Title: Galilean and Carrollian relativities in noncommutative spacetime models

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Series: Quantum Gravity

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Abstract: I describe the non-relativistic câ†’âˆž and ultra-relativistic câ†’0 limits of the kappa-deformed symmetries, with and without a cosmological constant. The corresponding kappa-Newtonian and kappa-Carrollian noncommutative spacetimes are also obtained. These constructions show the non-trivial interplay between the quantum deformation parameter kappa, the curvature parameter Lambda and the speed of light parameter c.

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Galilean and Carrollian relativities in noncommutative space time models

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- A. Ballesteros, G. Gubitosi, I. Gutierrez-Sagredo, F. Herranz PLB 805 (2020)

See also:

Lorentzian Snyder spacetimes and their Galilei and Carroll limits from projective geometry

Angel Ballesteros (Burgos U.), Giulia Gubitosi (Burgos U.), Francisco J. Herranz (Burgos U.) (Dec 30, 2019)

Published in: Class.Quant.Grav. 37 (2020) 19, 195021 • e-Print: 1912.12878 [hep-th]

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Possible Kinematics

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The κ-Newtonian and κ-Carrollian algebras and their noncommutative spacetimes

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Ann. Inst. Henri Poincaré, Section A :
Vol. III, n° 1, 1965, p. 1-12. *Physique théorique.*

Une nouvelle limite non-relativiste du groupe de Poincaré

par

Jean-Marc LÉVY-LEBLOND
(Laboratoire de Physique Théorique, Orsay).

The κ-(A)dS noncommutative spacetime

Angel Ballesteros (Burgos U.), Ivan Gutierrez-Sagredo (Burgos U.), Francisco J. Herranz (Burgos U.) (May 29, 2019)

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The κ-(A)dS quantum algebra in (3+1) dimensions

Ángel Ballesteros (Burgos Polytechnic U.), Francisco J. Herranz (Burgos U.), Fabio Musso (Rome U.), Pedro Naranjo (Burgos U.) (Dec 9, 2016)

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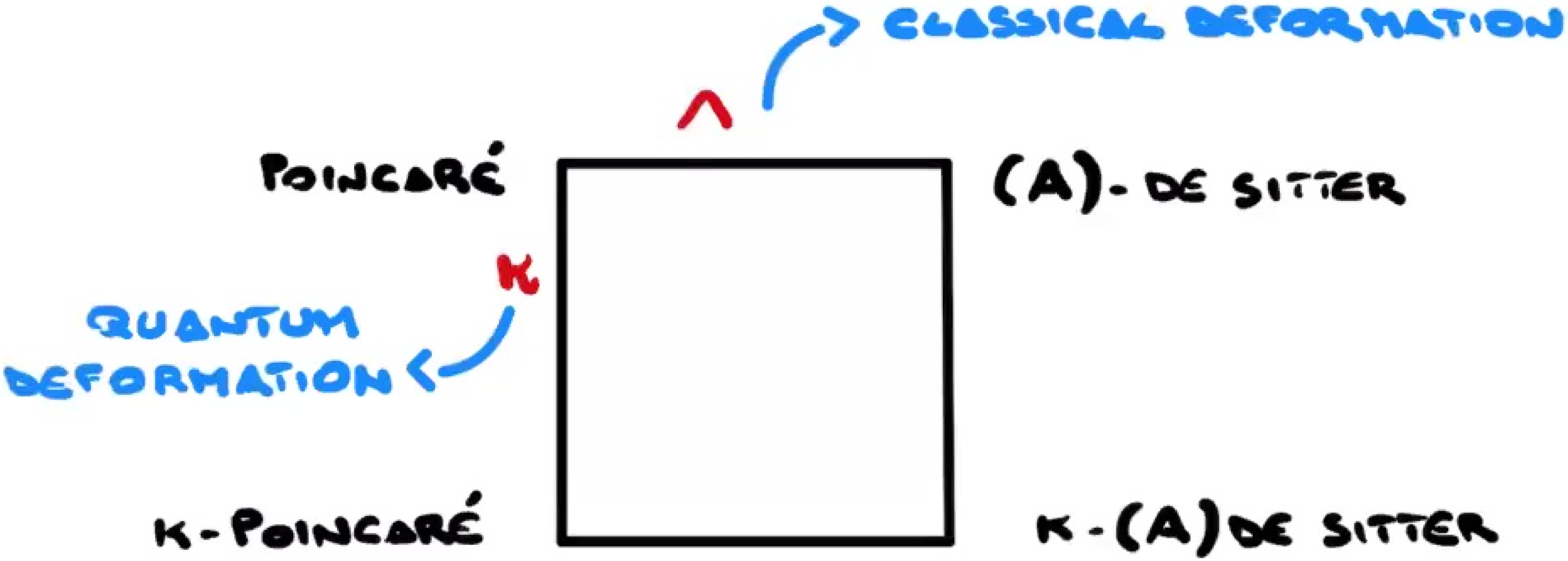
Deformations of the Poincaré relativistic symmetries



Λ deformation: large spacetime intervals (COSMOLOGY)

κ deformation: tiny length scales / high energies
$\kappa \sim E_p$ (PLANCK SCALE)

