Title: Division algebraic symmetry breaking

Speakers: Cohl Furey, Mia Hughes

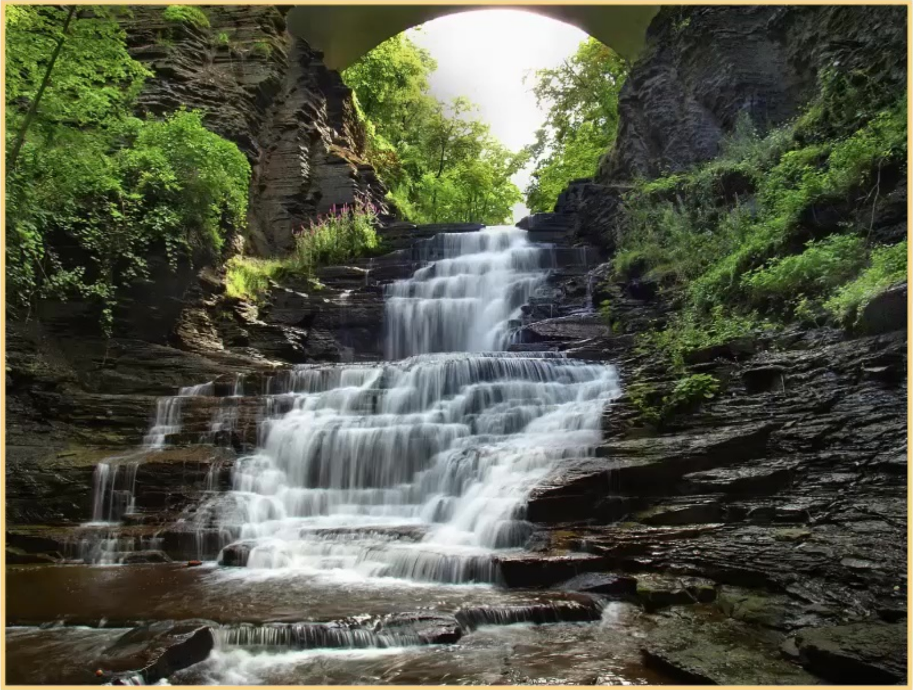
Collection: Octonions and the Standard Model

Date: March 15, 2021 - 12:00 PM

URL: http://pirsa.org/21030013

Abstract: Can the 32C-dimensional algebra R(x)C(x)H(x)O offer anything new for particle physics? Indeed it can. Here we identify a sequence of complex structures within R(x)C(x)H(x)O which sets in motion a cascade of breaking symmetries: Spin(10) -> Pati-Salam -> Left-Right symmetric -> Standard model + B-L (both pre- and post-Higgs-mechanism). These complex structures derive from the octonions, then from the quaternions, then from the complex numbers. Finally, we describe a left-right symmetric Higgs system which exhibits, we believe for the first time, an explicit demonstration of quaternionic triality.

Division algebraic symmetry breaking



N. Furey
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Humboldt-Universität zu Berlin

Based on

N.F. and M.J. Hughes,
One generation of standard model fermions as a single copy of RCHO,
in preparation

N.F. and M.J. Hughes,
Division algebraic symmetry breaking
in preparation

Recorded seminars
First: Rutgers University Mathematics, 29th October 2020
Recent: Perimeter Institute, **pirsa.org/21020027/,** 22nd February 2021

Division algebraic symmetry breaking

* **1973 Günaydin & Gürsey**

$g_2 \rightarrow su(3)_C$

* **1999 Dixon**

$so(1,9) \oplus su(2) \rightarrow so(1,3) \oplus g_{sm}$

* **2016-8 Dubois-Violette & Todorov**

$f_4 \rightarrow su(3) \oplus su(3)$ / $so(9) \rightarrow g_{sm}$

* **2019 Krasnov**

$so(9) \rightarrow g_{sm}$ / $so(5) \rightarrow u(2)$

* **2020 Boyle**

$so(10) \rightarrow u(3) \oplus su(2) \oplus su(2)$
LR symmetric

* **2020 F & H**

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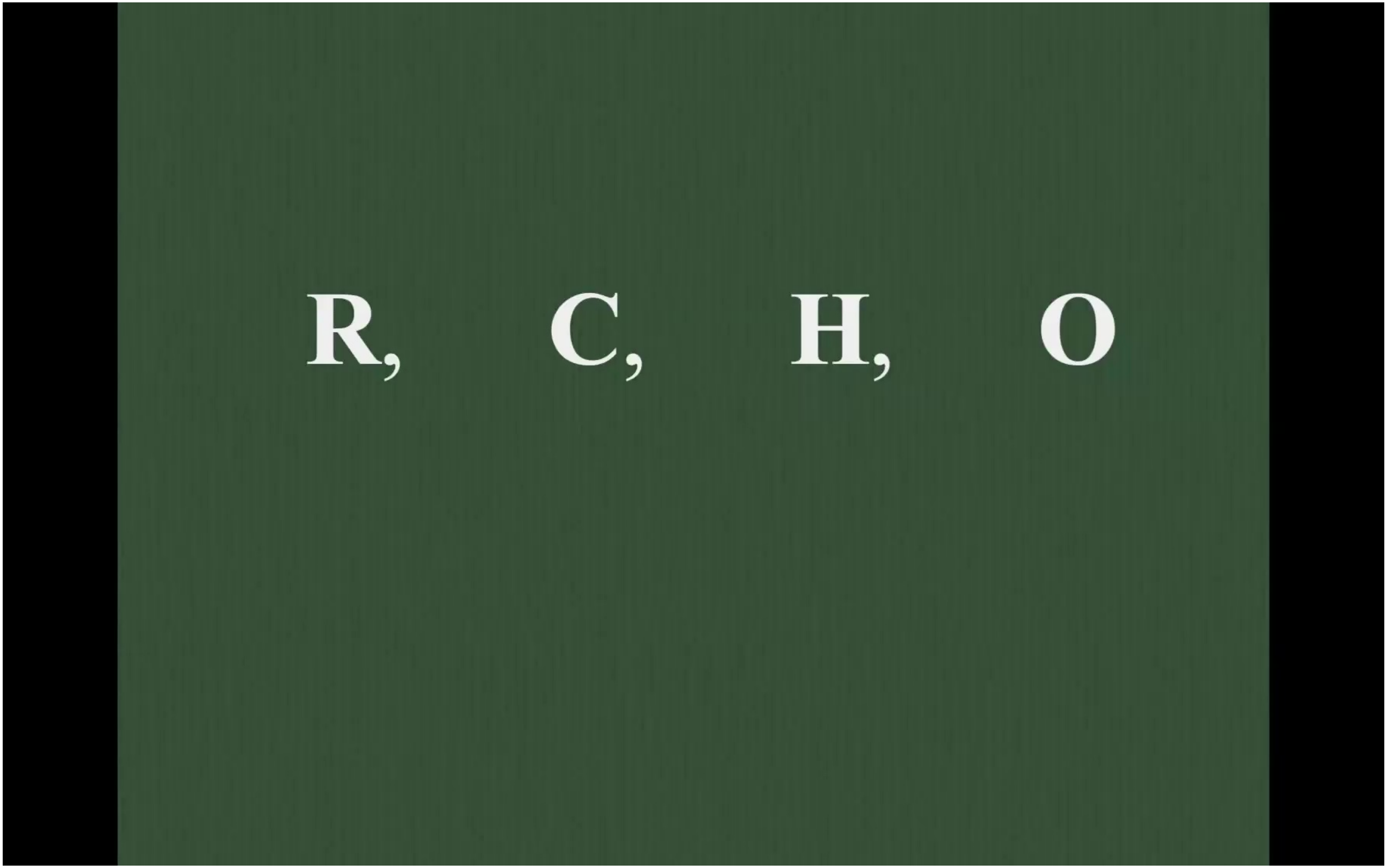
$so(10) \rightarrow u(3) \oplus su(2) \oplus su(2)$
LR symmetric

