Title: Probing quantum nature of Newtonian gravity with optomechanics

Speakers: Haixing Miao

Series: Strong Gravity

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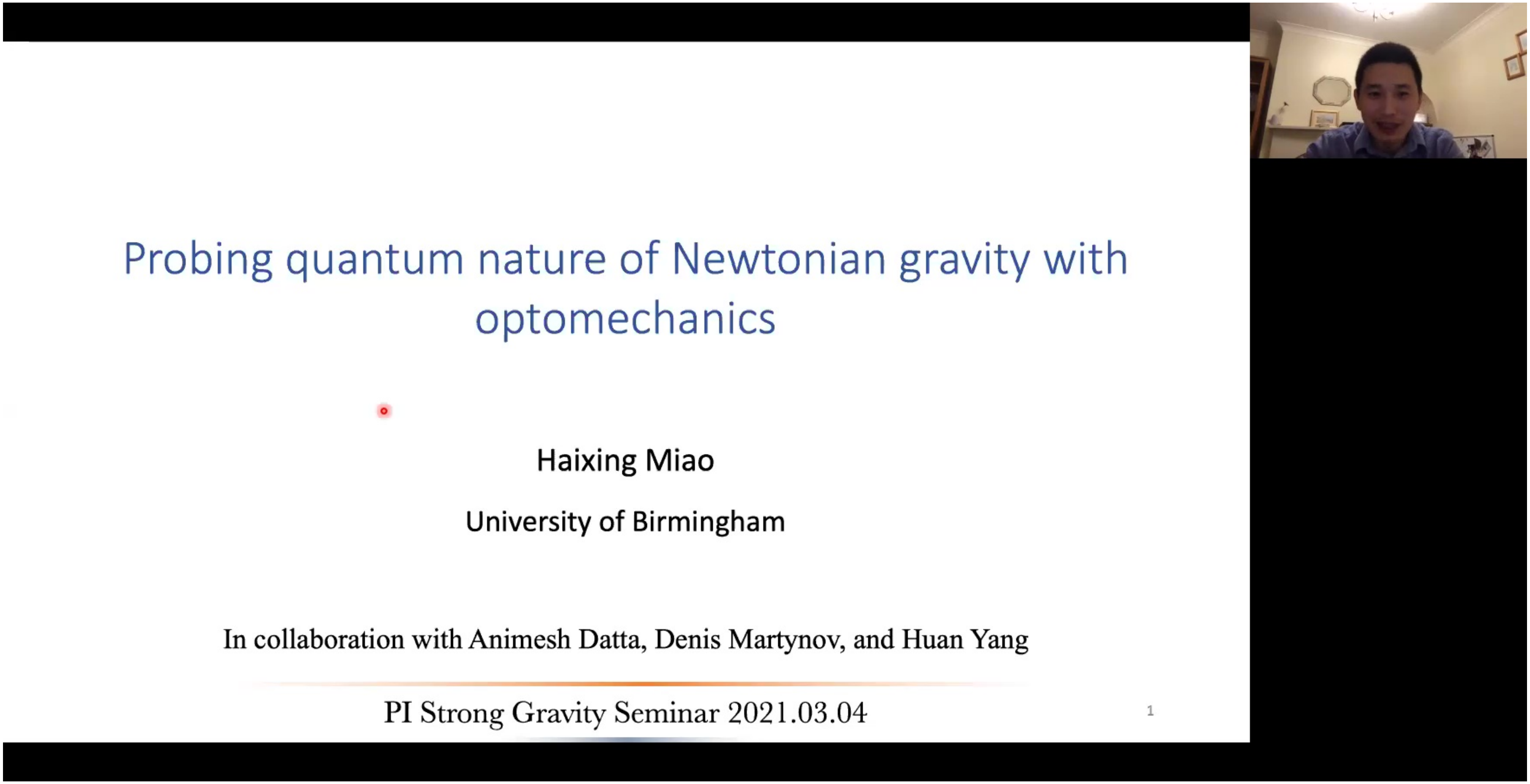
Abstract: Motivated by Feynman's early proposal, several schemes have been proposed recently to explore the quantum nature of gravity using table-top experiments. The key idea behind them is to study whether gravity can lead to non-classical quantum correlations between two objects. These experiments, if successful, can test different models of quantum gravity in the Newtonian limit. In this talk, I will discuss one such scheme based upon optomechanics that couples light to mechanical oscillators mediated by quantum radiation pressure. The non-classicality of gravity is witnessed by the quantum squeezing of light.



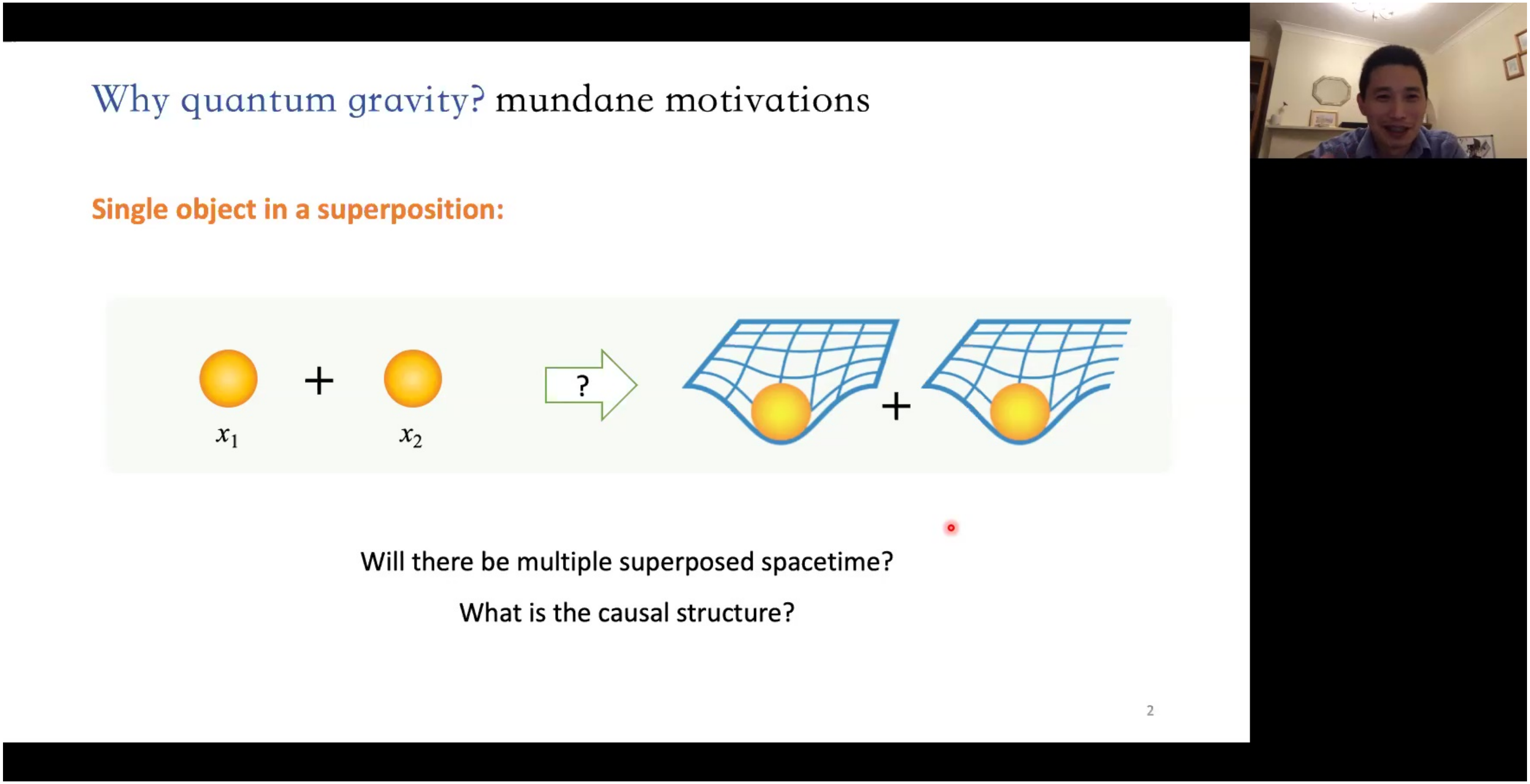
Probing quantum nature of Newtonian gravity with optomechanics

Haixing Miao

University of Birmingham

In collaboration with Animesh Datta, Denis Martynov, and Huan Yang

PI Strong Gravity Seminar 2021.03.04

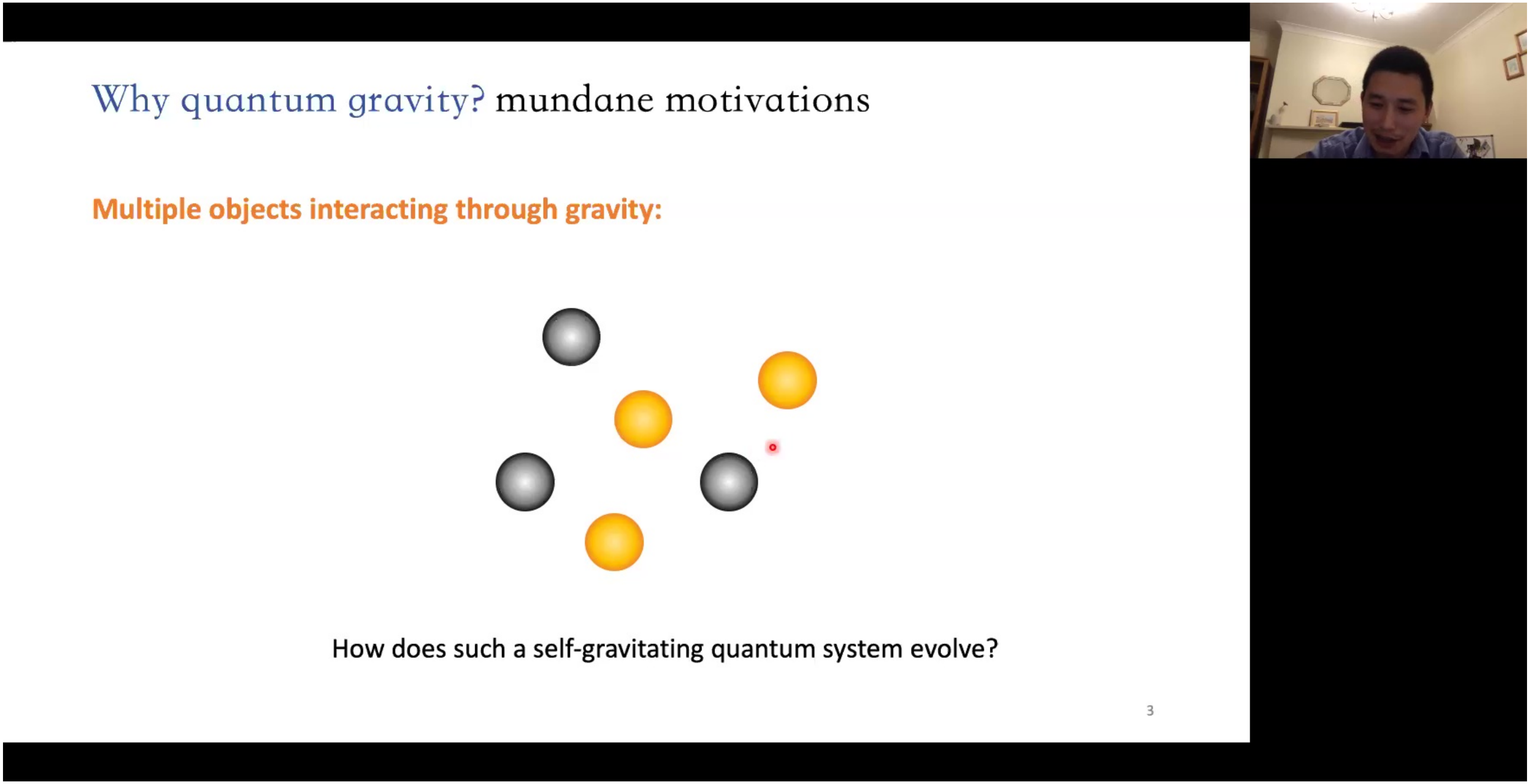


Why quantum gravity? mundane motivations

Single object in a superposition:

Will there be multiple superposed spacetime?

What is the causal structure?

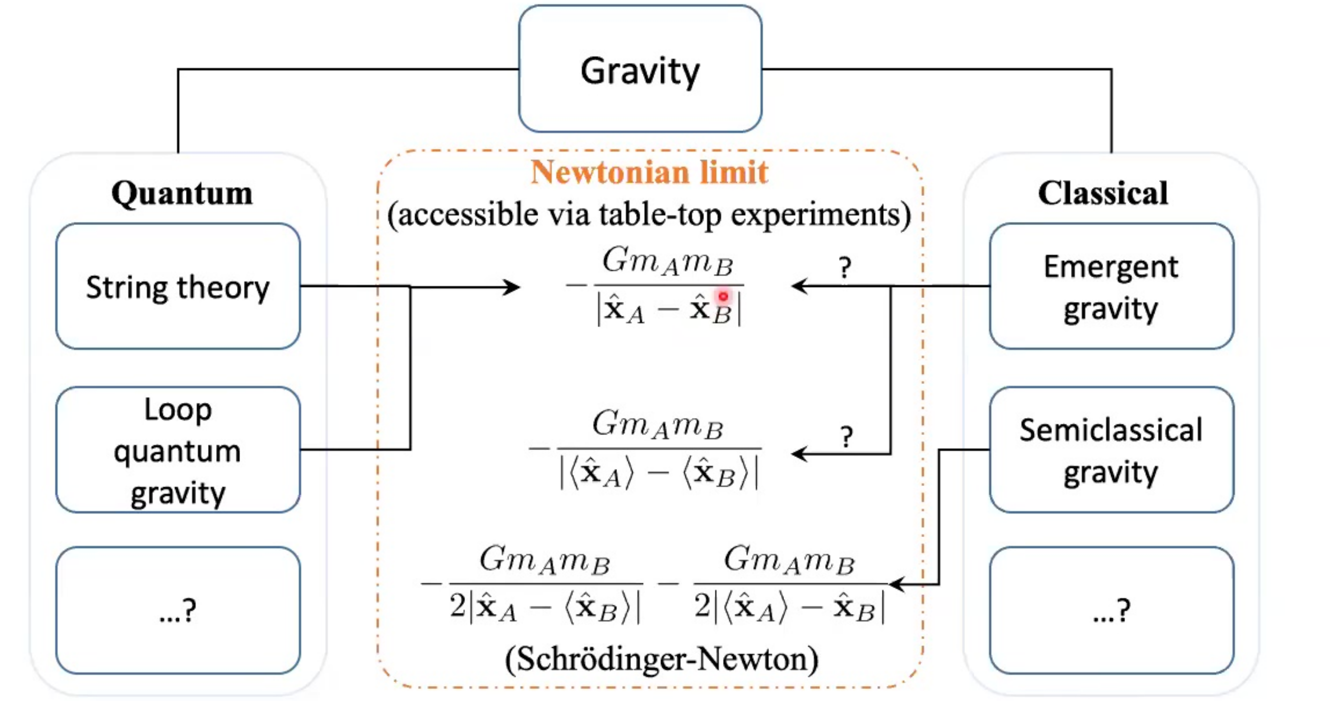


Why quantum gravity? mundane motivations

Multiple objects interacting through gravity:

How does such a self-gravitating quantum system evolve?

