Title: Clifford algebra of the Standard Model

Speakers: Ivan Todorov

Collection: Octonions and the Standard Model

Date: March 22, 2021 - 12:00 PM

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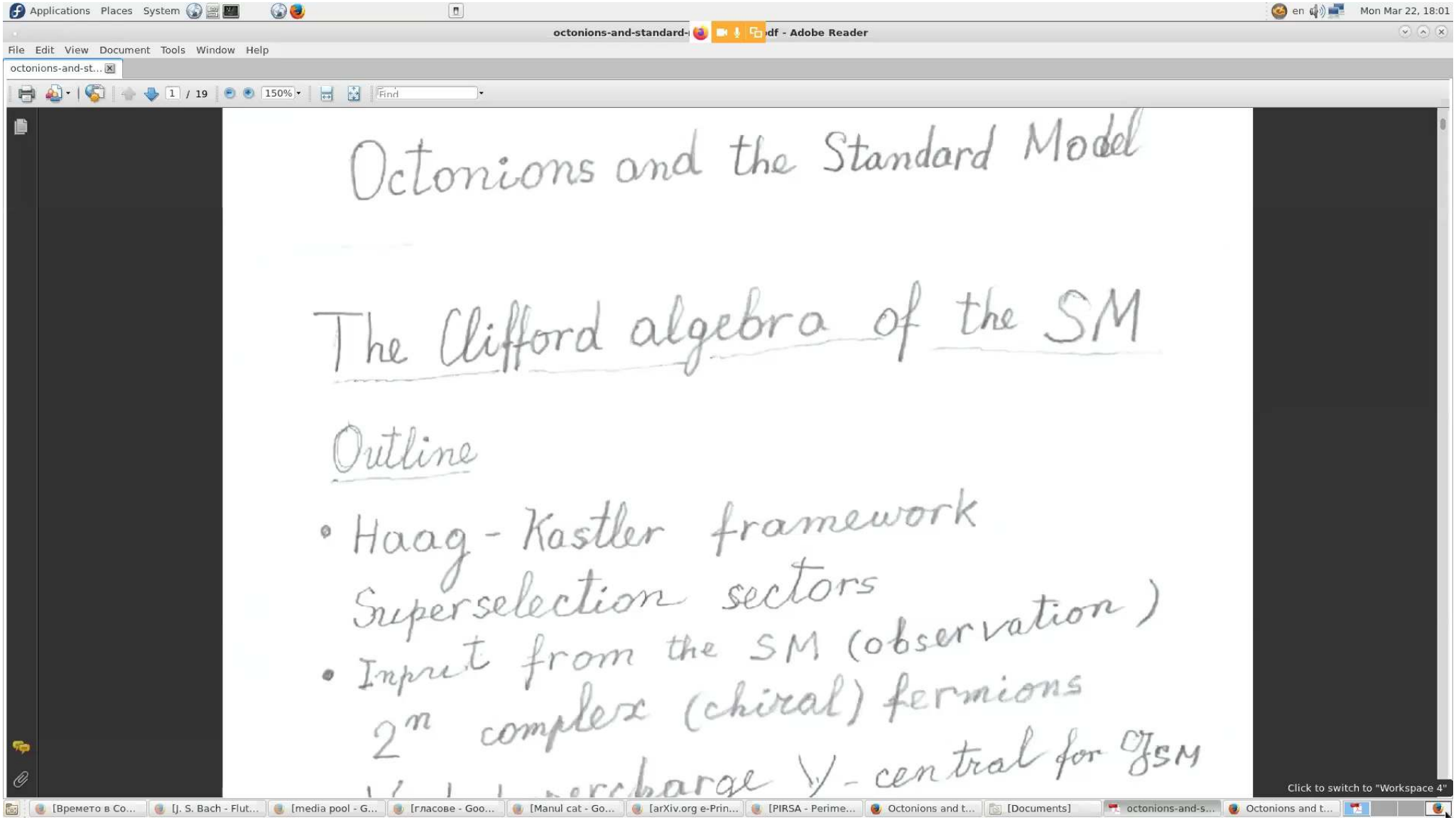
Abstract: We explore the Z2 graded product C`10 = C`4âŠ—Ë†C`6 (introduced by Furey) as a finite quantum algebra of the Standard Model of particle physics. The gamma matrices generating C`10 are expressed in terms of left multiplication by the imaginary octonion units and the Pauli matrices. The subgroup of Spin(10) that fixes an imaginary unit (and thus allows to write O = CâŠ—C 3 expressing the quark-lepton splitting) is the Pati-Salam group GP S = Spin(4) Ã— Spin(6)/Z2 âŠ, Spin(10). If we identify the preserved imaginary unit with the C`6 pseudoscalar Ï‰6 = Î³1...Î³6, Ï‰2 6 = â^'1 (cf. the talk of Furey and Hughes), then Pex = 1 2 (1 â^' iÏ‰6) will play the role of the projector on the extended particle subspace including the right-handed (sterile) neutrino. We express the generators of C`4 and C`6 in terms of fermionic oscillators aÎ±, aâ^— Î±, Î± = 1, 2 and bj , bâ^— j , j = 1, 2, 3 describing flavour and colour, respectively. The internal space observable algebra (an analog of the algebra of real functions on space-time) is then defined as the Jordan subalgebra of hermitian elements of the complexified Clifford algebra C âŠ— C`10 that commute with the weak hypercharge 1 2 Y = 1 3 P3 j=1 b â^— j bj â^' 1 2 P2 Î±=1 a â^— Î±aÎ±. We only distinguish particles from antiparticles if they have different eigenvalues of Y . Thus the sterile neutrino and antineutrino (with Y = 0) are allowed to mix into Majorana neutrinos. Restricting C`10 to the particle subspace which consists of leptons with Y < 0 and quarks with Y > 0 allows a natural definition of the Higgs field $\hat{I}'_{\imath}$, the scalar of Quillenâ€™s superconnection, as an element of C`1 4, the odd part of the first factor in C`10. As an application we express the ratio mH mW of the Higgs and the W-boson masses in terms of the cosine of the theoretical Weinberg angle.
The talk is based on the paper arXiv:2010.15621v3 a copy of which including minor corrections is attached.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Octonions and the Standard Model
The Clifford algebra of the SM
Outline
• Haag - Kastler framework
Superselection sectors
• Input from the SM (observation)
2^n complex (chiral) fermions
- central for $g_{SM}
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2^n complex (chiral) fermions