$so(10) \rightarrow so(6) \oplus so(4)$
Pati-Salam


R, C, H, O

special → ubiquitous

R, C, H, O

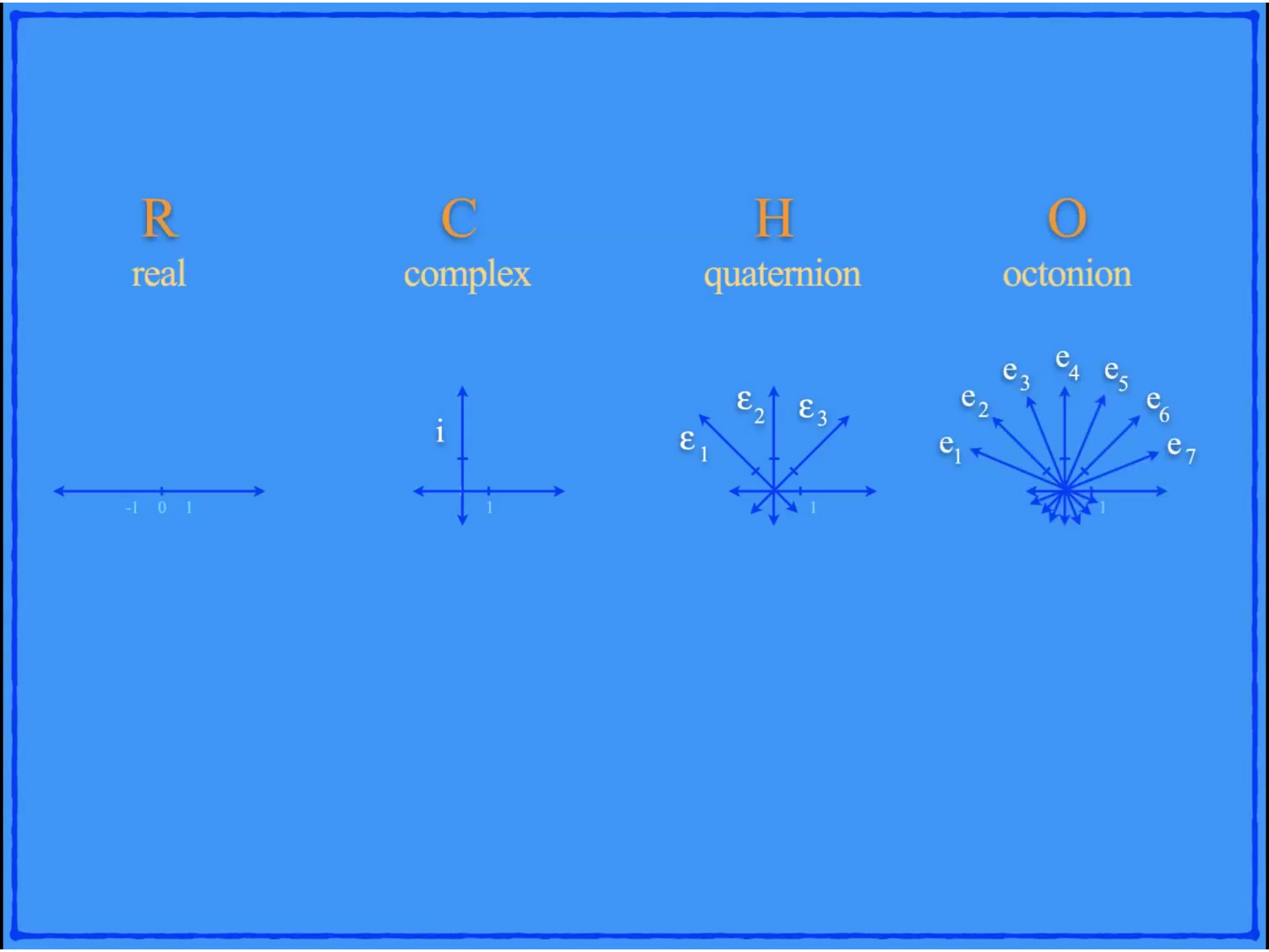
R, → everywhere

C, → quantum

H, → special relativity

O → strong nuclear

electromagnetism

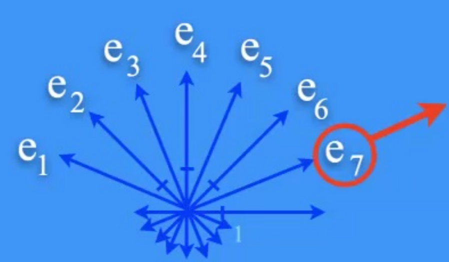


R
real
-1
0
1
C
complex
i
1
H
quaternion
ε1
ε2
ε3
1
O
octonion
e1
e2
e3
e4
e5
e6
e7
1

Why isolate one imaginary unit?

$\mathbb{O}$

octonion



Octonion

Left multiplication

$$e_{j_1}(e_{j_2}(e_{j_3}(e_{j_4}(e_{j_5}(e_{j_6}\cdot))))) = \pm(e_{j_7}\cdot)$$

$$\vdots$$

$$e_{j_1}(e_{j_2}\cdot) \qquad e_{j_1}(e_{j_3}\cdot) \qquad \dots \qquad e_{j_5}(e_{j_6}\cdot)$$

$$(e_{j_1}\cdot) \qquad (e_{j_2}\cdot) \qquad \dots \qquad (e_{j_6}\cdot)$$

$$1$$

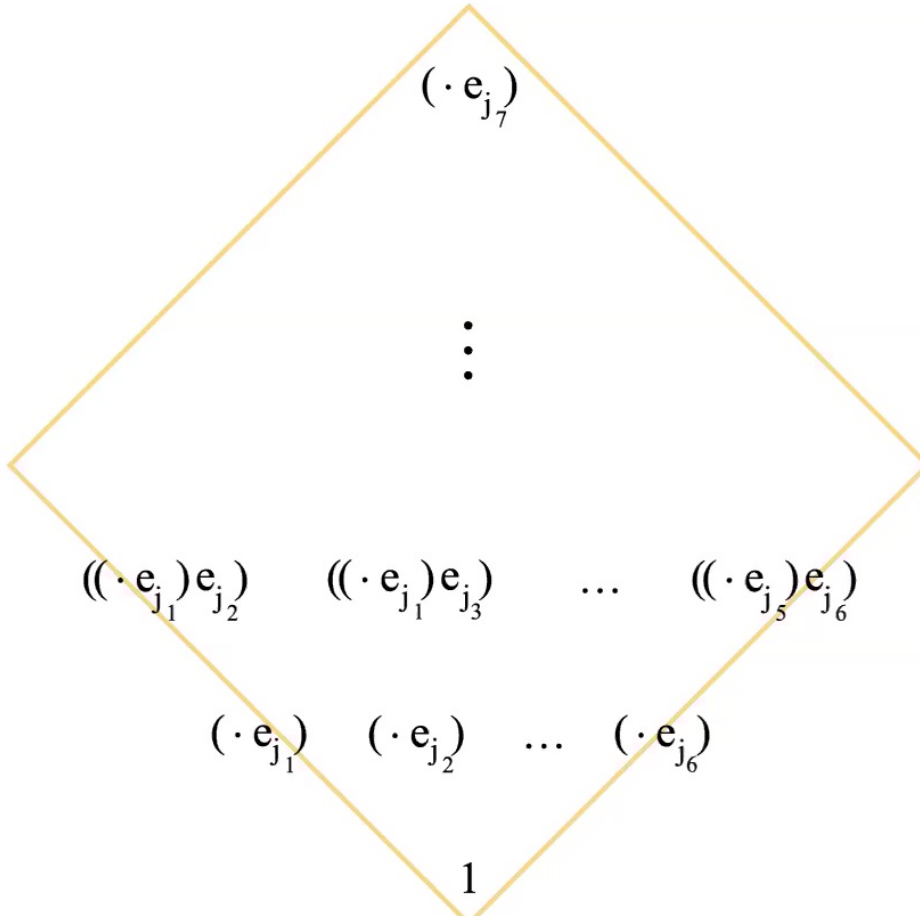
Octonion

Right multiplication

$\Rightarrow$ Cl(0,6)