Deformations of the Poincaré relativistic symmetries

Λ → CLASSICAL DEFORMATION

POINCARÉ — (A)-DE SITTER

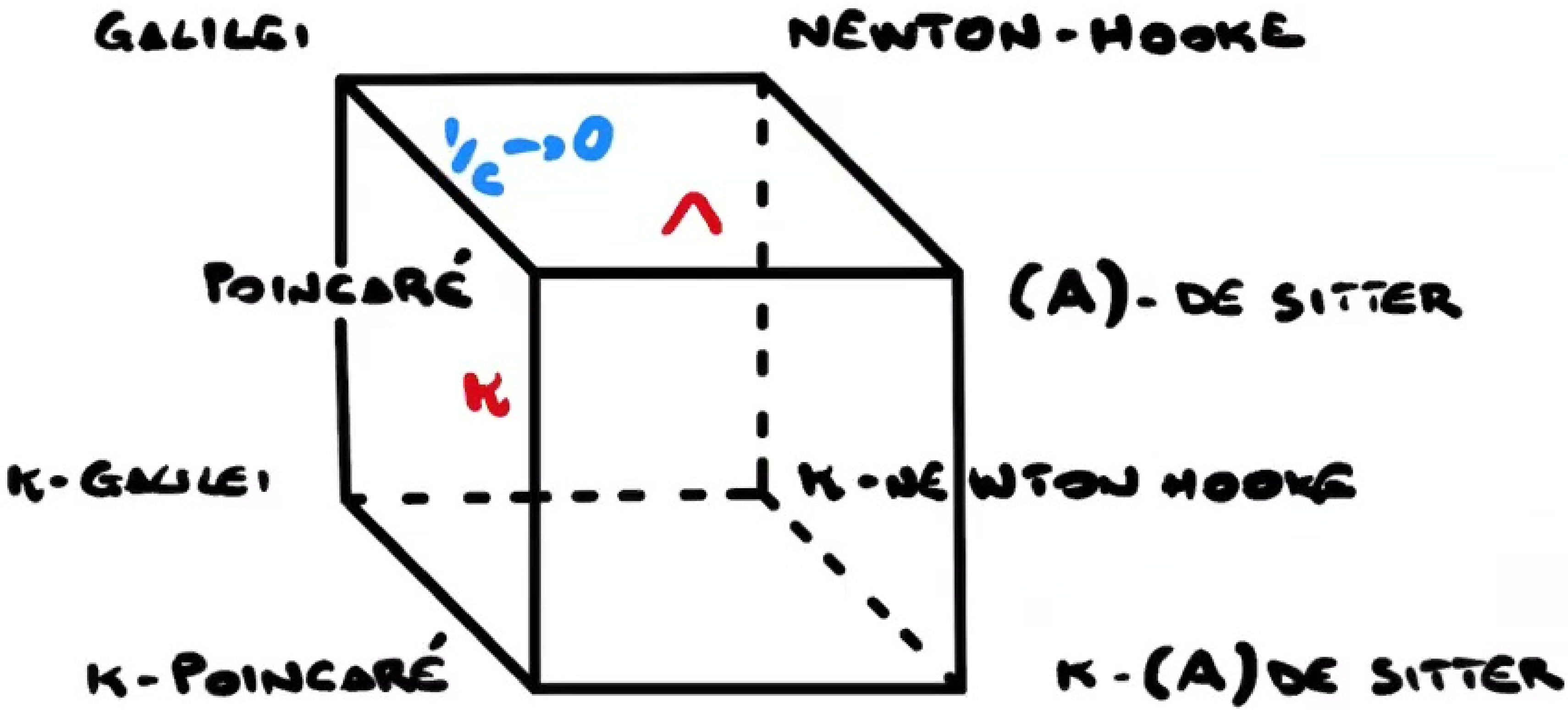
κ → QUANTUM DEFORMATION

NON-TRIVIAL INTERPLAY BETWEEN κ AND Λ

κ-POINCARÉ — κ-(A)DE SITTER

κ, Λ deformation: relevant for high-energy probes travelling cosmological distances

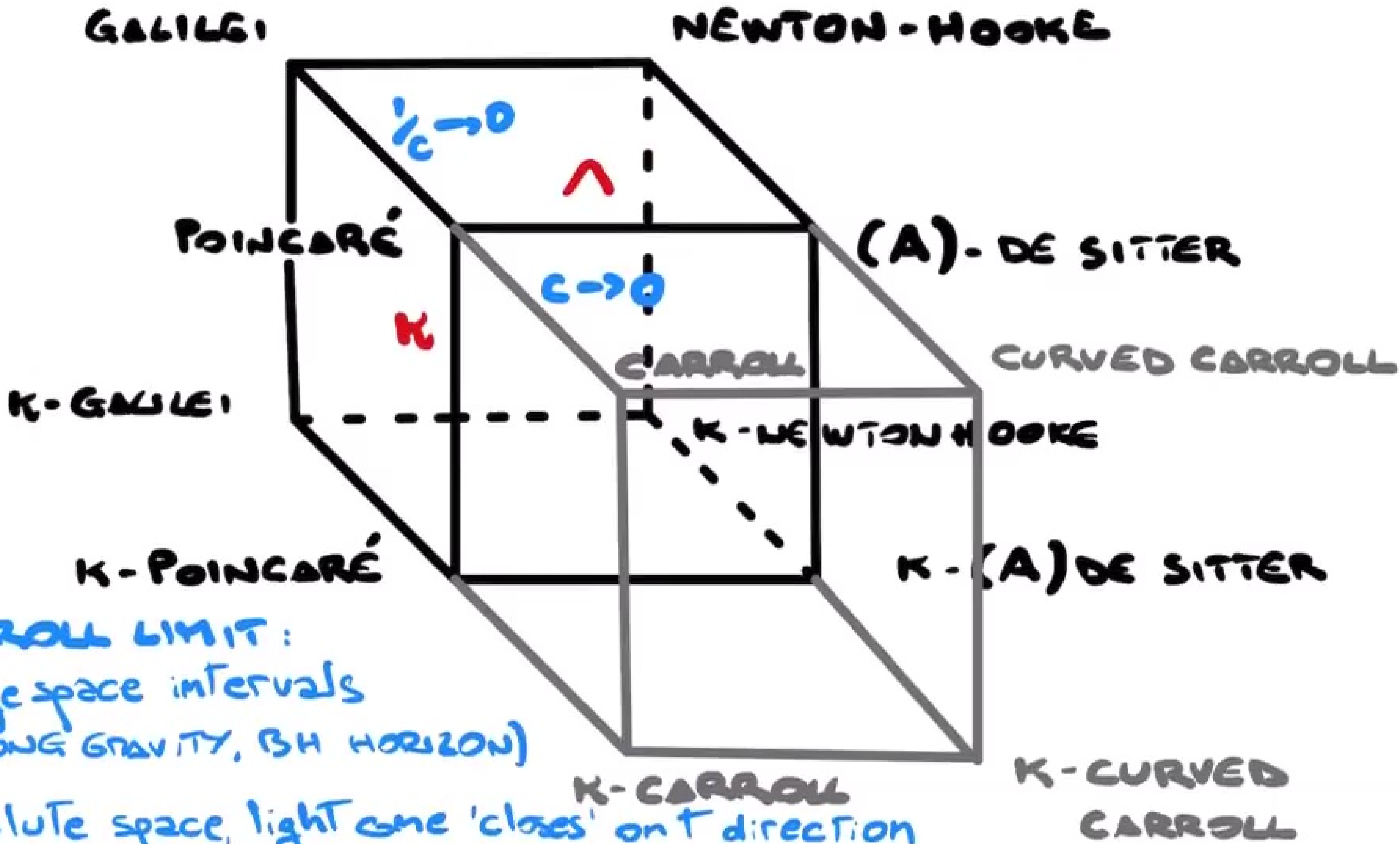
Speed of light contraction: the Galilei limit



GALILEI LIMIT: small velocities (NON-RELATIVISTIC QM)

