Title: Hybrid fracton phases: Parent orders for liquid and non-liquid quantum phases

Speakers: Wenjie Ji

Series: Quantum Matter

Date: March 02, 2021 - 3:30 PM

URL: http://pirsa.org/21030017

Abstract: In this work, we introduce and study "hybrid" fracton orders, especially though a family of exactly solvable models. The hybrid fracton orders exhibit both the phenomenology of a conventional 3d topological ordered phase and a fracton phase. There are simple yet non-trivial fusion and braiding between the excitations between the two kinds. One example is the hybrid order of the Z2 topological order with the Z2 Xcube order, in which the fracton excitations fuse into the toric code charge, and in turn, the flux loop of the toric code can fuse into various lineon excitations. In the same way there is a hybrid ordered phase of Haah's code and the 3d toric code. Proliferating certain gapped excitations in these hybrid orders can drive a phase transition into either a fracton order or a conventional 3d topological phase.

Reference. ArXiv 2102.09555

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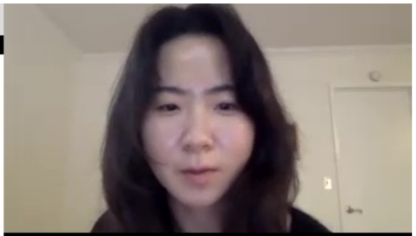

Hybrid Fracton Phases

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Sagar Vijay (UC Santa Barbara)

arXiv: 2102.09555

March 2, 2021 @ Perimeter Institute

Plan

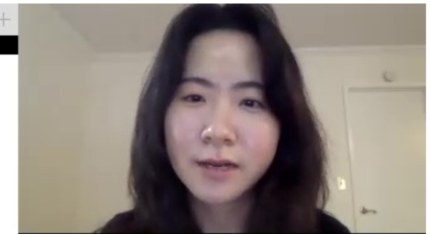
- Introduction : Topological ordered phases and their exotic cousins - fracton orders .

    Any model that exhibits the phenomenology of the two?

- The hybrid model of Z_2 toric code and Z_2 model

- Symmetry fractionalization and general construction

- A parent order of the Z_2 topological order and the Z_2 fracton order



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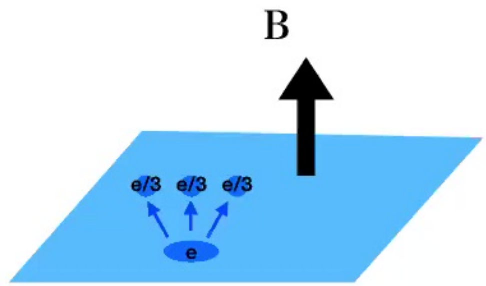
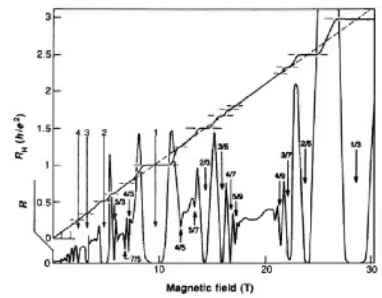
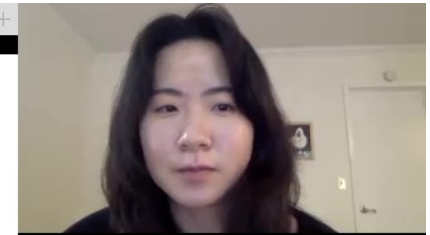
Topological ordered phases

Rich in many ways ...

- fractionalized excitations
- Degenerate ground states
- Excitations detect each other remotely
- Stable againt *any* local perturbations







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Topological ordered phases

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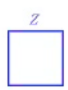
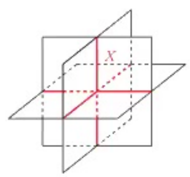
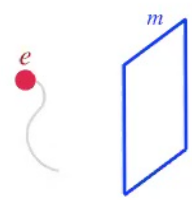
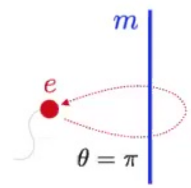
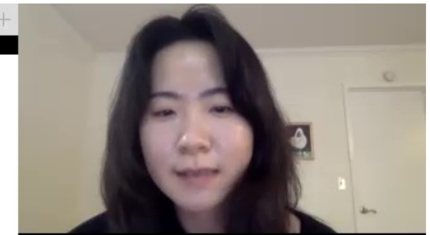
Lattice models : 3d Z_2 topological order

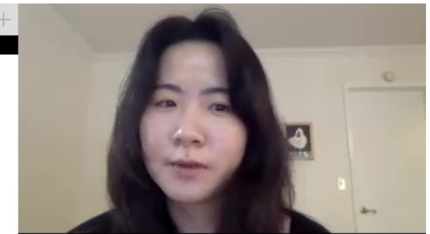
$H_{Toric\ code} = -\sum_p$ 

$-\sum_v$ 









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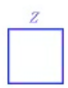
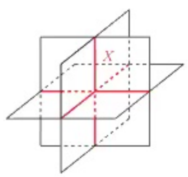
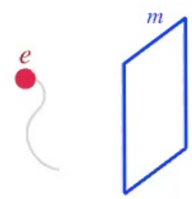
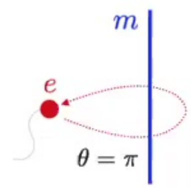
$H_{Toric\ code} = -\sum_p$  $-\sum_v$ 