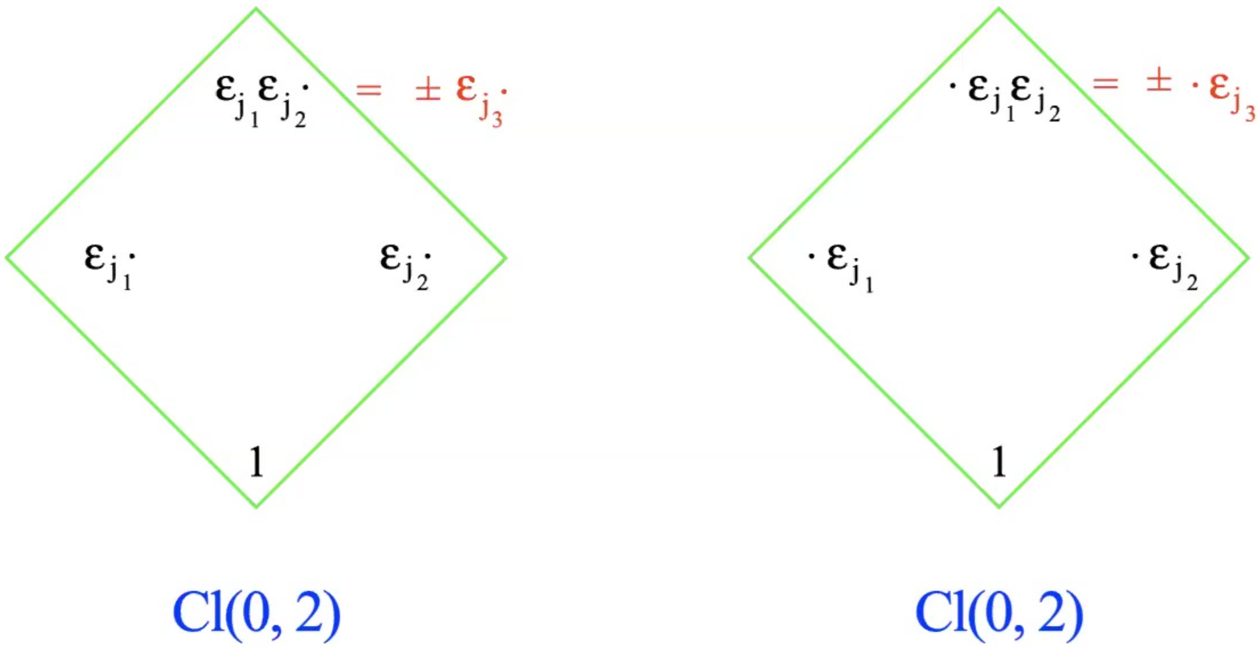
$$(\cdot\, e_7) = 1/2\Big(\; e_1(e_3\cdot) + e_2(e_6\cdot) + e_4(e_5\cdot) - (e_7\cdot)\Big)$$

$$=: \; (E_7\cdot)$$



Quaternion

Left and **Right** multiplication



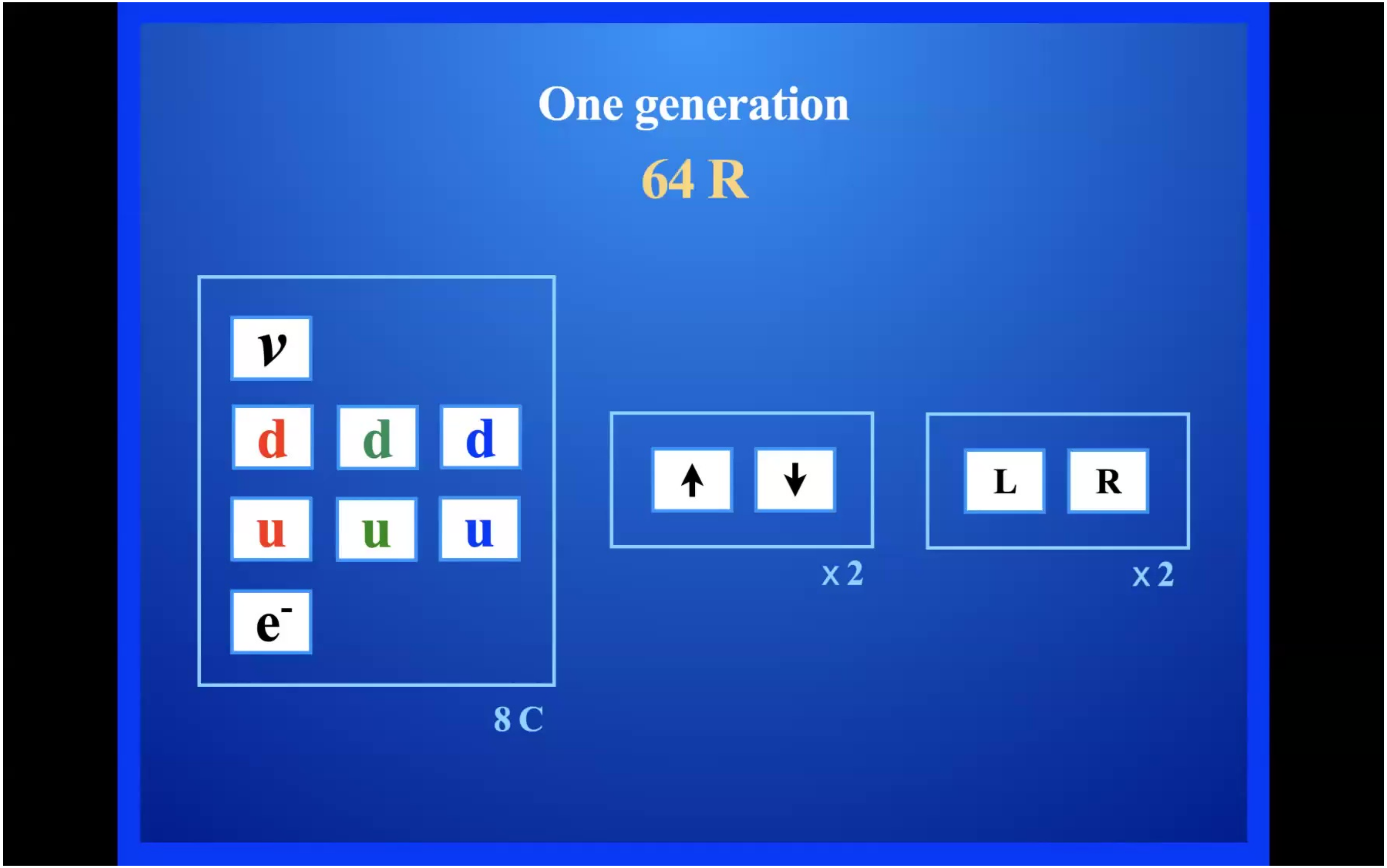
$$O \Rightarrow Cl(0,6)$$

$$H \Rightarrow Cl(0,2) \otimes Cl(0,2)$$

L mult ~ R mult			Isolated
✓	O	$\Rightarrow$ Cl(0, 6)	e_{j_7}
X	H	$\Rightarrow$ Cl(0, 2) $\otimes$ Cl(0,2)	ε_{j_3}
✓	C	$\Rightarrow$ Cl(0, 1)	i
✓	R	$\Rightarrow$ Cl(0, 0)	-

$$(\cdot\, e_7) = 1/2\Big(e_1(e_3\cdot) + e_2(e_6\cdot) + e_4(e_5\cdot) - (e_7\cdot)\Big)$$

$$=: (E_7\cdot)$$


One generation
64 R
ν
d
d
d
u
u
u
e⁻
8 C
↑
↓
x 2
L
R
x 2

$$\mathbf{R} \otimes \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

$$= \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

CHO → CHO ← CHO

↓

$\mathbb{C}l(10)$

$so(10)$	$so(6)\oplus so(4)$	$su(3)\oplus su(2)\oplus su(2)\oplus u(1)$	$su(3)\oplus su(2)\oplus u(1)$
Spin(10)	Pati-Salam	LR symmetric	Standard Model

Anything new beyond Cl(10)?

