Title: Quantum preparation games

Speakers: Mirjam Weilenmann

Series: Perimeter Institute Quantum Discussions

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Abstract: To analyze the performance of adaptive measurement protocols for the detection and quantification of state resources, we introduce the framework of quantum preparation games. A preparation game is a task whereby a player sequentially sends a number of quantum states to a referee, who probes each of them and announces the measurement result. The measurement setting at each round, as well as the final score of the game, are decided by the referee based on the past history of settings and measurement outcomes. We show how to compute the maximum average score that a player can achieve under very general constraints on their preparation devices and provide practical methods to carry out optimizations over n-round preparation games. We apply our general results to devise new adaptive protocols for entanglement detection and quantification. Given a set of experimentally available local measurement settings, we provide an algorithm to derive, via convex optimization, optimal n-shot protocols for entanglement detection using these settings. We also present families of adaptive protocols for multiple-target entanglement detection with arbitrarily many rounds. Surprisingly, we find that there exist instances of entanglement detection problems with just one target entangled state where the optimal adaptive protocol supersedes all non-adaptive alternatives.

Quantum Preparation Games

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joint work with Edgar A. Aguilar and Miguel Navascués

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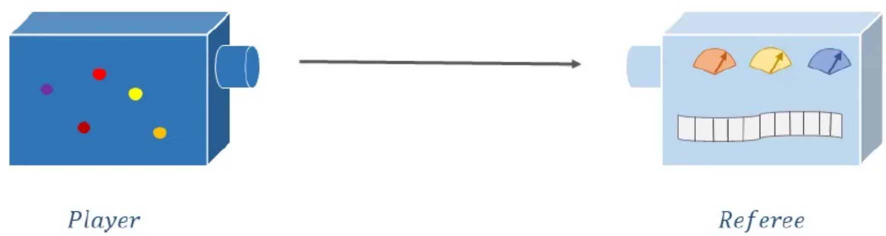
ÖAW

Presentation based on arXiv:2011.02216

Motivation: Certifying and Quantifying Quantum Resources

Picture credits: Christian Murzek.

Quantum Preparation Games – General Setting



Basic setting: player prepares resources, referee scores player's resources after n rounds

Player's strategy $\mathcal{P}$

- Prepare quantum systems from $\mathcal{C}$

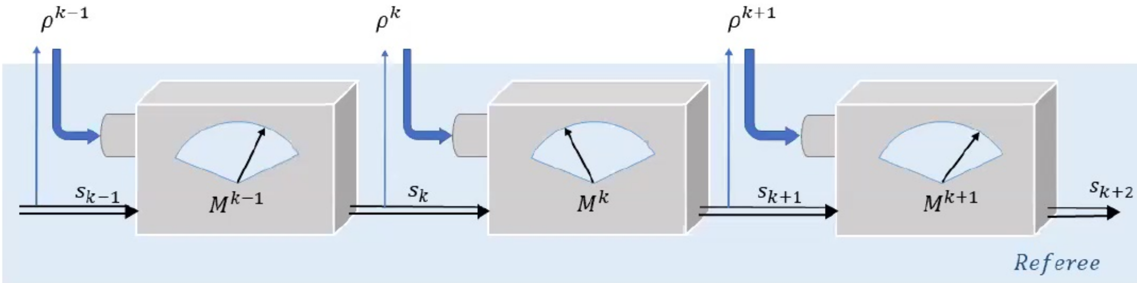
Protocols: Referee's strategies $(\mathcal{M}, \mathcal{S}, g)$

- Measuring devices $\mathcal{M}$
- Classical memory with states $\mathcal{S}$
- Scoring rule g

Expected score for a player with strategy $\mathcal{P}$

$$G(\mathcal{P}) = \sum_{s \in \mathcal{S}} p(s|\mathcal{P}) \langle g(s) \rangle .$$

Quantum Preparation Games – Referee



- Recursive computation (and optimisation) of the score of a player:

$$\mu_s^{(n)} = \max_{\rho \in \mathcal{C}} \sum_{s'} \mathrm{tr}(M_{s'|s}^{(n)} \rho) \langle g(s') \rangle,$$

$$\mu_s^{(k)} = \max_{\rho \in \mathcal{C}} \sum_{s'} \mathrm{tr}(M_{s'|s}^{(k)} \rho) \mu_{s'}^{(k+1)},$$

$$G_{\max} = \mu^{(1)}.$$

- Maxwell-demon games: full information about previous states of the experiment in memory $\mathcal{S}_k$.