Weak hypercharge Y - central for G_{SM}

- Early attempts: GUTs, Octonions (1973-)
- $Cl_{10} = Cl_4 \hat{\otimes} Cl_6$ Fermi oscillators

$a_\alpha^{(*)} = (a_\alpha, a_\alpha^*)$, $\alpha = 1, 2$) flavour, $b_j^{(*)}$, $j = 1,2,3$ - colour

- Sterile neutrino J_{SV} donot separate (anti)particles
- Simple components of the observable (Jordan) algebra

Aside: Jordan algebra with symmetry generators

- Higgs field as odd part of a superconnection
- Extended supercurvature satisfying Bianchi identity
- Gauge boson mass spectrum; the Higgs mass
- Concluding remarks. 2010.15621v3

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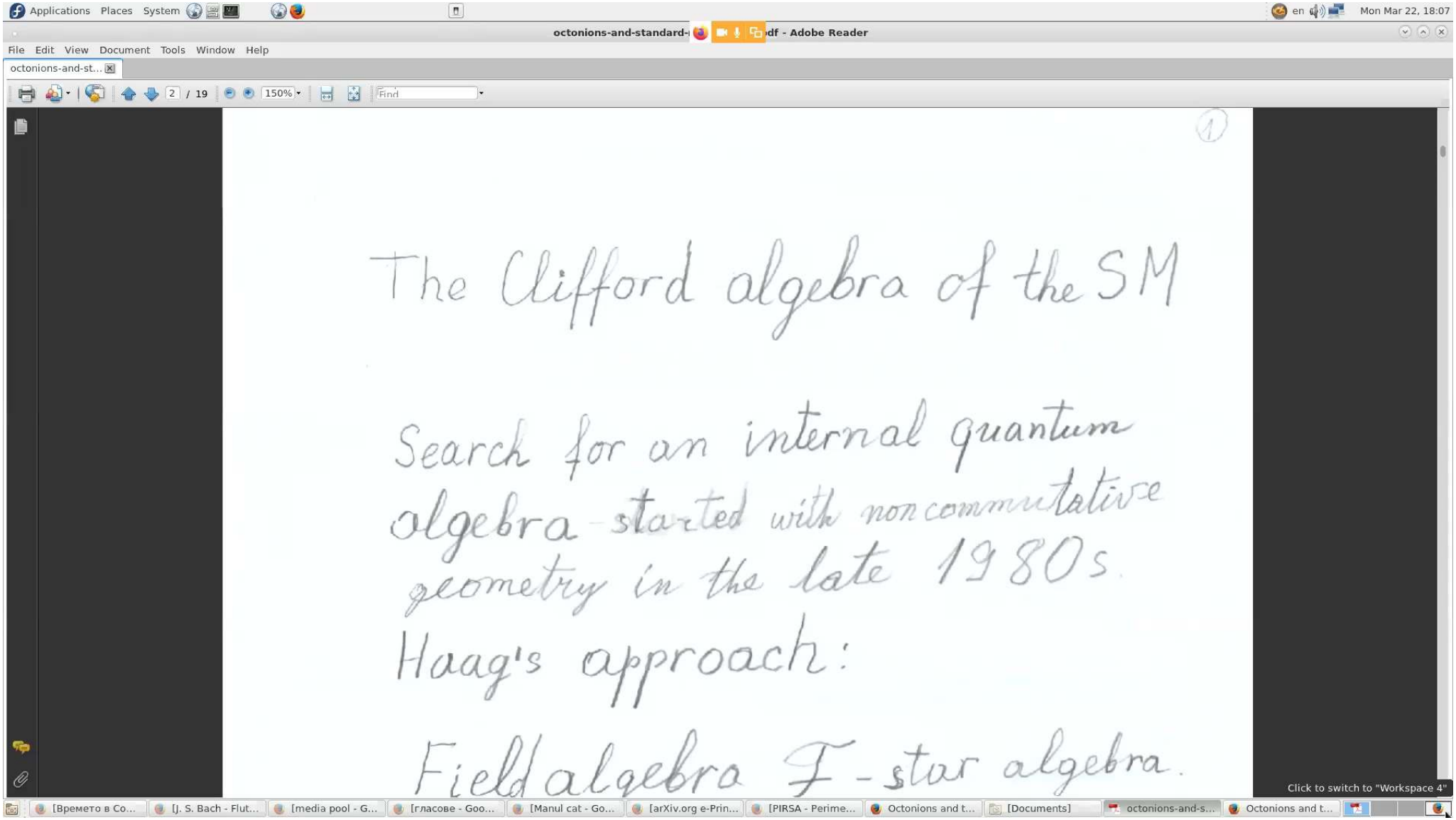
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The Clifford algebra of the SM
Search for an internal quantum algebra started with noncommutative geometry in the late 1980s.
Haag's approach:
Field algebra F - star algebra.
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Field algebra $\mathcal{F}$ - star algebra.

Superselection rules:

Hermitian elements which commute with all observables.

Algebra of observables $\mathcal{O} \subset \mathcal{F}$

Superselection sectors - labeled by eigenvalues of the superselection charges. No coherent superposition

Superselection rules:

Hermitian elements which commute with all observables.

Algebra of observables $\mathcal{O} \subset \mathcal{F}$

Superselection sectors – labeled by eigenvalues of the superselection charges. No coherent superposition between different sectors.

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Input from the Standard Model:

- Fundamental fermions 2^n

4 leptons ν_L e_L, ν_R e_R (8 with the antileptons)

4 quarks in 3 colours $12 + 4 = 2^4$

16 fermions of each generation

$\Rightarrow$ Spinor representations of Spin(2n) (or Spin(p,q))

- Complex - chiral representations

$\Rightarrow$ Spin(p,q) with $p - q = \pm 2 \mod (8)$

Spin(n) $n = 2, 6, 10$

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$\Rightarrow$ Spin(p,q) with $p - q = \pm 2 \mod (8)$