Spin(10)

$r_{ij}\ \gamma_i\ \gamma_j$

X no substructure

X $16 \oplus 16^*$

old way

Spin(10)

$r_{ij}\ \gamma_i\ \gamma_j$

✗ no substructure

✓ $16 \oplus 16$

old way

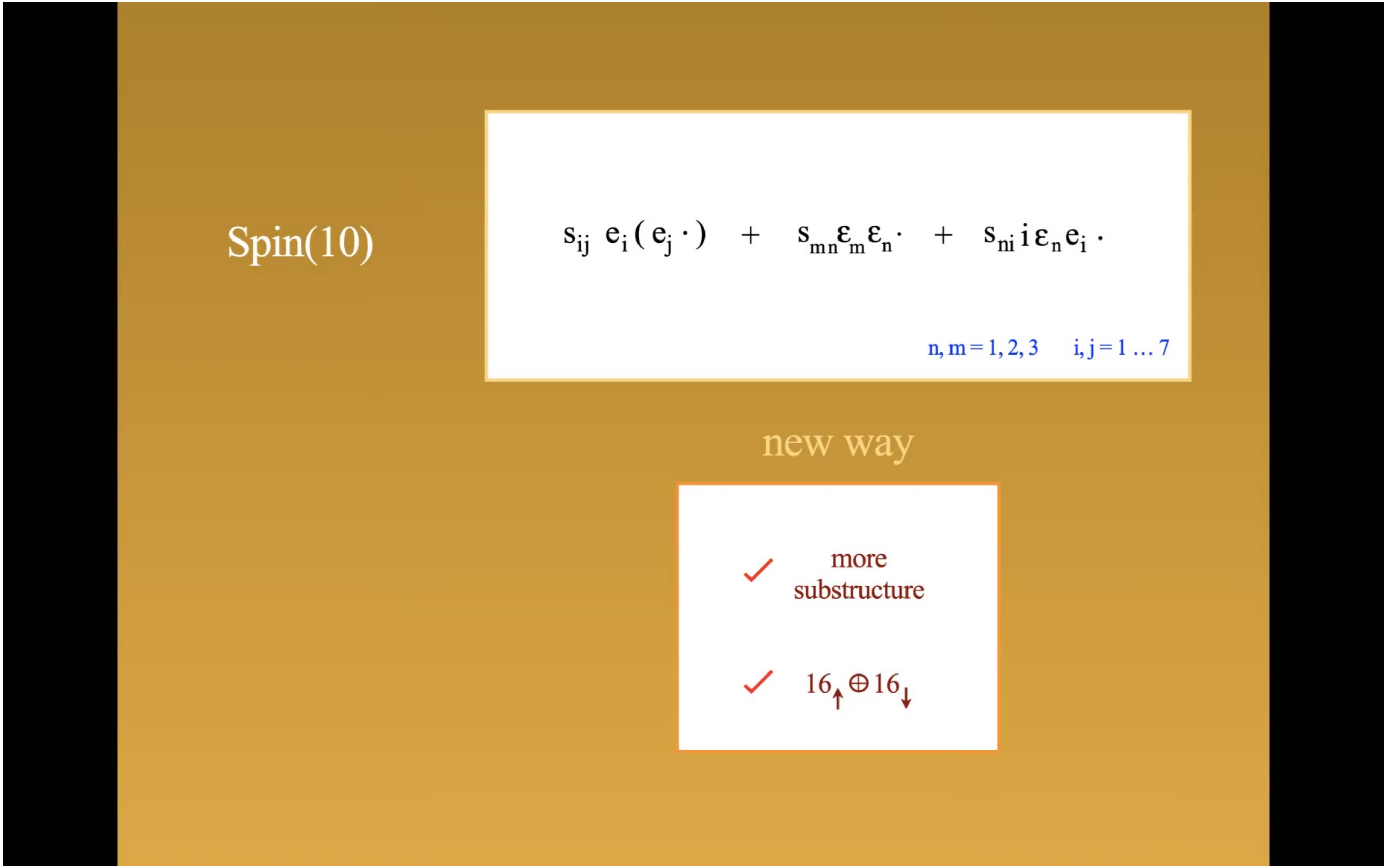
Spin(10)

$$s_{ij}\ e_i\,(e_j\,\cdot) \quad + \quad s_{mn}\varepsilon_m\varepsilon_n\cdot \quad + \quad s_{ni}\, i\, \varepsilon_n e_i\ \cdot$$

real

$n, m = 1, 2, 3 \qquad i, j = 1 \ldots 7$

new way


Spin(10)
$s_{ij}\; e_i (e_j \cdot) \; + \; s_{mn} \varepsilon_m \varepsilon_n \cdot \; + \; s_{ni}\, i\, \varepsilon_n e_i \cdot$
n, m = 1, 2, 3 i, j = 1 … 7
new way
more substructure
$16_{\uparrow} \oplus 16_{\downarrow}$

Spin(10)

$$r''_{ij}\, e_i(e_j\,\cdot) \;+\; r_n\, P_L\, \varepsilon_n\cdot \;+\; r'_n\, P_R\, \varepsilon_n\cdot$$

$$+\; r_{ni}\; i\,\varepsilon_n e_i\,\cdot \;+\; r''_i\, e_i(e_7\,\cdot)$$

real

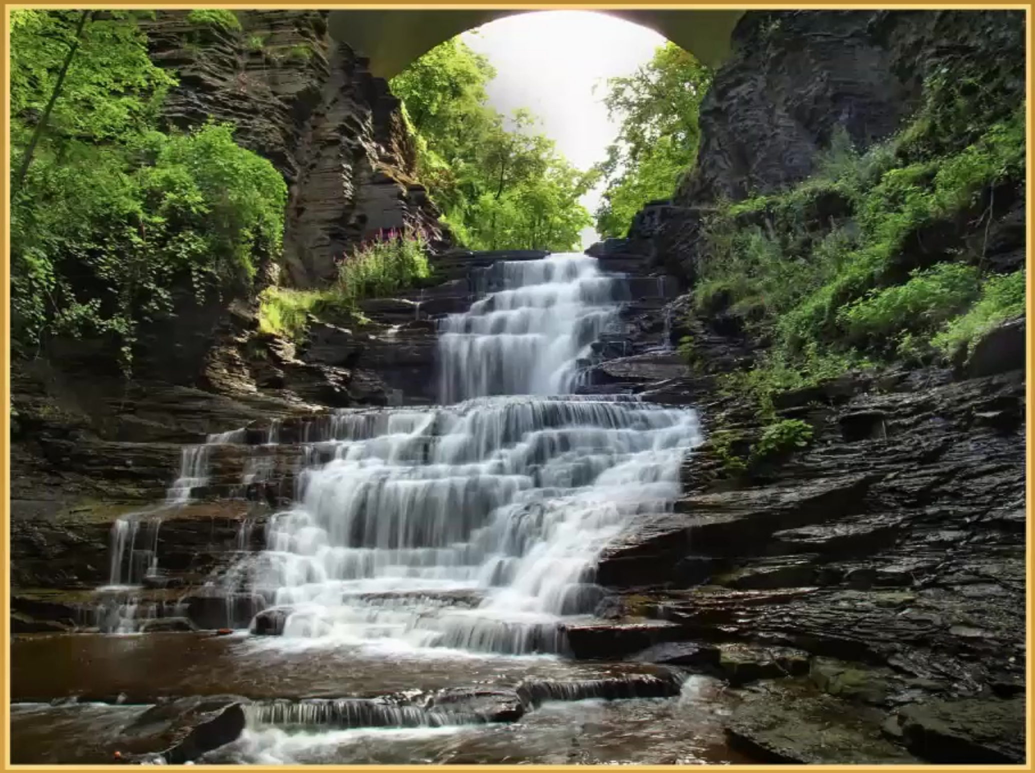
$n = 1, 2, 3 \qquad i, j = 1 \ldots 6$

new way

Complex structure

Generalizes notion of multiplying by complex i.

Cascade of complex structures



Spin(10)

$$r''_{ij}\, e_i (e_j \cdot) \;+\; r_n P_L \varepsilon_n \cdot \;+\; r'_n P_R \varepsilon_n \cdot$$

$$+\; r_{ni}\, i\, \varepsilon_n e_i \cdot \;+\; r''_i e_i (e_7 \cdot)$$

$n = 1, 2, 3 \qquad i, j = 1 \dots 6$

Spin(10)

$$r''_{ij}\, e_i (e_j \cdot) \; + \; r_n P_L \varepsilon_n \cdot \; + \; r'_n P_R \varepsilon_n \cdot$$

$$+ \; r_{ni}\, i\, \varepsilon_n e_i \cdot \; + \; r''_i e_i (e_7 \cdot)$$

$n = 1, 2, 3 \quad i, j = 1 \ldots 6$

$e_7 \Psi$ $\Downarrow$

so(6) su(2) su(2)

$$r''_{ij}\, e_i (e_j \cdot) \; + \; r_n P_L \varepsilon_n \cdot \; + \; r'_n P_R \varepsilon_n \cdot$$

