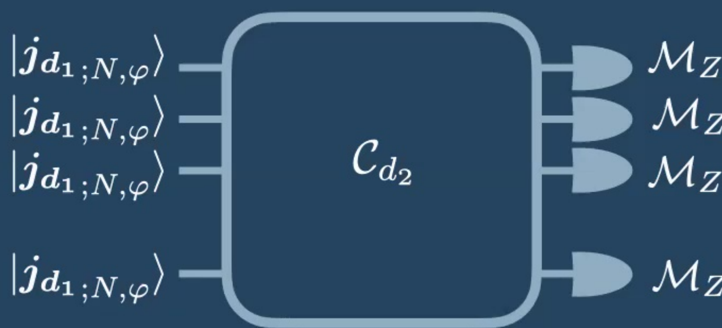
$$\{e^{i\hat{q}_k^2/2}, e^{-i\alpha\hat{p}_k}, e^{i\alpha\hat{q}_k}, e^{i\hat{q}_k\hat{q}_l}, e^{\frac{i\pi}{4}(\hat{p}_k^2+\hat{q}_k^2)}, \mathrm{HM}\} \qquad \alpha = \sqrt{2\pi/d_1}/a$$

Main results

- Results: N-fold rotation-symmetric codes (i)

Pauli eigenstates:
RSB qudit dimension d_1

$j = 0, \ldots, d_1 - 1$



Pauli measurements:
Phase measurements (PM)

RSB Clifford group dimension d_2

$$\{e^{i\frac{2\pi}{a d_1 N}\hat{n}}, e^{i\frac{\pi}{d_1}\left(\frac{\hat{n}^2}{N^2} - \beta\frac{\hat{n}}{aN}\right)}, e^{i\frac{2\pi}{d_1 N^2}\hat{n}_k\hat{n}_l}, \text{PM}\}$$

$$\beta = \begin{cases} 0 & : d_1 a^2 \text{ even} \\ 1 & : d_1 a^2 \text{ odd} \end{cases}$$

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Discrete-variable systems: qubits and qudits

- Qubits: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$
  - Pauli group single qubit: $\mathcal{P}_2 \;=\; \{\pm i^u X^v Z^w \;:\; u, v, w \in \mathbb{Z}_2\}$
  - Pauli group for n qubits: $\mathcal{P}_2^n = \bigotimes_{j=1}^{n} \mathcal{P}_2$
  - Clifford group: $\mathcal{C}_2^n = \{Q : QUQ^\dagger \in \mathcal{P}_2^n \quad \forall \quad U \in \mathcal{P}_2^n\}$

    $C_2^n = \langle H, S, \textsc{cnot}\rangle$

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Discrete-variable systems: qubits and qudits

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- Qudits: $|\psi\rangle = \sum_{j=0}^{d-1} \alpha_j |j\rangle$

  - Pauli group single qudit: $\mathcal{P}_d = \{\omega_D^u X_d^v Z_d^w : v, w \in \mathbb{Z}_d, u \in \mathbb{Z}_D\}$

$$\omega_d = e^{2\pi i/d} \qquad D = \begin{cases} d, & \text{for odd } d \\ 2d, & \text{for even } d \end{cases}$$

  - Pauli operators:

$$X_d = \sum_{j=0}^{d-1} |j + 1 \bmod d\rangle \langle j|$$

$$Z_d = \sum_{j=0}^{d-1} \omega_d^j |j\rangle \langle j|$$

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Discrete-variable systems: qubits and qudits

- Qudits:
  - Clifford group: $\mathcal{C}_d^n = \{Q : QUQ^\dagger \in \mathcal{P}_d^n \quad \forall \quad U \in \mathcal{P}_d^n\}$

    $\mathcal{C}_d^n = \langle F_d, S_d, \mathrm{SUM}_d \rangle$

  - Qudit Clifford gates:

$$F_d = \frac{1}{\sqrt{d}} \sum_{j,k=0}^{d-1} \omega_d^{jk} |k\rangle \langle j|$$

$$S_d = \sum_{j=0}^{d-1} \omega_d^{j^2/2} \eta_d^{-j} |j\rangle \langle j| \qquad \mathrm{SUM}_d^{(k,l)} = \sum_{i,j=0}^{d-1} |i\rangle^{(k)}\, {}^{(k)}\langle i| \otimes |i+j\rangle^{(l)}\, {}^{(l)}\langle j|$$

$$\eta_d = \omega_D \omega_{2d}^{-1}$$

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DV systems: qubits into qudits

- Logical qubit encoded in a physical qudit

  - States: $|\boldsymbol{j_{d_1}}\rangle = \frac{1}{\sqrt{a_2}} \sum_{k=0}^{a_2-1} |(kd_1 + j)a_1\rangle_{d_2}$ $\quad d_2 = d_1 a_1 a_2$

  - Operators: $\boldsymbol{X_{d_1}} = X_{d_2}^{a_1}$, $\boldsymbol{Z_{d_1}} = Z_{d_2}^{a_2}$

    In our work:
    symmetric encodings

  - Qubit example: $|\mathbf{0_2}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_8 + |4\rangle_8)$ $\quad d = 8$, $a_1 = a_2 = 2$

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DV systems: qubits into qudits

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- Logical qubit encoded in a physical qudit
  - States: $|\boldsymbol{j_{d_1}}\rangle = \frac{1}{\sqrt{a_2}} \sum_{k=0}^{a_2-1} |(kd_1 + j)a_1\rangle_{d_2}$ $\quad d_2 = d_1 a_1 a_2$

    RHS: Stabilizer state in d_2

  - Operators: $\boldsymbol{X_{d_1}} = X_{d_2}^{a_1}$, $\boldsymbol{Z_{d_1}} = Z_{d_2}^{a_2}$

    In our work:
    symmetric encodings

  - Qubit example: $|\boldsymbol{0_2}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_8 + |4\rangle_8)$ $\quad d = 8$, $a_1 = a_2 = 2$

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Efficient simulation of DV systems

- Gottesman-Knill theorem:

A quantum circuit based only on:

(i) Qubits initialised in a Pauli eigenstate

(ii) Clifford group operations

(iii) Pauli measurements

can be simulated efficiently with a classical computer

$$C_2^n = \langle H, S, \text{CNOT} \rangle$$

- Extensions to odd prime dimension d

$$\mathcal{C}_d^n = \langle F_d, S_d, \text{SUM}_d \rangle$$

- Extensions to any dimension

N. de Beaudrap, Quantum Information & Computation 13, 73 (2013)

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Continuous-variable systems: qubits and qudits

- **Bosonic codes:** storing error-correctable quantum information in bosonic modes
  - Translation-symmetric codes: **GKP codes**
  - Rotation-symmetric bosonic codes: **RSB codes**

- **Hilbert space of infinite dimension**

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Continuous-variable systems: qubits and qudits

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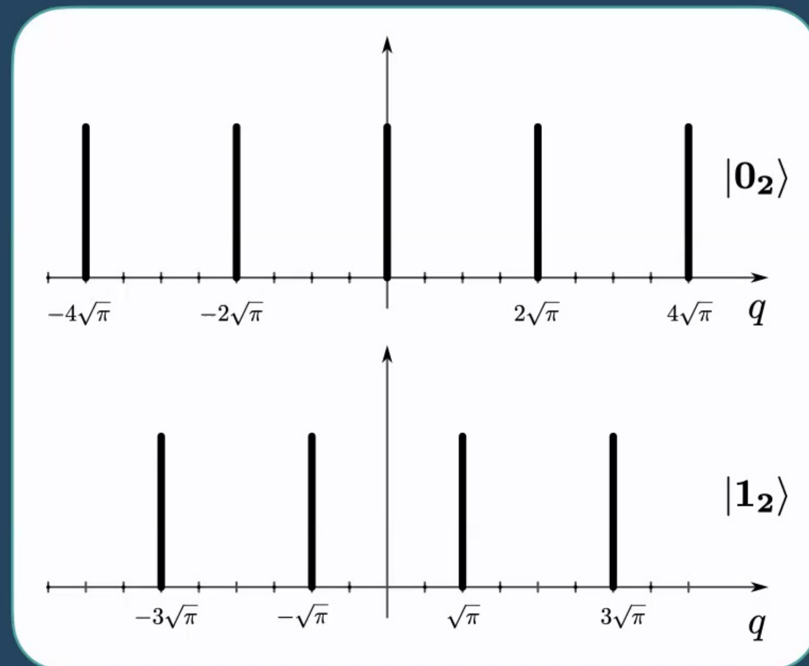
- Translation-symmetric codes: **GKP codes**
  - States (ideal):

$$|\boldsymbol{j_2}\rangle = \sum_{s\in\mathbb{Z}} |\sqrt{\pi}(j+2s)\rangle_{\hat{q}}$$

$$|\boldsymbol{j_d}\rangle = \sum_{s\in\mathbb{Z}} |\alpha(j+ds)\rangle_{\hat{q}}$$

$$\alpha = \sqrt{\frac{2\pi}{d}} \qquad j = 0, \ldots, d-1$$



30 D. Gottesman, et al, PRA 64, 012310 (2001)

Continuous-variable systems: qubits and qudits

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- Translation-symmetric codes: **GKP codes**
  - Operations:

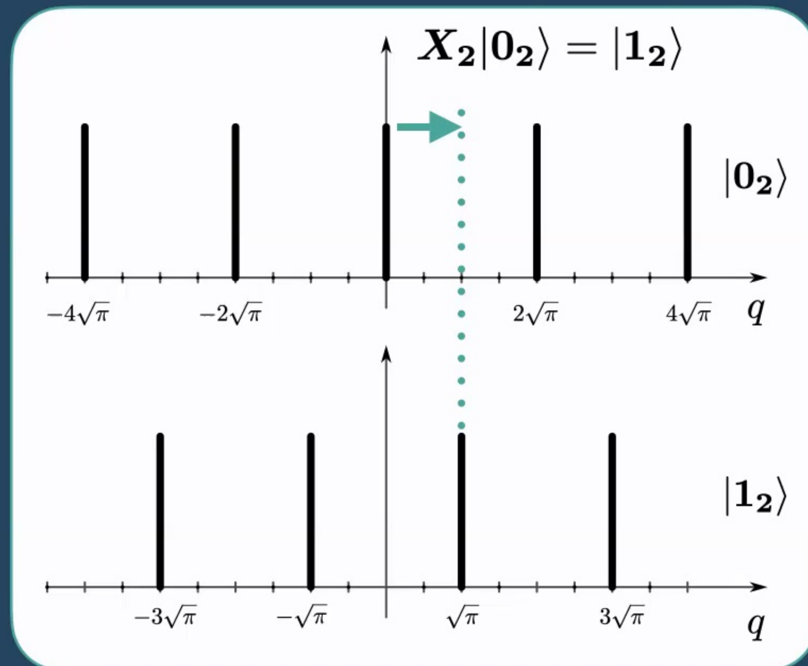
$$F = e^{i\frac{\pi}{4}\left(\hat{p}^2+\hat{q}^2\right)}$$