Spin(n) $n = 2, 6, 10$

$n = 2$ - complex numbers $\mathbb{C}$

Dirac equation = Cauchy - Riemann eqs

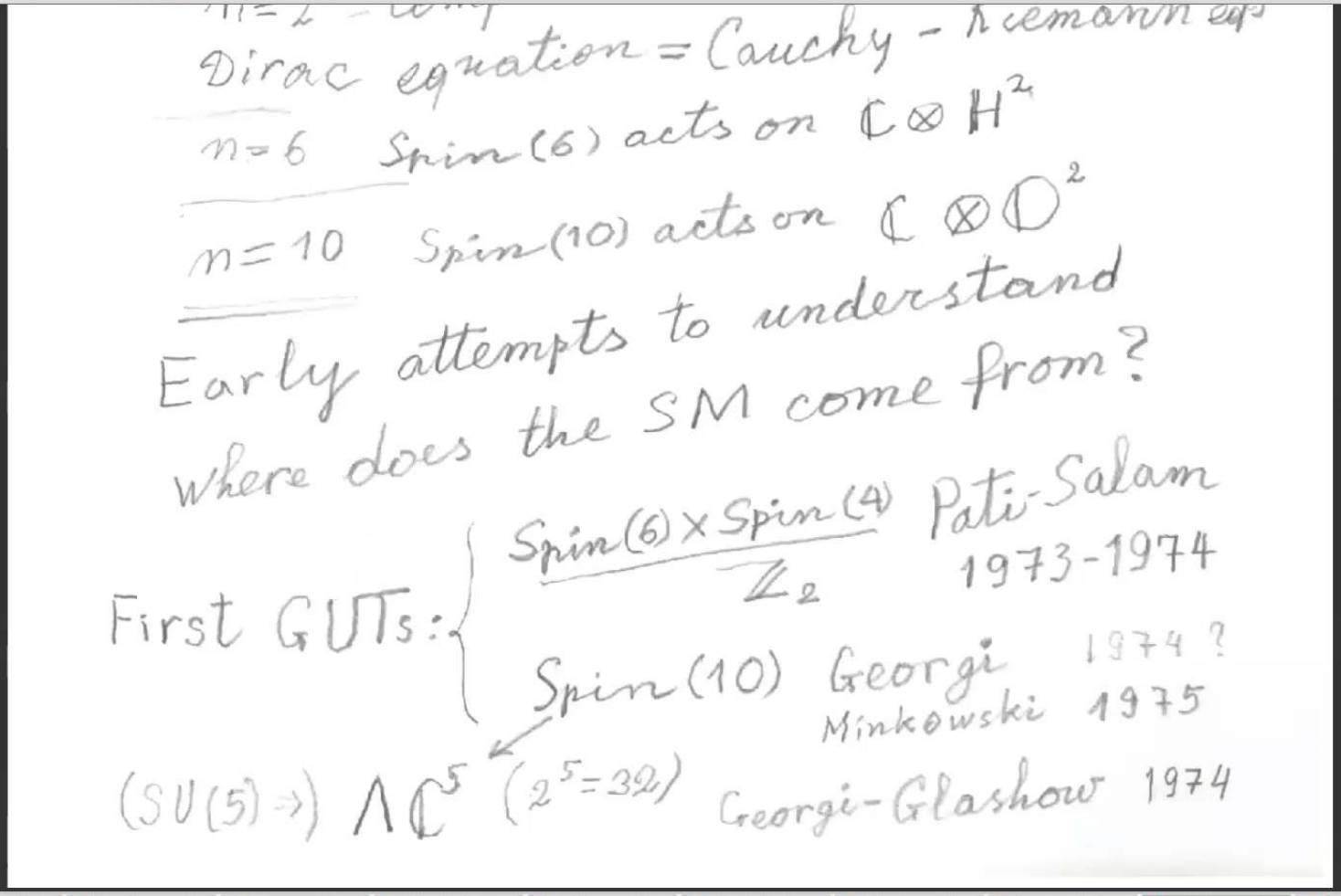
$n = 6$ Spin(6) acts on $\mathbb{C} \otimes \mathbb{H}^2$

$n = 10$ Spin(10) acts on $\mathbb{C} \otimes \mathbb{O}^2$

Early attempts to understand

where does the SM come from?

Dt:-Salam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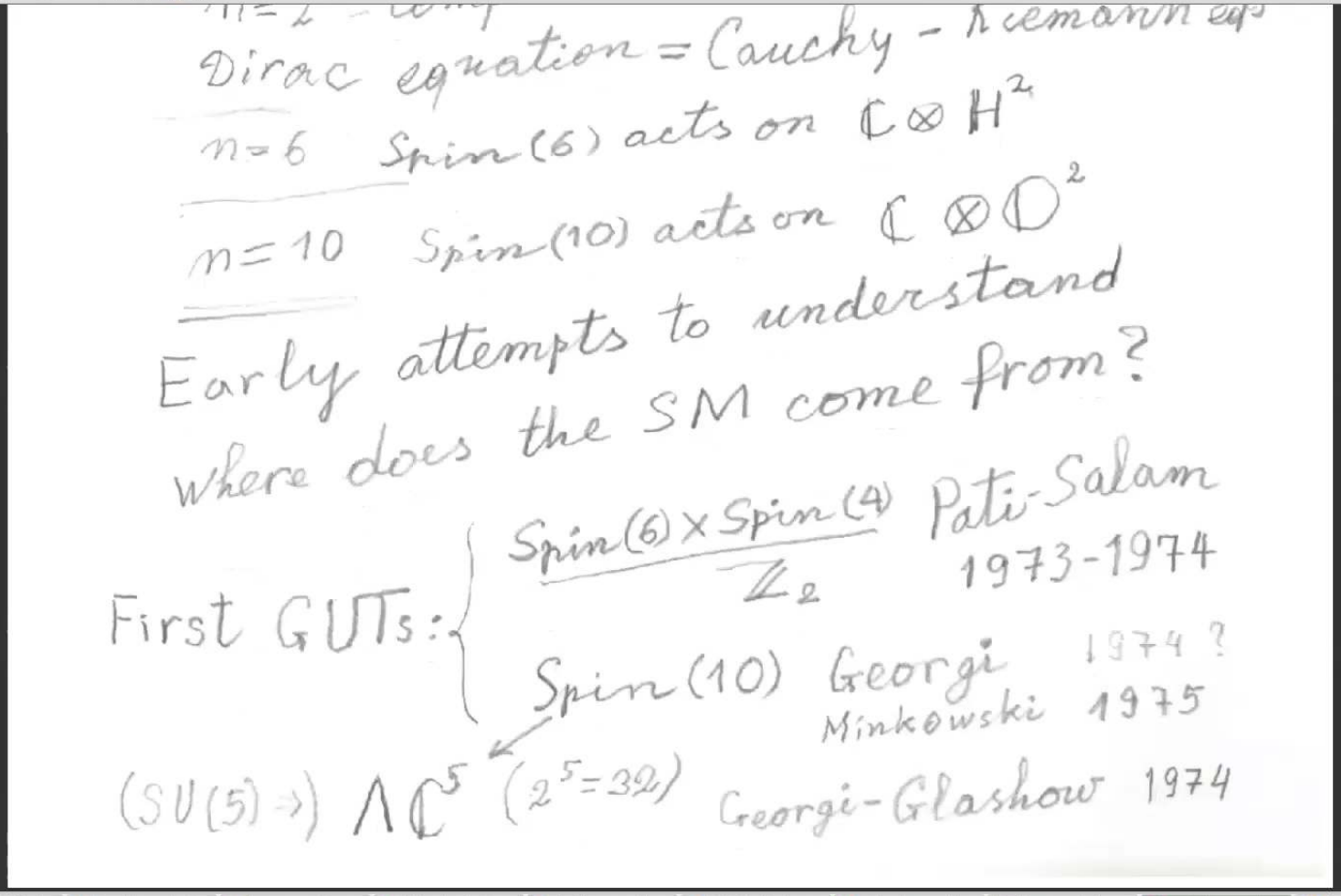
Early attempts to understand where does the SM come from?
First GUTs:
$\frac{Spin(6)\times Spin(4)}{\mathbb{Z}_2}$ Pati-Salam 1973-1974
Spin(10) Georgi 1974 ?
Minkowski 1975
$(SU(5)\to)\ \Lambda\mathbb{C}^5$ $(2^5=32)$ Georgi-Glashow 1974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One can obtain an extension G of the gauge group $G_{SM} = S(U(2)\times U(3))$ of the Standard Model as the stabilizer of the lepton-quark splitting of the octonions in J_2^8