Preparation Games for Quantifying Resources from Gradient Descent

Preparation game $(\mathcal{M}, \mathcal{S}, g)$ with memory requirements growing as $O(k^N)$ with round number k.

- Choose one out of N ± 1-outcome measurements $M_1(k), M_2(k), \ldots, M_N(k)$ according to some probability distribution.
- Counter of positive vs. negative outcomes for each measurement $(s_1(k), \ldots, s_N(k)) \in \{-k, \ldots k\}^N$.
- Scoring rule in terms of final memory state $g(s_1(n), s_2(n), \ldots, s_N(n))$.

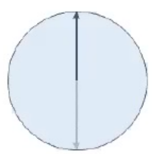
Example: Preparation Game for Quantifying 2-qubit Entanglement

Task: Find game to quantify the entanglement of $|\psi_\theta\rangle = \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$ in n-rounds.

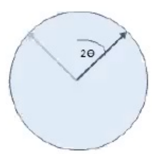
- Alice and Bob perform adaptive measurements ,

$$M_1(k) = \left\{ \frac{\mathbb{I} + W(\theta)}{2}, \frac{\mathbb{I} - W(\theta)}{2} \right\},$$

$\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$



$\{|+\rangle\langle +|, |-\rangle\langle -|\}$



$$|\psi_\theta\rangle = \cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$$
$$= |+\rangle \left(\frac{\cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle}{\sqrt{2}} \right) + |-\rangle \left(\frac{\cos(\theta)\,|0\rangle - \sin(\theta)\,|1\rangle}{\sqrt{2}} \right)$$

$$W(\theta) = \frac{1}{2}\,[\ |0\rangle\langle 0| \otimes Z + |1\rangle\langle 1| \otimes (-Z)$$
$$+ |+\rangle\langle +| \otimes (\sin(2\theta)X + \cos(2\theta)Z) + |-\rangle\langle -| \otimes (-\sin(2\theta)X + \cos(2\theta)Z)]$$

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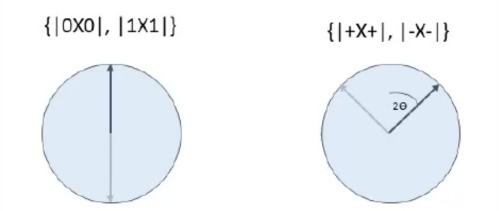

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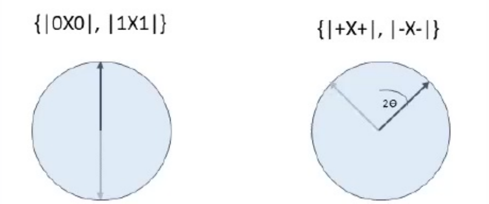
- Alice and Bob perform adaptive measurements for $\theta = \theta(s_2(k))$,

$$M_1(k) = \left\{ \frac{\mathbb{I} + W(\theta)}{2}, \frac{\mathbb{I} - W(\theta)}{2} \right\}, \quad M_2(k) = \left\{ \frac{\mathbb{I} + \frac{\partial}{\partial\theta} W(\theta)}{2}, \frac{\mathbb{I} - \frac{\partial}{\partial\theta} W(\theta)}{2} \right\}.$$