$$S = e^{i\frac{\hat{q}^2}{2}}$$

$$C_Z^{(k,l)} = e^{-i\hat{q}_k\hat{q}_l}$$

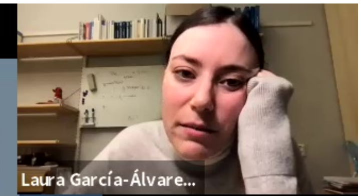
$$Z_2 = e^{i\hat{q}\sqrt{\pi}}$$

$$X_2 = e^{-i\hat{p}\sqrt{\pi}}$$



31 D. Gottesman, et al, PRA 64, 012310 (2001)

Continuous-variable systems: qubits and qudits



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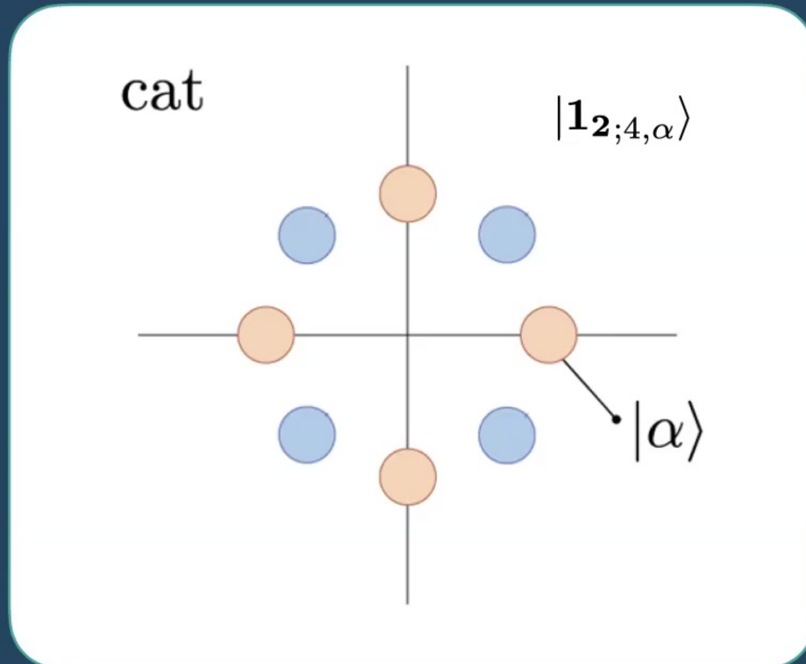
- Rotation-symmetric codes: **RSB codes**

$$\hat{R}_N = e^{i\frac{2\pi}{N}\hat{n}}$$

- States (ideal):

$$|\boldsymbol{j_2}_{;N,\varphi}\rangle = \frac{1}{\sqrt{\mathcal{N}_j}} \sum_{m=0}^{2N-1} (-1)^{jm} e^{i\frac{m\pi}{N}\hat{n}} |\varphi\rangle$$

$$\boldsymbol{Z_2} = \hat{R}_{2N} = e^{i\frac{\pi}{N}\hat{n}}$$



32 A. Grimsmo, J. Combes, B. Baragiola, PRX 10, 011058 (2020)

Continuous-variable systems: qubits and qudits

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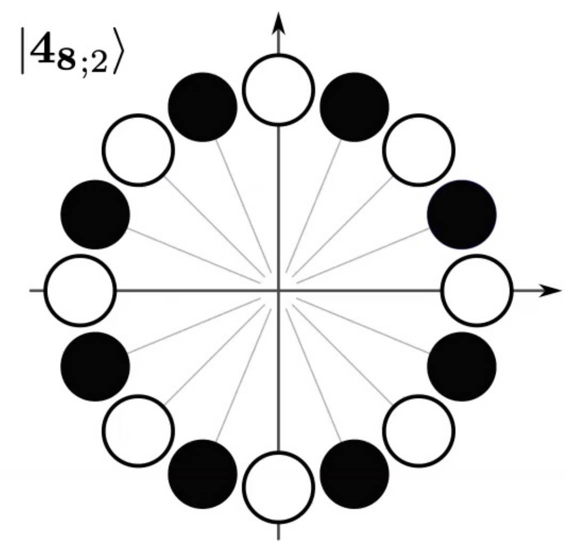
- Rotation-symmetric codes: **RSB codes**

$$\hat{R}_M = e^{i\frac{2\pi}{M}\hat{n}}$$

- States (ideal):

$$|\boldsymbol{j}_{\boldsymbol{d};M,\varphi}\rangle = \frac{1}{\sqrt{\mathcal{N}_j}} \sum_{m=0}^{dM-1} \omega_d^{-jm} e^{i\frac{2\pi}{dM}m\hat{n}} |\varphi\rangle$$

$$\boldsymbol{Z}_{\boldsymbol{d}} = \hat{R}_{dM} = e^{i\frac{2\pi}{dM}\hat{n}}$$



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Continuous-variable systems: qubits and qudits

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- Rotation-symmetric codes: **RSB codes**
  - Operations:

$$S_d = e^{i\frac{\pi}{d}\left(\frac{\hat{n}^2}{M^2} - \beta\frac{\hat{n}}{M}\right)} \qquad \beta = \begin{cases} 0 & : d \text{ even} \\ 1 & : d \text{ odd} \end{cases}$$

$$C_Z{}^{(k_N, s_M)} = e^{i\frac{2\pi}{dNM}\hat{n}_k\hat{n}_s}$$

$$F_d|\psi_d\rangle = \frac{1}{\sqrt{d}}\sum_{j,k=0}^{d-1} e^{i\frac{2\pi}{d}kj}\alpha_k|j_d\rangle$$

$$|\psi_d\rangle = \sum_{k=0}^{d-1}\alpha_k|k_d\rangle$$

Derivation: Action of operations on the logical encoded states expressed in the Fock basis

Continuous-variable systems: qubits and qudits

- Rotation-symmetric codes: **RSB codes**
  - Operations:

$$S_d = e^{i\frac{\pi}{d}\left(\frac{\hat{n}^2}{M^2} - \beta\frac{\hat{n}}{M}\right)} \qquad \beta = \begin{cases} 0 & : d \text{ even} \\ 1 & : d \text{ odd} \end{cases}$$

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Wigner negative operations

Derivation: Action of operations on the logical encoded states expressed in the Fock basis

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CV systems: qubits into qudits

- Recall: DV qubits into qudits

$$|j_{d_1}\rangle = \frac{1}{\sqrt{a_2}} \sum_{k=0}^{a_2-1} |(kd_1 + j)a_1\rangle_{d_2} \qquad d_2 = d_1 a_1 a_2$$

- CV ideal cases:
  - GKP: Infinite energy ideal GKP states
  - RSB: Orthogonal rotated primitive states $\quad \langle\varphi|Z_d^s|\varphi\rangle = 0$

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CV systems: qubits into qudits