$$\mathbb{O} = \mathbb{C} \oplus \mathbb{C}^3 \to G_{SM} \subset G \subset Spin(10)$$

J_2^8: Michel Dubois-Violette, I.T. 1806.09450

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Michel Dubois-Violette, I.T. 1806.09450

Spin(9): Kirill Krasnov 1912.11282v2

Spin(10)$\subset E_6$ Latham Boyle 2006.16265

Here, in 2010.15621 v3,
we postulate superselection of the
weak hypercharge Y, generator of centre of g_{SM}.
It appears naturally in the picture
of Fermi oscillator – CAR
Equivalent to complex structure D-V 1992
Equivalent to $Cl_{10} = Cl_4 \hat{\otimes} Cl_6$, Cl_4 a_α $\alpha=1,2$ flavour; Cl_6 b_j $j=1,2,3$ colour

Cohl Furey 1806.00612

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Weak isospin $I = I^L$

$$I_+ = a_1^* a_2 , \quad I_- = a_2^* a_1 , \quad 2I_3 = a_1^* a_1 - a_2^* a_2$$

$$B - L = \frac{1}{3} \sum_{j=1}^{3} [b_j^*, b_j]$$

Hypercharge = difference of number operators:

$$\frac{1}{2} Y = \frac{1}{3} \sum_{j=1}^{3} b_j^* b_j - \frac{1}{2} \sum_{\alpha=1}^{2} a_\alpha^* a_\alpha$$

$U(1)_Y$: $b_j \to e^{-i\frac{\varphi}{3}} b_j$, $b_j^* \to e^{i\frac{\varphi}{3}} b_j^*$

$a_\alpha \to e^{i\frac{Y}{2}} a_\alpha, \quad a^*_\alpha \to e^{-i\frac{Y}{2}} a^*_\alpha$

Observables and generators of the gauge Lie algebra are $U(1)_Y$ invariant

$\Longrightarrow \mathcal{G}_{SM} \subset \mathcal{G} = u(2) \oplus u(3) \subset so(10)$

SM gauge symmetry extended by $U(1)_{B-L}$.

$[a^*_\alpha, a_\beta]$, $[b^*_j, b_k]$ are $U(1)_Y$ invariant.

$a_\alpha \to e^{i\frac{\varphi}{2}} a_\alpha, \quad a_\alpha^* \to e^{-i\frac{\varphi}{2}} a_\alpha^*$

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6

Unexpected invariants:

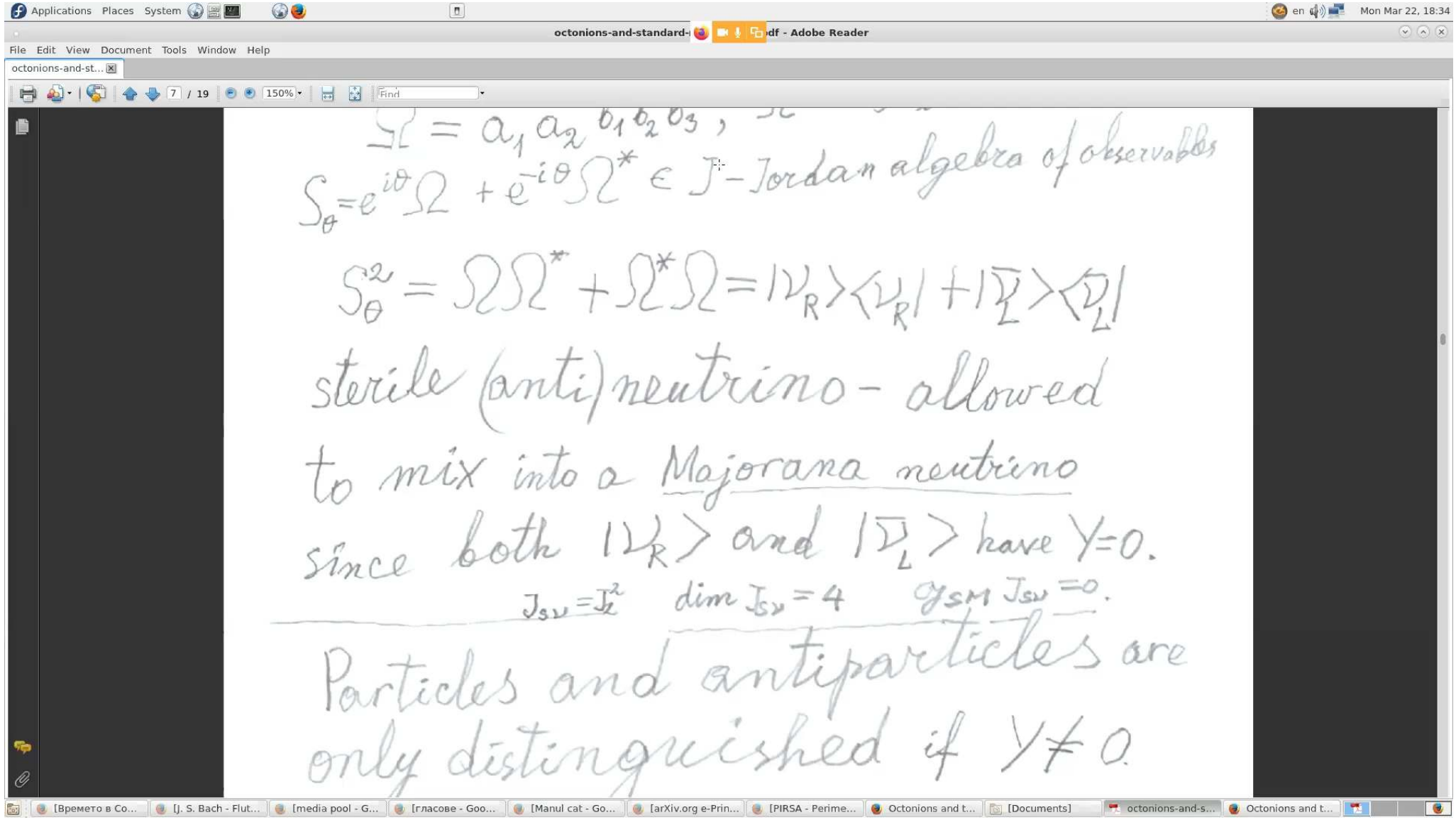
$\Omega = a_1 a_2 b_1 b_2 b_3$, $\Omega^* = b_3^* b_2^* b_1^* a_2^* a_1^*$, $\Omega^2 = 0 = \Omega^{*2}$