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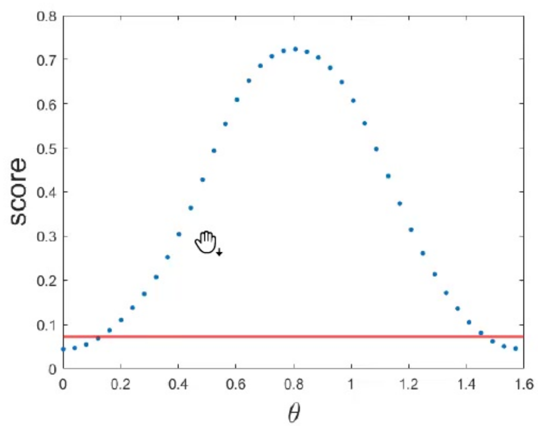
$$+ |+\rangle\langle +| \otimes (\sin(2\theta)X + \cos(2\theta)Z) + |-\rangle\langle -| \otimes (-\sin(2\theta)X + \cos(2\theta)Z)]$$

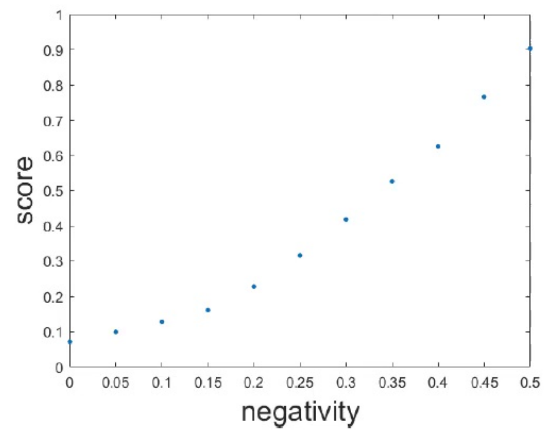
- Final score: $g(s_1(n), \theta = \theta(s_2(n))) = h\left(\cos^2(\theta)\right) \Theta\left(s_1(n) - \delta(\theta)\right)$

Quantifying 2-qubit Entanglement in 40 Rounds

Player preparing states $|\psi_\theta\rangle\langle\psi_\theta|^{\otimes n}$



Player preparing states $\sigma \in \mathcal{E}_\mathcal{N}$



Negativity $\mathcal{N}(\sigma) = \frac{\|\sigma^{T_B}\|_1 - 1}{2}$.

Preparation Games for Performing Hypothesis Tests

Task: certify entanglement of a state ρ in n rounds; distinguish $\mathcal{E}_{\mathrm{ENT}} = \{\rho^{\otimes n}\}$ from $\mathcal{E}_{\mathrm{SEP}}$.

- Binary final outcome $s \in \{\mathrm{ent}, \mathrm{sep}\}$ with $g(\mathrm{ent}) = 1$, $g(\mathrm{sep}) = 0$.
- Quality of the protocol given by the worst-case errors

$$e_I = \max_{\mathcal{P} \in \mathcal{E}_{\mathrm{SEP}}} p(\mathrm{ent}|\mathcal{P})$$

$$e_{II} = \max_{\mathcal{P} \in \mathcal{E}_{\mathrm{ENT}}} p(\mathrm{sep}|\mathcal{P})$$

Maximal expected score for separable strategy

Goal: design optimal protocols $(\mathcal{M}, \mathcal{S}, g)$ for ρ taking restrictions on $\mathcal{M}$ of the referee into account.

Optimising Quantum Preparation Games

- Solving problems of the following type (1-round version)

$$\min_{(M_s)_s, e_{II}} \; e_{II}$$

$$\text{s.t. } 1 - \sum_s \text{tr}(M_s \rho)\langle g(s)\rangle \leq e_{II},$$

$$\sum_s \text{tr}(M_s \sigma)\langle g(s)\rangle \leq e_I \quad \forall \sigma \in \text{SEP},$$

$$(M_s)_s \in \mathcal{M},$$

$p(\text{sep}|\rho)$

$p(\text{ent}|\sigma)$

Optimising Quantum Preparation Games

- Solving problems of the following type (1-round version)

$$\begin{aligned} &\min_{(M_s)_s, e_{II}} \; e_{II} \\ \text{s.t. } & 1 - \sum_s \mathrm{tr}(M_s \rho)\langle g(s)\rangle \leq e_{II}, \\ & e_I \mathbb{I} - \sum_s M_s \langle g(s)\rangle \in \mathrm{SEP}^*, \\ & (M_s)_s \in \mathcal{M}, \end{aligned}$$

- For SEP, the dual to the Doherty-Parillo-Spedalieri hierarchy approximates SEP^* (from the inside).