absolute time, light-cone 'opens' along $t=0$

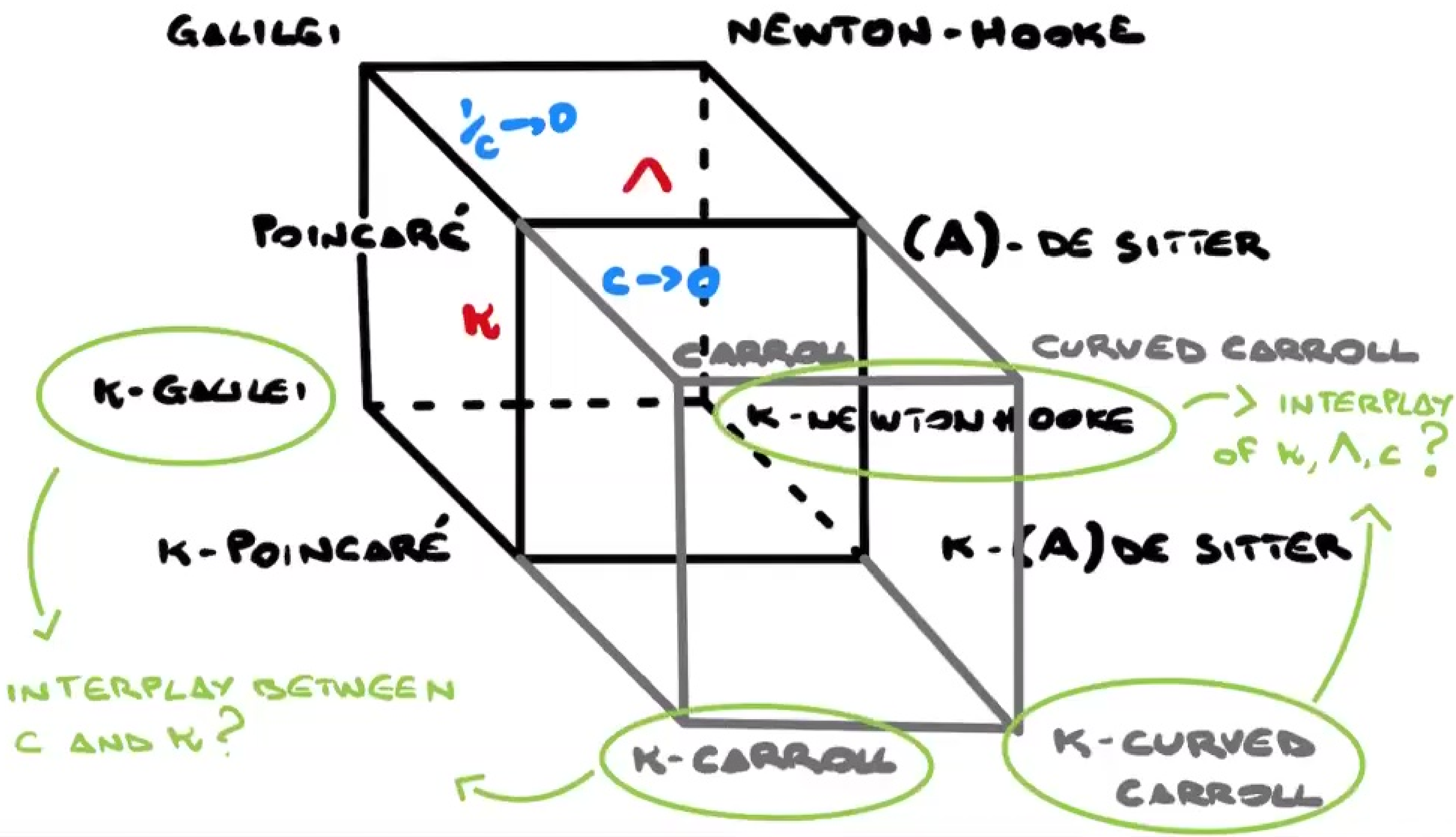
Another speed of light contraction: the Carroll limit



CARROLL LIMIT:
large space intervals
(STRONG GRAVITY, BH HORIZON)

absolute space, light cone 'closes' on t direction

Another speed of light contraction: the Carroll limit



(Anti-)de Sitter relativity as a classical deformation of Poincaré relativity

$$[J_a, J_b] = \varepsilon_{abc} J_c$$
$$[J_a, P_b] = \varepsilon_{abc} P_c$$
$$[J_a, K_b] = \varepsilon_{abc} K_c$$
$$[K_a, P_0] = P_a$$
$$[K_a, P_b] = \delta_{ab} P_0$$
$$[K_a, K_b] = -\varepsilon_{abc} J_c$$
$$[P_0, P_a] = 0$$
$$[P_a, P_b] = 0$$
$$[P_0, J_a] = 0$$

$$\mathcal{C} = P_0^2 - \vec{P}^2$$

POINCARÉ ALGEBRA

J_a: rotations

P_a: spatial translations

P_0: time translation

K_a: boosts

(Anti-)de Sitter relativity as a classical deformation of Poincaré relativity

$$[J_a, J_b] = \epsilon_{abc} J_c$$
$$[J_a, P_b] = \epsilon_{abc} P_c$$
$$[J_a, K_b] = \epsilon_{abc} K_c$$
$$[K_a, P_0] = P_a$$
$$[K_a, P_b] = \delta_{ab} P_0$$
$$[K_a, K_b] = -\epsilon_{abc} J_c$$
$$[P_0, P_a] = -\Lambda K_a$$
$$[P_a, P_b] = \Lambda \epsilon_{abc} J_c$$
$$[P_0, J_a] = 0$$

$$\mathcal{C} = P_0^2 - \vec{P}^2 + \Lambda(\vec{K}^2 - \vec{J}^2)$$

$\Lambda > 0$: $SO(3,2)$ ANTI-DESITTER

$\Lambda < 0$: $SO(4,1)$ DESITTER

$\Lambda \to 0$ contraction is related to the automorphism

$$\mathcal{PT}(P_0, P_a, K_a, J_a) = (-P_0, -P_a, K_a, J_a)$$

(spacetime contraction)

[Poincaré recovered for $P_a \to \epsilon P_a$, $\epsilon \to 0$]

Quantum deformation of Poincaré symmetries

$$[J_a, J_b] = \epsilon_{abc} J_c$$
$$[J_a, P_b] = \epsilon_{abc} P_c$$
$$[J_a, K_b] = \epsilon_{abc} K_c$$
$$[K_a, P_0] = P_a$$
$$[K_a, P_b] = \delta_{ab}\left[\frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right) + \frac{\vec{P}^2}{2\kappa}\right] - \frac{P_a P_b}{\kappa}$$
$$[K_a, K_b] = -\epsilon_{abc} J_c$$
$$[P_0, P_a] = 0$$
$$[P_a, P_b] = 0$$
$$[P_0, J_a] = 0$$

$$\mathcal{C} = \left[2\kappa \sinh\left(\frac{P_0}{2\kappa}\right)\right]^2 - \vec{P}^2 e^{P_0/\kappa}$$

κ- POINCARÉ ALGEBRA

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0$$
$$\Delta P_a = P_a \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_a$$
$$\Delta J_a = J_a \otimes \mathbb{1} + \mathbb{1} \otimes J_a$$
$$\Delta K_a = K_a \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes K_a + \frac{\epsilon_{abc}}{\kappa} P_b \otimes J_c$$

COPRODUCTS 'ACTION ON PRODUCTS OF FUNCTIONS'

Quantum deformation of Poincaré symmetries

r-matrix: $r = \frac{1}{\kappa}\left(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3\right)$

co-commutator: $\delta(X) := [X \otimes \mathbb{1} + \mathbb{1} \otimes X, r]$, X: STANDARD POINCARÉ GENERATORS

$$\delta(P_0) = \delta(J_a) = 0$$
$$\delta(P_a) = \frac{1}{\kappa} P_a \wedge P_0$$
$$\delta(K_a) = \frac{1}{\kappa}\left(K_a \wedge P_0 + \epsilon_{abc} P_b \wedge J_c\right)$$

dual Lie algebra: $\{p_0, p_a, k_a, j_a\}$

$$[j_a, j_b] = 0 \qquad [j_a, p_b] = \frac{1}{\kappa}\epsilon_{abc} k_c \qquad [j_a, p_0] = 0$$
$$[j_a, k_b] = 0 \qquad [k_a, p_0] = \frac{1}{\kappa} k_a \qquad [p_a, p_0] = \frac{1}{\kappa} p_a$$
$$[k_a, k_b] = 0 \qquad [k_a, p_b] = 0 \qquad [p_a, p_b] = 0$$

dual group element: $G^* = \exp[-J_i\, e(j_i)]\, \exp[-K_i\, e(k_i)]\, \exp[P_a\, e(p_a)]\, \exp[P_0\, e(p_0)]$

The grouplaw determines the coproducts of the Poisson-Hopf algebra $\{P_0, \vec{P}, \vec{K}, \vec{J}\}$