$S_\theta = e^{i\theta}\Omega + e^{-i\theta}\Omega^* \in J$ - Jordan algebra of observables

$S_\theta^2 = \Omega\Omega^* + \Omega^*\Omega = |\nu_R\rangle\langle\nu_R| + |\bar{\nu}_L\rangle\langle\bar{\nu}_L|$

sterile (anti) neutrino - allowed to mix into a Majorana neutrino

i.e. both $|\nu_R\rangle$ and $|\bar{\nu}_L\rangle$ have $Y=0$.


$\Omega = a_1 a_2 b_1 b_2 b_3$,
$S_\theta = e^{i\theta}\Omega + e^{-i\theta}\Omega^* \in J^3$ - Jordan algebra of observables
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sterile (anti) neutrino - allowed
to mix into a Majorana neutrino
since both $|\nu_R\rangle$ and $|\bar{\nu}_L\rangle$ have $Y=0$.
$J_{SM} = J_L^2$ dim $J_{SM} = 4$ $g_{SM}\, J_{SM} = 0$.
Particles and antiparticles are
only distinguished if $Y \neq 0$.

Particles and antiparticles are only distinguished if $Y \neq 0$.

Projector on the 15 dimensional particle subspace:

$P = l + q$ $\quad l (= l^2)$ projects on 3 leptons:

ν_L, e_L $(Y=-1)$; e_R $(Y=-2)$

$q = q_1 + q_2 + q_3$ $\quad q_j$ - coloured quark in 4 flavours

7

Elementary projectors:

$$\pi_\alpha = a_\alpha a_\alpha^*, \quad \pi_\alpha' = a_\alpha^* a_\alpha, \quad \alpha = 1, 2$$

$$P_j = b_j b_j^*, \quad P_j' = b_j^* b_j, \quad j = 1, 2, 3.$$

$$\pi_\alpha^2 = \pi_\alpha, \quad \pi_\alpha'^2 = \pi_\alpha', \quad \pi_\alpha \pi_\alpha' = 0, \quad \pi_\alpha + \pi_\alpha' = 1$$

$$P_j^2 = P_j, \quad P_j'^2 = P_j', \quad P_j + P_j' = 1, \quad P_j P_j' = 0.$$

In terms of these

$$q_{ijk} = P_i P_j' P_k' \quad \text{where } (ijk) \in P_{3+}(1,2,3)$$

$$\pi_\alpha = a_\alpha a_\alpha^*, \quad \pi_\alpha' = a_\alpha^* a_\alpha, \quad \alpha = 1, 2$$

$$p_j = b_j b_j^*, \quad p_j' = b_j^* b_j, \quad j = 1, 2, 3.$$

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In terms of these

$$q_j = p_j p_k' p_\ell' \quad \text{where } (j,k,\ell) \in \text{Perm}(1,2,3)$$

$$\ell = p_1 p_2 p_3 (1 - \pi_1 \pi_2) = p_1 p_2 p_3 - |\nu_R\rangle\langle\nu_R|$$

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$\ell = \pi_1 \pi_2 P_3 (1 - \pi_1 \pi_2) = P_1 P_2 P_3 - |\nu_R\rangle\langle\nu_R|$

Primitive projectors (leptons):

$|\nu_L\rangle\langle\nu_L| = \bar{\pi}_1 \bar{\pi}_2 P_1 P_2 P_3 = \bar{\pi}_1 \bar{\pi}_2 \ell$ Products of 5 elementary

$|e_L\rangle\langle e_L| = \pi_1 \bar{\pi}_2 \ell$, $|e_R\rangle\langle e_R| = \pi_1 \pi_2 \ell$

Jordan algebra J of superselected observables $J = J_P \oplus J_{\bar{P}} \oplus J_{S\nu}$

Particles + antiparticle + sterile neutrino

D-V.T. 2003.06591 have taken $P_0 = \ell_0 + \varrho$, $\ell_0 = P_1 P_2 P_3 = \ell + |\nu_R\rangle$

$P P_0 = P_{..}$

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(8)

Decomposition of the particle Jordan algebra into simple (irreducible) components J_r^d

r = rank, d = degree (labeled by Y)

$J_p = J_2^2 \oplus J_6^2$ (left $\oplus\ \mathbb{R}e_R \oplus J_3^2 \oplus J_3^2$ right

$Y = -1\ \ 1/3$ chiral); $-2\ \ \frac{4}{3}\ \ \frac{2}{3}$ chiral

Sterile neutrino form a simple component $J_{S\nu} = J_2^2$, $C_{SM} J_{S\nu} = 0$

9

Aside: What give us (special) Jordan algebras?