More fundamentally,

Long range entangled
topological entanglement entropy $S_{TEE} = -2 \log D$





[Kitaev-Preskill, Levin-Wen, '05]

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A new topological order: fracton phases

Similarly in several ways: degenerate ground states, topological

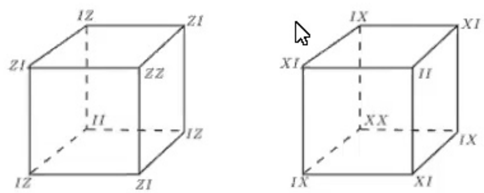
Yet exotic

Excitations with restricted mobilities

log (ground state degeneracy) ~ L

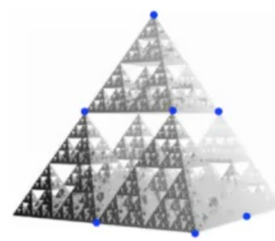
Example. Haah's code [Haah-Preskill, '11]

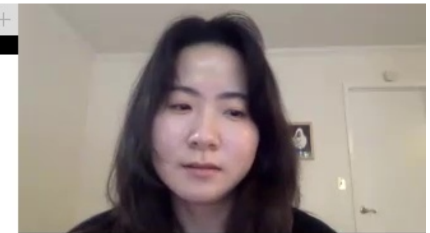
$$H = -\sum_c A_c - \sum_c B_c$$



A_c B_c

Immobile excitations





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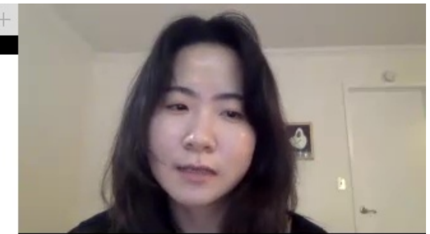


An exotic topological order: fracton phases

Similarly in several ways: degenerate ground states, topological

Yet exotic

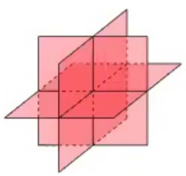
Excitations with restricted mobilities

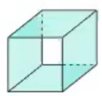
log (ground state degeneracy) ~ L

Example. Z_2 X-cube model [Vigay-Fu, '16]

on a 3D cubic lattice, one qubit on each plaquette

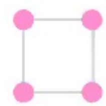
$$H = -\sum_v A_v - \sum_c (B_c^x + B_c^y + B_c^z)$$





$$A_v = \prod_{p \supset v} X_p \qquad B_c^x = \prod_{p = p_{xy}, p_{xz} \subset c} Z_p$$

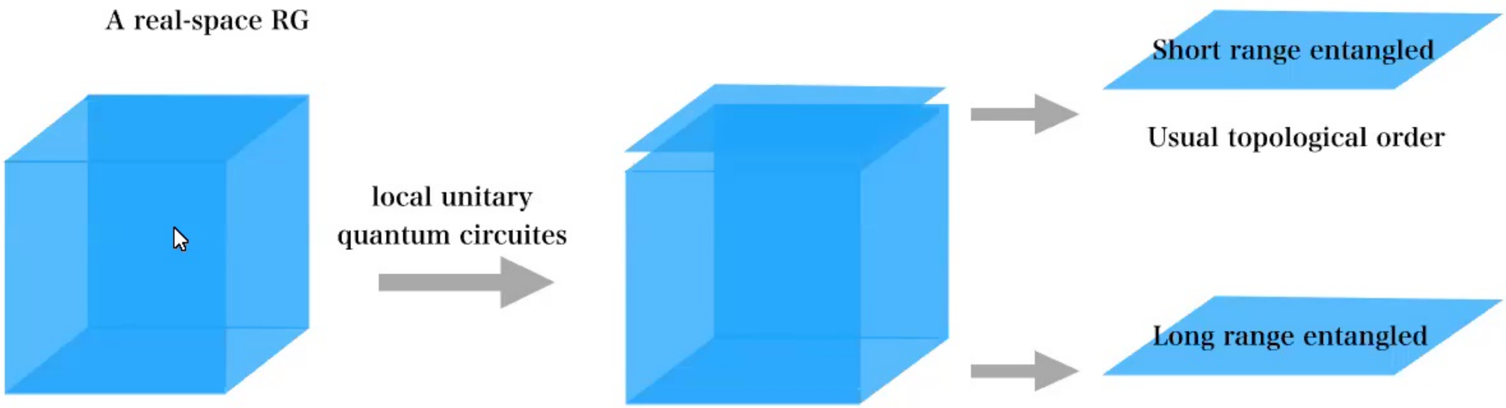
four excitations, each immobile



Excitation that can move only along one direction

$$log_2 \, \mathrm{GSD} \sim 2(L_x + L_y + L_z) - 3$$

Are they the same?



[Shirley-Slage-Xie, '18]

How are they related ?

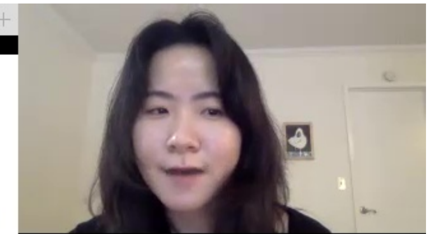
- Phases relate two orders ?

Strongly coupled stacks of lower-dimensional topological phases
$\longrightarrow$ either fracton orders or 3D topological orders [Vijay '17, Prem-Huang-Song-Hermele '19 , ...]

- Two orders coexist ?

lattice models with excitations that are immobile, non-Abelian as well as mobile excitations.