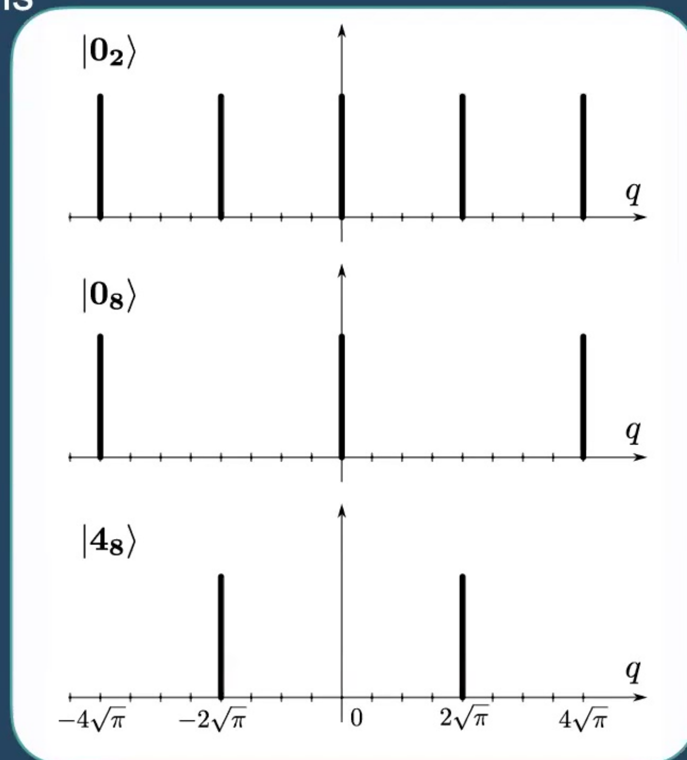
- GKP: Identification based on the wavefunctions

$$|j_{d_1}\rangle = \sum_{k=0}^{a-1} |(aj + ad_1 k)_{d_2}\rangle$$

$$d_2 = d_1 a^2$$

- Qubit:

$$|0_2\rangle = |0_8\rangle + |4_8\rangle$$



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CV systems: qubits into qudits

- GKP: Identification based on the wavefunctions

$$|j_{d_1}\rangle = \sum_{k=0}^{a-1} |(aj + ad_1 k)_{d_2}\rangle$$

$$d_2 = d_1 a^2$$

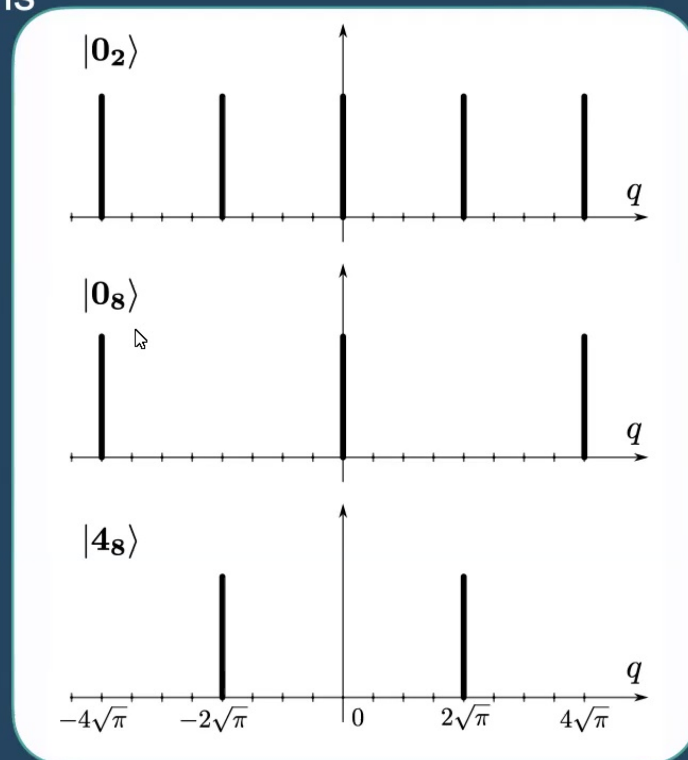
- Qubit:

$$|0_2\rangle = |0_8\rangle + |4_8\rangle$$

Larger set of displacements:

$$Z_8 = e^{i\hat{q}\sqrt{\frac{2\pi}{8}}},$$

$$X_8 = e^{-i\hat{p}\sqrt{\frac{2\pi}{8}}}$$



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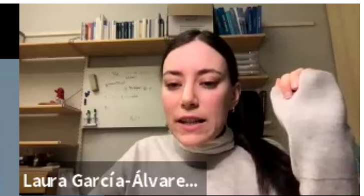
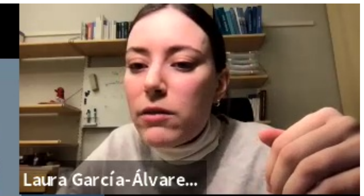




CV systems: qubits into qudits

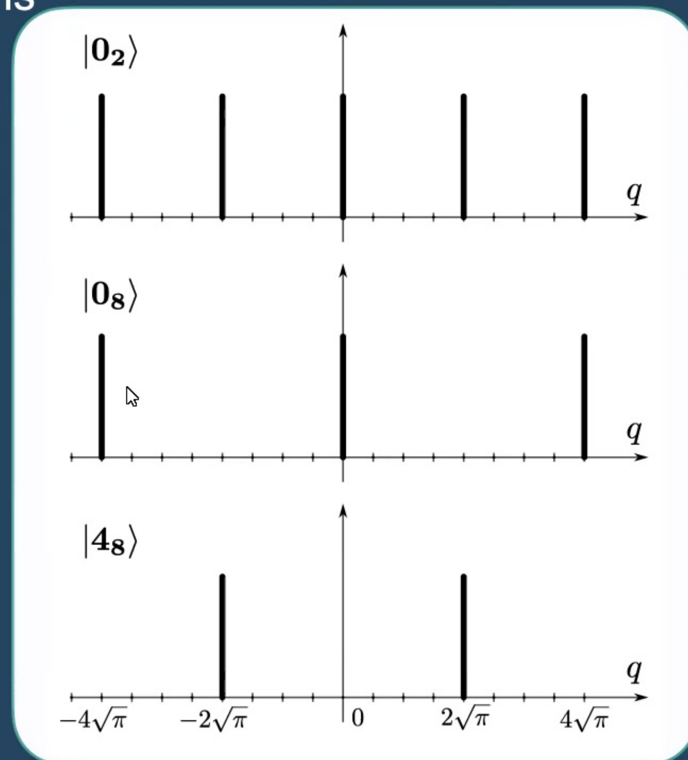
- GKP: Identification based on the wavefunctions