$X \circ Y = Y \circ X \qquad X^2 = X \circ X, \; X^{n+1} = X^n \circ X$

Power associativity; partially ordered $X^2 > 0$ for $X \neq 0$. For finite dimensional $\mathrm{tr}\, X^2 > 0$ (if $X \neq 0$).

Observables in J play a dual role: they generate symmetry:

$$\frac{dX}{dt} = \frac{1}{i\hbar}[X, H] =: X \alpha H = -H \alpha X$$

"α-product"

Two products symmetric and skewsymmetric

hermitian matrices

$\dim J_r^2 = r^2$

J_r^2 are all special = Jordan subalgebras of associative algebras with Jordan product

$$X \circ Y = \frac{1}{2}[X,Y]_+ = \frac{1}{2}(XY + YX)$$

(10)

$Cl_{10} = R[32]$ – there are 32 primitive idempotents in Cl_{10} corresponding to first generation (anti) fermions.

Particle subspace (including a sterile neutrino CO^2 { Bryant 2011.05568
Krasnov 2021

The Higgs field

(10)

$Cl_{10} = R[32]$ – there are 32 primitive idempotents in Cl_{10} corresponding to first generation (anti) fermions.

Particle subspace (including a sterile neutrino CO^2 { Bryant 2011.05568
Krasnov 2021

The Higgs field

Finite quantum algebra was

first envisaged in the framework of noncommutative geometry (Connes; Kerner, Madore, Dubois-Violette, 1988–) with the idea to give a geometric meaning of the Higgs boson.

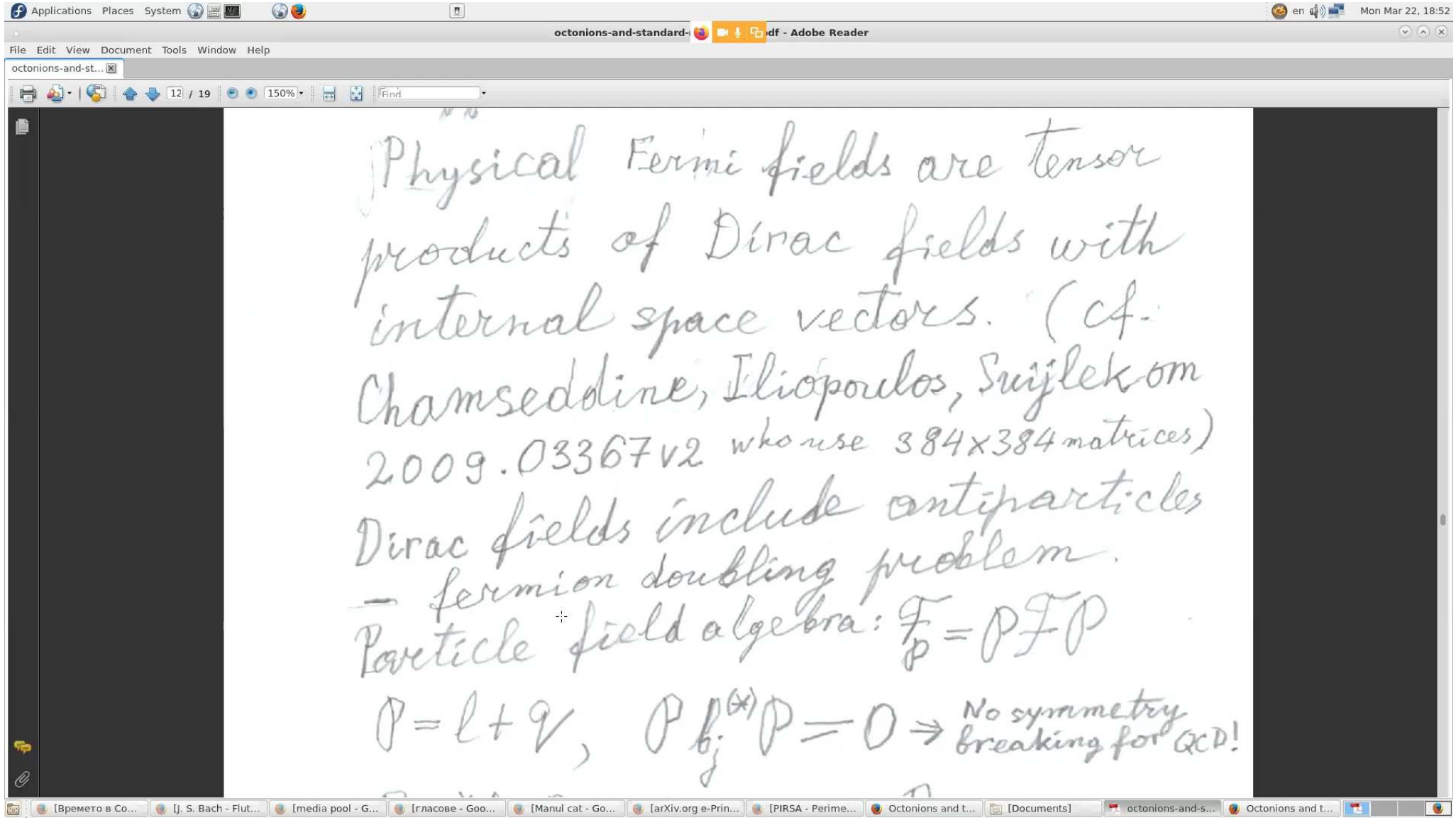
The Higgs field finds a natural place in the odd part of a Z_2-graded Clifford algebra.