$n = 1, 2, 3 \quad i, j = 1 \ldots 6$

$e_7 \Psi \Downarrow$

Pati-Salam

$$\underset{so(6)}{r''_{ij}\, e_i (e_j \cdot)} + \underset{su(2)}{r_n P_L \varepsilon_n \cdot} + \underset{su(2)}{r'_n P_R \varepsilon_n \cdot}$$

$n = 1, 2, 3 \qquad i, j = 1 \ldots 6$

$E_7 \Psi \Downarrow$

$$\underset{u(3)}{r'''_{ij}\, e_i (e_j \cdot)} + \underset{su(2)}{r_n P_L \varepsilon_n \cdot} + \underset{su(2)}{r'_n P_R \varepsilon_n \cdot}$$

$n = 1, 2, 3 \qquad i, j = 1 \ldots 6$

$$E_7 \Psi \Downarrow$$

LR symmetric

u(3)		su(2)		su(2)
$r''''_{ij} e_i (e_j \cdot)$	$+$	$r_n P_L \varepsilon_n \cdot$	$+$	$r'_n P_R \varepsilon_n \cdot$

$n = 1, 2, 3 \qquad i, j = 1 \ldots 6$

$$\varepsilon_3 \Psi_R \Downarrow$$

Standard model
+ B-L

u(3)		su(2)		u(1)
$r''''_{ij} e_i (e_j \cdot)$	$+$	$r_n P_L \varepsilon_n \cdot$	$+$	$u(1)_Y \cdot$

$n = 1, 2, 3 \qquad i, j = 1 \ldots 6$

$$\varepsilon_3 \Psi_R \quad \Downarrow$$

Standard model + B-L

u(3)		su(2)		u(1)
$r'''_{ij}\, e_i\,(e_j \cdot)$	$+$	$r_n\, P_L\, \varepsilon_n \cdot$	$+$	$u(1)_Y \cdot$