$$|j_{d_1}\rangle = \sum_{k=0}^{a-1} |(aj + ad_1 k)_{d_2}\rangle$$

$$d_2 = d_1 a^2$$

- Qubit:

$$|0_2\rangle = |0_8\rangle + |4_8\rangle$$



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CV systems: qubits into qudits

- GKP: Identification based on the wavefunctions

$$|j_{d_1}\rangle = \sum_{k=0}^{a-1} |(aj + ad_1 k)_{d_2}\rangle$$

$$d_2 = d_1 a^2$$

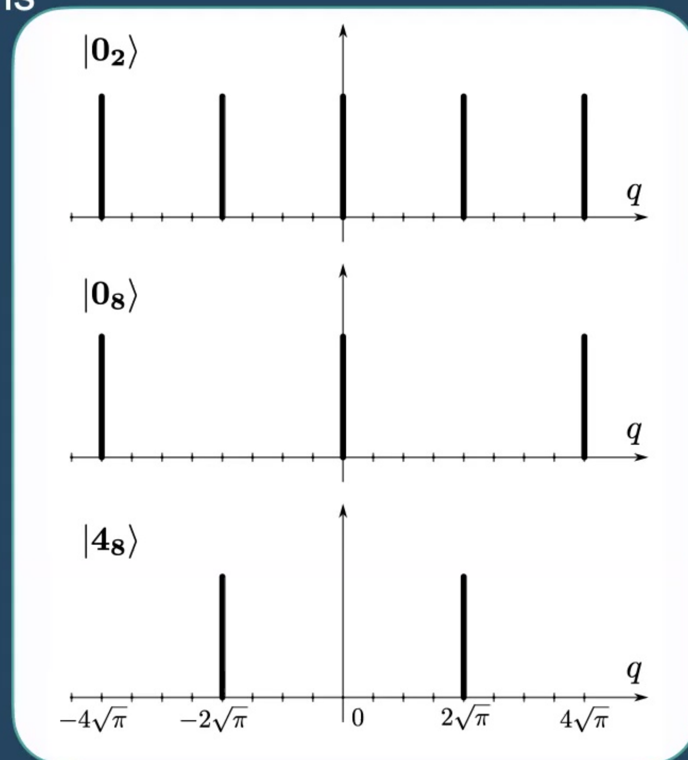
- Qubit:

$$|0_2\rangle = |0_8\rangle + |4_8\rangle$$

Larger set of displacements:

$$Z_8 = e^{i\hat{q}\sqrt{\frac{2\pi}{8}}},$$

$$X_8 = e^{-i\hat{p}\sqrt{\frac{2\pi}{8}}}$$



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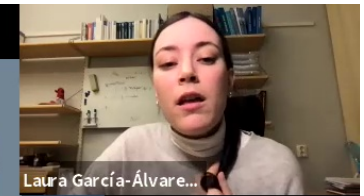
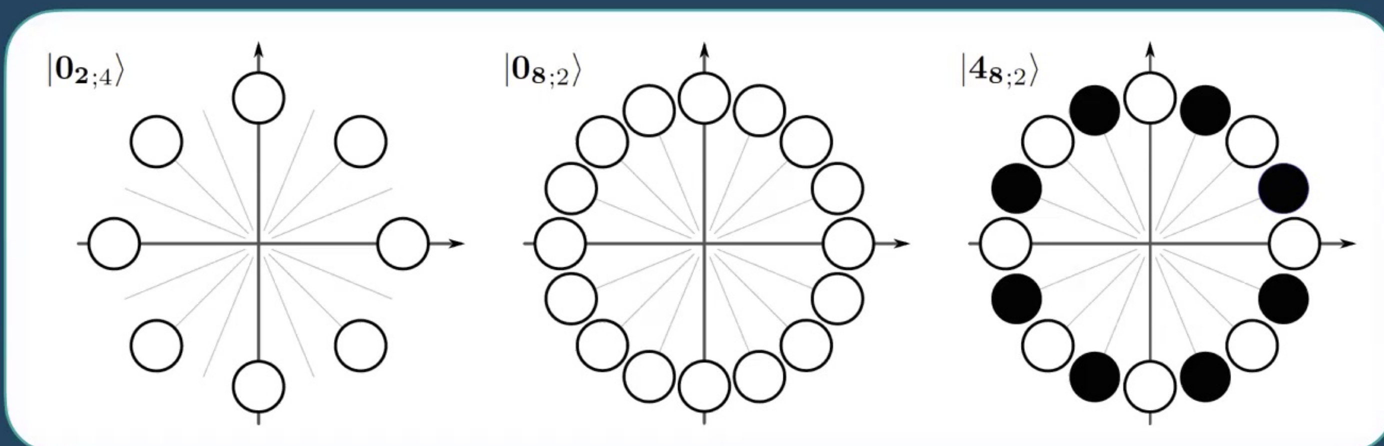
CV systems: qubits into qudits



- RSB (i): Identification based on the wavefunctions

$$|j_{d_1;N}\rangle = \frac{1}{\sqrt{a}} \sum_{t=0}^{a-1} |(aj + ad_1 t)_{d_2;\frac{N}{a}}\rangle \qquad d_2 = d_1 a^2 \qquad \begin{cases} N \geq a \\ M = N/a \end{cases}$$