A simplified overview



Gravity

Quantum

String theory

Loop quantum gravity

...?

Newtonian limit

(accessible via table-top experiments)

$$-\frac{Gm_Am_B}{|\hat{\mathbf{x}}_A - \hat{\mathbf{x}}_B|}$$

?

$$-\frac{Gm_Am_B}{|\langle\hat{\mathbf{x}}_A\rangle - \langle\hat{\mathbf{x}}_B\rangle|}$$

?

$$-\frac{Gm_Am_B}{2|\hat{\mathbf{x}}_A - \langle\hat{\mathbf{x}}_B\rangle|} - \frac{Gm_Am_B}{2|\langle\hat{\mathbf{x}}_A\rangle - \hat{\mathbf{x}}_B|}$$

(Schrödinger-Newton)

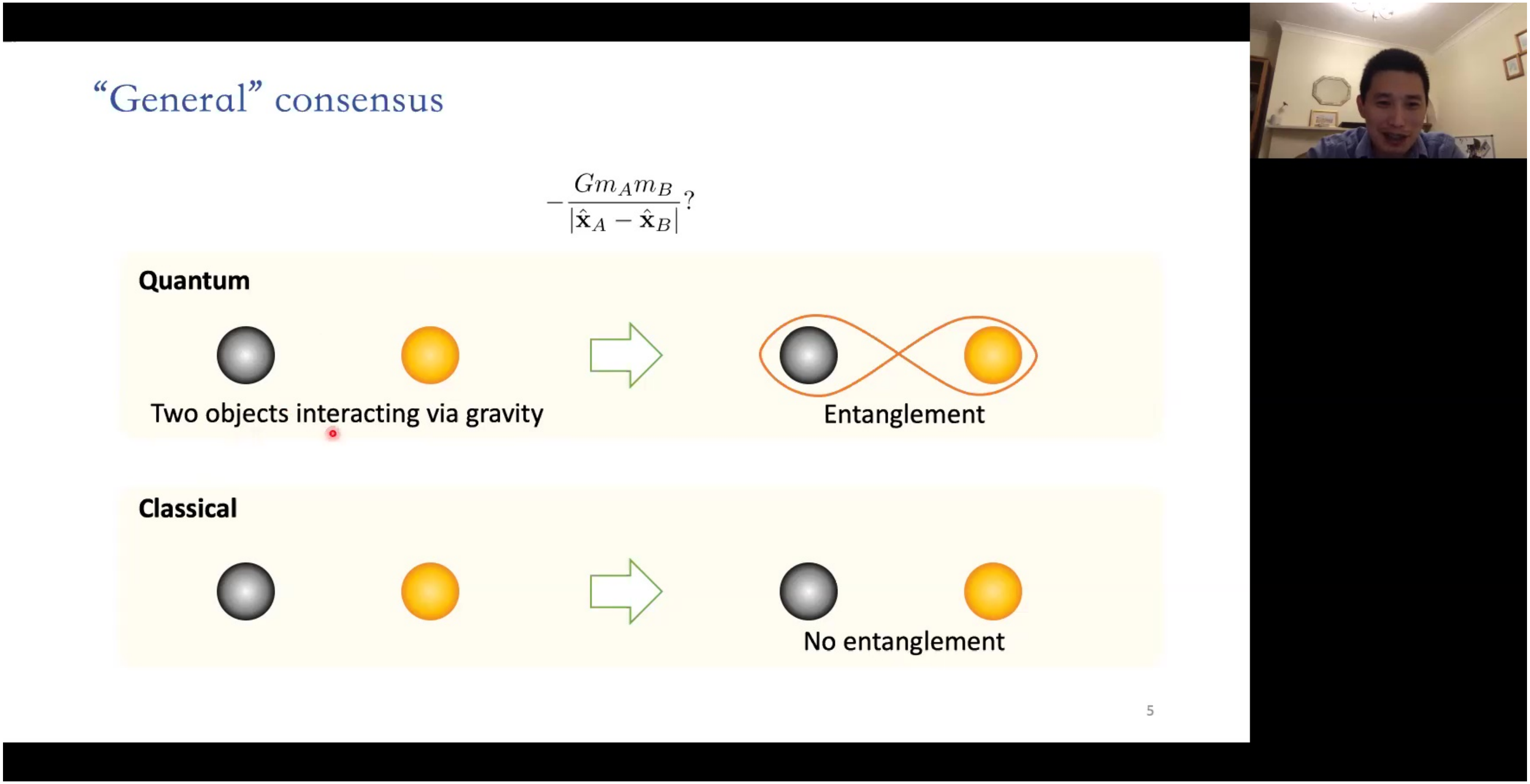
Classical

Emergent gravity

Semiclassical gravity

...?

Two objects interacting via gravity



“General” consensus

$$-\frac{Gm_A m_B}{|\hat{\mathbf{x}}_A - \hat{\mathbf{x}}_B|}?$$

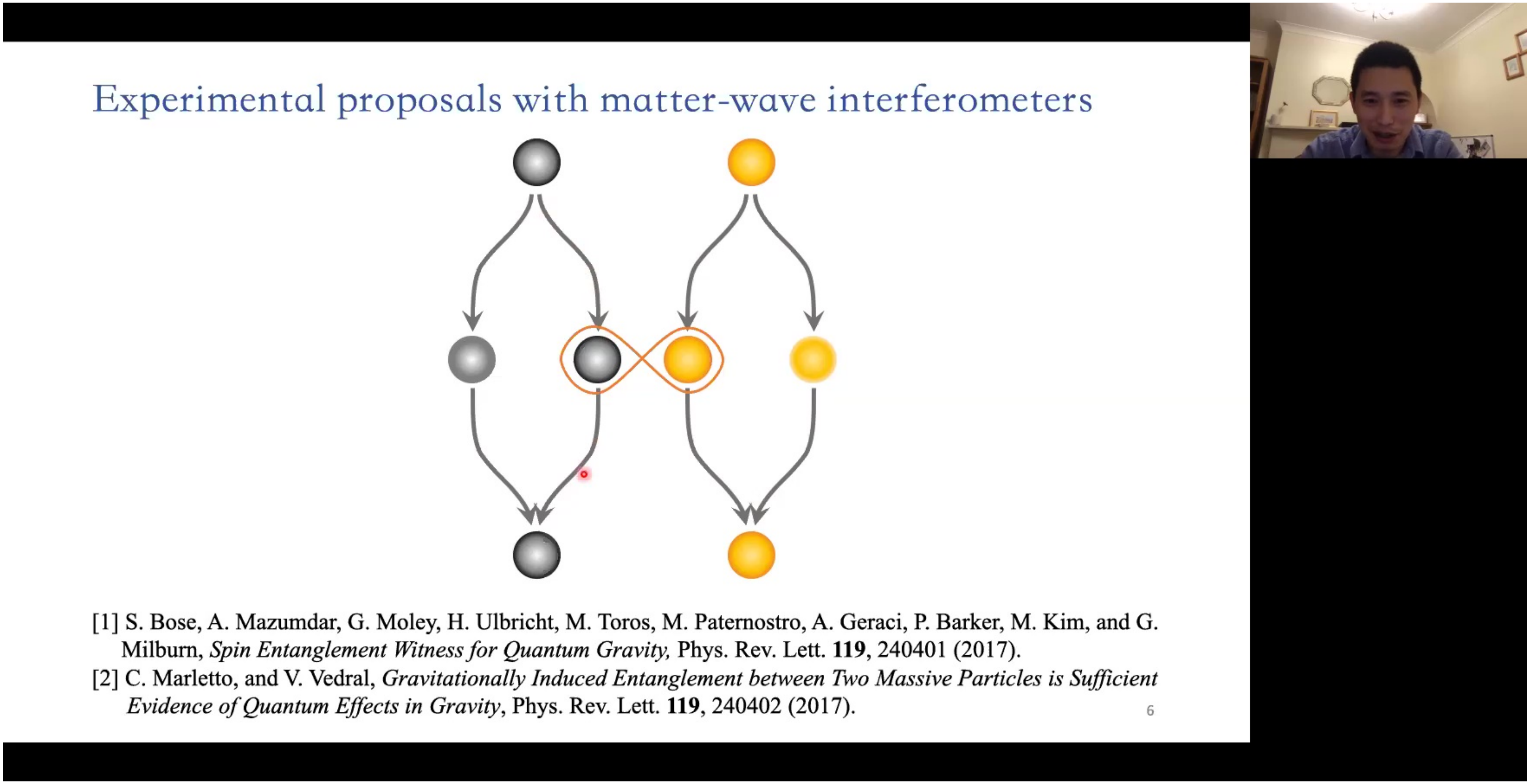
Quantum

Two objects interacting via gravity

Entanglement

Classical

No entanglement



Experimental proposals with matter-wave interferometers

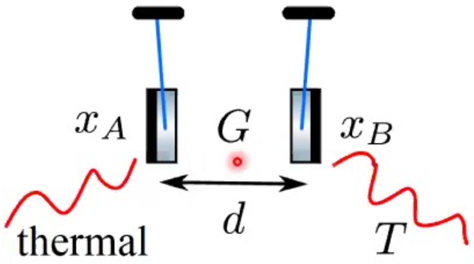
[1] S. Bose, A. Mazumdar, G. Moley, H. Ulbricht, M. Toros, M. Paternostro, A. Geraci, P. Barker, M. Kim, and G. Milburn, *Spin Entanglement Witness for Quantum Gravity,* Phys. Rev. Lett. **119**, 240401 (2017).

[2] C. Marletto, and V. Vedral, *Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity*, Phys. Rev. Lett. **119**, 240402 (2017).

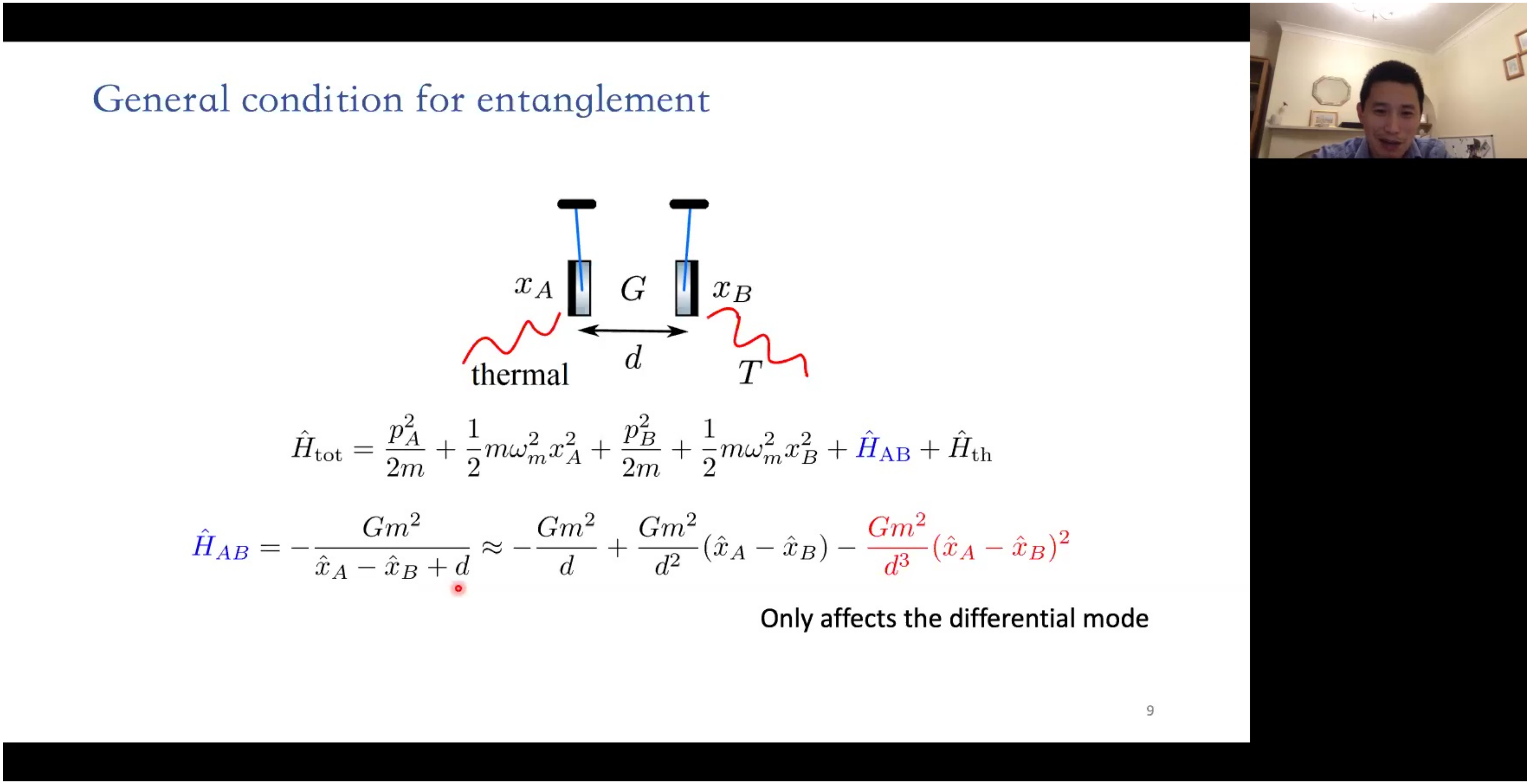
How difficult is it to observe

gravity-mediated entanglement?