[Bulmash, Barkeshli, Prem, Williamson, Stephen, Rubio, Dua, Aasen, Slage, ... '19-'20]



How are they related ?

- Phases relate two orders ?

Strongly coupled stacks of lower-dimensional topological phases
$\longrightarrow$ **either fracton orders or 3D topological orders** **[Ma Xie]**

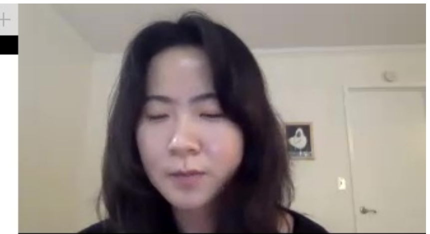
- Two orders coexist ?

lattice models with excitations that are immobile, non-Abelian as well as mobile excitations.

[Bulmash, Hemele, ...]

- More straightforword relations?

We find models that hybrid two orders, coexist, and not separable.



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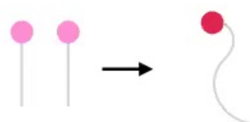
A hybrid order

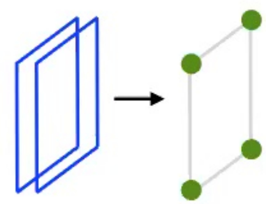
Excitations reminecent of both orders

Not separable

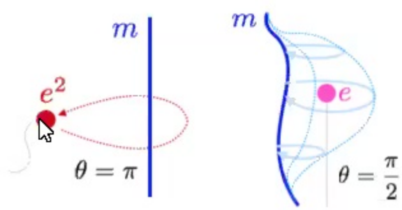
fracton × fracton = mobile charge



loop × loop = lineons at corners



Remotely mutually detectable



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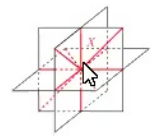
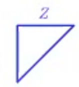
Fractonic Hybrid X-cube model

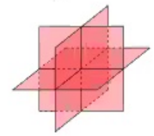
$$H_{stack} = H_{Toric\ code} + H_{X-cube}$$

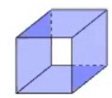
$$H_{Toric\ code} = -\sum_{v} A_v - \sum_{p} B_p$$

$$H_{X-cube} = -\sum_{v} A_v^{XC} - \sum_{c} \sum_{r=x,y,z} B_{c,r}$$

$\mathbf{A}_v =$ 

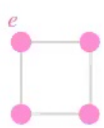
$\mathbf{B}_p =$ 

$\mathbf{A}_v^{XC} =$ 

$\mathbf{B}_{c,x} =$ 

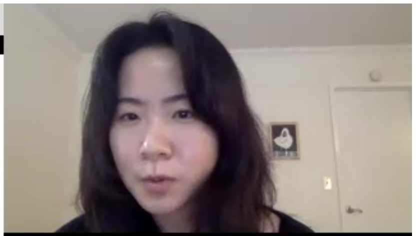
$A_v = -1$

$B_p = -1$



$A_v^{XC} = -1$

$B_{c,r} = -1$



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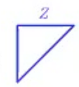
Fractonic Hybrid X-cube model

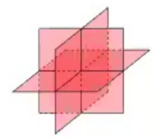
$$H_{hybrid} = H'_{Toric\ code} + H'_{X-cube}$$

$$H'_{Toric\ code} = -\sum_v (A'_v + A'^{\dagger}_v) - \sum_p B_p$$

$$H'_{X-cube} = -\sum_v A^{XC}_v - \sum_c \sum_{r=x,y,z} (B'_{c,r} + B'^{\dagger}_{c,r})$$

$\mathbf{A}'_v =$

$\mathbf{B}_p =$ 

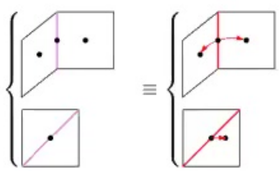
$\mathbf{A}^{XC}_v =$ 

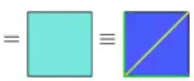
$\mathbf{B}'_{c,x} =$

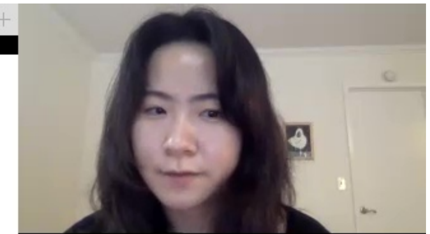
$$X_e \to \boldsymbol{\xi}_e = X_e \prod_{p\in n(e)} \mathrm{CNOT}_{e,p},$$

$$Z_p \to \boldsymbol{\zeta}_p = Z_p S_{(ij)} S^{\dagger}_{(ik)} S_{(il)},$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$\boldsymbol{\xi}_e =$ 

$\boldsymbol{\zeta}_p =$ 



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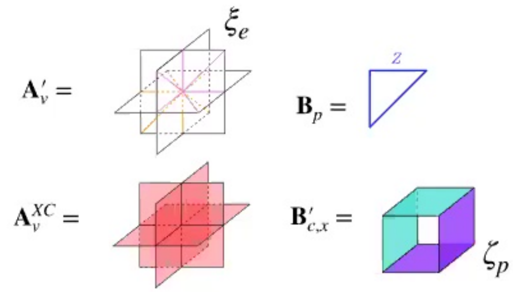

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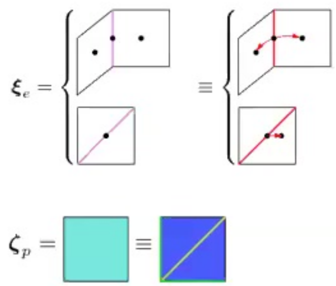