…lay between curvature and quantum effects: the k-(A)dS algebra

$$
\begin{aligned}
&\{J_1,J_2\} = \frac{e^{2z\sqrt{\omega}J_3}-1}{2z\sqrt{\omega}} - \frac{z\sqrt{\omega}}{2}\left(J_1^2+J_2^2\right), && \{J_1,J_3\} = -J_2, && \{J_2,J_3\} = J_1,\\
&\{J_1,P_1\} = z\sqrt{\omega}J_1P_2, && \{J_1,P_2\} = P_3 - z\sqrt{\omega}J_1P_1, && \{J_1,P_3\} = -P_2,\\
&\{J_2,P_1\} = -P_3 + z\sqrt{\omega}J_2P_2, && \{J_2,P_2\} = -z\sqrt{\omega}J_2P_1, && \{J_2,P_3\} = P_1,\\
&\{J_3,P_1\} = P_2, && \{J_3,P_2\} = -P_1, && \{J_3,P_3\} = 0,\\
&\{J_1,K_1\} = z\sqrt{\omega}J_1K_2, && \{J_1,K_2\} = K_3 - z\sqrt{\omega}J_1K_1, && \{J_1,K_3\} = -K_2,\\
&\{J_2,K_1\} = -K_3 + z\sqrt{\omega}J_2K_2, && \{J_2,K_2\} = -z\sqrt{\omega}J_2K_1, && \{J_2,K_3\} = K_1,\\
&\{J_3,K_1\} = K_2, && \{J_3,K_2\} = -K_1, && \{J_3,K_3\} = 0,\\
&\{K_a,P_0\} = P_a, && \{P_0,P_a\} = \omega K_a, && \{P_0,J_a\} = 0,
\end{aligned}
$$

$$
\begin{aligned}
\{K_1,P_1\} &= \frac{1}{2z}\left(\cosh(2z\sqrt{\omega}J_3) - e^{-2zP_0}\right) + \frac{z^3\omega^2}{4}e^{-2z\sqrt{\omega}J_3}\left(J_1^2+J_2^2\right)^2 + \frac{z}{2}\left(P_2^2+P_3^2-P_1^2\right)\\
&\quad + \frac{z\omega}{2}\left[K_2^2+K_3^2-K_1^2+J_1^2\left(1-e^{-2z\sqrt{\omega}J_3}\right)+J_2^2\left(1+e^{-2z\sqrt{\omega}J_3}\right)\right],\\
\{K_2,P_2\} &= \frac{1}{2z}\left(\cosh(2z\sqrt{\omega}J_3) - e^{-2zP_0}\right) + \frac{z^3\omega^2}{4}e^{-2z\sqrt{\omega}J_3}\left(J_1^2+J_2^2\right)^2 + \frac{z}{2}\left(P_1^2+P_3^2-P_2^2\right)\\
&\quad + \frac{z\omega}{2}\left[K_1^2+K_3^2-K_2^2+J_1^2\left(1+e^{-2z\sqrt{\omega}J_3}\right)+J_2^2\left(1-e^{-2z\sqrt{\omega}J_3}\right)\right],\\
\{K_3,P_3\} &= \frac{1-e^{-2zP_0}}{2z} + \frac{z}{2}\left[(P_1+\sqrt{\omega}K_2)^2+(P_2-\sqrt{\omega}K_1)^2-P_3^2-\omega K_3^2\right]\\
&\quad + z\omega e^{-2z\sqrt{\omega}J_3}\left(J_1^2+J_2^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\{P_1,K_2\} &= z\left(P_1P_2+\omega K_1K_2-\sqrt{\omega}P_3K_3+\omega J_1J_2e^{-2z\sqrt{\omega}J_3}\right),\\
\{P_2,K_1\} &= z\left(P_1P_2+\omega K_1K_2+\sqrt{\omega}P_3K_3+\omega J_1J_2e^{-2z\sqrt{\omega}J_3}\right),\\
\{P_1,K_3\} &= \frac{1}{2}\sqrt{\omega}J_1\left(1-e^{-2z\sqrt{\omega}J_3}\left[1-z^2\omega\left(J_1^2+J_2^2\right)\right]\right)+z\left(P_1P_3+\omega K_1K_3+\sqrt{\omega}K_2P_3\right),\\
\{P_3,K_1\} &= \frac{1}{2}\sqrt{\omega}J_1\left(1-e^{-2z\sqrt{\omega}J_3}\left[1-z^2\omega\left(J_1^2+J_2^2\right)\right]\right)+z\left(P_1P_3+\omega K_1K_3-\sqrt{\omega}P_2K_3\right),\\
\{P_2,K_3\} &= \frac{1}{2}\sqrt{\omega}J_2\left(1-e^{-2z\sqrt{\omega}J_3}\left[1-z^2\omega\left(J_1^2+J_2^2\right)\right]\right)+z\left(P_2P_3+\omega K_2K_3-\sqrt{\omega}K_1P_3\right),\\
\{P_3,K_2\} &= \frac{1}{2}\sqrt{\omega}J_2\left(1-e^{-2z\sqrt{\omega}J_3}\left[1-z^2\omega\left(J_1^2+J_2^2\right)\right]\right)+z\left(P_3P_2+\omega K_3K_2+\sqrt{\omega}P_1K_3\right),
\end{aligned}
$$

$$
\begin{aligned}
\{K_1,K_2\} &= -\frac{\sinh(2z\sqrt{\omega}J_3)}{2z\sqrt{\omega}} - \frac{z\sqrt{\omega}}{2}\left(J_1^2+J_2^2+2K_3^2\right) - \frac{z^3\omega^{3/2}}{4}e^{-2z\sqrt{\omega}J_3}\left(J_1^2+J_2^2\right)^2,\\
\{K_1,K_3\} &= \frac{1}{2}J_2\left(1+e^{-2z\sqrt{\omega}J_3}\left[1+z^2\omega\left(J_1^2+J_2^2\right)\right]\right)+z\sqrt{\omega}K_2K_3,\\
\{K_2,K_3\} &= -\frac{1}{2}J_1\left(1+e^{-2z\sqrt{\omega}J_3}\left[1+z^2\omega\left(J_1^2+J_2^2\right)\right]\right)-z\sqrt{\omega}K_1K_3,
\end{aligned}
$$

$$
\begin{aligned}
\{P_1,P_2\} &= -\omega\frac{\sinh(2z\sqrt{\omega}J_3)}{2z\sqrt{\omega}} - \frac{z\sqrt{\omega}}{2}\left(2P_3^2+\omega(J_1^2+J_2^2)\right) - \frac{z^3\omega^{5/2}}{4}e^{-2z\sqrt{\omega}J_3}\left(J_1^2+J_2^2\right)^2,\\
\{P_1,P_3\} &= \frac{1}{2}\omega J_2\left(1+e^{-2z\sqrt{\omega}J_3}\left[1+z^2\omega\left(J_1^2+J_2^2\right)\right]\right)+z\sqrt{\omega}P_2P_3,\\
\{P_2,P_3\} &= -\frac{1}{2}\omega J_1\left(1+e^{-2z\sqrt{\omega}J_3}\left[1+z^2\omega\left(J_1^2+J_2^2\right)\right]\right)-z\sqrt{\omega}P_1P_3. \qquad (3.5)
\end{aligned}
$$

$$
r = \frac{1}{\kappa}\left(K_1\wedge P_1 + K_2\wedge P_2 + K_3\wedge P_3 + \sqrt{\omega}J_1\wedge J_2\right)
$$