General condition for entanglement (using two pendula)



Gravitational interaction (steady-state) in the presence of thermal decoherence

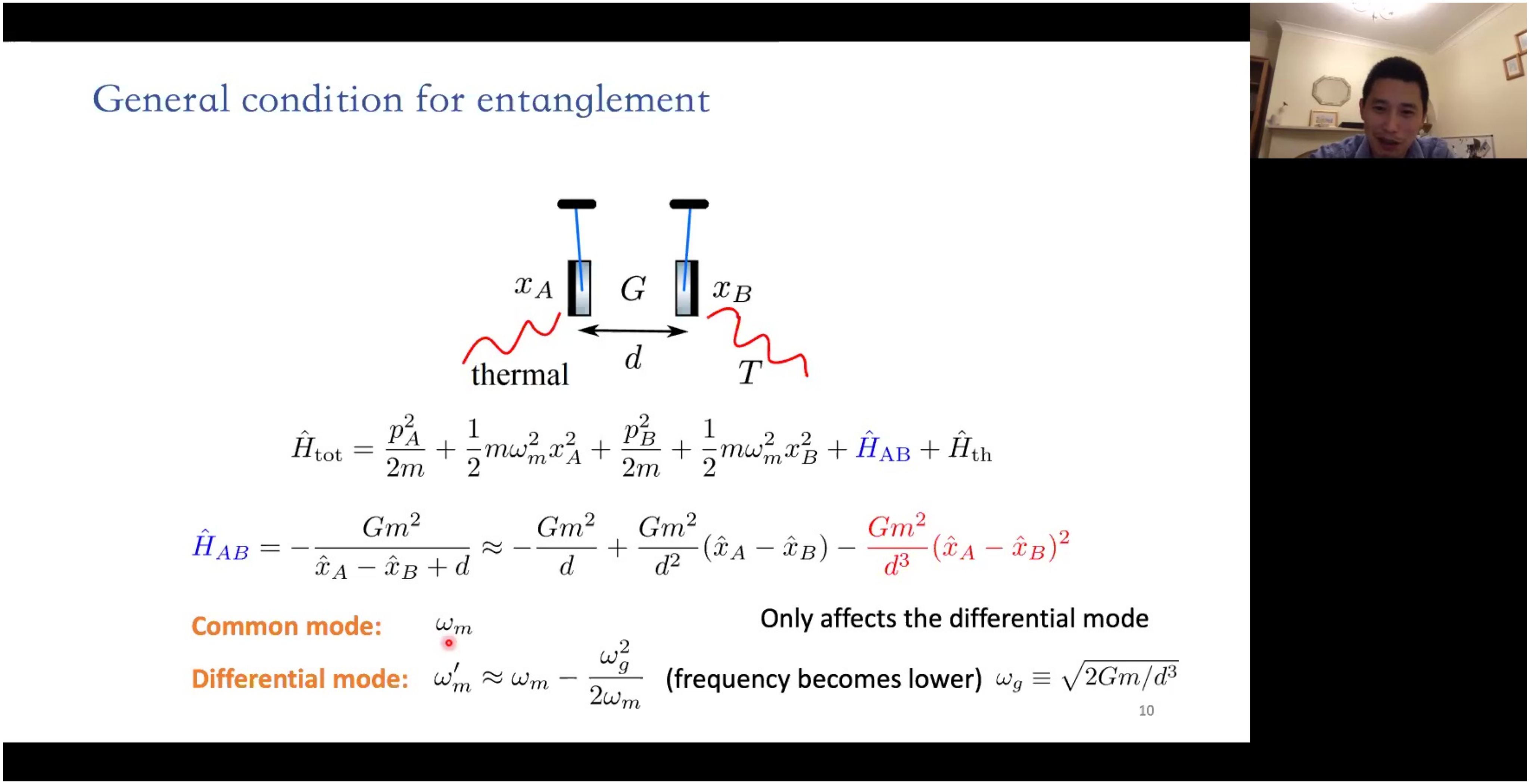


General condition for entanglement

$$\hat{H}_{\text{tot}} = \frac{p_A^2}{2m} + \frac{1}{2}m\omega_m^2 x_A^2 + \frac{p_B^2}{2m} + \frac{1}{2}m\omega_m^2 x_B^2 + \hat{H}_{\text{AB}} + \hat{H}_{\text{th}}$$

$$\hat{H}_{AB} = -\frac{Gm^2}{\hat{x}_A - \hat{x}_B + d} \approx -\frac{Gm^2}{d} + \frac{Gm^2}{d^2}(\hat{x}_A - \hat{x}_B) - \frac{Gm^2}{d^3}(\hat{x}_A - \hat{x}_B)^2$$

Only affects the differential mode



General condition for entanglement

$$\hat{H}_{\text{tot}} = \frac{p_A^2}{2m} + \frac{1}{2}m\omega_m^2 x_A^2 + \frac{p_B^2}{2m} + \frac{1}{2}m\omega_m^2 x_B^2 + \hat{H}_{\text{AB}} + \hat{H}_{\text{th}}$$

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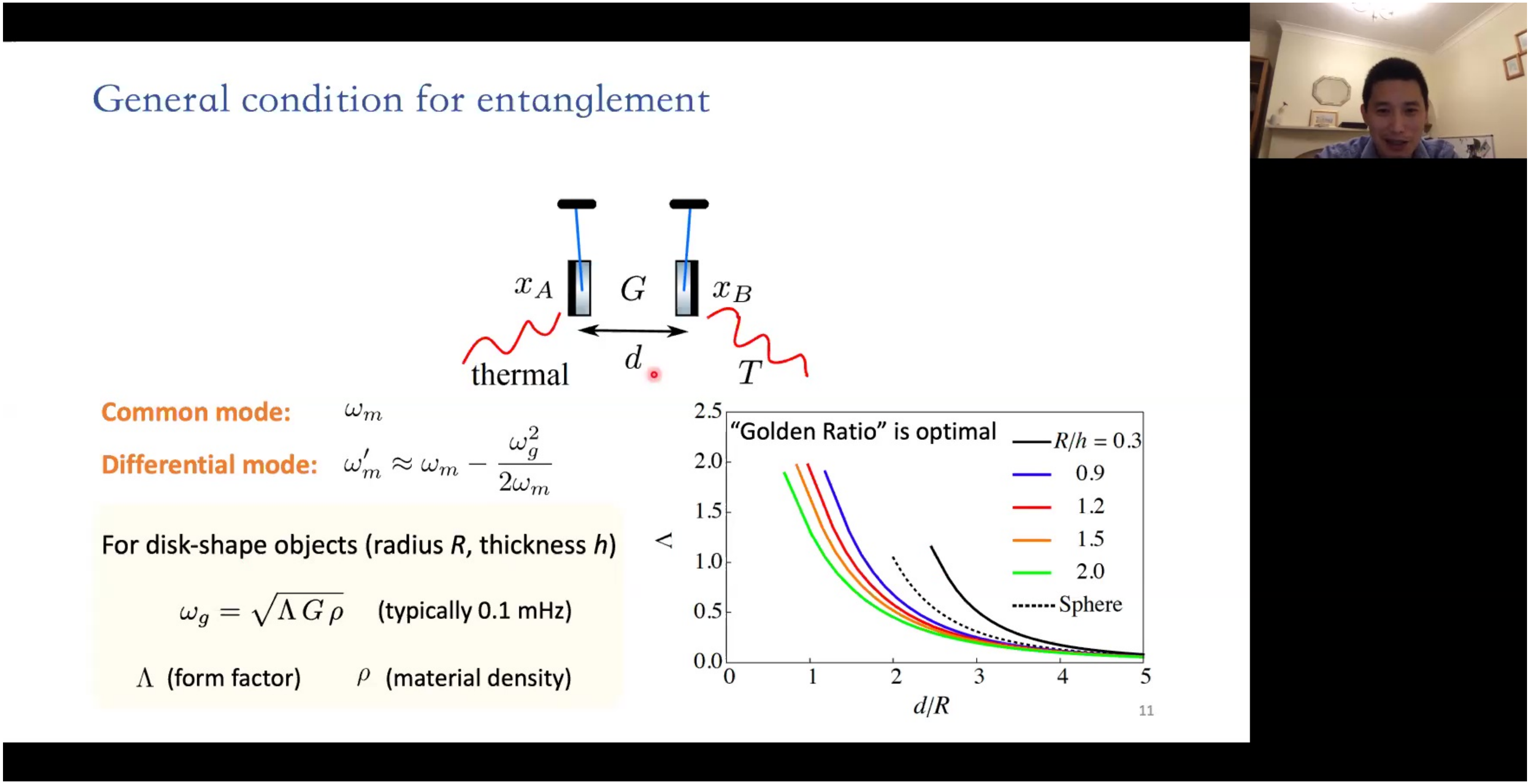
Only affects the differential mode

Common mode: ω_m

Differential mode: $\omega'_m \approx \omega_m - \frac{\omega_g^2}{2\omega_m}$ (frequency becomes lower) $\omega_g \equiv \sqrt{2Gm/d^3}$



General condition for entanglement

Common mode: ω_m

Differential mode: $\omega'_m \approx \omega_m - \dfrac{\omega_g^2}{2\omega_m}$

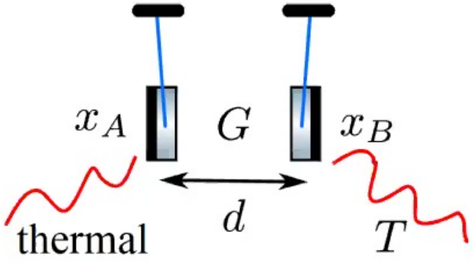
For disk-shape objects (radius *R*, thickness *h*)

$$\omega_g = \sqrt{\Lambda\, G\, \rho}$$ (typically 0.1 mHz)

Λ (form factor) ρ (material density)

"Golden Ratio" is optimal

General condition for entanglement



$$n_{\rm th}(\omega) = \frac{1}{2}\coth\left(\frac{\hbar\omega}{2k_BT}\right)$$

Common mode: ω_m

Differential mode: $\omega_m' \approx \omega_m - \dfrac{\omega_g^2}{2\omega_m}$

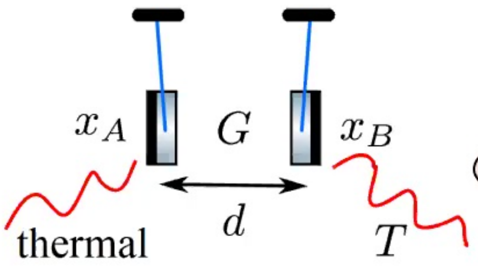
Common and differential modes @ thermal equilibrium

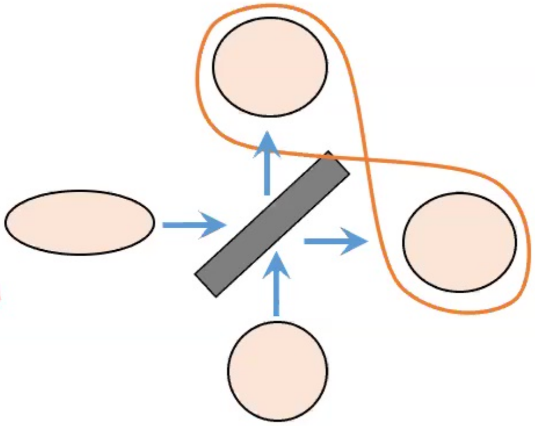
$$\langle x^2\rangle = x_q^2\,[2n_{\rm th}(\omega)+1] \qquad x_q^2 \equiv \frac{\hbar}{2m\omega}$$

$$\langle p^2\rangle = p_q^2\,[2n_{\rm th}(\omega)+1] \qquad p_q^2 \equiv \frac{\hbar m\omega}{2}$$

$$(\omega = \omega_m \text{ or } \omega_m')$$

General condition for entanglement





[Quantum optics analogy]

Common mode: ω_m

Differential mode: $\omega'_m \approx \omega_m - \dfrac{\omega_g^2}{2\omega_m}$

Common and differential modes @ thermal equilibrium

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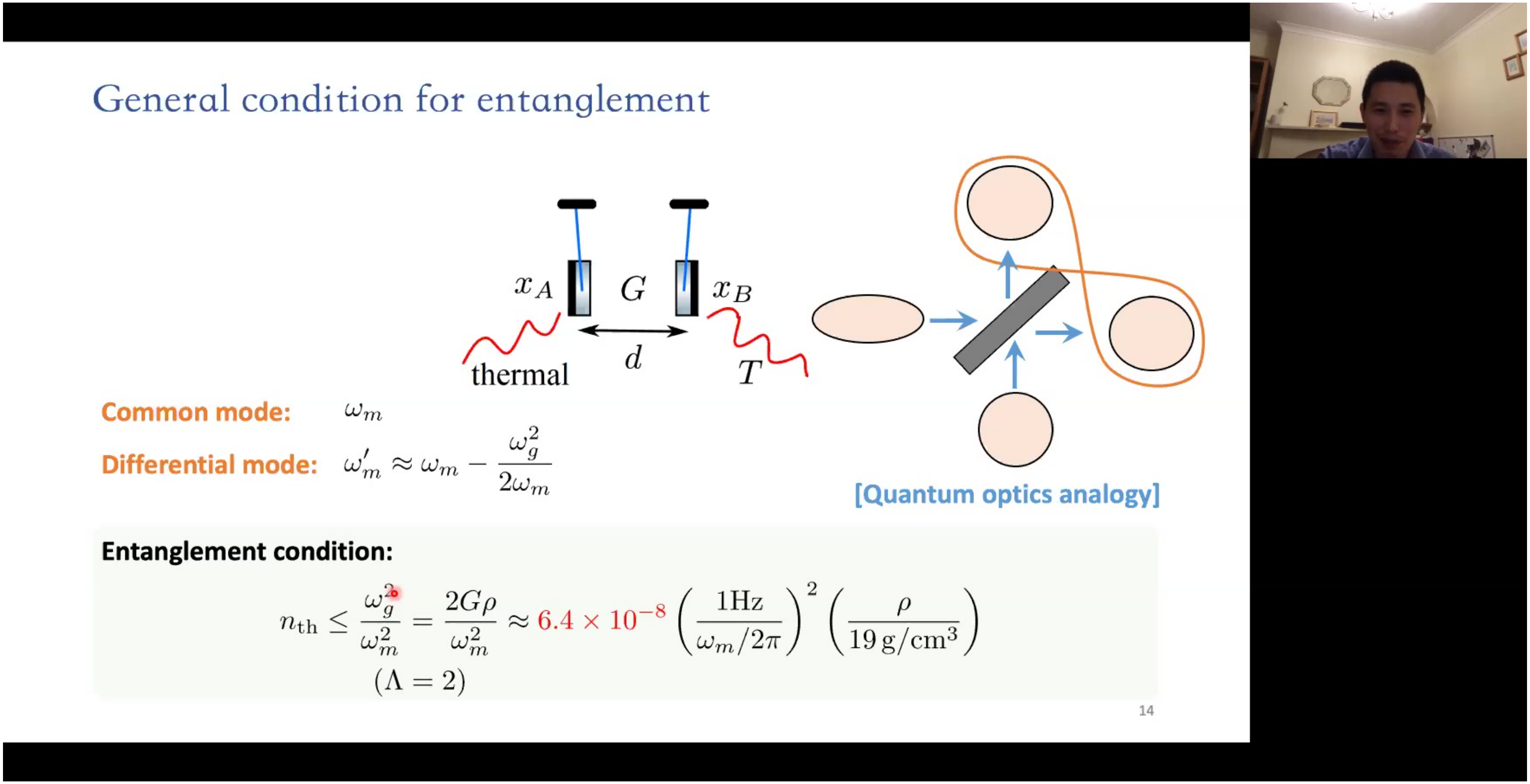


General condition for entanglement

Common mode: ω_m

Differential mode: $\omega'_m \approx \omega_m - \dfrac{\omega_g^2}{2\omega_m}$

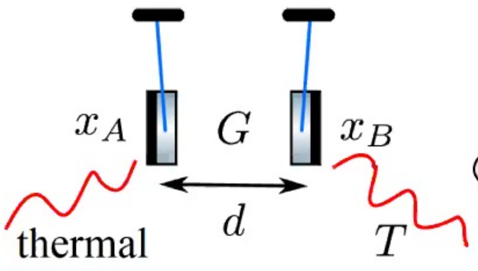
[Quantum optics analogy]

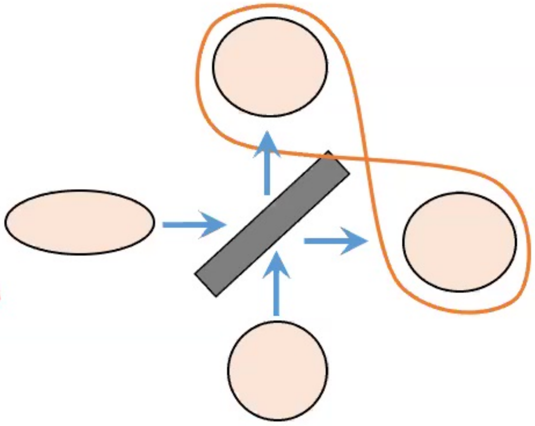
Entanglement condition:

$$n_{\rm th} \le \frac{\omega_g^2}{\omega_m^2} = \frac{2G\rho}{\omega_m^2} \approx 6.4 \times 10^{-8} \left(\frac{1\mathrm{Hz}}{\omega_m/2\pi}\right)^2 \left(\frac{\rho}{19\,\mathrm{g/cm^3}}\right)$$

$$(\Lambda = 2)$$

General condition for entanglement





[Quantum optics analogy]