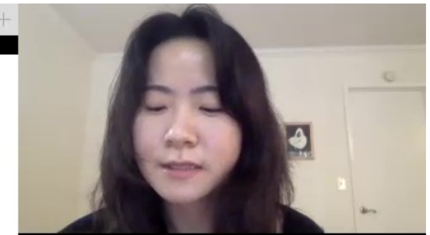

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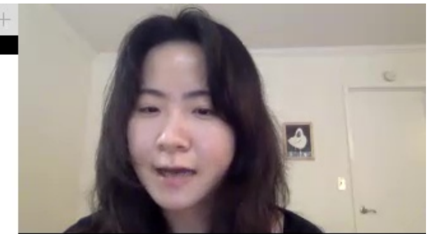

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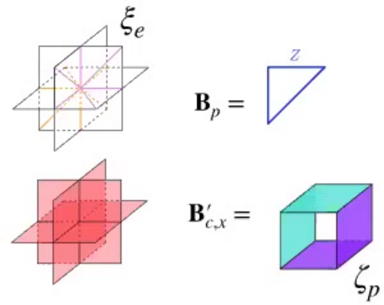
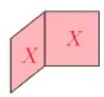
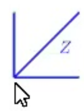
$\mathbf{A}'_v =$, $\mathbf{B}_p =$, $\mathbf{A}^{XC}_v =$, $\mathbf{B}'_{c,x} =$



<u>Generating algebra</u> **hybrid two lattice gauge fields**

$$< Z_e,\ \xi_e,\ \zeta_p,\ X_p >$$

$\xi_e^2 =$  or  $\zeta_p^2 =$ 

$$\xi_e Z_e = -Z_e \xi_e \qquad X_p \zeta_p = -\zeta_p X_p$$

for some neighboring (e,p) $\xi_e \zeta_p = \pm i \zeta_p \xi_e$

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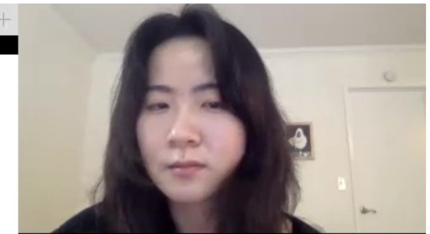
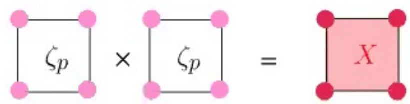


Fusion rules

Mobilities of excitations inherite from the toric code and X-cube models.

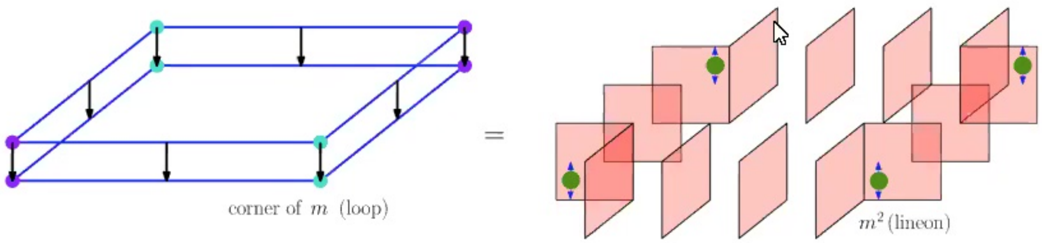


$$A_v^{XC} = -1 \quad A_v = i \qquad\qquad A_v = -1$$

$$e \times e = e^2$$

Fusion rules

Mobilities of excitations inherite from the toric code and X-cube models.



$$B_{c,x} = B_{c,y} = i$$

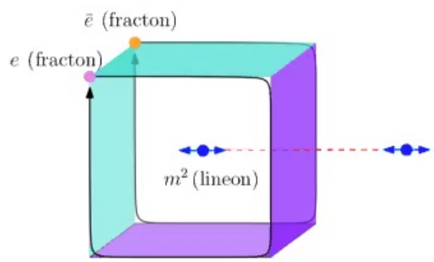
$$B_{c,x} = B_{c,y} = -1$$

$$m \times m = m^2$$

Braiding

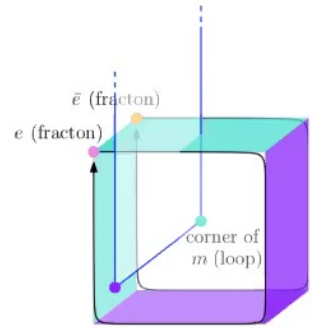
The statistical phase between e^a and m^b : $\theta_{ab} = e^{i\frac{\pi}{2}ab}$