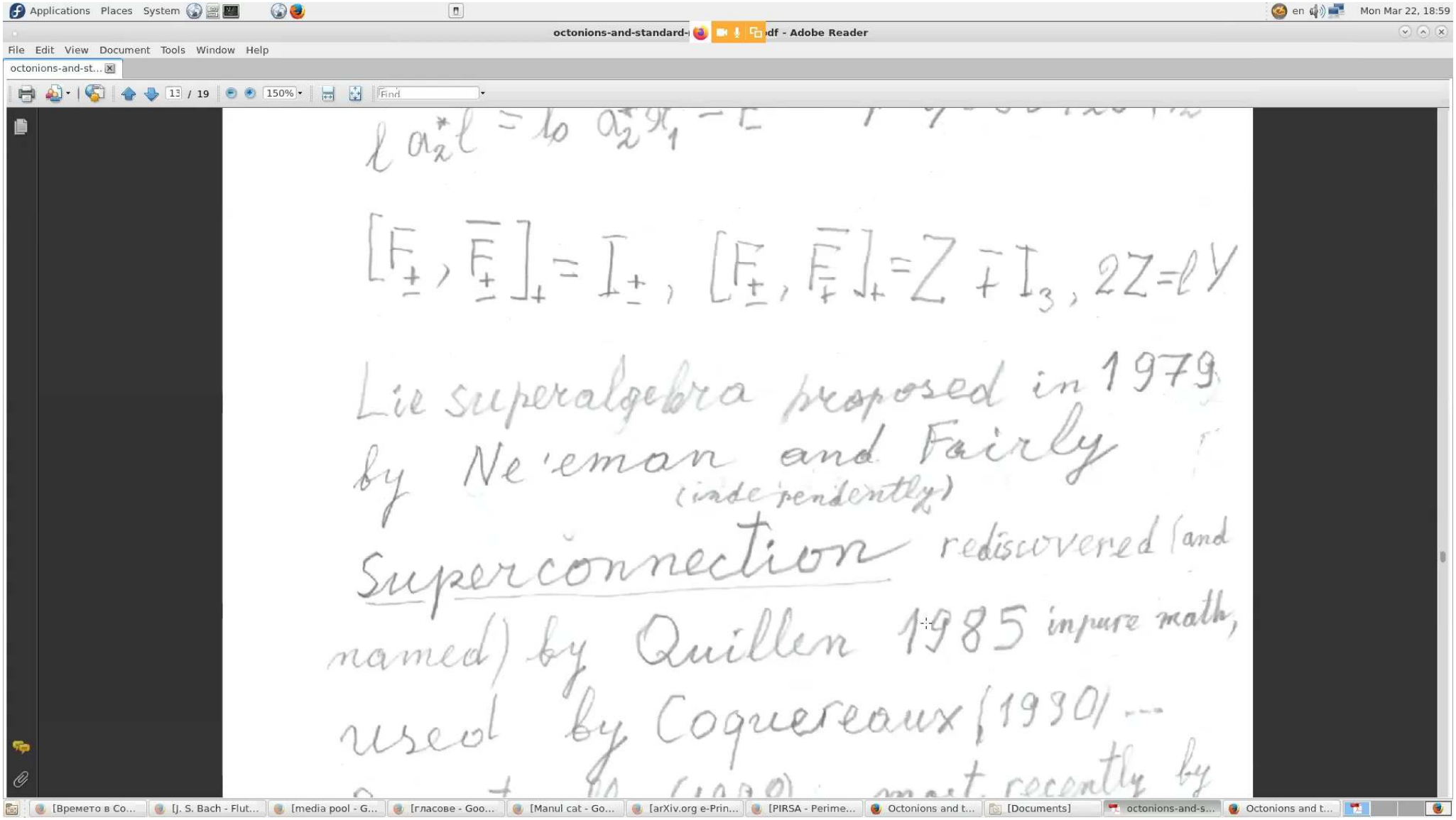
(11)

$Cl_{10} = Cl_{10}^0 \oplus Cl_{10}^1$, $g_C \subset spin(10) \subset Cl_{10}^0$

Higgs $\in Cl_{10}^1$, $g_C \subset Cl_{10}^0$

Physical Fermi fields are tensor products of Dirac fields with internal space vectors. (cf. Chamseddine, Iliopoulos, Suijlekom 2009.03367v2 who use 384×384 matrices)


Physical Fermi fields are tensor products of Dirac fields with internal space vectors. (cf. Chamseddine, Iliopoulos, Suijlekom 2009.03367v2 who use 384×384 matrices)
Dirac fields include antiparticles — fermion doubling problem.
Particle field algebra: $\mathcal{F}_P = PFP$
$P = l + q$, $P f_j^{(*)} P = 0 \Rightarrow$ No symmetry breaking for QCD!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$[F_\pm, \bar{F}_\pm]_+ = I_\pm$, $[F_\pm, \bar{F}_\mp]_+ = Z \mp I_3$, $2Z = \ell Y$
Lie superalgebra proposed in 1979
by Ne'eman and Fairlie
(independently)
Superconnection rediscovered (and
named) by Quillen 1985 in pure math,
used by Coquereaux (1990) ...
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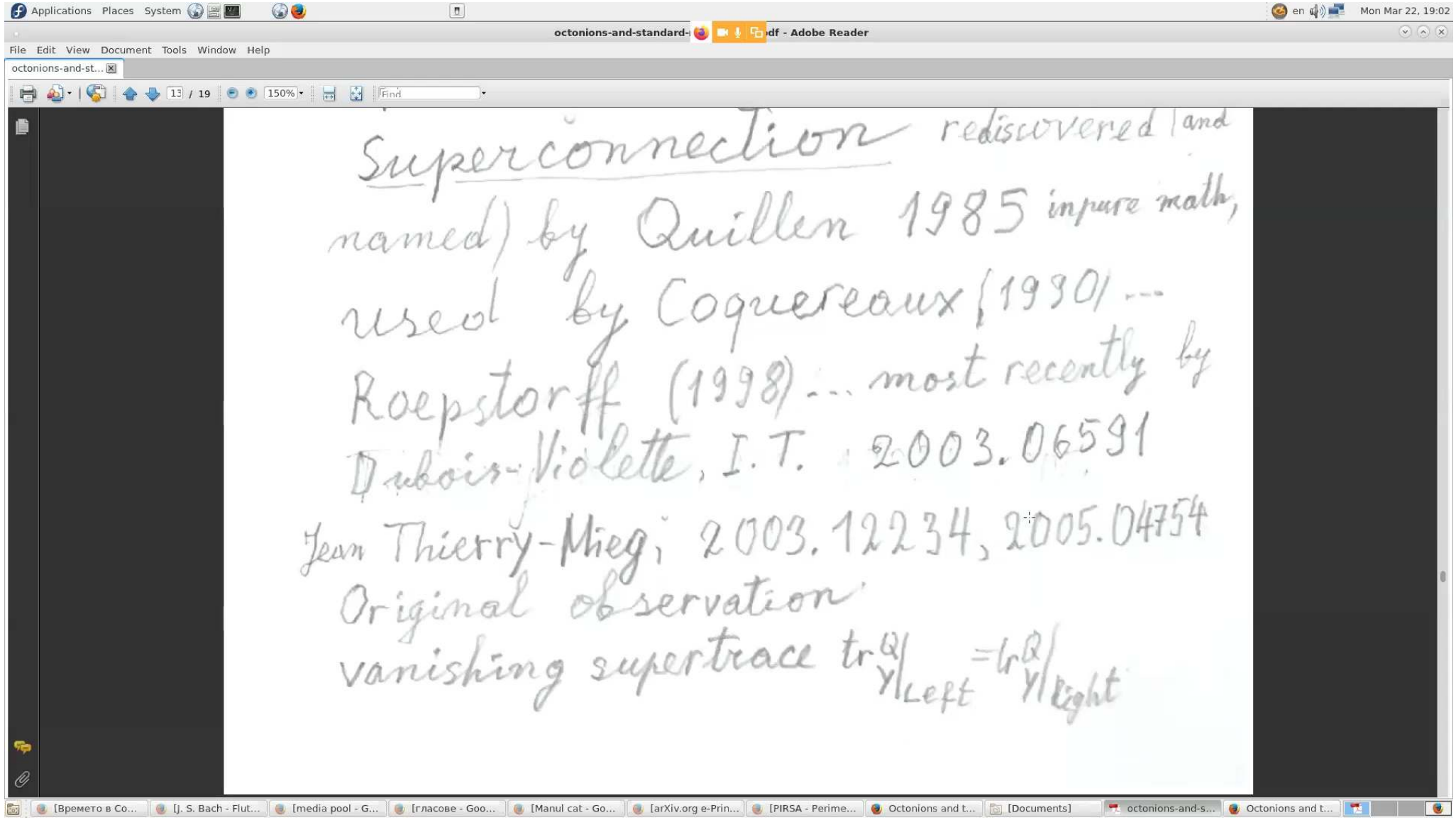
by Ne'eman and Regge (independently)