$$
\left[\sqrt{\omega} := -\Lambda\right]
$$

NON-TRIVIAL
ROTATION
SECTOR
DUE TO κ-Λ
INTERPLAY

$$
\begin{aligned}
&\Delta(P_0) = P_0\otimes 1 + 1\otimes P_0, \qquad \Delta(J_3) = J_3\otimes 1 + 1\otimes J_3,\\
&\Delta(J_1) = J_1\otimes e^{z\sqrt{\omega}J_3} + 1\otimes J_1, \qquad \Delta(J_2) = J_2\otimes e^{z\sqrt{\omega}J_3} + 1\otimes J_2
\end{aligned}
$$

$$
\begin{aligned}
\Delta(P_1) &= P_1\otimes\cosh(z\sqrt{\omega}J_3) + e^{-zP_0}\otimes P_1 - \sqrt{\omega}K_2\otimes\sinh(z\sqrt{\omega}J_3)\\
&\quad - z\sqrt{\omega}P_3\otimes J_1 + z\omega K_3\otimes J_2 + z^2\omega\left(\sqrt{\omega}K_1 - P_2\right)\otimes J_1J_2e^{-z\sqrt{\omega}J_3}\\
&\quad - \frac{1}{2}z^2\omega\left(\sqrt{\omega}K_2 + P_1\right)\otimes\left(J_1^2 - J_2^2\right)e^{-z\sqrt{\omega}J_3},\\
\Delta(P_2) &= P_2\otimes\cosh(z\sqrt{\omega}J_3) + e^{-zP_0}\otimes P_2 + \sqrt{\omega}K_1\otimes\sinh(z\sqrt{\omega}J_3)\\
&\quad - z\sqrt{\omega}P_3\otimes J_2 - z\omega K_3\otimes J_1 - z^2\omega\left(\sqrt{\omega}K_2 + P_1\right)\otimes J_1J_2e^{-z\sqrt{\omega}J_3}\\
&\quad - \frac{1}{2}z^2\omega\left(\sqrt{\omega}K_1 - P_2\right)\otimes\left(J_1^2 - J_2^2\right)e^{-z\sqrt{\omega}J_3},\\
\Delta(P_3) &= P_3\otimes 1 + e^{-zP_0}\otimes P_3 + z\left(\omega K_2 + \sqrt{\omega}P_1\right)\otimes J_1e^{-z\sqrt{\omega}J_3}\\
&\quad - z\left(\omega K_1 - \sqrt{\omega}P_2\right)\otimes J_2e^{-z\sqrt{\omega}J_3},
\end{aligned}
$$

$$
\begin{aligned}
\Delta(K_1) &= K_1\otimes\cosh(z\sqrt{\omega}J_3) + e^{-zP_0}\otimes K_1 + P_2\otimes\frac{\sinh(z\sqrt{\omega}J_3)}{\sqrt{\omega}}\\
&\quad - zP_3\otimes J_2 - z\sqrt{\omega}K_3\otimes J_1 - z^2\left(\omega K_2 + \sqrt{\omega}P_1\right)\otimes J_1J_2e^{-z\sqrt{\omega}J_3}\\
&\quad - \frac{1}{2}z^2\left(\omega K_1 - \sqrt{\omega}P_2\right)\otimes\left(J_1^2 - J_2^2\right)e^{-z\sqrt{\omega}J_3},\\
\Delta(K_2) &= K_2\otimes\cosh(z\sqrt{\omega}J_3) + e^{-zP_0}\otimes K_2 - P_1\otimes\frac{\sinh(z\sqrt{\omega}J_3)}{\sqrt{\omega}}\\
&\quad + zP_3\otimes J_1 - z\sqrt{\omega}K_3\otimes J_2 - z^2\left(\omega K_1 - \sqrt{\omega}P_2\right)\otimes J_1J_2e^{-z\sqrt{\omega}J_3}\\
&\quad + \frac{1}{2}z^2\left(\omega K_2 + \sqrt{\omega}P_1\right)\otimes\left(J_1^2 - J_2^2\right)e^{-z\sqrt{\omega}J_3},\\
\Delta(K_3) &= K_3\otimes 1 + e^{-zP_0}\otimes K_3 + z(\sqrt{\omega}K_1 - P_2)\otimes J_1e^{-z\sqrt{\omega}J_3}\\
&\quad + z(\sqrt{\omega}K_2 + P_1)\otimes J_2e^{-z\sqrt{\omega}J_3}.
\end{aligned}
$$

$$
\left[z = 1/\kappa\right]
$$

The Galilean limit of (A)dS (Inönü-Wigner contraction)

$P_a \to \frac{1}{c} P_a \,, \; K_a \to \frac{1}{c} K_a$

$$\mathcal{C} = P_0^2 - \vec{P}^2 c^2 + \Lambda(\vec{K}^2 c^2 - \vec{J}^2)$$

$$-\frac{\mathcal{C}}{c^2} \xrightarrow[c\to\infty]{} \vec{P}^2 - \Lambda \vec{K}^2$$

$$\left.\begin{aligned}
[J_a, J_b] &= \varepsilon_{abc} J_c \\
[J_a, P_b] &= \varepsilon_{abc} P_c \\
[J_a, K_b] &= \varepsilon_{abc} K_c \\
[K_a, P_0] &= P_a \\
[K_a, P_b] &= \delta_{ab} P_0 / c^2 \\
[K_a, K_b] &= -\varepsilon_{abc} J_c / c^2 \\
[P_0, P_a] &= -\Lambda K_a \\
[P_a, P_b] &= \Lambda \varepsilon_{abc} J_c / c^2 \\
[P_0, J_a] &= 0
\end{aligned}\right\} \xrightarrow{c\to\infty}$$

$c \to \infty$ contraction is related to the automorphism $\mathcal{S}(P_0, P_a, K_a, J_a) = (P_0, -P_a, -K_a, J_a)$ (speed-space contraction)

$$[K_a, P_b] = 0$$

$$[K_a, K_b] = 0$$

$$[P_a, P_b] = 0$$

Carrollian limit of (A)dS (Inönü-Wigner contraction)

$P_0 \to c P_0$, $K_a \to c K_a$

$$\mathcal{C} = \frac{P_0^2}{c^2} - \vec{P}^2 + \Lambda\left(\frac{\vec{K}^2}{c^2} - \vec{J}^2\right)$$

$$c^2 \mathcal{C} \xrightarrow[c \to 0]{} P_0^2 + \Lambda \vec{K}^2$$

$$
\left.
\begin{aligned}
[J_a, J_b] &= \varepsilon_{abc} J_c \\
[J_a, P_b] &= \varepsilon_{abc} P_c \\
[J_a, K_b] &= \varepsilon_{abc} K_c \\
[K_a, P_0] &= P_a \, c^2 \\
[K_a, P_b] &= \delta_{ab} P_0 \\
[K_a, K_b] &= -\varepsilon_{abc} J_c \, c^2 \\
[P_0, P_a] &= -\Lambda K_a \\
[P_a, P_b] &= \Lambda \varepsilon_{abc} J_c \\
[P_0, J_a] &= 0
\end{aligned}
\right\}
\xrightarrow{c \to 0}
\begin{aligned}
[K_a, P_0] &= 0 \\
[K_a, K_b] &= 0
\end{aligned}
$$