- Qubit: $|\mathbf{0}_{\mathbf{2};4}\rangle = \frac{1}{\sqrt{2}} |\mathbf{0}_{\mathbf{8};2}\rangle + |\mathbf{4}_{\mathbf{8};2}\rangle$



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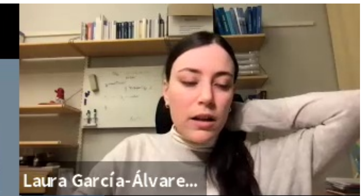
CV systems: qubits into qudits



- RSB (i):

$$|\boldsymbol{j_{d_1;N}}\rangle = \frac{1}{\sqrt{a}}\sum_{t=0}^{a-1}|(\boldsymbol{aj + ad_1 t})_{\boldsymbol{d_2;\frac{N}{a}}}\rangle \qquad d_2 = d_1 a^2 \qquad \begin{cases} N \geq a \\ M = N/a \end{cases}$$

- RSB (ii):

$$|\boldsymbol{u^k_{d_2;M,\varphi}}\rangle = \frac{1}{\sqrt{M}}\sum_{\ell=0}^{M-1} e^{i\frac{2\pi}{d_2 M}(\ell d_2 - k)\hat{n}}|\varphi\rangle$$

$$\boldsymbol{X_d}|\boldsymbol{u^k_d}\rangle = \omega^k_d|\boldsymbol{u^k_d}\rangle$$

$$|\boldsymbol{u^0_d}\rangle = |+_{\boldsymbol{d}}\rangle$$

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CV systems: qubits into qudits

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- RSB (i):

$$|\boldsymbol{j_{d_1;N}}\rangle = \frac{1}{\sqrt{a}} \sum_{t=0}^{a-1} |\boldsymbol{(aj + ad_1 t)_{d_2;\frac{N}{a}}}\rangle \qquad d_2 = d_1 a^2 \qquad \begin{cases} N \geq a \\ M = N/a \end{cases}$$

- RSB (ii):

$$|\boldsymbol{u^k_{d_2;M,\varphi}}\rangle = \frac{1}{\sqrt{M}} \sum_{\ell=0}^{M-1} e^{i\frac{2\pi}{d_2 M}(\ell d_2 - k)\hat{n}} |\varphi\rangle$$

$$\boldsymbol{X_d}|\boldsymbol{u^k_d}\rangle = \omega_d^k |\boldsymbol{u^k_d}\rangle$$

$$|\boldsymbol{u^0_d}\rangle = |\boldsymbol{+_d}\rangle$$

$$|\boldsymbol{0_{d_1;N,\varphi}}\rangle = |\boldsymbol{+_{d_2;M,\varphi}}\rangle \qquad \begin{matrix} \forall d_1, d_2 \\ M = d_1 N \end{matrix}$$

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CV systems: qubits into qudits

	Codespace of dimension d_1	Codespace of dimension d_2	Conditions
GKP	$\lvert \boldsymbol{j_{d_1}} \rangle$	$\sum_{k=0}^{a-1} \lvert \boldsymbol{(aj + ad_1 k)_{d_2}} \rangle$	$d_2 = d_1 a^2$
RSB	$\lvert \boldsymbol{j_{d_1}}_{;N} \rangle$	$\frac{1}{\sqrt{a}} \sum_{t=0}^{a-1} \lvert \boldsymbol{(aj + ad_1 t)_{d_2}}_{;M} \rangle$	$d_2 = d_1 a^2$ $M = N/a$
RSB	$\lvert \boldsymbol{0_{d_1}}_{;N} \rangle$	$\lvert \boldsymbol{+_{d_2}}_{;M} \rangle$	$\forall d_1, d_2$ $M = d_1 N$

Efficient simulation of CV systems

- Basic idea:
  - (i) Encode **qubits into qudits** → extended set of logical gates in CV
  - (ii) Identify families of circuits as **Clifford circuits**
  - (iii) Use **existing no-go theorems** of DV quantum computation

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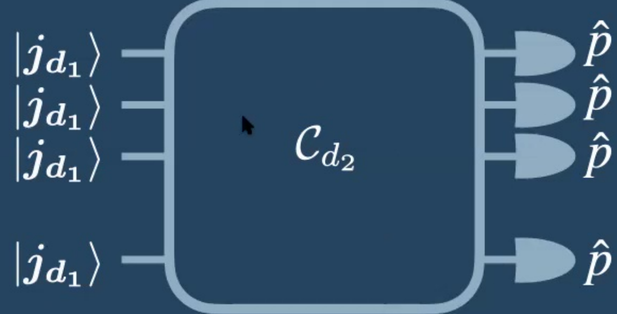
Main results

- Results: Translation-symmetric codes

Encoded Pauli eigenstates:
GKP qudit dimension d_1

$j = 0, \ldots, d_1 - 1$



Pauli measurements:
Homodyne
measurements (HM)

GKP Clifford group dimension d_2

$$\{e^{i\hat{q}_k^2/2}, e^{-i\alpha\hat{p}_k}, e^{i\alpha\hat{q}_k}, e^{i\hat{q}_k\hat{q}_l}, e^{\frac{i\pi}{4}(\hat{p}_k^2+\hat{q}_k^2)}, \mathrm{HM}\} \qquad \alpha = \sqrt{2\pi/d_1}/a$$

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Main results

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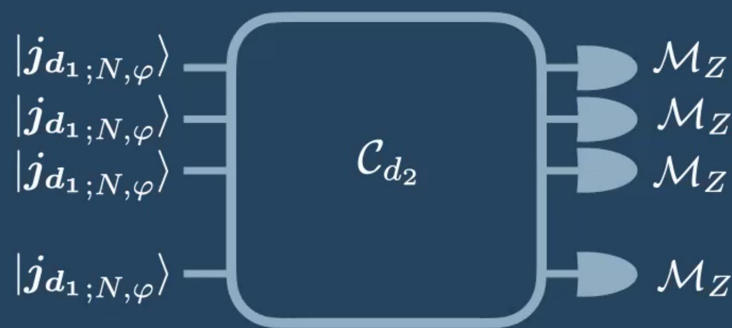
- Results: N-fold rotation-symmetric codes (i)