Doherty, Parillo, Spedalieri, PRL 88, 2002 and PRA 69, 2004.

Optimising Quantum Preparation Games

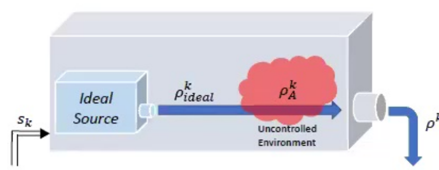
- Solving problems of the following type (1-round version)

$$\begin{aligned} &\min_{(M_s)_s, e_{II}} \quad e_{II} \\ \text{s.t. } & 1 - \sum_s \mathrm{tr}(M_s \rho)\langle g(s)\rangle \leq e_{II} \qquad \forall \rho \in E, \\ & e_I \mathbb{I} - \sum_s M_s \langle g(s)\rangle \in \mathrm{SEP}^*, \\ & (M_s)_s \in \mathcal{M}, \end{aligned}$$

- For SEP, the dual to the Doherty-Parillo-Spedalieri hierarchy approximates SEP^* (from the inside).
- Applies to cases where

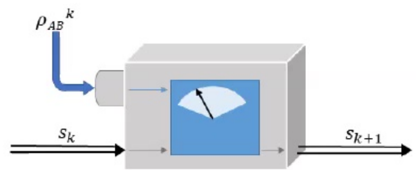
$$E = \{\rho' \mid \rho' \geq 0,\ \mathrm{tr}(\rho') = 1,\ \|\rho' - \rho\|_1 \leq \epsilon\}.$$

- Applies to finitely correlated strategies where correlations between rounds may build up in the uncontrolled environment.

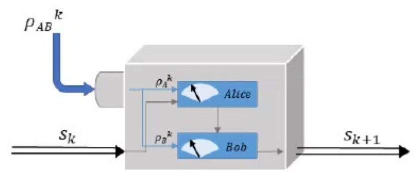


Entanglement Certification with Various Types of Measurements

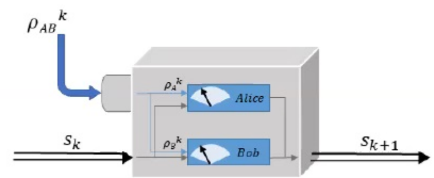
Global measurements



Adaptive measurements



Non-adaptive measureme



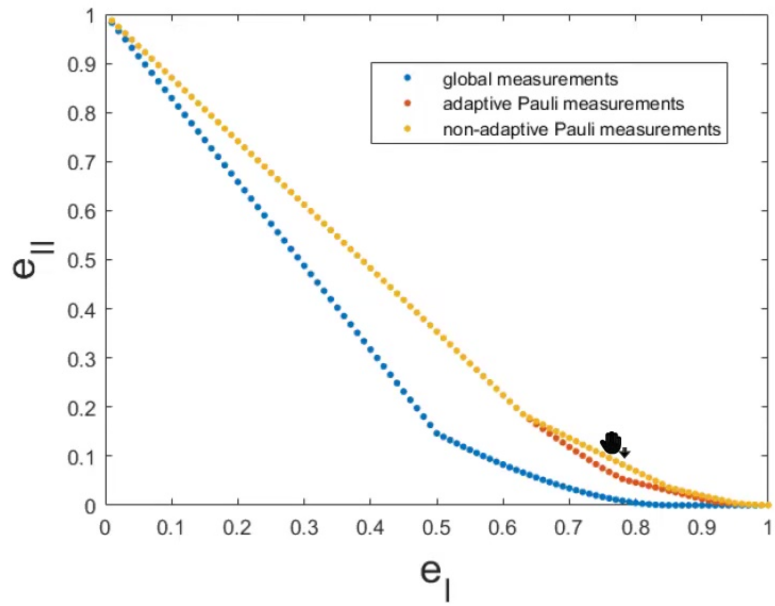
- Global measurements: any POVM $(M_s)_s$.
- Adaptive Pauli measurements $M_s = \sum_{x,y} P(x,y,s|a,b)\sigma_{x,a} \otimes \sigma_{y,b}$ s.t.