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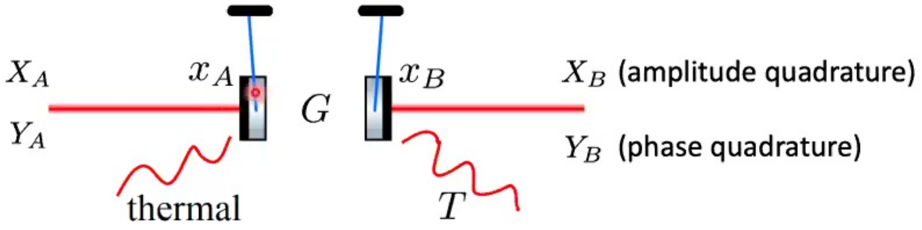
$$(\Lambda = 2)$$

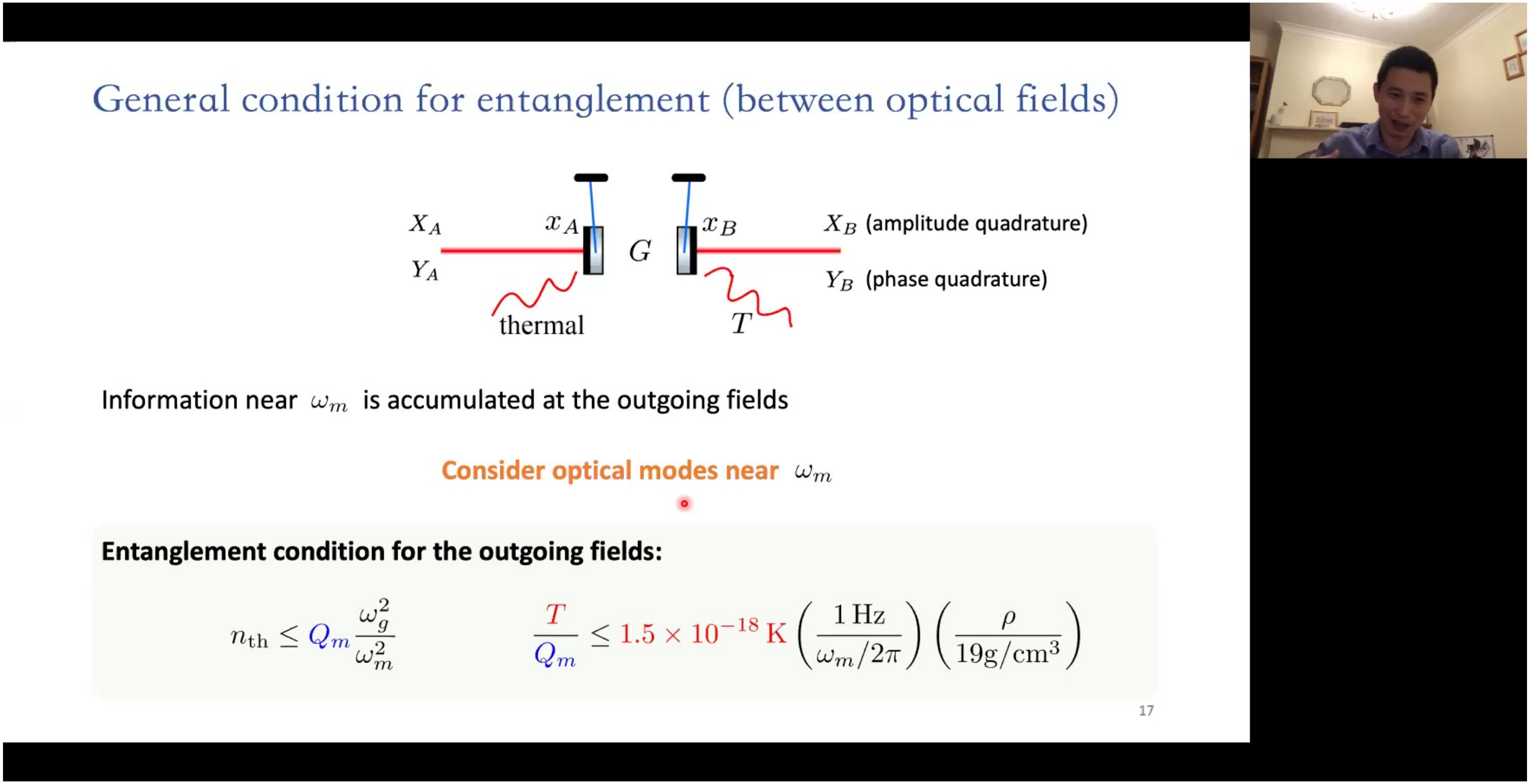
$$n_{\rm th} \sim 6.3 \times 10^{12} \left(\frac{T}{300\,\mathrm{K}}\right) \left(\frac{1\mathrm{Hz}}{\omega_m/2\pi}\right)$$

General condition for entanglement (between optical fields)



Information near ω_m is accumulated at the outgoing fields

Consider optical modes near ω_m



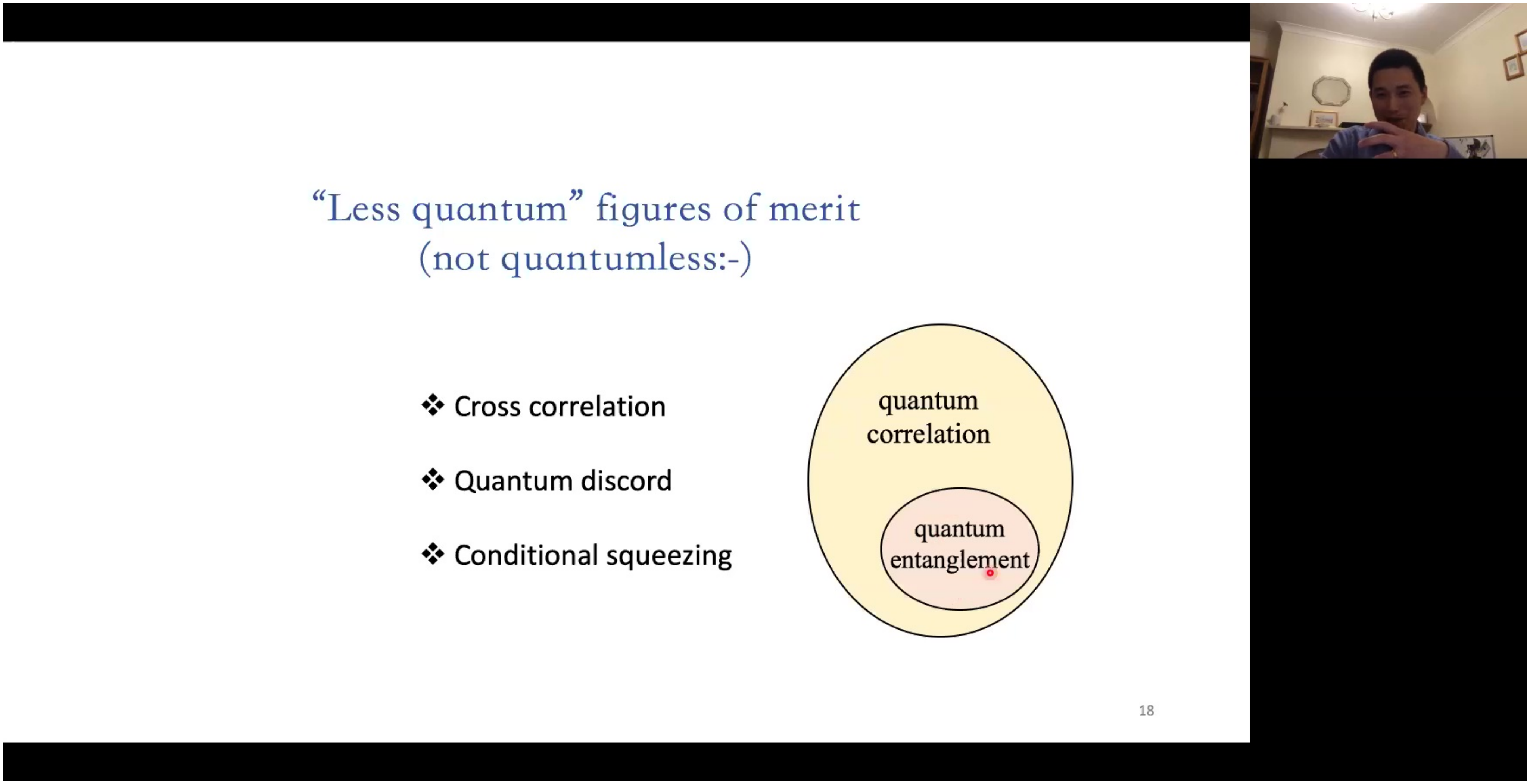
General condition for entanglement (between optical fields)

Information near ω_m is accumulated at the outgoing fields

Consider optical modes near ω_m

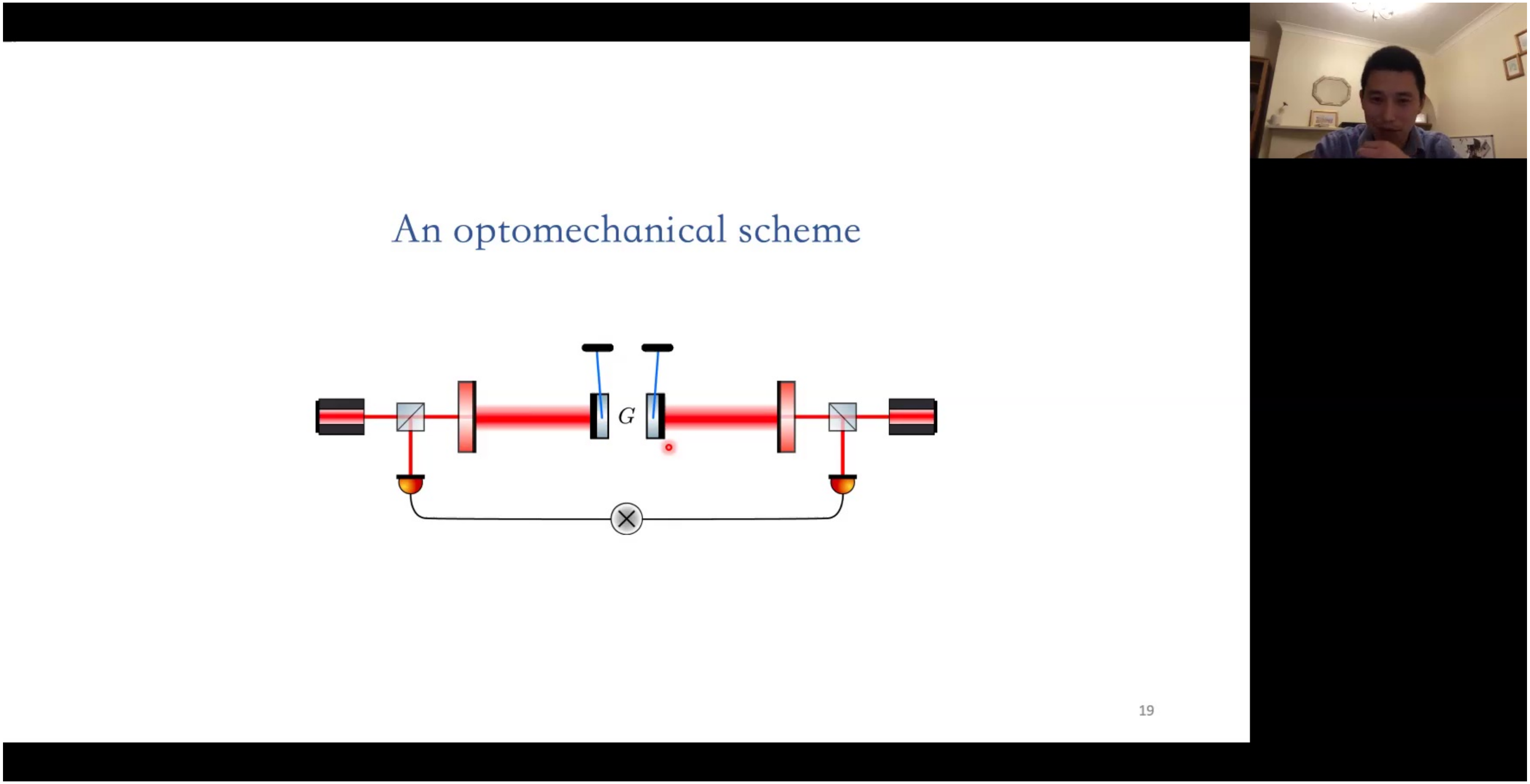
Entanglement condition for the outgoing fields:

$$n_{\rm th} \le Q_m \frac{\omega_g^2}{\omega_m^2} \qquad \frac{T}{Q_m} \le 1.5 \times 10^{-18}\,\mathrm{K} \left(\frac{1\,\mathrm{Hz}}{\omega_m/2\pi}\right) \left(\frac{\rho}{19\mathrm{g/cm^3}}\right)$$

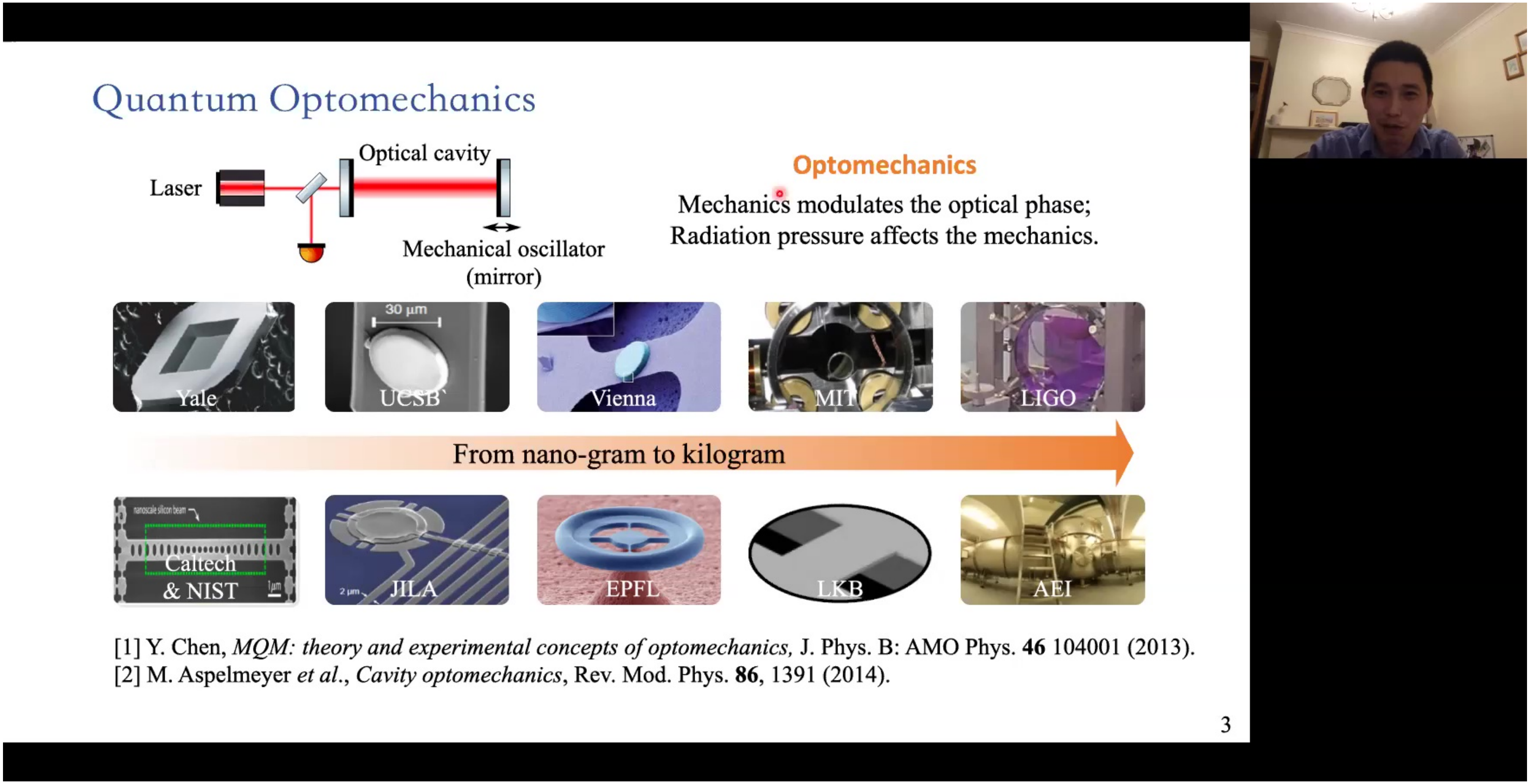


“Less quantum” figures of merit (not quantumless:-)