Pauli eigenstates:
RSB qudit dimension d_1

$j = 0, \ldots, d_1 - 1$



Pauli measurements:
Phase measurements (PM)

RSB Clifford group dimension d_2

$$\{e^{i\frac{2\pi}{ad_1N}\hat{n}}, e^{i\frac{\pi}{d_1}\left(\frac{\hat{n}^2}{N^2}-\beta\frac{\hat{n}}{aN}\right)}, e^{i\frac{2\pi}{d_1N^2}\hat{n}_k\hat{n}_l}, \mathrm{PM}\}$$

$$\beta = \begin{cases} 0 & : d_1a^2 \text{ even} \\ 1 & : d_1a^2 \text{ odd} \end{cases}$$

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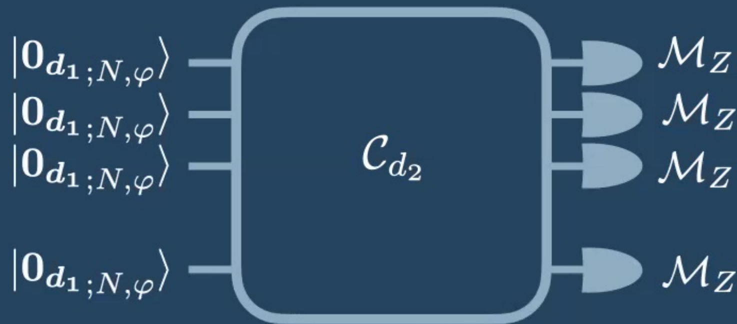
Main results

- Results: N-fold rotation-symmetric codes (ii)

Pauli eigenstates:
RSB qudit dimension d_1



Pauli measurements:
Phase measurements (PM)

RSB Clifford group dimension d_2

$$\{e^{i\frac{2\pi}{d_2 d_1 N}\hat{n}}, e^{i\frac{\pi}{d_2 d_1}\left(\frac{\hat{n}^2}{d_1 N^2}-\beta\frac{\hat{n}}{N}\right)}, e^{i\frac{2\pi}{d_2 {d_1}^2 N^2}\hat{n}_k\hat{n}_l}, \mathrm{PM}\}$$

$$\beta = \begin{cases} 0 & : d_2 \text{ even} \\ 1 & : d_2 \text{ odd} \end{cases}$$

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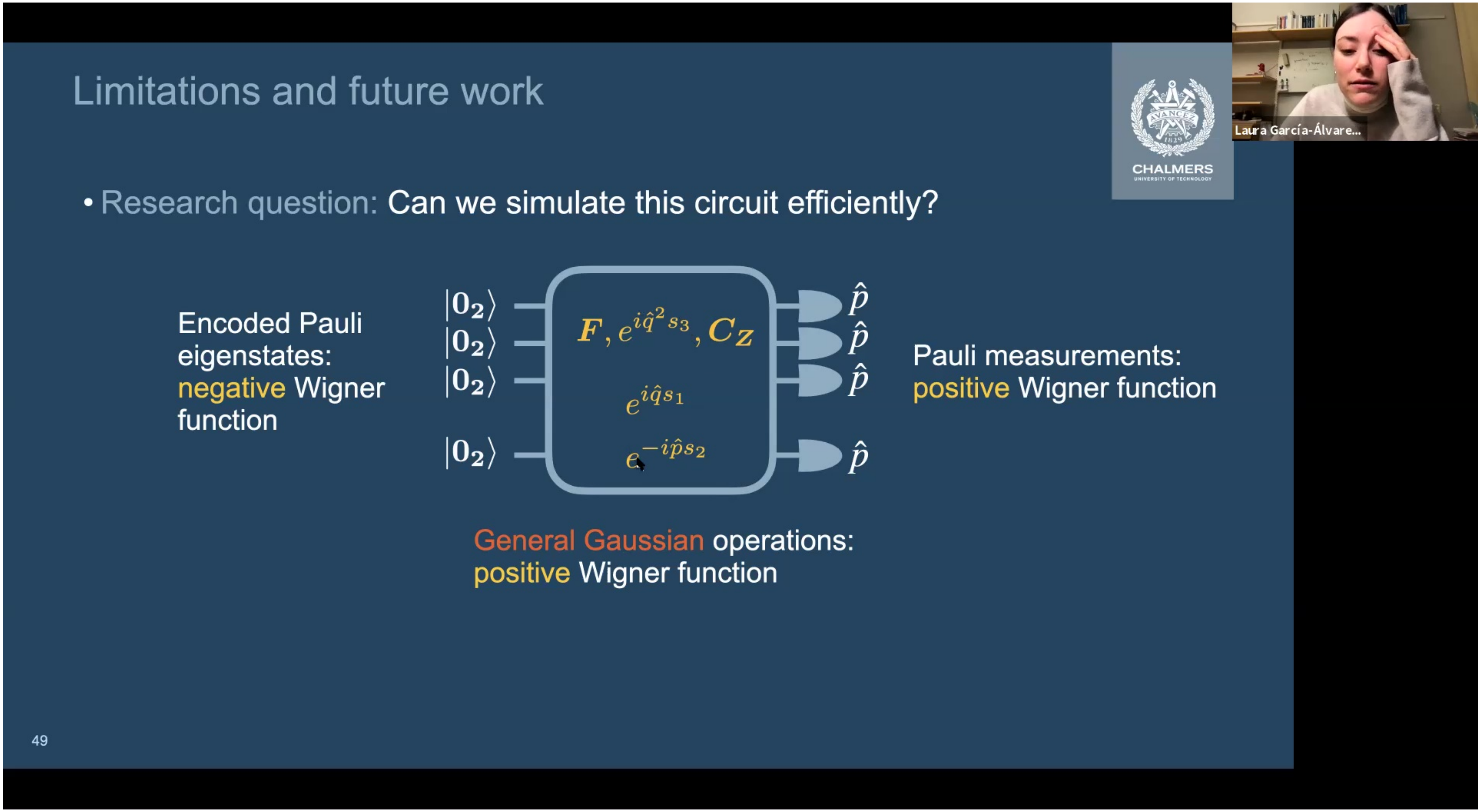
Summary