$$\sum_{y,s} P(x,y,s|a,b) = P(x) \quad \text{and} \quad \sum_{s} P(x,y,s|a,b) = P(x,y|a).$$

- Non-adaptive Pauli measurements $M_s = \sum_{x,y} P(x,y,s|a,b)\sigma_{x,a} \otimes \sigma_{y,b}$ s.t.

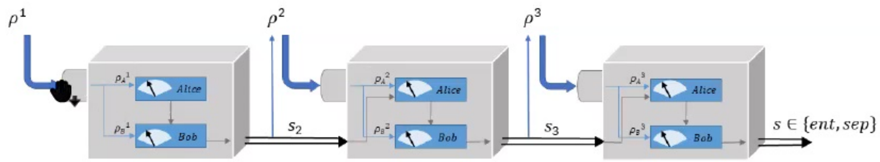
$$\sum_{s} P(x,y,s|a,b) = P(x,y).$$

Single-shot Entanglement Certification for $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |1+\rangle)$



Multi-round Maxwell Demon Games

Example: 3-round entanglement detection of $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |1+\rangle)$ with adaptive Pa[...] measurements.



$$M_s = \sum_{x_1, y_1, x_2, y_2, x_3, y_3} P(x_1 y_1 \ldots y_3 s | a_1 b_1 \ldots b_3)\, \sigma_{x_1, a_1} \otimes \sigma_{y_1, b_1} \otimes \cdots \otimes \sigma_{y_3, b_3}.$$

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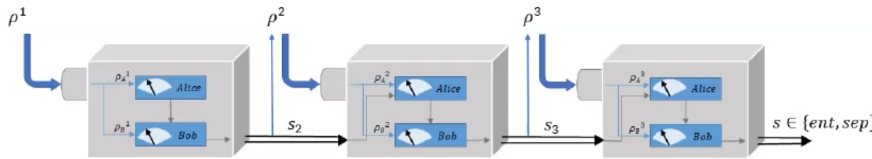
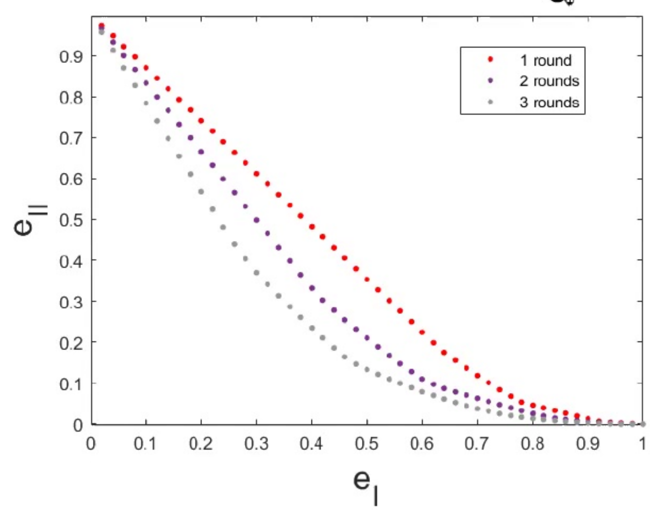
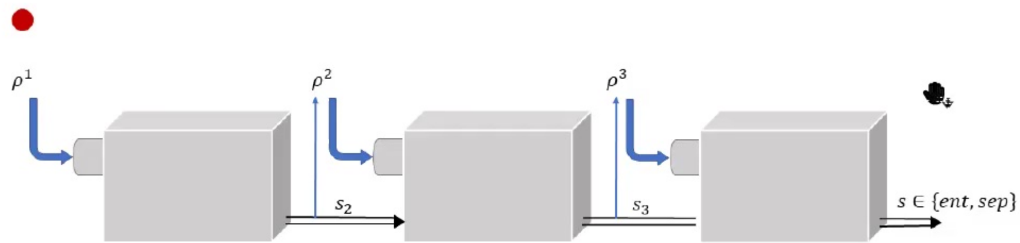
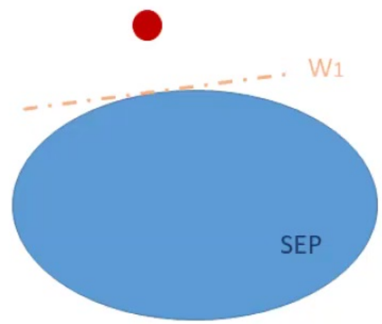