$c \to 0$ contraction is related to the automorphism

$$\tau(P_0, P_a, K_a, J_a) = (-P_0, P_a, -K_a, J_a)$$

(speed-time contraction)

	GALILEI	RELATIVISTIC	CARROLL	
$[J_a, J_b]$	$\epsilon_{abc} J_c$	$\epsilon_{abc} J_c$	$\epsilon_{abc} J_c$	ROTATION INVARIANCE UNAFFECTED
$[J_a, P_b]$	$\epsilon_{abc} P_c$	$\epsilon_{abc} P_c$	$\epsilon_{abc} P_c$	
$[J_a, K_b]$	$\epsilon_{abc} K_c$	$\epsilon_{abc} K_c$	$\epsilon_{abc} K_c$	
$[J_a, P_0]$	0	0	0	
$[K_a, P_0]$	P_a	P_a	$\boxed{0}$	ABSOLUTE SPACE
$[K_a, P_b]$	$\boxed{0}$	$\delta_{ab} P_0$	$\delta_{ab} P_0$	ABSOLUTE TIME
$[K_a, K_b]$	$\boxed{0}$	$-\epsilon_{abc} J_c$	$\boxed{0}$	NON-RELATIVISTIC BOOSTS
$[P_0, P_a]$	$-\Lambda K_a$	$-\Lambda K_a$	$-\Lambda K_a$	TRANSLATIONS UNAFFECTED IF $\Lambda = 0$
$[P_a, P_b]$	0	$\Lambda \epsilon_{abc} J_c$	$\Lambda \epsilon_{abc} J_c$	

Galilei and Carroll spacetimes

GALILEI | **(A)dS** | **CARROLL**

$$N_\Lambda^{3+1} = N_\Lambda^{(3+1)}/ISO(3) \qquad dS_\Lambda^{3+1} = SO_\Lambda^{(3+1)}/SO(3,1) \qquad C_\Lambda^{3+1} = C_\Lambda^{(3+1)}/ISO(3)$$

$$ISO(3) = \mathrm{span}\{\vec{K},\vec{J}\} \qquad SO(3,1) = \mathrm{span}\{\vec{K},\vec{J}\} \qquad ISO(3) = \mathrm{span}\{\vec{K},\vec{J}\}$$

$$s_4^2 - \Lambda s_0^2 = 1 \xleftarrow[c\to\infty]{s_i \to c s_i} \underline{s_4^2 - \Lambda s_0^2 + \Lambda(s_1^2 + s_2^2 + s^2) = 1} \xrightarrow[c\to 0]{s_0 \to \frac{1}{c} s_0} s_4^2 + \Lambda \vec{s}^{\,2} = 1$$

$$d\sigma_\Lambda^2 = \frac{ds_0^2}{1+\Lambda s_0^2} \qquad ds_\Lambda^2 = \frac{\Lambda(s_0 ds_0 - \vec{s}\cdot d\vec{s})^2}{1+\Lambda s_0^2 - \Lambda \vec{s}^{\,2}} + ds_0^2 - d\vec{s}^{\,2} \qquad d\sigma_\Lambda^2 = \frac{\Lambda(\vec{s}\cdot d\vec{s})^2}{1-\Lambda\vec{s}^{\,2}} + d\vec{s}^{\,2}$$

$$d\sigma_\Lambda'^2 = \lim_{c\to\infty}(-c^2 ds_\Lambda^2) = d\vec{s}^{\,2} \qquad\qquad d\sigma_\Lambda'^2 = \lim_{c\to 0}\left(\frac{1}{c^2} ds_\Lambda^2\right) = ds_0^2$$

METRIC ON FOLIATION $s_0 = \mathrm{const}$

$\Lambda > 0$: EXPANDING NEWTON-HOOKE

$\Lambda < 0$: OSCILLATING NH

$\Lambda > 0$: P_0 TRAJECTORIES ARE NON-COMPACT, $\vec{P}$ TRAJECTORIES ARE COMPACT

$\Lambda < 0$: P_0 TRAJECTORIES ARE COMPACT, $\vec{P}$ TRAJECTORIES ARE NON-COMPACT

METRIC ON FOLIATION $s_i = \mathrm{const}$

$\Lambda > 0$: sphere S^3

$\Lambda < 0$: hyperbolic space H^3

Contraction of quantum algebras

Contraction of quantum algebras

- The contraction procedure of a quantum algebra might require a rescaling of the quantum deformation parameter along with the generators (Lie Bialgebra Contraction - LBC) in order to obtain meaningful structures

- Two options:

  1. LBC at the level of the r-matrix (coboundary)
  2. LBC at the level of the co-commutators (fundamental)

The Galilean limit of k-Poincaré

$P_a \to \frac{1}{c} P_a \quad , K_a \to \frac{1}{c} K_a \quad (*)$

1.

$r = \frac{c^2}{\kappa}(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3)$ ⟵ well-behaved for $c \to \infty$ if $\kappa \to \frac{\kappa}{c^2}$: $r = \frac{1}{\kappa}(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3)$

however $\delta(X) = [X \otimes \mathbb{1} + \mathbb{1} \otimes X, r] = 0$ (trivial) → leads to classical Galilei algebra

2. working directly at the level of the co-commutators:

$\delta(P_0) = \delta(J_a) = 0$ (they are invariant under the rescaling (*))

$\delta(P_a) = \frac{1}{\kappa} P_a \wedge P_0$

$\delta(K_a) = \frac{1}{\kappa}(K_a \wedge P_0 + \varepsilon_{abc} P_b \wedge J_c)$

well-defined for $c \to \infty$ without rescaling κ

The Galilean limit of k-Poincaré

$[J_a, J_b] = \epsilon_{abc} J_c$

$[J_a, P_b] = \epsilon_{abc} P_c$

$[J_a, K_b] = \epsilon_{abc} K_c$

$[J_a, P_0] = 0$

$[K_a, P_b] = \frac{\delta_{ab}}{c^2}\left[\frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right) + c^2 \frac{\vec{P}^2}{2\kappa}\right] - \frac{c^2}{c^2}\frac{P_a P_b}{\kappa}$

$[K_a, K_b] = -\frac{\epsilon_{abc}}{c^2} J_c$

$[P_0, P_a] = 0$

$[P_a, P_b] = 0$

$\xrightarrow{c \to \infty}$

$[K_a, P_b] = \frac{\delta_{ab}}{2\kappa}\vec{P}^2 - \frac{P_a P_b}{\kappa}$

$[K_a, K_b] = 0$

$$\mathcal{C} = \left[2\kappa \sinh\left(\frac{P_0}{2\kappa}\right)\right]^2 - e^{P_0/\kappa} c^2 \vec{P}^2$$

$$-\frac{\mathcal{C}}{c^2} \xrightarrow{c \to \infty} e^{P_0/\kappa} \vec{P}^2$$

UNMODIFIED

$$\begin{cases} \Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 \\ \Delta P_a = P_a \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_a \\ \Delta J_a = J_a \otimes \mathbb{1} + \mathbb{1} \otimes J_a \\ \Delta K_a = K_a \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes K_a + \frac{\epsilon_{abc}}{\kappa} P_b \otimes J_c \end{cases}$$

The Carrollian limit of k-Poincaré

$P_0 \to c P_0$, $K_a \to c K_a$ (*)

$r = \frac{1}{c\kappa}(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3) \longleftarrow$ well-behaved for $c \to 0$ if $\kappa \to c\kappa$: $r = \frac{1}{\kappa}(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3)$