- Wigner negativity is a necessary but not sufficient resource for quantum advantage
- Identify as Clifford circuits Wigner negative CV architectures
    - Review and derive qudit operations for GKP and RSB codes
    - Encode qubits into qudits by comparing wavefunctions
- Discrete families of CV architectures with large Wigner negativity can be efficiently simulated


Limitations and future work
• Research question: Can we simulate this circuit efficiently?
Encoded Pauli eigenstates: negative Wigner function
$|0_2\rangle$
$|0_2\rangle$
$|0_2\rangle$
$|0_2\rangle$
$F, e^{i\hat{q}^2 s_3}, C_Z$
$e^{i\hat{q}s_1}$
$e^{-i\hat{p}s_2}$
$\hat{p}$
$\hat{p}$
$\hat{p}$
$\hat{p}$
Pauli measurements: positive Wigner function
General Gaussian operations: positive Wigner function
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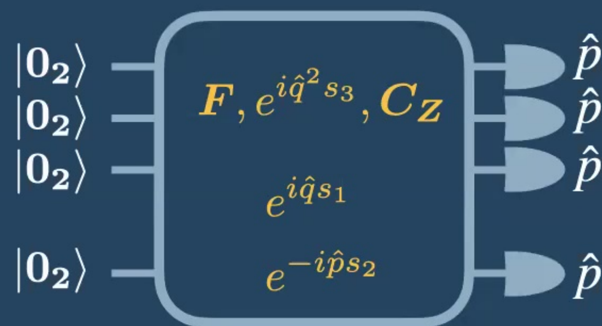
Laura García-Álvare...

Limitations and future work

- Research question: Can we simulate this circuit efficiently?

Encoded Pauli eigenstates: negative Wigner function



Pauli measurements: positive Wigner function

General Gaussian operations: positive Wigner function

- Question still open for general Gaussian circuits
- Results limited to ideal GKP and RSB codes: finite energy?

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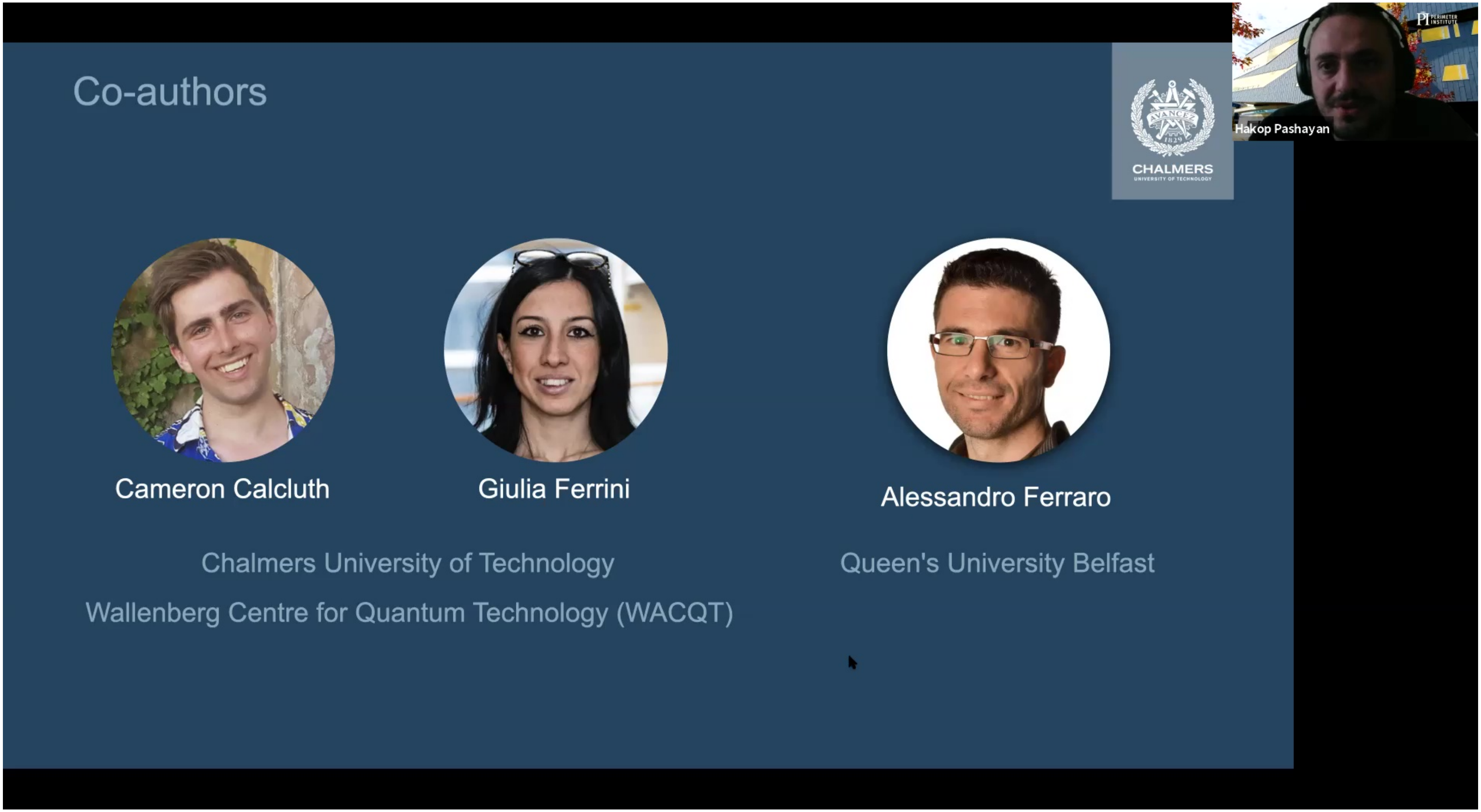
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