$$M_s = \sum_{x_1, y_1, x_2, y_2, x_3, y_3} P(x_1 y_1 \dots y_3 s | a_1 b_1 \dots b_3)\, \sigma_{x_1, a_1} \otimes \sigma_{y_1, b_1} \otimes \cdots \otimes \sigma_{y_3, b_3}.$$



Adaptiveness in Maxwell Demon Games





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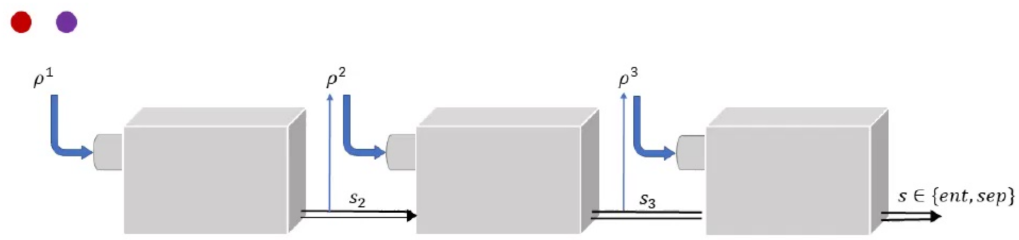

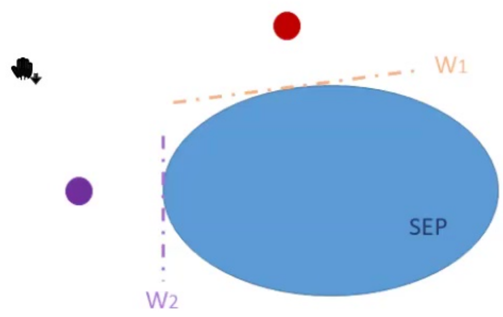

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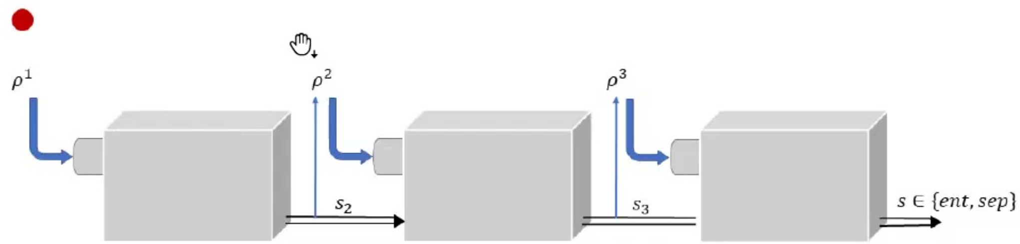

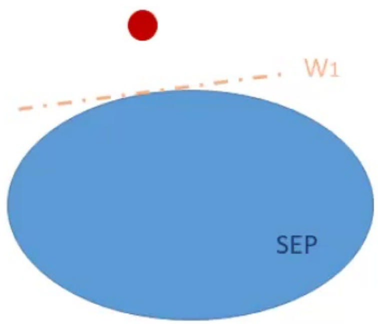

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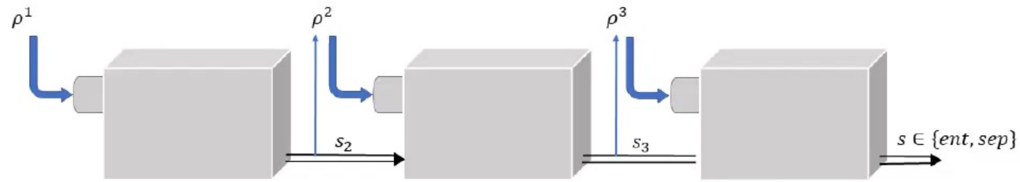