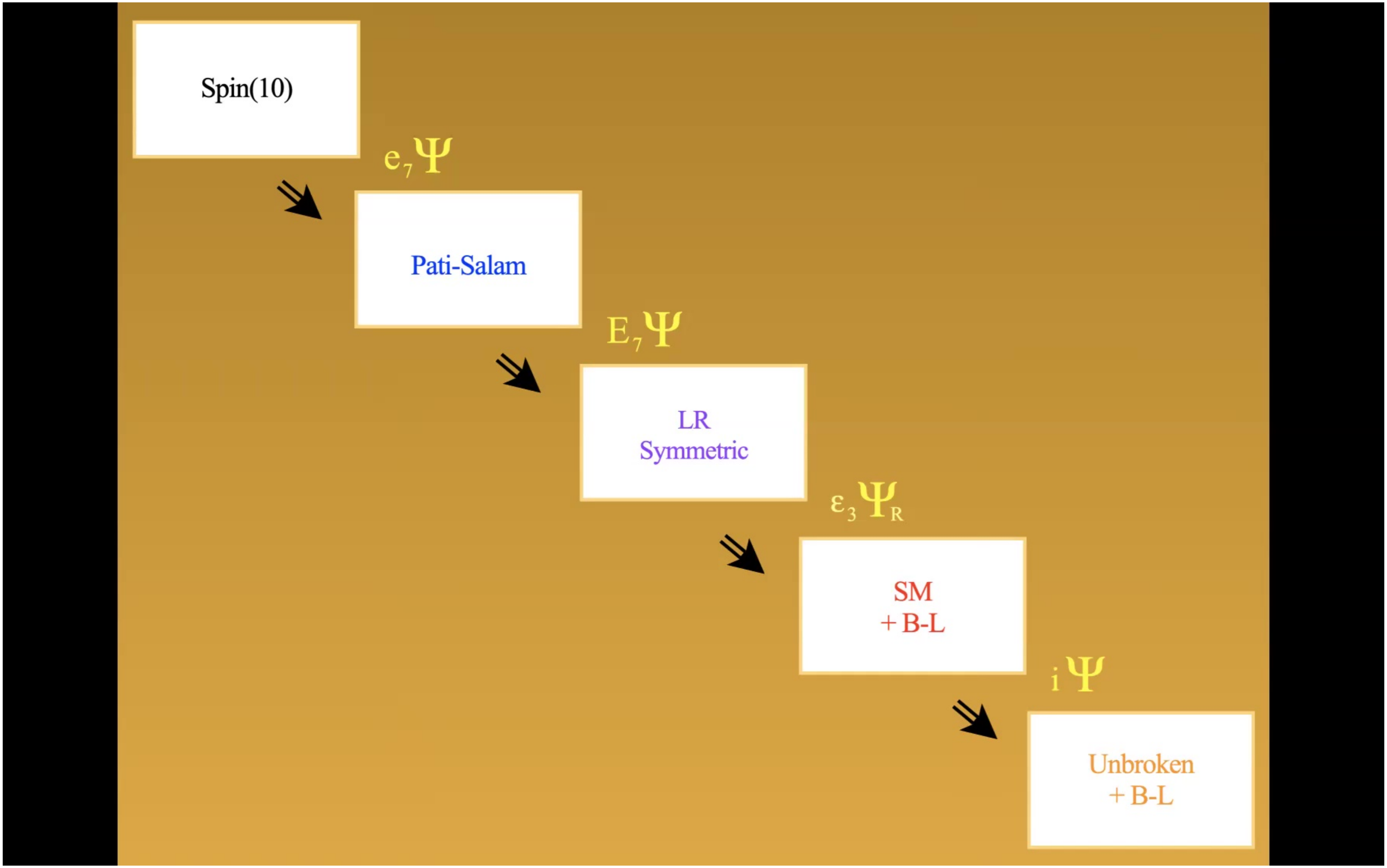
$n = 1, 2, 3 \qquad i, j = 1 \ldots 6$

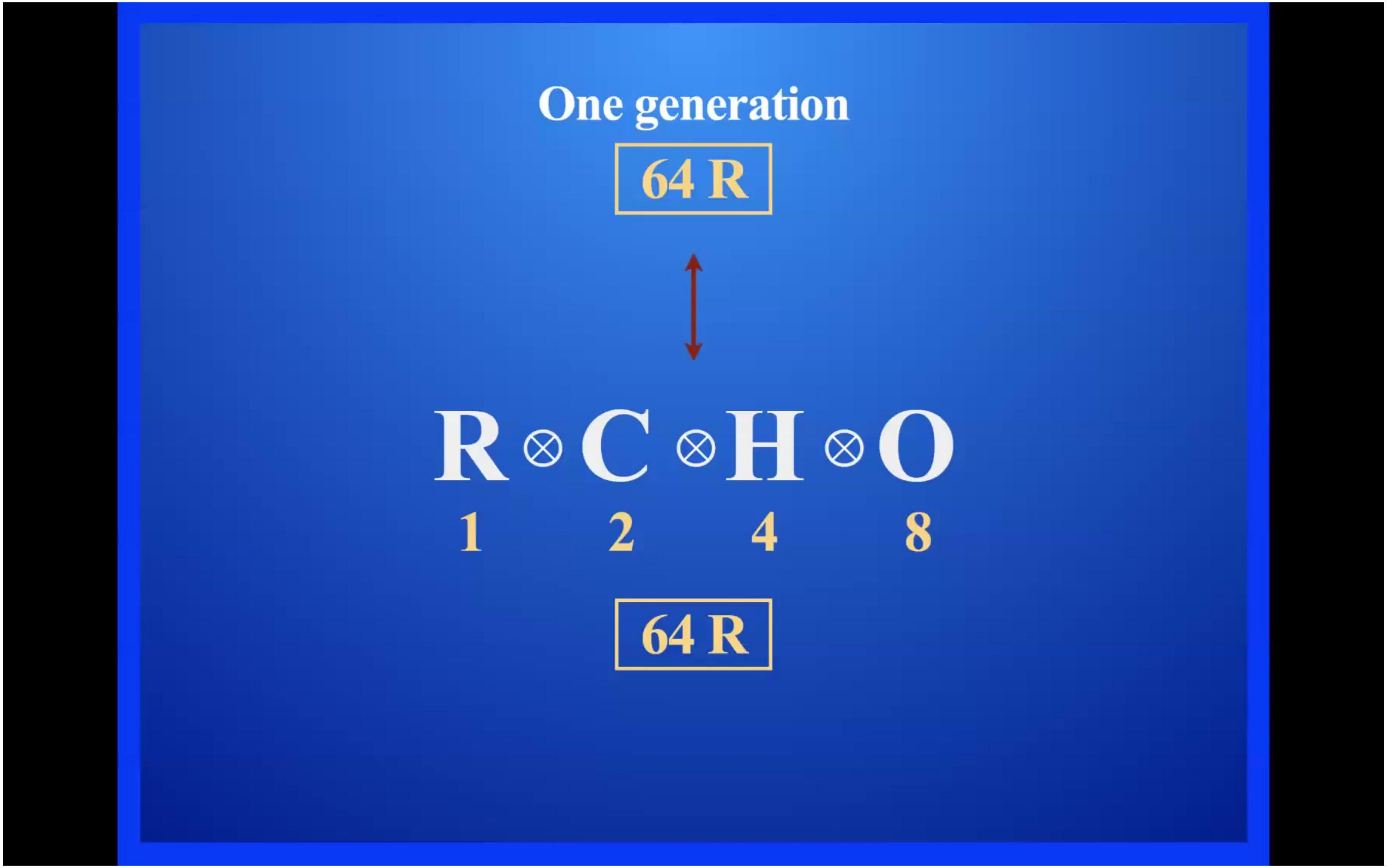
$$i\,\Psi \quad \Downarrow$$

Unbroken + B-L

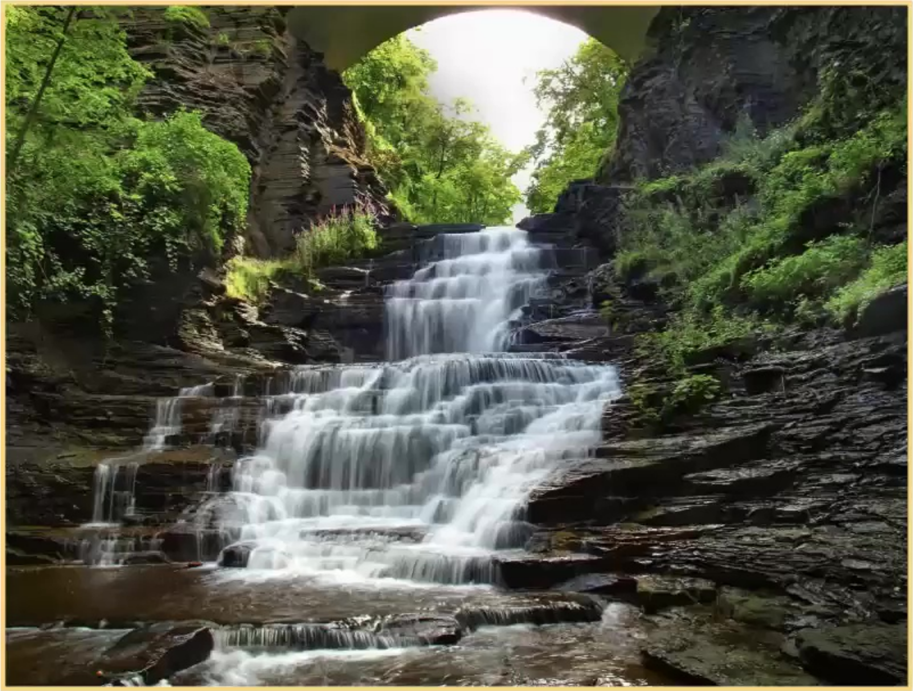
u(3)		u(1)
$r'''_{ij}\, e_i\,(e_j \cdot)$	$+$	$u(1)_{EM} \cdot$

$i, j = 1 \ldots 6$


Spin(10)
e7Ψ
Pati-Salam
E7Ψ
LR
Symmetric
ε3ΨR
SM
+ B-L
iΨ
Unbroken
+ B-L


One generation
64 R
R ⊗ C ⊗ H ⊗ O
1 2 4 8
64 R

Summary



CHO → CHO ← CHO

↓

$\mathbb{C}l(10)$

so(10)	so(6) ⊕ so(4)	su(3) ⊕ su(2) ⊕ su(2) ⊕ u(1)	su(3) ⊕ su(2) ⊕ u(1)
Spin(10)	Pati-Salam	LR symmetric	Standard Model

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$so(10)$	$\rightarrow$	$so(6) \oplus so(4)$ Pati-Salam	$\rightarrow$	$u(3) \oplus su(2) \oplus su(2)$ LR symmetric
	$\rightarrow$	$g_{sm} \oplus u(1)$ SM + B-L	$\rightarrow$	$u(3) \oplus u(1)$ Unbroken + B-L

* **2021 Todorov**

$so(10) \rightarrow u(3) \oplus su(2) \oplus su(2)$
LR symmetric

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Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

Mia Hughes

Triality Recap

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ SO(10) Model

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Pati-Salam Model

The Higgs

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

Mia Hughes

March 15, 2021

mia.j.hughes@gmail.com

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

Mia Hughes

Non-degenerate Trilinear Forms

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

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The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Pati-Salam Model

The Higgs

- In general, the division-algebraic multiplication rule is:

$$e_a\, e_b = t_{abc}\, e_c$$

- There is a natural inner product on each algebra:

$$\langle \psi^* \chi \rangle := \tfrac{1}{2}(\psi^* \chi + \chi^* \psi) = \psi_a \chi_a$$

- So if we multiply two elements of a division algebra $\mathbb{K}$:

$$v\psi = (v_a e_a)(\psi_b e_b) = v_a \psi_b\, t_{abc}\, e_c$$

and take the inner product with a third:

$$\tfrac{1}{2}(\chi^*(v\psi) + (v\psi)^* \chi) = v_a \psi_b \chi_c\, t_{abc}$$

Non-degenerate Trilinear Forms

- So each normed division algebra $\mathbb{K}$ gives rise to a trilinear form, called a "normed triality":

$$t(v, \psi, \chi) = \langle \chi^* v \, \psi \rangle = v_a \psi_b \chi_c \, t_{abc}$$

- Because of the division algebra property, this trilinear form is non-degenerate (and because of the normed property, the triality is "normed")

- Such trilinear forms are extremely rare: there's one for each normed division algebra, and that's it!

Triality Algebras

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

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Triality Recap

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ SO(10) Model

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Pati-Salam Model

The Higgs

- If we treat the 3 inputs to t as if they belong to different spaces, can we rotate each of them in such a way that preserves the trilinear form?

$$t(v,\psi,\chi) = t(O_v v, O_\psi \psi, O_\chi \chi)$$

$$O_v,\ O_\psi,\ O_\chi \in \mathrm{O}(\dim[\mathbb{K}])$$

- This gives the automorphism groups of the trialities, whose Lie algebras are:

$$\mathfrak{tri}(\mathbb{R}) = \varnothing$$
$$\mathfrak{tri}(\mathbb{C}) = \mathfrak{u}(1) \oplus \mathfrak{u}(1)$$
$$\mathfrak{tri}(\mathbb{H}) = \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$$
$$\mathfrak{tri}(\mathbb{O}) = \mathfrak{so}(8)$$

Triality Triples

- Each of the triality algebras consists of $\mathfrak{so}(\dim[\mathbb{K}])$ plus a possible extra part:

$$\mathfrak{tri}(\mathbb{R}) = \mathfrak{so}(1) = \varnothing$$
$$\mathfrak{tri}(\mathbb{C}) = \mathfrak{so}(2) \oplus \mathfrak{u}(1)$$
$$\mathfrak{tri}(\mathbb{H}) = \mathfrak{so}(4) \oplus \mathfrak{su}(2)$$
$$\mathfrak{tri}(\mathbb{O}) = \mathfrak{so}(8)$$

- The three inputs to the triality transform as the vector, spinor and conjugate spinor of $\mathfrak{so}(\dim[\mathbb{K}])$:

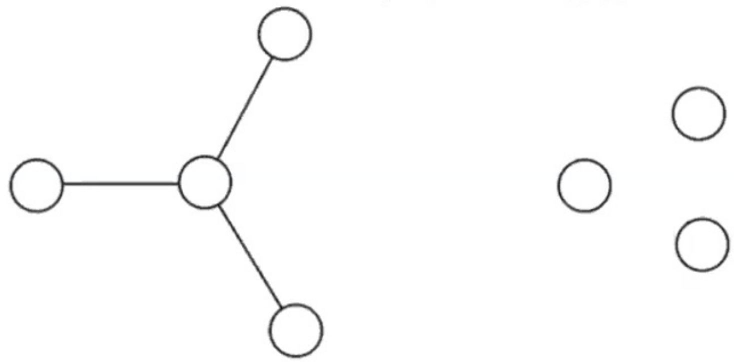
$$v \in V, \qquad \psi \in S_+, \qquad \chi \in S_-,$$