Superconnection rediscovered (and named) by Quillen 1985 in pure math, reused by Coquereaux (1990) ... Roepstorff (1998) ... most recently by Dubois-Violette, I.T. 2003.06591 Jean Thierry-Mieg, 2003.12234, 2005.04754

Original observation

vanishing supertrace $tr^{Q}_{Y|Left} = tr^{Q}_{Y|Right}$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Superconnection

$$\mathbb{D} = \chi D + \Phi, \quad D = d + A = dx^\mu(\partial_\mu + A_\mu)$$

$$iA_\mu = W_\mu + B_\mu + G_\mu \ (\in Cl_{10}^0), \ \Phi \in Cl_4^1$$

$$\pi_a = a_\alpha a_\alpha^* \qquad \pi_b' = a_\alpha^* a_\alpha$$

$$\chi = [a_1^*, a_1][a_2^*, a_2] = (\pi_1' - \pi_1)(\pi_2' - \pi_2)$$

projection of chirality on Cl_4

$$P_L = \pi_1'\pi_2 + \pi_1\pi_2', \quad P_R = \pi_1\pi_2 + \pi_1'\pi_2'$$

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$\Leftrightarrow$ associativity: $\mathbb{D}\mathbb{D}^2 = \mathbb{D}^2\mathbb{D}$

$\mathbb{D}^2$ - canonical curvature

The bosonic Lagrangian is the square of the curvature. The square of the canonical curvature gives a Higgs potential with minimum at $\Phi = 0 \Rightarrow$ no symmetry braking. Happily the Bianchi identity is still

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the Bianchi identity is still
satisfied if we add to $\mathbb{D}^2$ a
constant matrix operator

$$\hat{m}^2 = m^2(\ell + s^2 q).$$

18

Higgs mass

$$V(\Phi) = \frac{1+6\beta^4}{2}(\bar{\Phi}\Phi)^2 - (1+6\beta^4)\bar{\Phi}\Phi m^2 + (1+6\beta^4)m^4$$

$$\Rightarrow m_H^2 = (1+6\beta^4)m^2 \quad \text{coefficient to } -\bar{\Phi}\Phi$$

$$m_W^2 = \frac{1+6\beta^2}{4}m^2$$

$$\Rightarrow m_H^2 = 4\frac{1+6\beta^4}{1+6\beta^2}m_W^2 = \frac{5}{2}m_W^2 = 4\cos^2\theta_W m_W^2$$

for $\beta^2 = \frac{1}{2}$

$$m_H = 2\cos\theta_W m_W = \sqrt{\frac{5}{2}}\, m_W$$

Satisfied within 1% accuracy.

(18)

Higgs mass

$$V(\Phi) = \frac{1+6\beta^4}{2}(\bar{\Phi}\Phi)^2 - (1+6\beta^4)\bar{\Phi}\Phi m^2 + (1+6\beta^4)m^4$$

$\Rightarrow$ $m_H^2 = (1+6\beta^4)m^2$ } coefficient to $-\bar{\Phi}\Phi$

$$m_W^2 = \frac{1+6\beta^2}{4}m^2$$

$$\Rightarrow m_H^2 = 4\frac{1+6\beta^4}{1+6\beta^2}m_W^2 = \frac{5}{2}m_W^2 = 4\cos^2\theta_W m_W^2$$

for $\beta^2 = \frac{1}{2}$

$$m_H = 2\cos\theta_W m_W = \sqrt{\frac{5}{2}}\,m_W$$

Satisfied within 1% accuracy.

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Concluding remarks

- $Cl_{10} = Cl_4 \hat{\otimes} Cl_6$ $\quad Spin(10) \subset Cl^0_{10}$, $Higgs \in Cl^1_4$
- $Fermions \in \mathbb{C}O^2$ but ν_R may mix with D_L.
- GUT relations at unification recovered.
- New relation $m_H^2 = 4\cos^2\theta_W m_W^2 = \frac{5}{2} m_W^2$.
- Open problem: three generations - CKM mixing matrix?
  The role of triality in Yukawa
  ... reduce the number

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- Open problem: three generations - CKM mixing matrix?
  The role of triality in Yukawa interactions. → Reduce the number of parameters?! Understand the mass spectrum of fundamental fermions?
- Extension of G_{SM} - Will we need more Higgses?