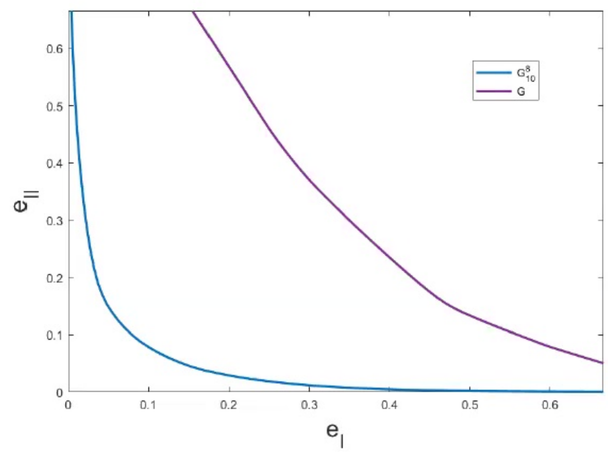
Surprise: Adaptiveness between rounds helps for entanglement detection of single states (e.g. for $|\phi\rangle$).

Beyond Maxwell Demon Games: Repetition of Optimal Protocols

- Construct G_v^m by playing a game G m times and accept if at least v of the individual games are won.

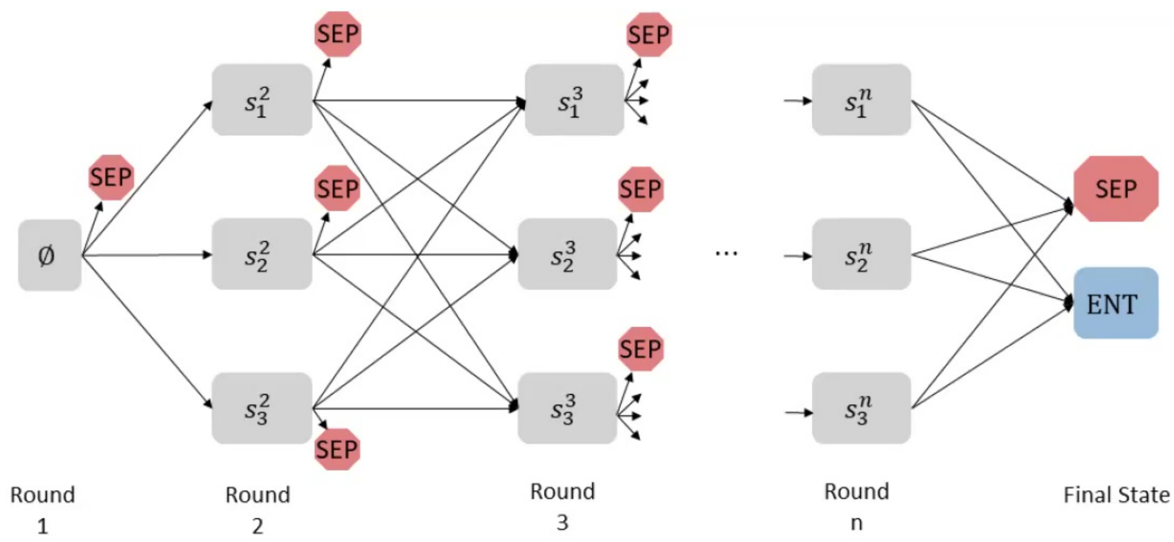
- Optimal score for a player restricted to strategies $\mathcal{P} \in \mathcal{S}$,

$$\max_{\mathcal{P}\in\mathcal{S}} G_v^m(\mathcal{P}) = \sum_{k=v}^{m} \binom{m}{k} G(\mathcal{P}^*)^k (1 - G(\mathcal{P}^*))^{n-k}.$$



Elkouss & Wehner, npj Quantum Information 2, 2016.