- Cross correlation
- Quantum discord
- Conditional squeezing



An optomechanical scheme



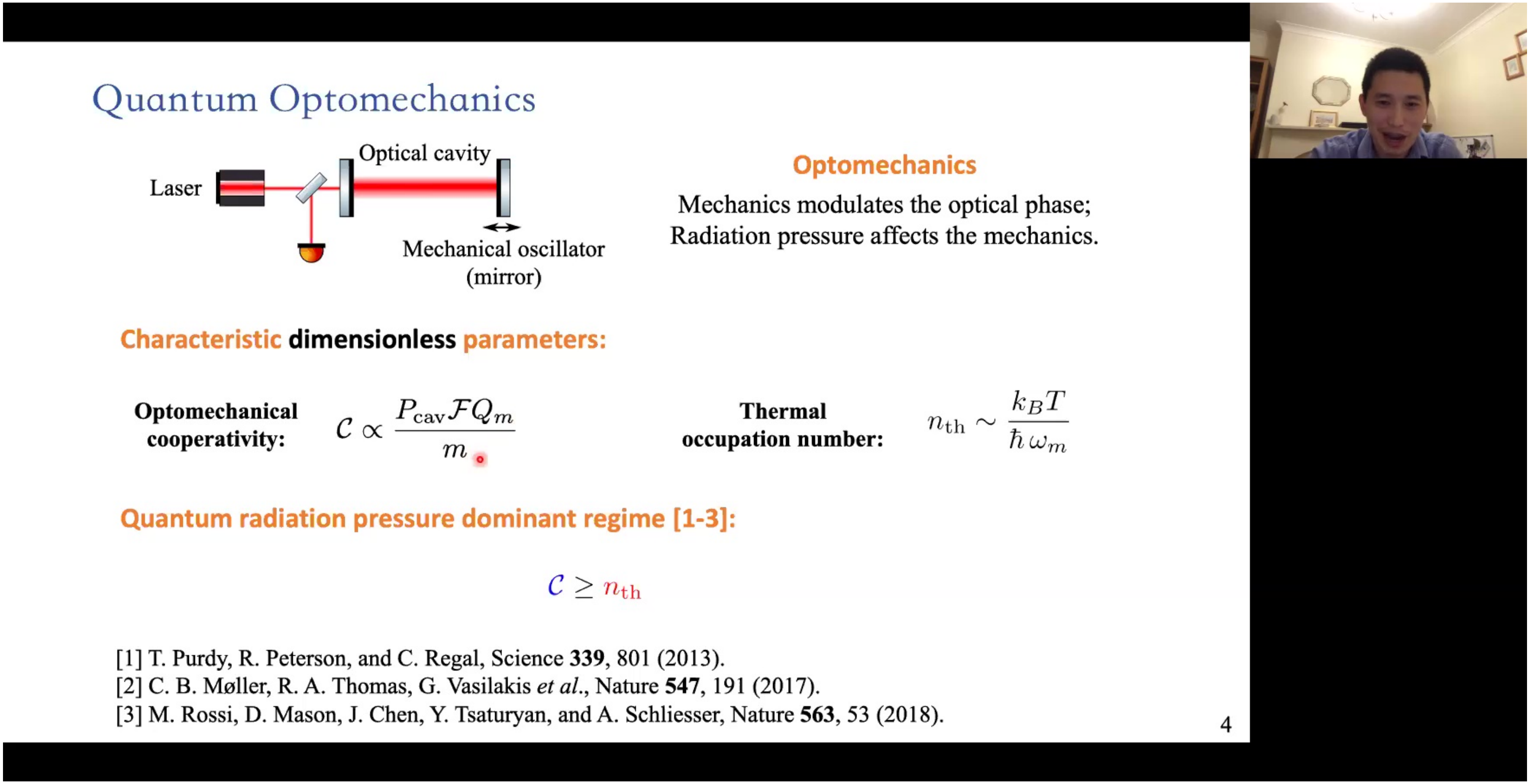
Quantum Optomechanics

Optomechanics

Mechanics modulates the optical phase;
Radiation pressure affects the mechanics.

From nano-gram to kilogram

[1] Y. Chen, *MQM: theory and experimental concepts of optomechanics,* J. Phys. B: AMO Phys. **46** 104001 (2013).
[2] M. Aspelmeyer *et al.*, *Cavity optomechanics*, Rev. Mod. Phys. **86**, 1391 (2014).



Quantum Optomechanics

Optomechanics

Mechanics modulates the optical phase;
Radiation pressure affects the mechanics.

Characteristic dimensionless parameters:

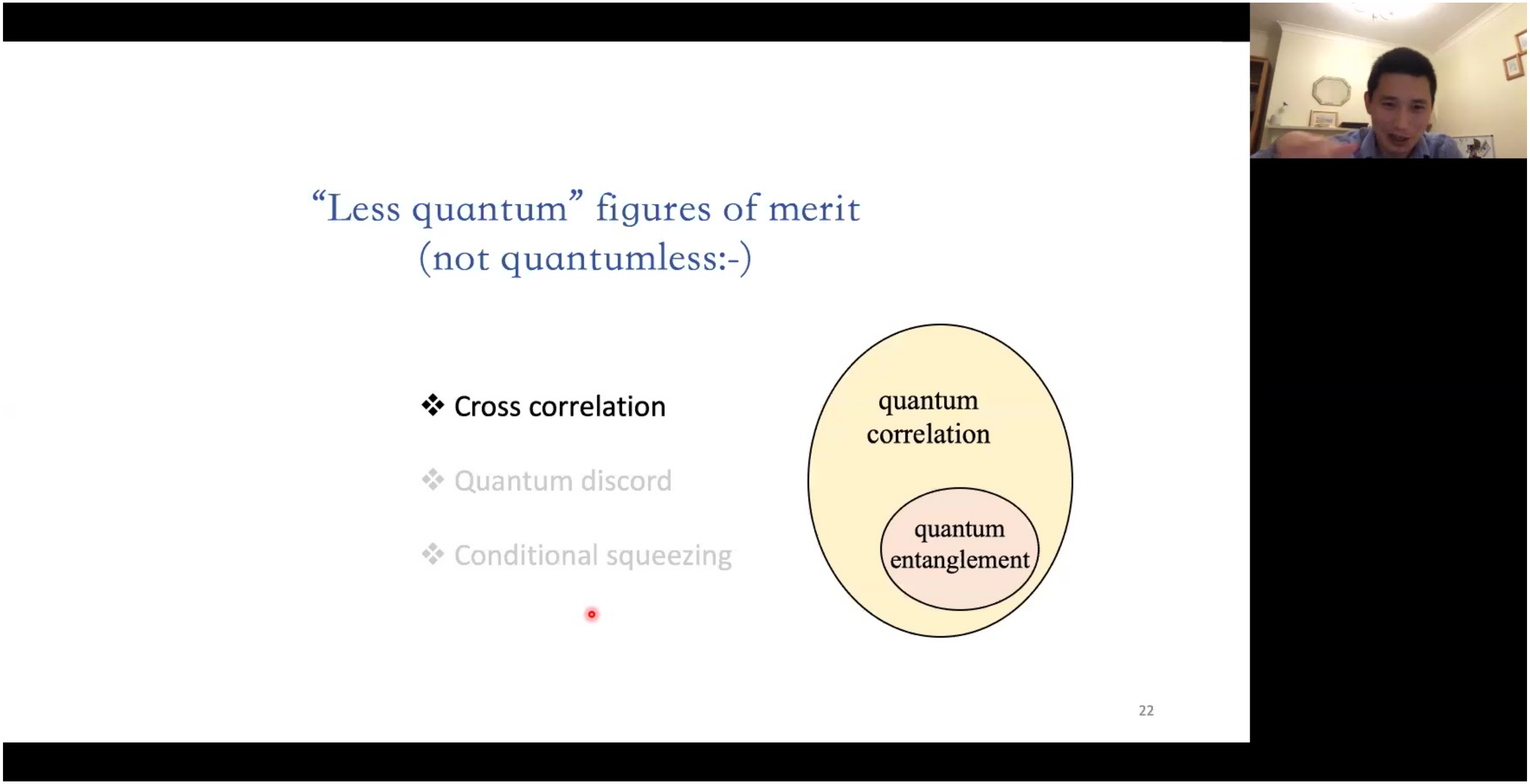
Optomechanical cooperativity: $$\mathcal{C} \propto \frac{P_{\mathrm{cav}} \mathcal{F} Q_m}{m}$$

Thermal occupation number: $$n_{\mathrm{th}} \sim \frac{k_B T}{\hbar\, \omega_m}$$

Quantum radiation pressure dominant regime [1-3]:

$$\mathcal{C} \geq n_{\mathrm{th}}$$

[1] T. Purdy, R. Peterson, and C. Regal, Science **339**, 801 (2013).
[2] C. B. Møller, R. A. Thomas, G. Vasilakis *et al.*, Nature **547**, 191 (2017).
[3] M. Rossi, D. Mason, J. Chen, Y. Tsaturyan, and A. Schliesser, Nature **563**, 53 (2018).

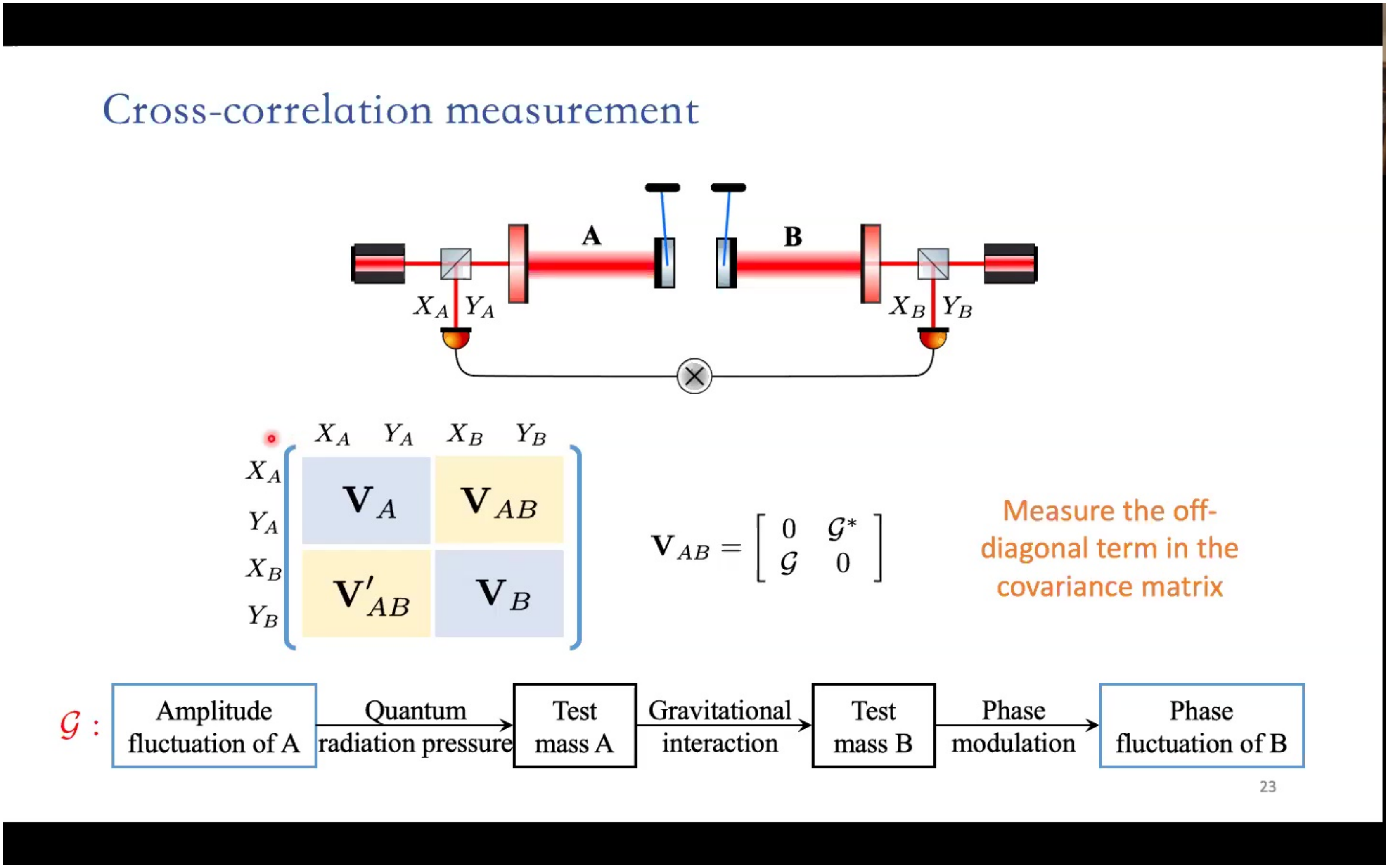


“Less quantum” figures of merit (not quantumless:-)