- And the structure constants are actually just the intertwiner ("Pauli matrices") between these three reps:

$$t_{abc} = (\sigma_a)_{bc}$$

Quaternionic Triality

- Triality is reflected in the symmetry of the famous Dynkin diagram for $\mathfrak{tri}(\mathbb{O}) = \mathfrak{so}(8)$, as well as its less famous cousin $\mathfrak{tri}(\mathbb{H}) = \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$



- In the $\mathbb{H}$ case, our trusty three reps transform as:

$$
\begin{aligned}
(\mathbf{2},\mathbf{2},\mathbf{1}): &\quad \delta v = \theta_1 v - v\theta_2, \\
(\mathbf{2},\mathbf{1},\mathbf{2}): &\quad \delta\psi = \theta_2\psi + \psi\theta_3, \\
(\mathbf{1},\mathbf{2},\mathbf{2}): &\quad \delta\chi = \theta_1\chi + \chi\theta_3, \qquad \theta_i \in \mathrm{Im}(\mathbb{H})
\end{aligned}
$$

$\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Formulation of the SO(10) Model

- There is a way to write nice-looking $\mathfrak{so}(10)$ transformations on an element of $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$:

$$\delta\psi = \tfrac{1}{4}\theta^{ij}e_i(e_j\,\psi) + \tfrac{1}{4}\theta^{mn}\epsilon_m\epsilon_n\,\psi + \tfrac{1}{2}\theta^{mi}\,i\,\epsilon_m e_i\,\psi$$

with with $j = 1, 2, \cdots, 7$ and $m = 1, 2, 3$

- So we have decomposed $\mathfrak{so}(10)$ as follows:

$$\mathfrak{so}(10) \cong \mathfrak{so}(7) \oplus \mathfrak{so}(3) + \mathrm{Im}(\mathbb{O}) \otimes \mathrm{Im}(\mathbb{H})$$

- Under this action ψ transforms as 2 copies of the **16** of $\mathfrak{so}(10)$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ Formulation of the SO(10) Model

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$\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Formulation of the SO(10) Model

- An element ψ of $\mathbb{A} := \mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ has 32 complex components – just right for a $(\mathbf{16}; \mathbf{2}, \mathbf{1})$ of $\mathfrak{so}(10) \oplus \mathfrak{sl}(2,\mathbb{C})$!
- In other words, an element of $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ is perfectly suited to house one generation of SM fermions
- But how do you transform them correctly?

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

Mia Hughes

Triality Recap

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ SO(10) Model

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Pati-Salam Model

The Higgs

Lorentz Symmetry

- While the $\mathfrak{so}(10)$ acts via left-multiplication,

$$\delta\psi = \tfrac{1}{4}\theta^{IJ}\sigma_I(\bar{\sigma}_J\psi),$$

the $\mathfrak{sl}(2,\mathbb{C})$ acts by right-multiplication by elements of $\mathbb{C}\otimes\text{Im}(\mathbb{H})$:

$$\delta\psi = \psi\,\theta_S, \qquad \theta_S \in \mathbb{C}\otimes\text{Im}(\mathbb{H})$$

- Of course this commutes with the $\mathfrak{so}(10)$, since left- and right-multiplication in $\mathbb{H}$ commute!

- So we do indeed have exactly a $(\mathbf{16};\mathbf{2},\mathbf{1})$ of $\mathfrak{so}(10)\oplus\mathfrak{sl}(2,\mathbb{C})$

Lagrangians

- Seeing ψ as an $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$-valued field in 4-d Minkowski space, we can define a Dirac operator

$$\partial\!\!\!/\psi := \partial_0\psi - \partial_1\psi\, i\epsilon_1 + \partial_2\psi\, i\epsilon_2 - \partial_3\psi\, i\epsilon_3$$

- Then to write the free kinetic terms all we need is

$$\mathcal{L} = \langle i\psi^\dagger \partial\!\!\!/\psi\rangle$$

- This is equivalent to the kinetic terms of 16 free 2-component complex Weyl spinors

- Taking gauge fields valued in $\mathfrak{so}(10)$ or any of its subalgebras, we can promote $\partial \to D$ and write appropriate gauge-invariant kinetic terms

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The $\mathbb{H}$iggs

1. Triality Recap
2. The $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ SO(10) Model
3. **The $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ Pati-Salam Model**
4. The $\mathbb{H}$iggs

The Pati-Salam Model

- The Pati-Salam model comes from breaking $\mathfrak{so}(10)$ into $\mathfrak{so}(6) \oplus \mathfrak{so}(4) \cong \mathfrak{su}(4)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R =: \mathfrak{g}_{PS}$
- This breaks ψ into two irreducible pieces:

$$\psi = \psi_+ + \psi_-,$$

where

$$\psi_+ \in (\mathbf{4}, \mathbf{2}, \mathbf{1}; \mathbf{2}, \mathbf{1}),$$
$$\psi_- \in (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}),$$

of $\mathfrak{su}(4) \oplus \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{sl}(2, \mathbb{C})$

- ψ_+ contains a generation of LH fermions and ψ_- contains the RH fermions

Something Smells like Triality

- Within $\mathfrak{sl}(2,\mathbb{C})$ there is $\mathfrak{su}(2)_{\text{spin}}$, the spatial rotation subgroup, so in fact we have a subalgebra $\mathfrak{tri}(\mathbb{H}) = \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{su}(2)_{\text{spin}} \subset \mathfrak{g}_{PS} \oplus \mathfrak{sl}(2,\mathbb{C})$!

- Under this subgroup ψ_+ and ψ_- transform as

$$\delta\psi_+ = \theta_L\psi_+ + \psi_+\theta_{spin},$$
$$\delta\psi_- = \theta_R\psi_+ + \psi_+\theta_{spin},$$

- So ψ_+ is just 4 lots of the $(\mathbf{2},\mathbf{1};\mathbf{2})$ and ψ_- is just 4 lots of the $(\mathbf{1},\mathbf{2};\mathbf{2})$! What about the $(\mathbf{2},\mathbf{2};\mathbf{1})$?!

How the Higgs fits into the $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ Model

- How do we represent the electroweak-breaking SM Higgs in the $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ model?

- It begins with an "H", so it should obviously be in $\mathbb{H}$...

- The SM Higgs h has 4 real d.o.f. and is a scalar w.r.t. $\mathfrak{su}(2)_{\text{spin}}$, so it really does make sense to make it a pure quaternion:

$$h \in \mathbb{H} \cong (\mathbf{1}, \mathbf{2}, \mathbf{2}; \mathbf{1}) \text{ of } \mathfrak{g}_{PS} \oplus \mathfrak{su}(2)_{\text{spin}}$$

- This decomposes into just the right rep for the SM Higgs under the decomposition of $\mathfrak{g}_{PS} \oplus \mathfrak{sl}(2, \mathbb{C})$ into the subalgebra $\mathfrak{g}_{SM} \oplus \mathfrak{sl}(2, \mathbb{C})$!

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The Triality Scalar

- At this point it's obvious how the quaternions' trilinear form fits into the picture... the Yukawa terms:

$$\mathcal{L}_{Yukawa} = k\langle\widetilde{\psi_+}\, h\, \psi_-\rangle,$$

where the tilde denotes simultaneous $\mathbb{H}$ and $\mathbb{O}$ conjugation

- Expanding out the octonionic part of the inner product splits this into the 4 colours:

$$\mathcal{L}_{Yukawa} = k\langle\widetilde{\psi_{+A}}\, h\, \psi_-{}^A\rangle,$$

with $A = 0, 1, 2, 3$ and $\psi_{+A},\ \psi_-{}^A \in \mathbb{C}\otimes\mathbb{H}$

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

Mia Hughes

Triality Recap

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ SO(10) Model

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Pati-Salam Model

The $\mathbb{H}$iggs

The Triality Scalar

- Further splitting ψ_{+A} and $\psi_{-}{}^{A}$ into their $\mathbb{C}$-real and $\mathbb{C}$-imaginary parts gives 8 copies of the pure quaternionic triality scalar!
- Of course once $\mathfrak{g}_{PS}$ is broken to $\mathfrak{g}_{SM}$ these terms will end up with different coupling constants, leading to the different masses of the SM fermions
- But at the $\mathfrak{g}_{PS}$ level it seems $\mathbb{H}$ triality can tie in nicely with the Pati-Salam model

Further Work: Octonionic Triality

- Of course it would be much more exciting to find $\mathbb{O}$ triality in the Standard Model
- Perhaps the $\mathfrak{g}_{PS} \to \mathfrak{g}_{SM}$ Higgs could be written as a (complex) octonion?
- What does the division-algebraic multiplication rule actually do for the fermions?