Multi-round Preparation Games with Restricted Memory



Weilenmann, Mirjam

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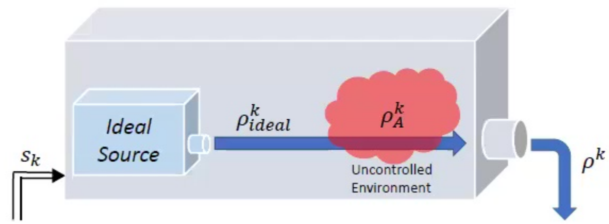


- Task: certify the entanglement of $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ in $n = 20$ rounds interacting (for $t = 0.1$) with an environment according to

$$H_I = a^\dagger \otimes (\mathbb{I} \otimes |0\rangle\langle 1| + |0\rangle\langle 1| \otimes \mathbb{I}) + a \otimes (\mathbb{I} \otimes |1\rangle\langle 0| + |1\rangle\langle 0| \otimes \mathbb{I}) .$$

- Optimisation problem (round by round see-saw optimisation)

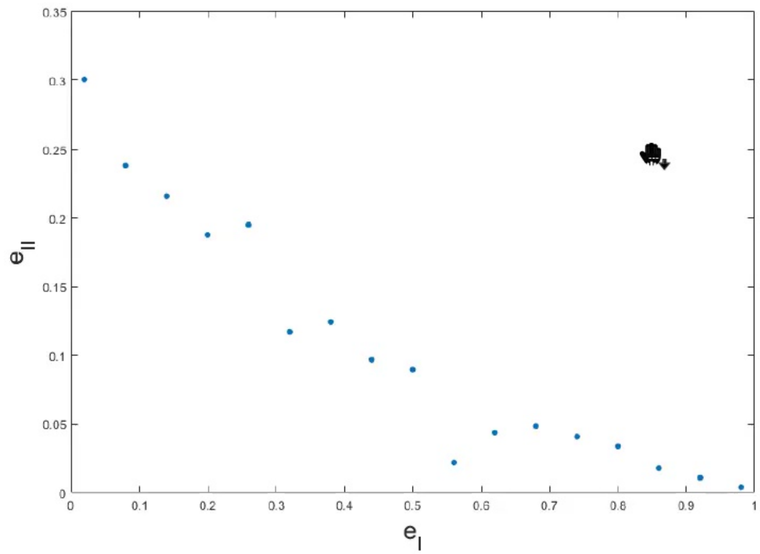
$$\min_{\{M^{(k)}_{s_k}\}_{k,s_k}, e_{II}} \quad e_{II}$$

$$\text{s.t.} \quad e_I \geq \text{tr}[M(\{M^{(k)}_{s_k}\}_{k,s_k})\sigma] \quad \forall \sigma \in \text{SEP}$$

$$e_{II} \geq 1 - \text{tr}[\Omega_\rho(\{M^{(k)}_{s_k}\}_{k,s_k})\rho_A] \quad \forall \rho_A$$

with recursive decomposition of Ω_ρ and of $M(\{M^{(k)}_{s_k}\}_{k,s_k})$ as before.

Example: Entanglement Certification when Interacting with an Environme



Summary and Open Questions

- Framework of preparation games allows us to analyse and optimise protocols.
- Optimal protocols for Maxwell demon games with few rounds.
- Various ways to construct protocols for larger round numbers.
- Implications for other resources ? (High-dimensional entanglement, multi-party entanglement, non-locality, magic states)
- Allow referee to conduct resourceful operations ?
- Application in NISQ devices (allow referee with quantum memory of fixed dimension) ?
- Asymptotic rates for Maxwell-demon games ?

Hu et al., Optimized detection of unfaithful high-dimensional entanglement, arXiv:2011.02217.

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Thank you!

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