- ❖ Cross correlation
- ❖ Quantum discord
- ❖ Conditional squeezing

quantum correlation

quantum entanglement



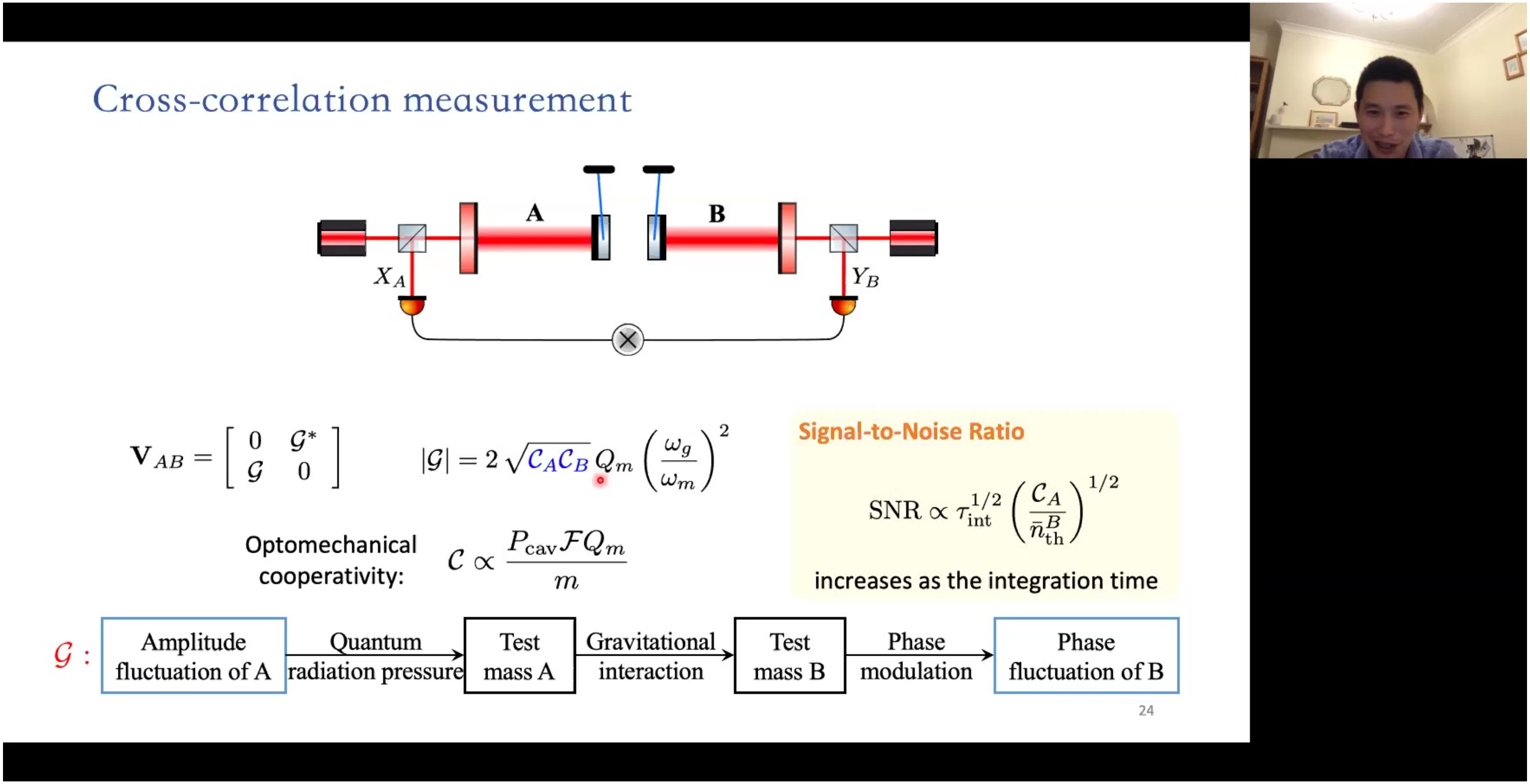
Cross-correlation measurement

$$\begin{array}{cc} & \begin{array}{cccc} X_A & Y_A & X_B & Y_B \end{array} \\ \begin{array}{c} X_A \\ Y_A \\ X_B \\ Y_B \end{array} & \begin{bmatrix} \mathbf{V}_A & \mathbf{V}_{AB} \\ \mathbf{V}'_{AB} & \mathbf{V}_B \end{bmatrix} \end{array}$$

$$\mathbf{V}_{AB} = \begin{bmatrix} 0 & \mathcal{G}^* \\ \mathcal{G} & 0 \end{bmatrix}$$

Measure the off-diagonal term in the covariance matrix

$\mathcal{G}$: Amplitude fluctuation of A $\xrightarrow{\text{Quantum radiation pressure}}$ Test mass A $\xrightarrow{\text{Gravitational interaction}}$ Test mass B $\xrightarrow{\text{Phase modulation}}$ Phase fluctuation of B



Cross-correlation measurement

$$\mathbf{V}_{AB} = \begin{bmatrix} 0 & \mathcal{G}^* \\ \mathcal{G} & 0 \end{bmatrix} \qquad |\mathcal{G}| = 2\sqrt{\mathcal{C}_A\mathcal{C}_B}\, Q_m \left(\frac{\omega_g}{\omega_m}\right)^2$$

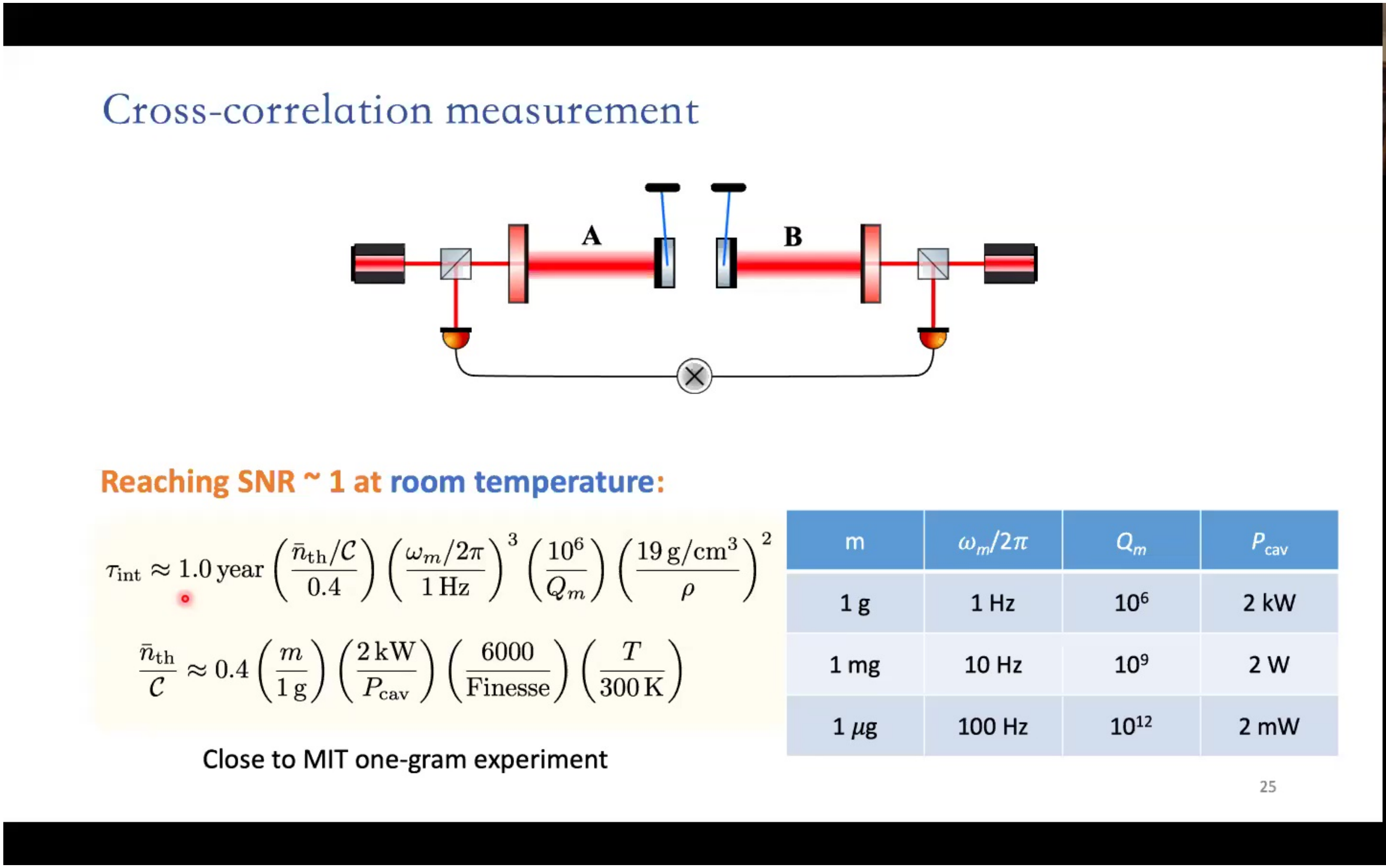
Optomechanical cooperativity: $\mathcal{C} \propto \frac{P_{\text{cav}}\mathcal{F}Q_m}{m}$

Signal-to-Noise Ratio

$$\text{SNR} \propto \tau_{\text{int}}^{1/2}\left(\frac{\mathcal{C}_A}{\bar{n}_{\text{th}}^B}\right)^{1/2}$$

increases as the integration time

$$\mathcal{G}:\ \boxed{\text{Amplitude fluctuation of A}} \xrightarrow{\text{Quantum radiation pressure}} \boxed{\text{Test mass A}} \xrightarrow{\text{Gravitational interaction}} \boxed{\text{Test mass B}} \xrightarrow{\text{Phase modulation}} \boxed{\text{Phase fluctuation of B}}$$



Cross-correlation measurement

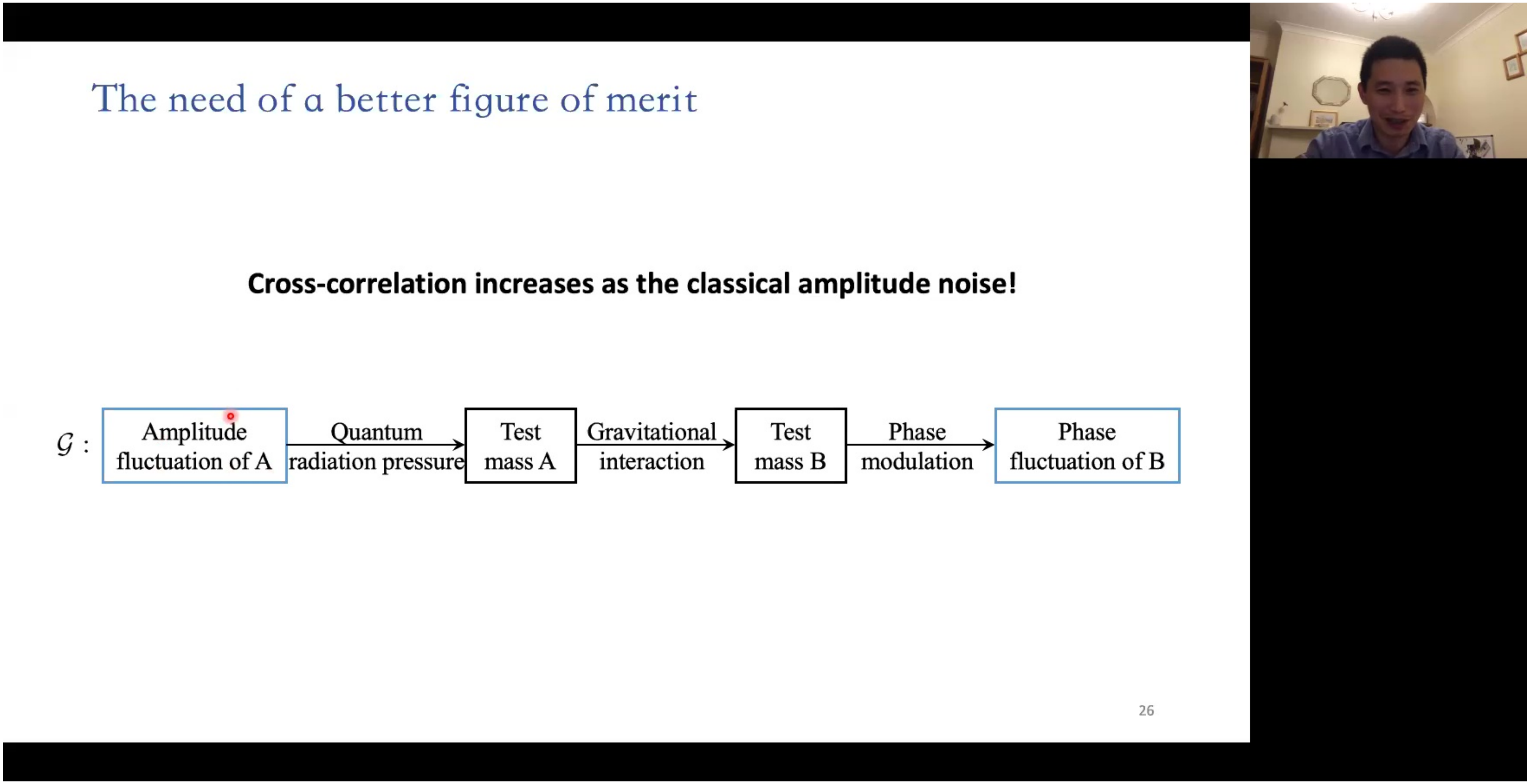
Reaching SNR ~ 1 at room temperature:

$$\tau_{\rm int} \approx 1.0\,\text{year}\left(\frac{\bar{n}_{\rm th}/\mathcal{C}}{0.4}\right)\left(\frac{\omega_m/2\pi}{1\,\text{Hz}}\right)^3\left(\frac{10^6}{Q_m}\right)\left(\frac{19\,\text{g/cm}^3}{\rho}\right)^2$$

$$\frac{\bar{n}_{\rm th}}{\mathcal{C}} \approx 0.4\left(\frac{m}{1\,\text{g}}\right)\left(\frac{2\,\text{kW}}{P_{\rm cav}}\right)\left(\frac{6000}{\text{Finesse}}\right)\left(\frac{T}{300\,\text{K}}\right)$$

Close to MIT one-gram experiment

m	$\omega_m/2\pi$	Q_m	$P_{\rm cav}$
1 g	1 Hz	10^6	2 kW
1 mg	10 Hz	10^9	2 W
1 μg	100 Hz	10^{12}	2 mW



The need of a better figure of merit

Cross-correlation increases as the classical amplitude noise!