The resulting co-commutators are in this case non-trivial

$$\left.\begin{aligned} &\delta(P_0) = \delta(J_a) = 0 \\ &\delta(P_a) = \frac{c}{\kappa} P_a \wedge \frac{P_0}{c} \\ &\delta(K_a) = \frac{c}{\kappa}\left(\frac{K_a}{c} \wedge P_0 + \varepsilon_{abc} P_b \wedge J_c\right) \end{aligned}\right\} \xrightarrow{c \to 0} \begin{aligned} &\delta(P_0) = \delta(J_a) = 0 \\ &\delta(P_a) = \frac{1}{\kappa} P_a \wedge P_0 \\ &\delta(K_a) = \frac{1}{\kappa} K_a \wedge P_0 \end{aligned}$$

[in this case both kinds of contractions work and give consistent results]

	κ-GALILEI	κ-POINCARÉ	κ-CARROLL	
$[J_a, J_b]$	$\epsilon_{abc} J_c$	$\epsilon_{abc} J_c$	$\epsilon_{abc} J_c$	
$[J_a, P_b]$	$\epsilon_{abc} P_c$	$\epsilon_{abc} P_c$	$\epsilon_{abc} P_c$	
$[J_a, K_b]$	$\epsilon_{abc} K_c$	$\epsilon_{abc} K_c$	$\epsilon_{abc} K_c$	
$[J_a, P_0]$	0	0	0	
$[K_a, P_0]$	P_a	P_a	$\boxed{0}$	ABSOLUTE SPACE
$[K_a, P_b]$	$\boxed{\delta_{ab}\frac{\vec{P}^2}{2\kappa} - \frac{P_a P_b}{\kappa}}$	$\delta_{ab}\left[\frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right) + \frac{\vec{P}^2}{2\kappa}\right] - \frac{P_a P_b}{\kappa}$	$\delta_{ab}\frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right)$	~~ABSOLUTE TIME~~
$[K_a, K_b]$	0	$-\epsilon_{abc} J_c$	0	
$[P_0, P_a]$	0	0	0	
$[P_a, P_b]$	0	0	0	

k-Galilei and k-Carroll spacetimes

Invariance under κ-Poincaré symmetries requires the introduction of NONCOMMUTATIVE spacetime coordinates:

$$[x^a, x^0] = \frac{1}{\kappa} x^a \qquad \text{κ-MINKOWSKI SPACETIME}$$

$$[x^a, x^b] = 0$$

The Galilean and Carroll limits are taken after rescaling the coordinates so that products $x \cdot P$ are invariant:

GALILEI	CARROLL
$x^0 \to x^0$	$x^0 \to \frac{x^0}{c}$
$x^a \to c\, x^a$	$x^a \to x^a$
$\kappa \to \kappa$	$\kappa \to c\kappa$
$\downarrow c \to \infty$	$\downarrow c \to 0$
$[x^a, x^0] = \frac{1}{\kappa} x^a$	$[x^a, x^0] = \frac{1}{\kappa} x^0$

SAME NONCOMMUTATIVITY AS IN κ-MINKOWSKI (LOST SEPARATION BETWEEN TIME AND SPACE OF THE CORRESPONDING CLASSICAL SPACETIMES)

The Galilean limit of k-(A)dS

$[\eta^2 := -\Lambda]$

The Galilean limit of k-(A)dS

$[\eta^2 := -\Lambda]$

• contraction procedure works as in the no-curvature case

[coproducts are again invariant under contraction]

$$
\begin{aligned}
&\{J_1,J_2\} = \frac{e^{2\eta J_3/\kappa}-1}{2\eta/\kappa} - \frac{\eta}{2\kappa}\left(J_1^2+J_2^2\right), && \{J_1,J_3\} = -J_2, && \{J_2,J_3\} = J_1, \\
&\{J_1,P_1\} = \tfrac{\eta}{\kappa}J_1P_2, && \{J_1,P_2\} = P_3 - \tfrac{\eta}{\kappa}J_1P_1, && \{J_1,P_3\} = -P_2, \\
&\{J_2,P_1\} = -P_3 + \tfrac{\eta}{\kappa}J_2P_2, && \{J_2,P_2\} = -\tfrac{\eta}{\kappa}J_2P_1, && \{J_2,P_3\} = P_1, \\
&\{J_3,P_1\} = P_2, && \{J_3,P_2\} = -P_1, && \{J_3,P_3\} = 0, \\
&\{J_1,K_1\} = \tfrac{\eta}{\kappa}J_1K_2, && \{J_1,K_2\} = K_3 - \tfrac{\eta}{\kappa}J_1K_1, && \{J_1,K_3\} = -K_2, \\
&\{J_2,K_1\} = -K_3 + \tfrac{\eta}{\kappa}J_2K_2, && \{J_2,K_2\} = -\tfrac{\eta}{\kappa}J_2K_1, && \{J_2,K_3\} = K_1, \\
&\{J_3,K_1\} = K_2, && \{J_3,K_2\} = -K_1, && \{J_3,K_3\} = 0, \\
&\{K_a,P_0\} = P_a, && \{P_0,P_a\} = \eta^2K_a, && \{P_0,J_a\} = 0.
\end{aligned}
$$

$$
\begin{aligned}
\{K_1,P_1\} &= \frac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right) + \frac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right), \\
\{K_2,P_2\} &= \frac{1}{2\kappa}\left(P_1^2+P_3^2-P_2^2\right) + \frac{\eta^2}{2\kappa}\left(K_1^2+K_3^2-K_2^2\right), \\
\{K_3,P_3\} &= \frac{1}{2\kappa}\left[(P_1+\eta K_2)^2+(P_2-\eta K_1)^2-P_3^2-\eta^2K_3^2\right],
\end{aligned}
$$

$$
\begin{aligned}
\{P_1,K_2\} &= \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2-\eta P_3K_3\right), \\
\{P_2,K_1\} &= \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2+\eta P_3K_3\right), \\
\{P_1,K_3\} &= \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right), \\
\{P_3,K_1\} &= \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3-\eta P_2K_3\right), \\
\{P_2,K_3\} &= \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3-\eta K_1P_3\right), \\
\{P_3,K_2\} &= \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3+\eta P_1K_3\right), \\
\{K_a,K_b\} &= -\frac{\eta}{\kappa}\epsilon_{abc}K_cK_3, \qquad \{P_a,P_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}P_cP_3.
\end{aligned}
$$

ROTATION SECTOR IS AGAIN DEFORMED (Λ,κ interplay)

NON-ZERO $[K_a,P_b]$ (κ, c interplay)

NON-ZERO $[K_a,K_b]$ (Λ,κ,c interplay)

The Carrollian limit of k-(A)dS

$[\eta^2 := -\Lambda]$

$$\{J_a, J_b\} = \epsilon_{abc} J_c, \quad \{J_a, P_b\} = \epsilon_{abc} P_c, \quad \{J_a, K_b\} = \epsilon_{abc} K_c,$$
$$\{K_a, P_0\} = 0, \quad \{K_a, K_b\} = 0, \quad \{P_0, J_a\} = 0,$$
$$\{P_0, P_a\} = \eta^2 K_a, \quad \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c,$$
$$\{K_a, P_b\} = \delta_{ab}\left(\frac{1 - e^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa}\mathbf{K}^2\right) - \frac{\eta^2}{\kappa} K_a K_b.$$