Example. a $e - \bar{e}$ fracton dipole braids with a lineon

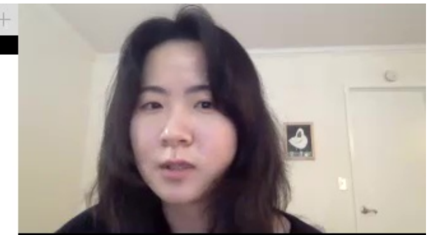


$\theta = -1$

a $e - \bar{e}$ fracton dipole braids with a corner of a m loop



$\theta = i$



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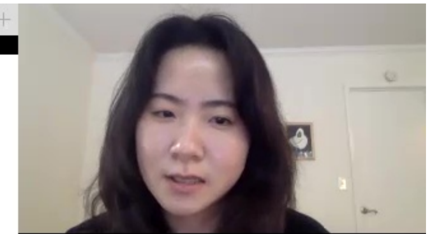
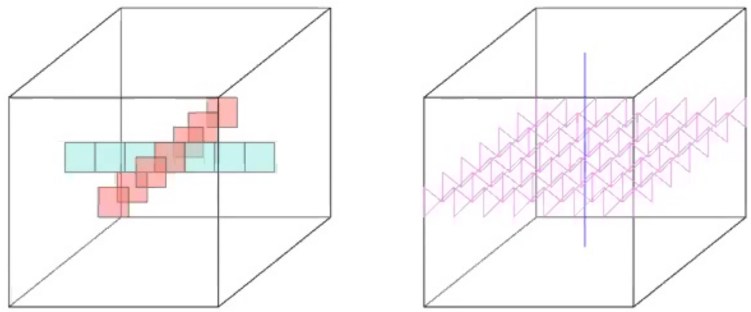


Topological ordered



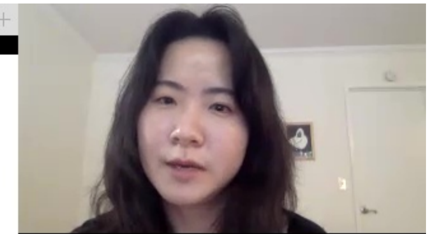
Groud state subspace = X-cube subsector × Toric code subsector

Example. $\left(\prod_{p\perp C}\xi_e\right)^2 = \prod_v A_v \to 1$

$\log_2$ **Ground state degeneracy =** $2(L_x + L_y + L_z)$

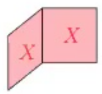
Stable against any local perturbations? Yes.

Why this map will intertwine the two orders?



$$< Z_e, X_e, Z_p, X_p >$$

$$< Z_e, \xi_e, \zeta_p, X_p >$$

$\xi_e^2 =$ 

or 

$\zeta_p^2 =$ 

$$\xi_e Z_e = - Z_e \xi_e \qquad X_p \zeta_p = - \zeta_p X_p$$

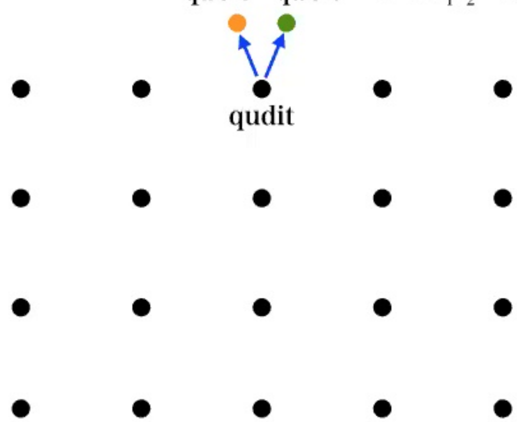
for some neighboring (e,p) $\quad \xi_e \zeta_p = \pm i \zeta_p \xi_e$

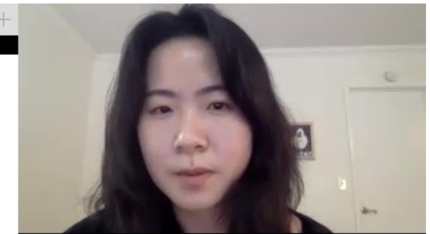
21PI | defense | Untitled | 21PI_draft

Where does the map come ?

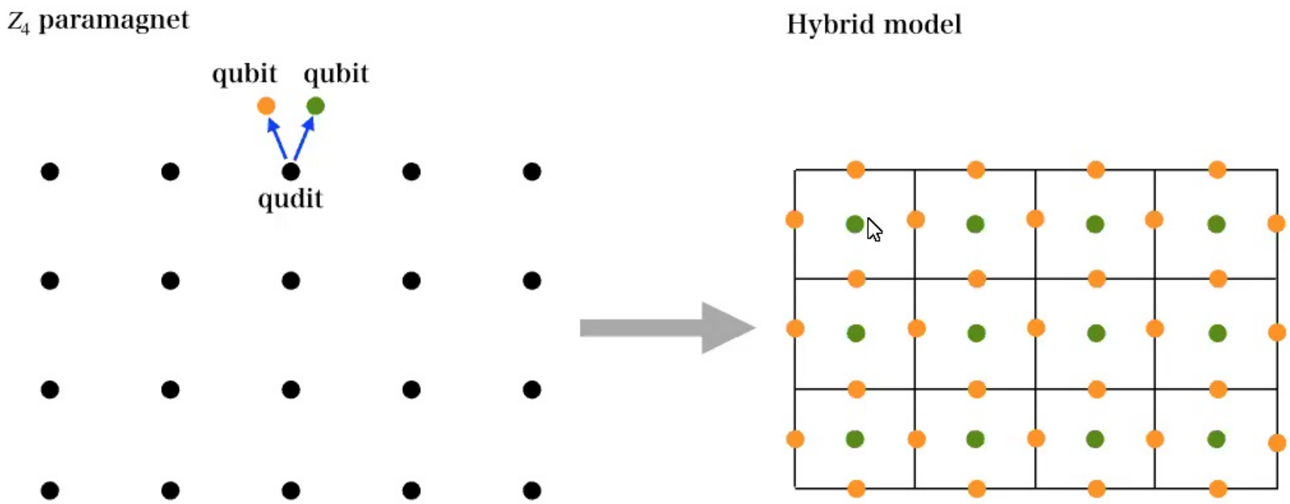
Z_4 paramagnet

qubit qubit $\mathscr{Z} = Z_1 S_2$ $\mathscr{X} = X_2 CNOT_{2,1}$

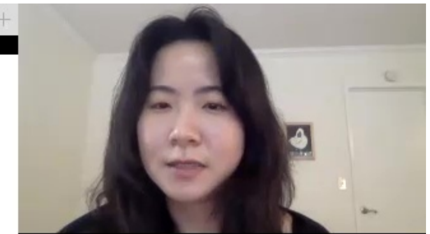




Where does the map come ?