$\mathcal{G}$:

Amplitude fluctuation of A

Quantum radiation pressure

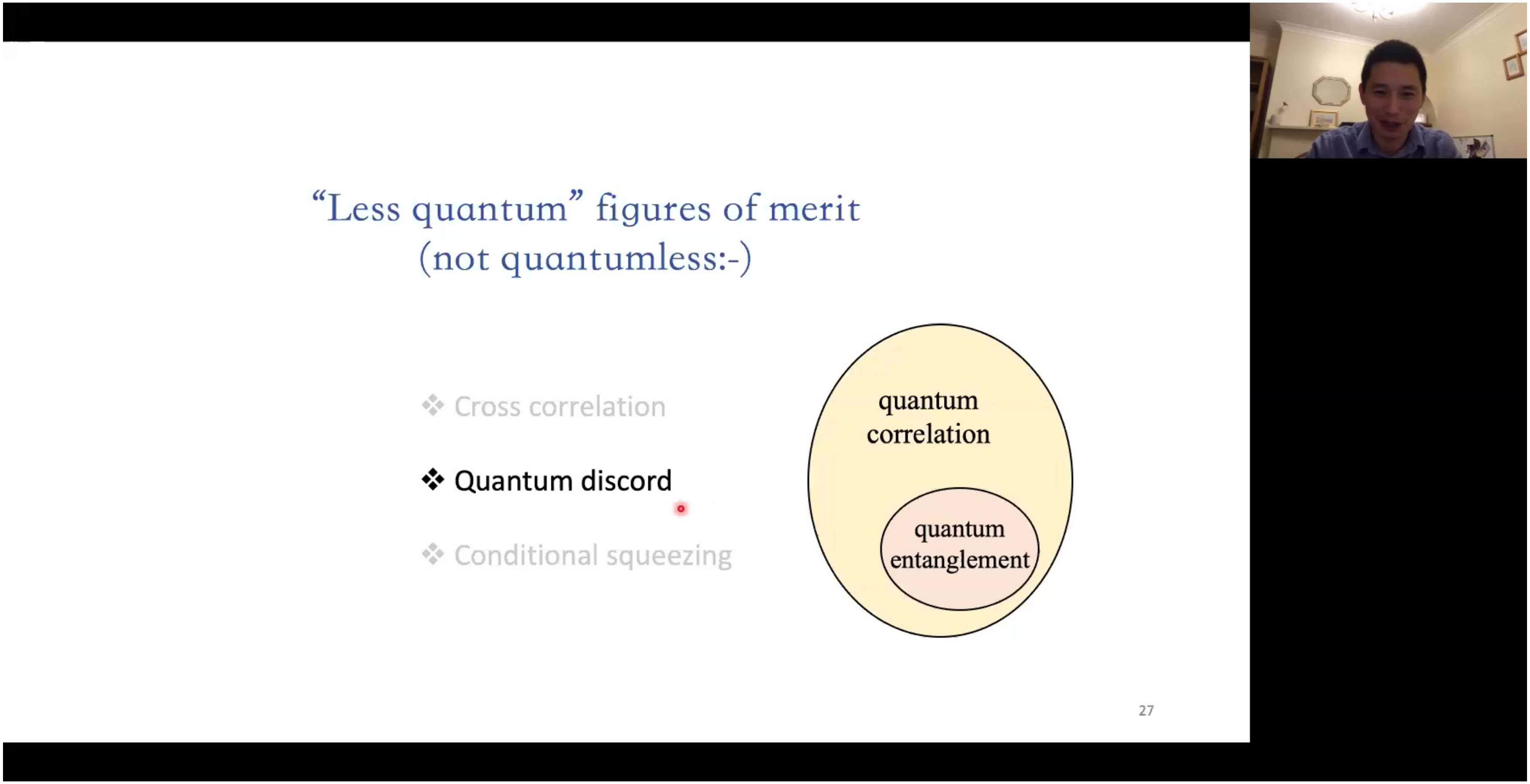
Test mass A

Gravitational interaction

Test mass B

Phase modulation

Phase fluctuation of B

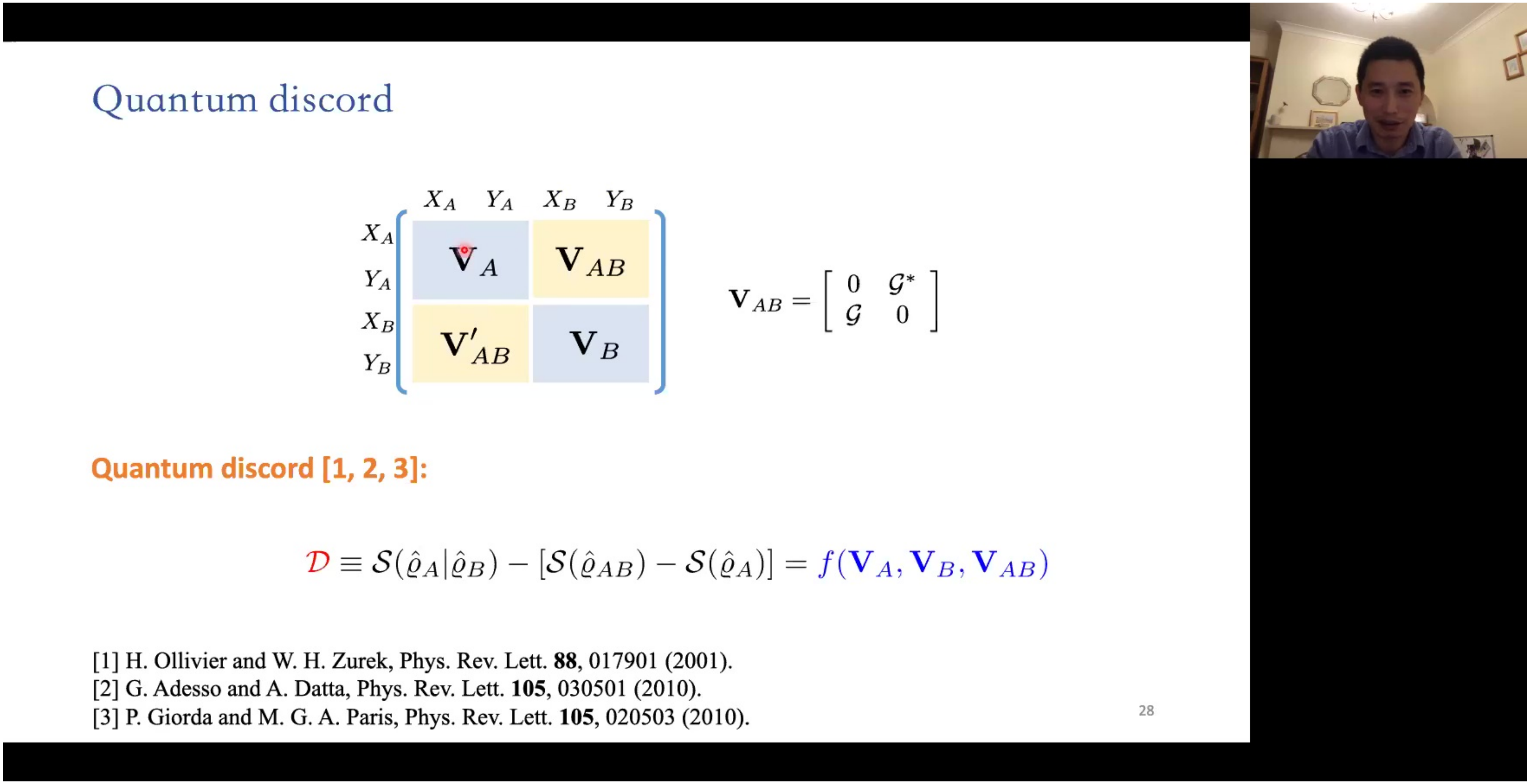


“Less quantum” figures of merit (not quantumless:-)

- ❖ Cross correlation
- ❖ **Quantum discord**
- ❖ Conditional squeezing

quantum correlation

quantum entanglement



Quantum discord

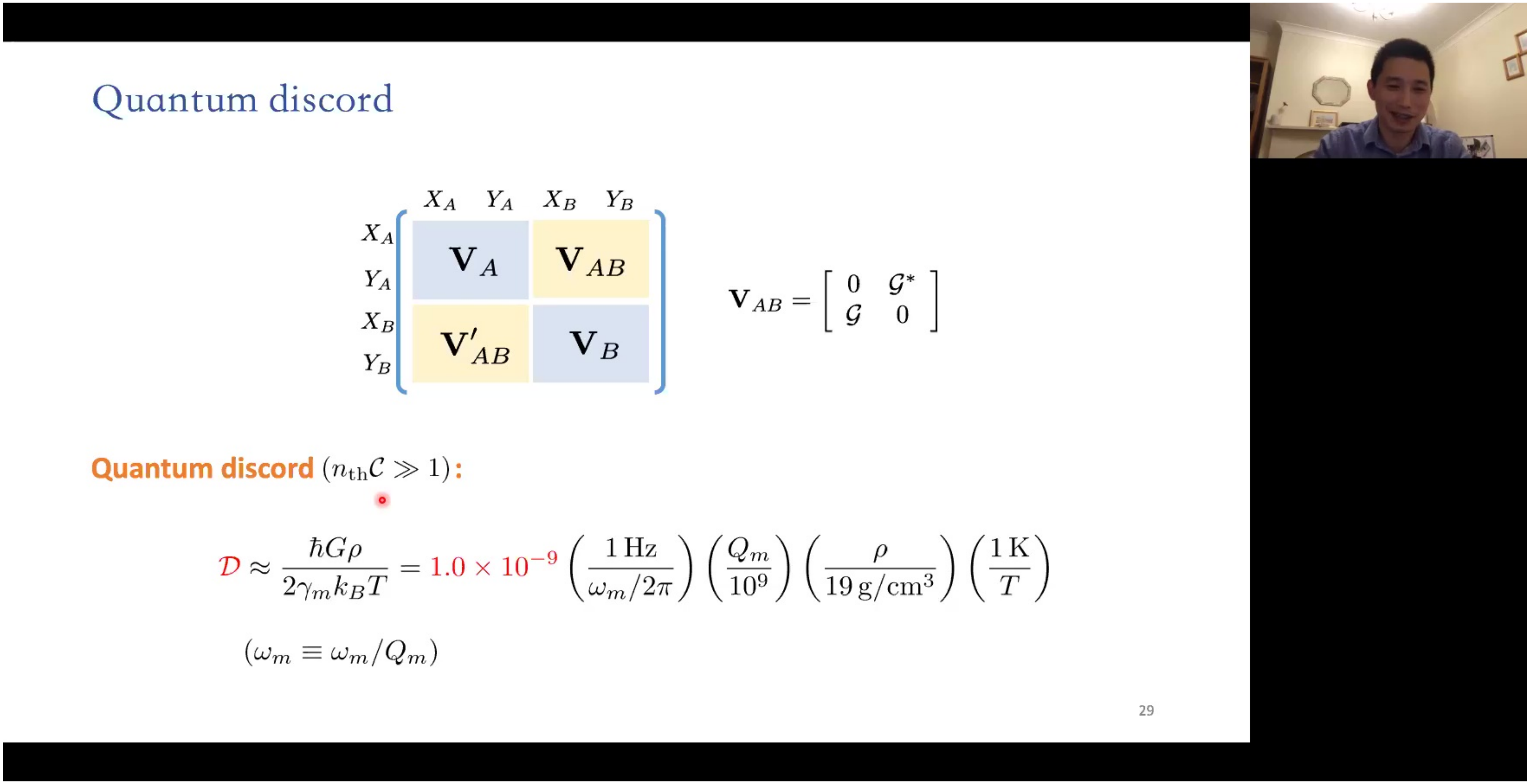
$$\begin{array}{c|cc} & X_A \quad Y_A & X_B \quad Y_B \\ \hline X_A, Y_A & \mathbf{V}_A & \mathbf{V}_{AB} \\ X_B, Y_B & \mathbf{V}'_{AB} & \mathbf{V}_B \end{array}$$

$$\mathbf{V}_{AB} = \begin{bmatrix} 0 & \mathcal{G}^* \\ \mathcal{G} & 0 \end{bmatrix}$$

Quantum discord [1, 2, 3]:

$$\mathcal{D} \equiv \mathcal{S}(\hat{\varrho}_A|\hat{\varrho}_B) - [\mathcal{S}(\hat{\varrho}_{AB}) - \mathcal{S}(\hat{\varrho}_A)] = f(\mathbf{V}_A, \mathbf{V}_B, \mathbf{V}_{AB})$$

[1] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[2] G. Adesso and A. Datta, Phys. Rev. Lett. **105**, 030501 (2010).
[3] P. Giorda and M. G. A. Paris, Phys. Rev. Lett. **105**, 020503 (2010).



Quantum discord

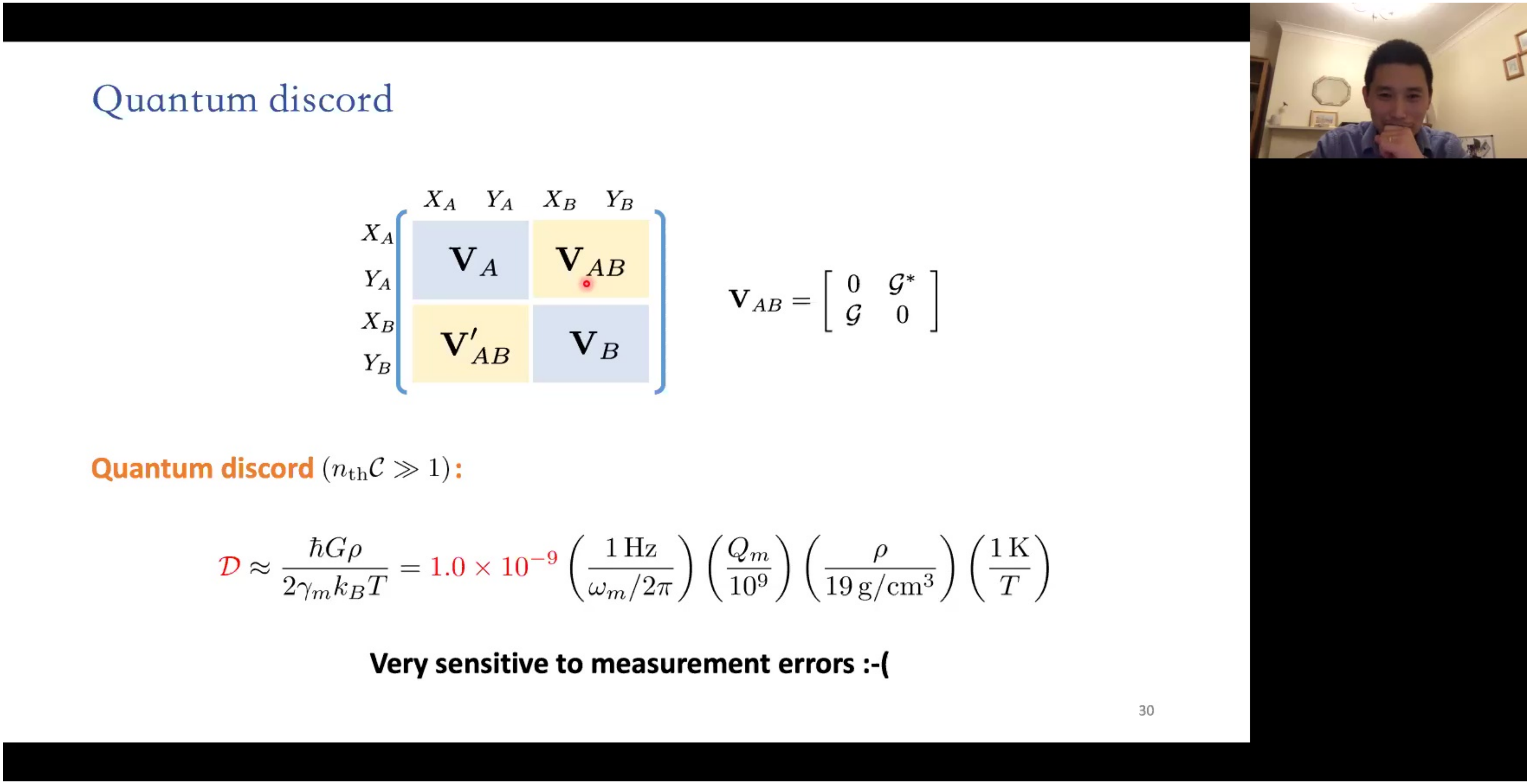
$$\begin{array}{c|cccc} & X_A & Y_A & X_B & Y_B \\ \hline X_A & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbf{V}_A$}} & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbf{V}_{AB}$}} \\ Y_A & & & & \\ X_B & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbf{V}'_{AB}$}} & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbf{V}_B$}} \\ Y_B & & & & \end{array}$$

$$\mathbf{V}_{AB} = \begin{bmatrix} 0 & \mathcal{G}^* \\ \mathcal{G} & 0 \end{bmatrix}$$