$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0, \qquad \Delta(J_a) = J_a \otimes 1 + 1 \otimes J_a,$$
$$\Delta(P_a) = P_a \otimes 1 + e^{-P_0/\kappa} \otimes P_a - \frac{\eta^2}{\kappa}\epsilon_{abc} K_b \otimes J_c,$$
$$\Delta(K_a) = K_a \otimes 1 + e^{-P_0/\kappa} \otimes K_a,$$

[coproducts are modified by this contraction]

- isotropy is restored
  (c contraction cancels effects of Λ-κ interplay)
- curvature effects only enter at second order in η
  (interplay between c contraction and curvature)

k-Galilei and k-Carroll curved spacetimes

$[\eta^2 = -\Lambda]$

- contraction procedure works as in the no-curvature case

[coproducts are again invariant under contraction]

$$\{J_1,J_2\} = \frac{e^{2\eta J_3/\kappa}-1}{2\eta/\kappa} - \frac{\eta}{2\kappa}\left(J_1^2+J_2^2\right), \qquad \{J_1,J_3\} = -J_2, \qquad \{J_2,J_3\} = J_1,$$

$$\{J_1,P_1\} = \tfrac{\eta}{\kappa}J_1P_2, \qquad \{J_1,P_2\} = P_3 - \tfrac{\eta}{\kappa}J_1P_1, \qquad \{J_1,P_3\} = -P_2,$$
$$\{J_2,P_1\} = -P_3 + \tfrac{\eta}{\kappa}J_2P_2, \qquad \{J_2,P_2\} = -\tfrac{\eta}{\kappa}J_2P_1, \qquad \{J_2,P_3\} = P_1,$$
$$\{J_3,P_1\} = P_2, \qquad \{J_3,P_2\} = -P_1, \qquad \{J_3,P_3\} = 0,$$
$$\{J_1,K_1\} = \tfrac{\eta}{\kappa}J_1K_2, \qquad \{J_1,K_2\} = K_3 - \tfrac{\eta}{\kappa}J_1K_1, \qquad \{J_1,K_3\} = -K_2,$$
$$\{J_2,K_1\} = -K_3 + \tfrac{\eta}{\kappa}J_2K_2, \qquad \{J_2,K_2\} = -\tfrac{\eta}{\kappa}J_2K_1, \qquad \{J_2,K_3\} = K_1,$$
$$\{J_3,K_1\} = K_2, \qquad \{J_3,K_2\} = -K_1, \qquad \{J_3,K_3\} = 0,$$
$$\{K_a,P_0\} = P_a, \qquad \{P_0,P_a\} = \eta^2 K_a, \qquad \{P_0,J_a\} = 0,$$

$$\{K_1,P_1\} = \frac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right) + \frac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right),$$
$$\{K_2,P_2\} = \frac{1}{2\kappa}\left(P_1^2+P_3^2-P_2^2\right) + \frac{\eta^2}{2\kappa}\left(K_1^2+K_3^2-K_2^2\right),$$
$$\{K_3,P_3\} = \frac{1}{2\kappa}\left[(P_1+\eta K_2)^2 + (P_2-\eta K_1)^2 - P_3^2 - \eta^2K_3^2\right],$$

ROTATION SECTOR IS AGAIN DEFORMED (Λ, κ interplay)

NON-ZERO $[K_a, P_b]$ (κ, c interplay)

$$\{P_1,K_2\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2-\eta P_3K_3\right),$$
$$\{P_2,K_1\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2+\eta P_3K_3\right),$$
$$\{P_1,K_3\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right),$$
$$\{P_3,K_1\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3-\eta P_2K_3\right),$$
$$\{P_2,K_3\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3-\eta K_1P_3\right),$$
$$\{P_3,K_2\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3+\eta P_1K_3\right),$$
$$\{K_a,K_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}K_cK_3, \qquad \{P_a,P_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}P_cP_3.$$

NON-ZERO $[K_a, K_b]$ (Λ, κ, c interplay)

The Carrollian limit of k-(A)dS

$[\eta^2 := -\Lambda]$

k-Galilei and k-Carroll curved spacetimes

$[\eta^2 = -\Lambda]$

κ-(A)dS

$$\{x^1, x^0\} = \frac{1}{\kappa} \frac{\tanh(\eta x^1)}{\eta \, ch^2(\eta x^2) ch^2(\eta x^3)}$$

$$\{x^2, x^0\} = \frac{1}{\kappa} \frac{\tanh(\eta x^2)}{\eta \, ch^2(\eta x^3)}$$

$$\{x^3, x^0\} = \frac{1}{\kappa} \frac{\tanh(\eta x^3)}{\eta}$$

$$\{x^1, x^2\} = -\frac{1}{\kappa} \frac{ch(\eta x^1)\tanh(\eta x^3)}{\eta}$$

$$\{x^1, x^3\} = \frac{1}{\kappa} \frac{ch(\eta x^1)\tanh(\eta x^2)\tanh(\eta x^3)}{\eta}$$

$$\{x^2, x^3\} = -\frac{1}{\kappa} \frac{\sinh(\eta x^1)\tanh(\eta x^3)}{\eta}$$

GALILEI $\longrightarrow$

$x^a \to c x^a$

$c \to \infty$

κ-MINKOWSKI

$$\boxed{\{x^a, x^0\} = \frac{1}{\kappa} x^a}$$

$$\{x^1, x^2\} = -\frac{\eta}{\kappa}(x^3)^2$$

$$\{x^1, x^3\} = \frac{\eta}{\kappa} x^2 x^3$$

$$\{x^2, x^3\} = -\frac{\eta}{\kappa} x^1 x^3$$

ANISOTROPY

CARROLL $\longrightarrow$

$x^0 \to \frac{x^0}{c}$

$\kappa \to c\kappa$

$c \to 0$

$\{x^a, x^0\} =$ AS IN κ-AdS

$\{x^a, x^b\} = 0$

ISOTROPY RESTORED

	κ-'GALILEI'	κ-(A)dS	κ-'CARROLL'
$[J_a, J_b]$	ANISOTROPY $\sim \frac{\Lambda}{\kappa}$	ANISOTROPY $\sim \frac{\Lambda}{\kappa}$	ISOTROPY
$[J_a, P_b]$			
$[J_a, K_b]$			
$[J_a, P_0]$	0	0	0
$[K_a, P_0]$	P_a	P_a	0
$[K_a, P_b]$	$\left[\frac{1}{\kappa}\right] \left[\frac{\sqrt{\Lambda}}{\kappa}\right]$	$\delta_{ab} P_0 + \left(\left[\frac{1}{\kappa}\right], \left[\frac{\sqrt{\Lambda}}{\kappa}\right]\right)$	$\delta_{ab} P_0 + \left(\left[\frac{1}{\kappa}\right], \left[\frac{\Lambda}{\kappa}\right]\right)$
$[K_a, K_b]$	$\left[\frac{\sqrt{\Lambda}}{\kappa}\right]$	$-\varepsilon_{abc} J_c + \left[\frac{\sqrt{\Lambda}}{\kappa}\right]$	0
$[P_0, P_a]$	$-\Lambda K_a$	$-\Lambda K_a$	$-\Lambda K_a$
$[P_a, P_b]$	$\left[\frac{\sqrt{\Lambda}}{\kappa}\right]$	$\Lambda \varepsilon_{abc} J_c + \left[\frac{\sqrt{\Lambda}}{\kappa}\right]$	$\Lambda \varepsilon_{abc} J_c$