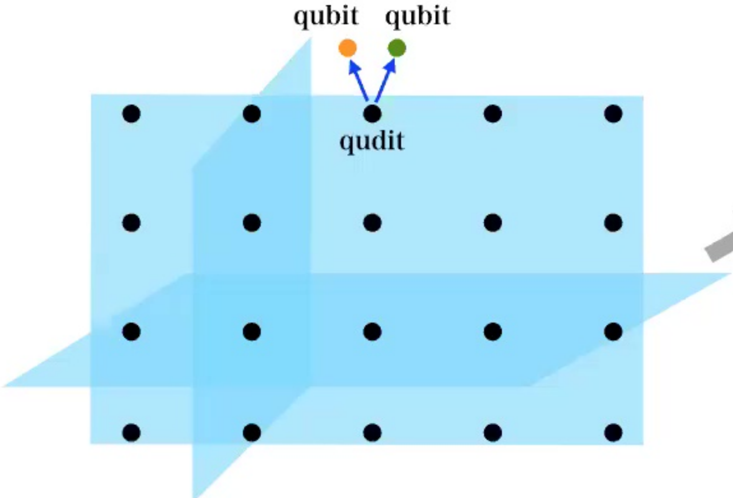
Step 1: Dualize one set of qubits to qubits on plaquettes
Step 2: Dualize the other set of qubits to qubits on edges



21PI defense Untitled 21PI_draft

Fractionalized symmetry on fracton orders

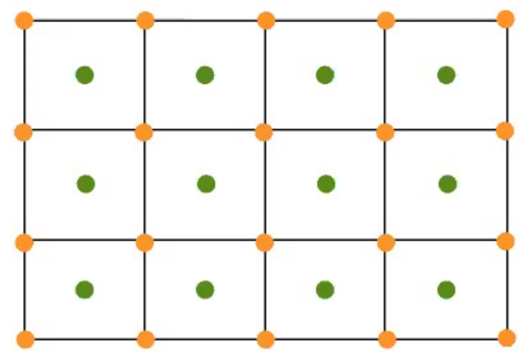
Z_4 paramagnet



Z_4 global symmetry

Z_2 planar symmetries

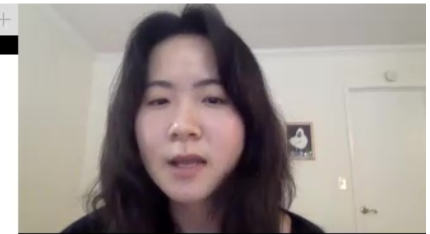
Step 1: Z_2 paramagnetic + X-cube model



Z_2 global symmetry

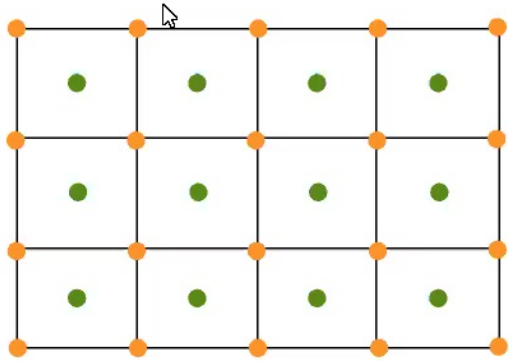
fractionalized on the X-cube model

$\rho \in H^2(Z_2, Z_2)$



Fractionalized symmetry on fracton orders

Step 1: Z_2 enriched X-cube model

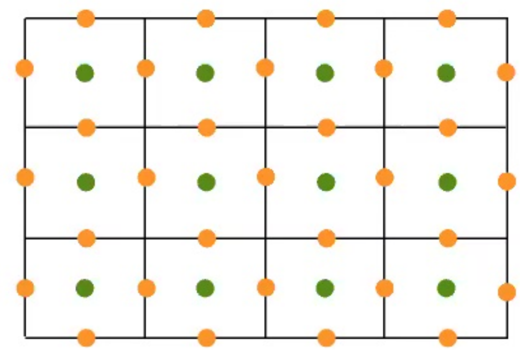


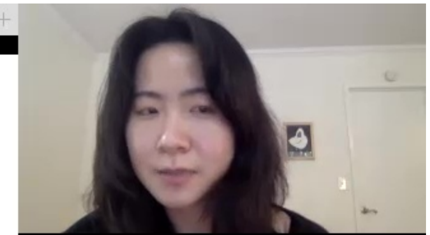
Z_2 global symmetry
fractionalized on the X-cube model

$\rho \in H^2(Z_2, Z_2)$

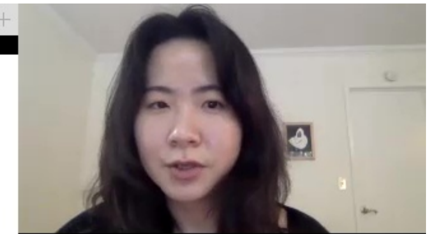
gauge / dualize
the fractionalized
Z_2 global symmetry