Quantum discord $(n_{\rm th}\mathcal{C} \gg 1)$**:**

$$\mathcal{D} \approx \frac{\hbar G\rho}{2\gamma_m k_B T} = 1.0 \times 10^{-9} \left(\frac{1\,\mathrm{Hz}}{\omega_m/2\pi}\right)\left(\frac{Q_m}{10^9}\right)\left(\frac{\rho}{19\,\mathrm{g/cm^3}}\right)\left(\frac{1\,\mathrm{K}}{T}\right)$$

$$(\omega_m \equiv \omega_m/Q_m)$$



Quantum discord

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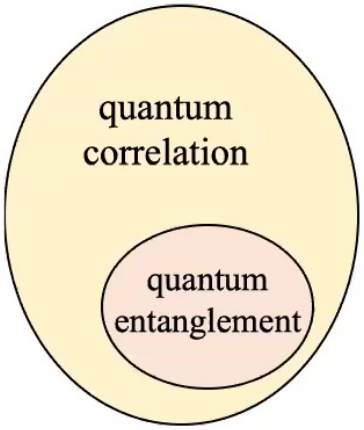
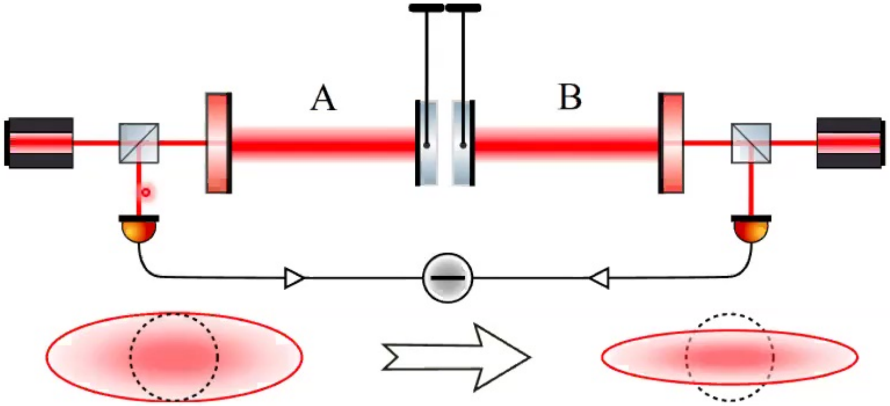
Very sensitive to measurement errors :-(

“Less quantum” figures of merit (not quantumless:-)

- Cross correlation
- Quantum discord
- **Conditional squeezing**



Conditional squeezing

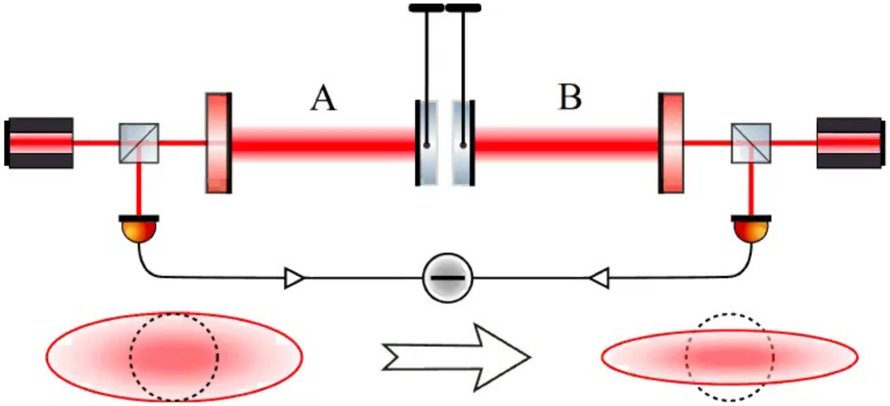


Amplitude fluctuation is at the vacuum level
(independent of mirror motion)

A squeezed state

H. Miao, D. Martynov, H. Yang, and A. Datta, Phys. Rev. A **101**, 063804 (2020).

Conditional squeezing

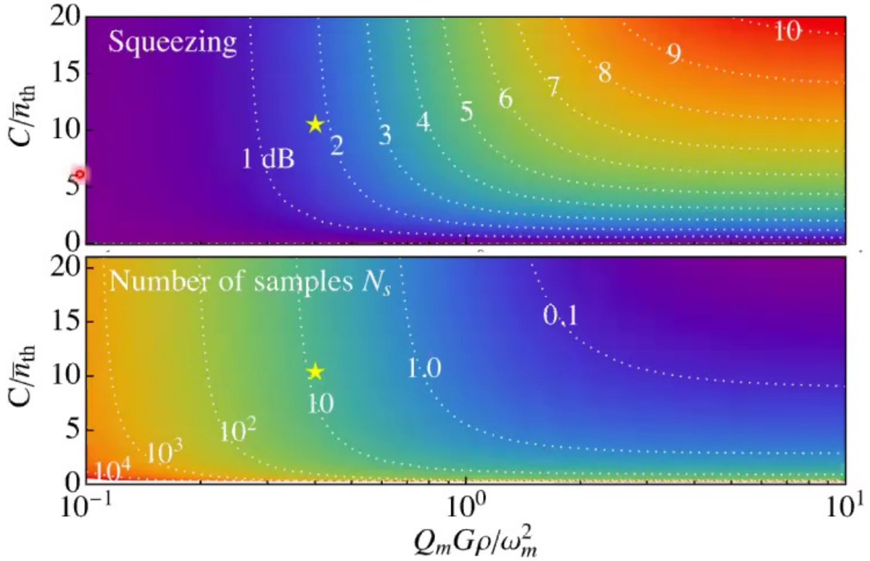


In the quantum-radiation-pressure dominant regime:

$$\mathrm{SQZ} = 10\log_{10}\left[1+\left(\frac{2Q_m G\rho}{\omega_m^2}\right)^2\right] \approx 2\,\mathrm{dB}\left(\frac{0.5\,\mathrm{Hz}}{\omega_m/2\pi}\right)^4\left(\frac{Q_m}{3\times10^6}\right)^2\left(\frac{\rho}{19\,\mathrm{g/cm^3}}\right)^2$$

$$\tau \approx 1\,\mathrm{year}\left(\frac{\omega_m/2\pi}{0.5\,\mathrm{Hz}}\right)^3\left(\frac{3\times10^6}{Q_m}\right)\left(\frac{19\,\mathrm{g/cm^3}}{\rho}\right)^2$$

Conditional squeezing



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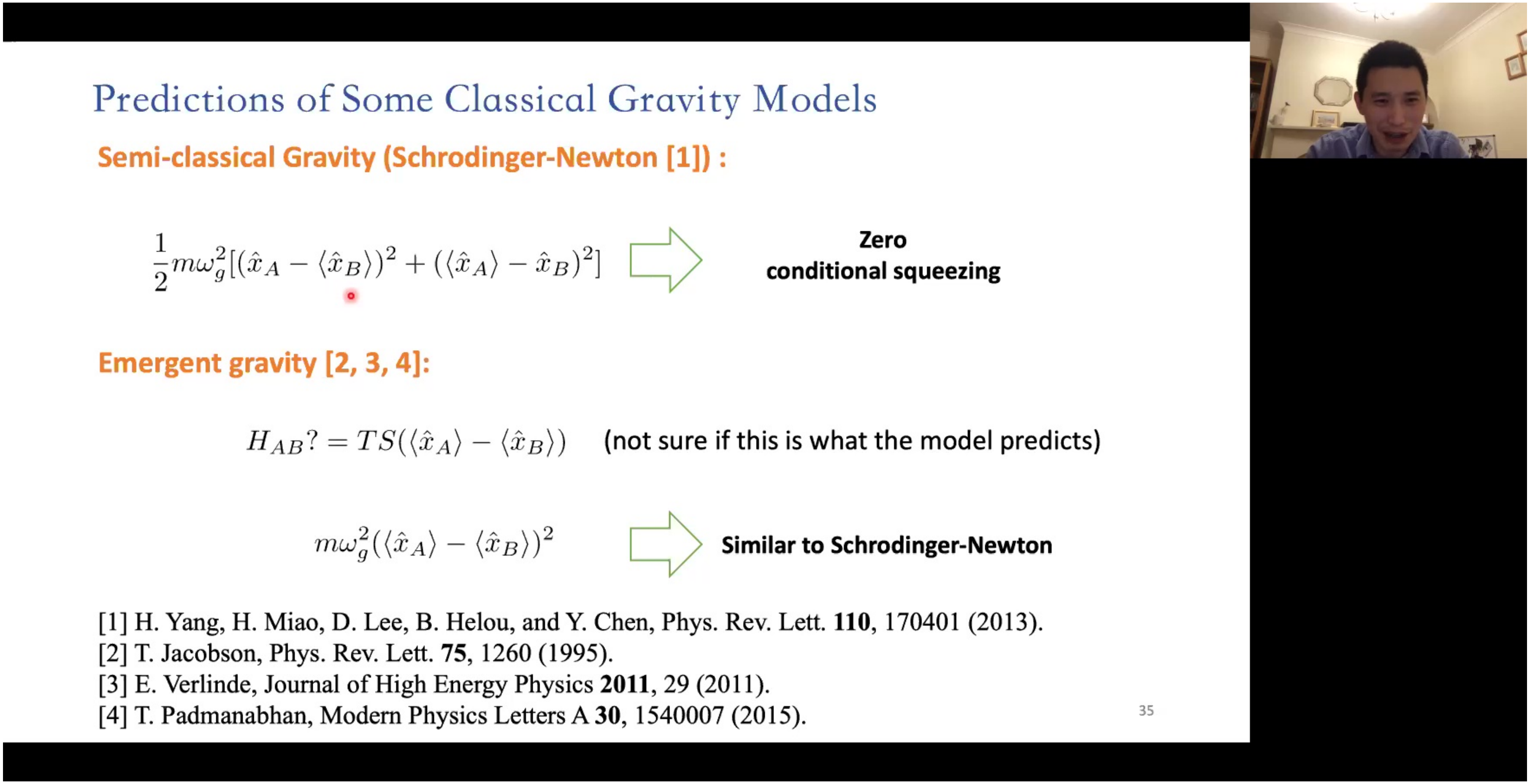


Predictions of Some Classical Gravity Models

Semi-classical Gravity (Schrodinger-Newton [1]) :

$$\frac{1}{2}m\omega_g^2[(\hat{x}_A - \langle\hat{x}_B\rangle)^2 + (\langle\hat{x}_A\rangle - \hat{x}_B)^2]$$

Zero conditional squeezing

Emergent gravity [2, 3, 4]:

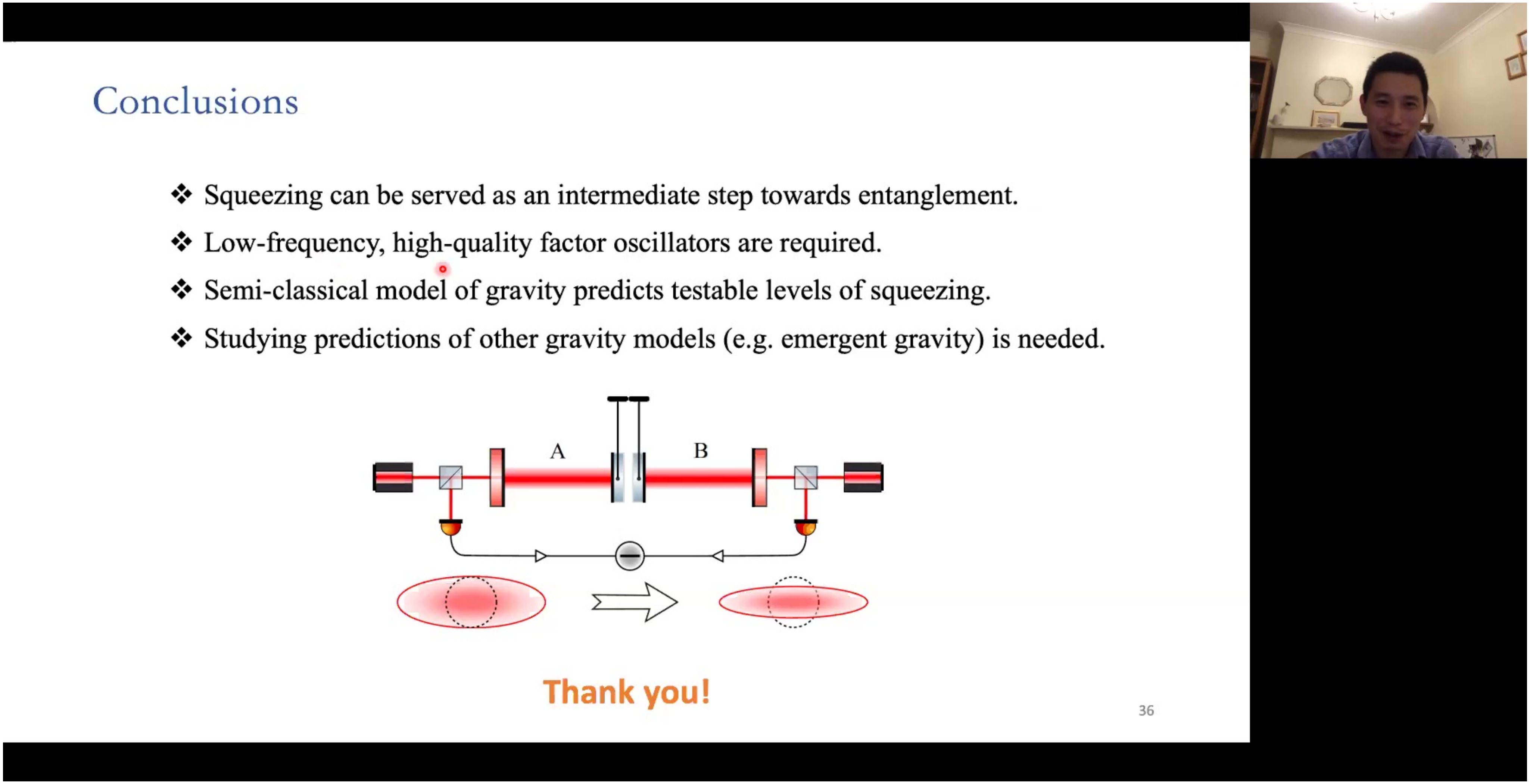
$$H_{AB}? = TS(\langle\hat{x}_A\rangle - \langle\hat{x}_B\rangle)$$

(not sure if this is what the model predicts)

$$m\omega_g^2(\langle\hat{x}_A\rangle - \langle\hat{x}_B\rangle)^2$$

Similar to Schrodinger-Newton

[1] H. Yang, H. Miao, D. Lee, B. Helou, and Y. Chen, Phys. Rev. Lett. **110**, 170401 (2013).
[2] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).
[3] E. Verlinde, Journal of High Energy Physics **2011**, 29 (2011).
[4] T. Padmanabhan, Modern Physics Letters A **30**, 1540007 (2015).



Conclusions

- ❖ Squeezing can be served as an intermediate step towards entanglement.
- ❖ Low-frequency, high-quality factor oscillators are required.
- ❖ Semi-classical model of gravity predicts testable levels of squeezing.
- ❖ Studying predictions of other gravity models (e.g. emergent gravity) is needed.

Thank you!