Step 2: Hybrid model of Z_2 toric code + X-cube model



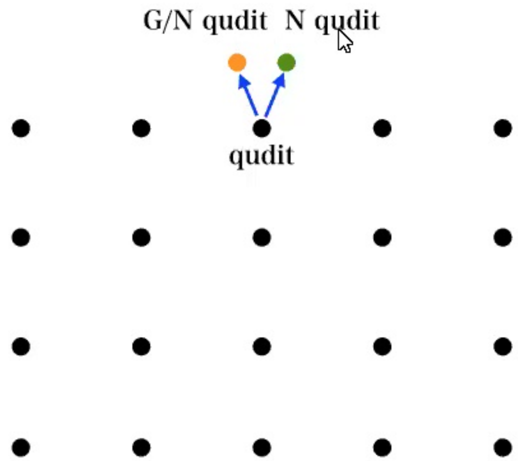


21PI defense Untitled 21PI_draft

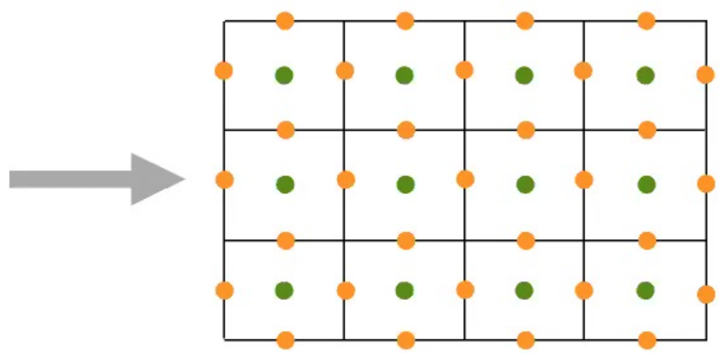


How general ?

G paramagnet



Hybrid model



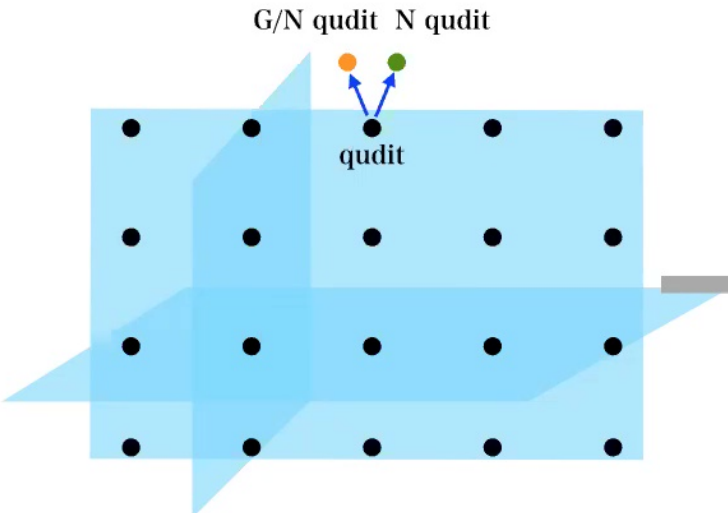
G global symmetry

N planar symmetries

21PI defense Untitled 21PI_draft

Lineonic hybrid model

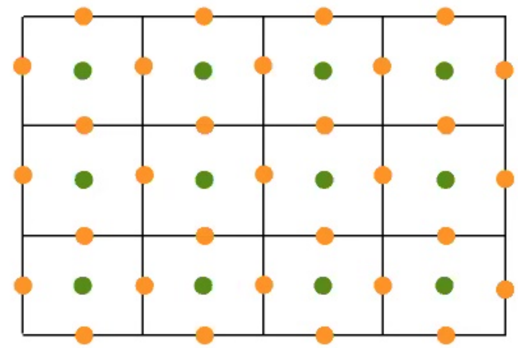
G paramagnet



G global symmetry
N planar symmetries

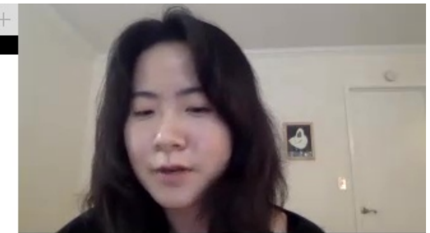
- charged under 2 out 3 planar symmetries

Hybrid model



- charge mobile along one direction (lineon)

lineon × lineon = mobile charge
loop × loop = fractons at corners



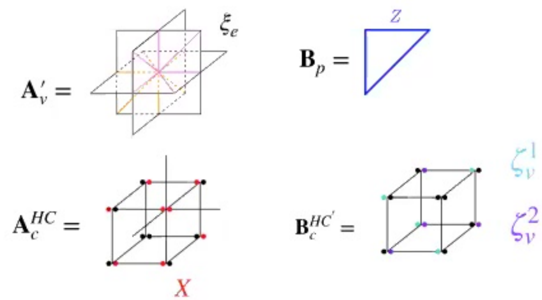
21PI defense Untitled 21PI_draft

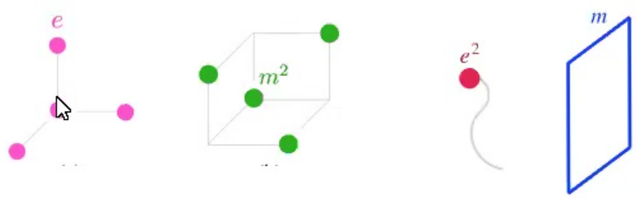
Hybrid Haah's model

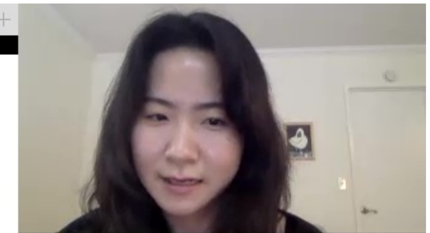
$$H_{hybrid} = H'_{Toric\ code} + H'_{X-cube}$$

$$H'_{Toric\ code} = -\sum_{v}(A'_v + A'^{\dagger}_v) - \sum_{p} B_p$$

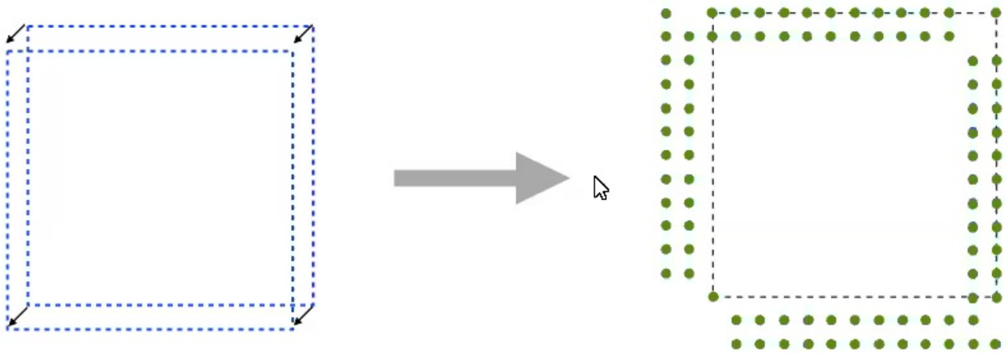
$$H'_{Haah's\ code} = -\sum_{c} A_c^{HC} - \sum_{c} B_c^{HC}$$



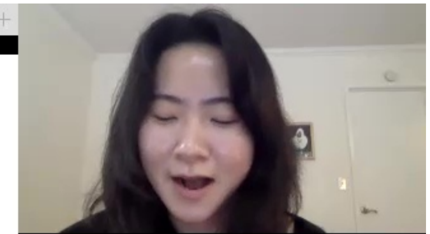




Hybrid Haah's model

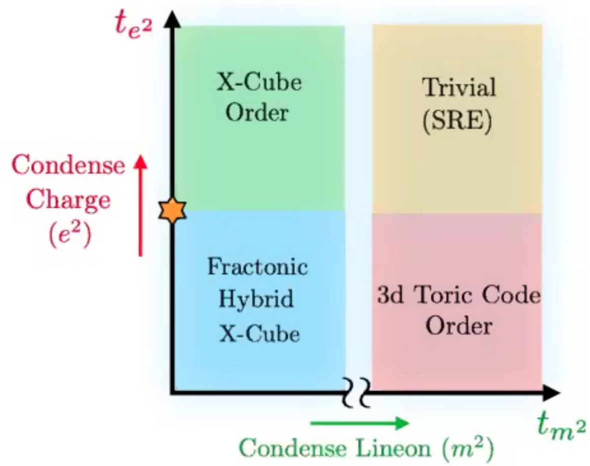


$$m \text{ loop} \times m \text{ loop} = m^2 \text{ fractons}$$

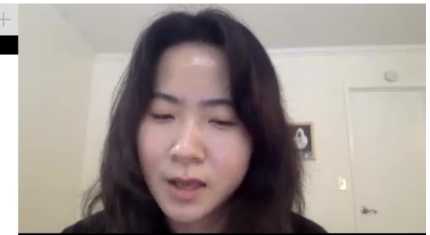


21PI | defense | Untitled | 21PI_draft

Parent order of Z2 gauge theory & Z2 Xcube order



$$H = H_{hybrid} - t_{e^2} \sum_e Z_e - t_{m^2} \sum_p X_p$$



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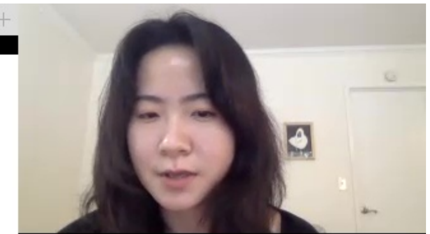
Conclusion: A family of hybrid orders

Excitations & Fusion Rules
Generating Set $= \{1, e, e^2, m, m^2\}$
$e^2 \equiv e \times e =$ mobile $\mathbb{Z}_2$ charge
$m =$ flux loop
$m^2 \times m^2 = e^2 \times e^2 = 1$
Hybrid Toric Code Layers
$e \equiv$ planon $\quad m^2 \equiv$ planon
$m \times m =$ planons (m^2) along loop
Fractonic Hybrid X-Cube
$e \equiv$ fracton $\quad m^2 \equiv$ lineon
$m \times m =$ lineons (m^2) at corners
Lineonic Hybrid X-Cube
$e \equiv$ lineon $\quad m^2 \equiv$ fracton
$m \times m =$ fractons (m^2) at corners
Hybrid Haah's Code
$e \equiv$ fracton $\quad m^2 \equiv$ fracton
$m \times m =$ fractons (m^2) along loop



Further questions

- Generalize to non-Abelian case
- All symmetry fractionalization ? $H^2(G, \mathscr{A})$
- Twisted hybrid fracton orders
- Topological quantum memory
- Unified description of liquid and non-liquid phases? Field theory descriptions ?

Thank you and keep staying well.