The End

Fun with the New $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ Model

Mia Hughes

Triality Recap

The $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ SO(10) Model

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The $\mathbb{H}$iggs

Thanks for listening!

Under investigation

Under investigation

Cl(0,8) ⟷

3 generations + 3 ν_R

Gauge bosons

Higgs bosons

Counting

Under investigation

256 R ↔ **256 R**

Cl(0,8) ↔ 3 generations + 3 ν_R

Gauge bosons

Higgs bosons

+12 R

Counting

Under investigation

Cl(8)

$$[P\, i\Lambda_j,\, P\, Cl(8)] + c.c.$$

$$Cl(8) \mapsto$$

$$(4\times\underline{\mathbf{8}}) \oplus (24\times\underline{\mathbf{3}}) \oplus (18\times\underline{\mathbf{1}}) \oplus$$

$$(4\times\underline{\mathbf{1}}) \oplus (24\times\underline{\mathbf{3}}^*) \oplus (18\times\underline{\mathbf{1}}) \oplus$$

Z boson $\boxed{(4\times\underline{\mathbf{1}})} \oplus \mathcal{C}_{36}$

$SU(3)_C$ decomposition

Under investigation

Cl(8)

$[P\, i\Lambda_j,\, P\, Cl(8)] + c.c.$

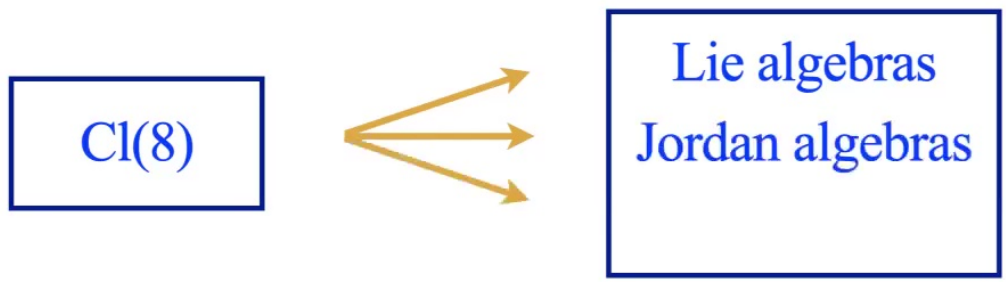
$$\sum_{i=1}^{n} P_i a\ P_i b + P_i b\ (P_i a)^\dagger$$

Multi-action

Under investigation

Cl(8)

$$\sum_{i=1}^{n} P_i a \; P_i b + P_i b \; (P_i a)^\dagger$$



Multi-action

Under investigation

$$\sum_{i=1}^{n} P_i a \; P_i b + P_i b \; (P_i a)^\dagger \qquad E_3$$

$$\sum_{i=1}^{n'} P_i' a \; P_i' b + P_i' b \; (P_i' a)^\dagger \qquad E_2$$

$$\sum_{i=1}^{n''} P_i'' a \; P_i'' b + P_i'' b \; (P_i'' a)^\dagger \qquad E_1$$

Multi-action

Under investigation

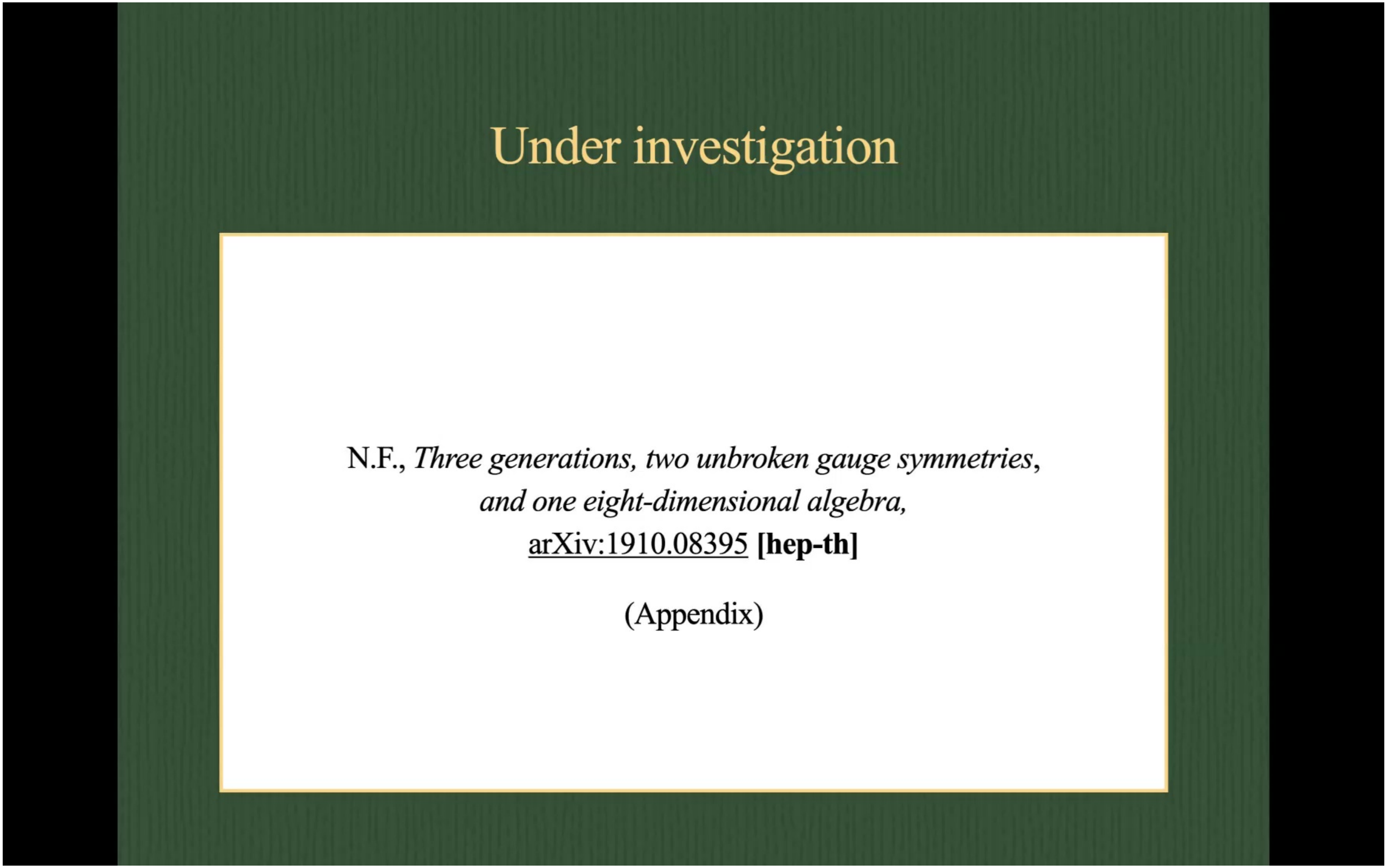
Coarse grain

$$\sum_{i=1}^{n} P_i a\; P_i b + P_i b\; (P_i a)^\dagger \qquad E_3$$

$$\sum_{i=1}^{n'} P_i' a\; P_i' b + P_i' b\; (P_i' a)^\dagger \qquad E_2$$

$$\sum_{i=1}^{n''} P_i'' a\; P_i'' b + P_i'' b\; (P_i'' a)^\dagger \qquad E_1$$

Multi-action


Under investigation
N.F., Three generations, two unbroken gauge symmetries,
and one eight-dimensional algebra,
arXiv:1910.08395 [hep-th]
(Appendix)

Under investigation

$$\sum_{i=1}^{n} P_i a \; P_i b + P_i b \; (P_i a)^\dagger \qquad E_3$$

Coarse grain ↑

$$\sum_{i=1}^{n'} P_i' a \; P_i' b + P_i' b \; (P_i' a)^\dagger \qquad E_2$$

$$\sum_{i=1}^{n''} P_i'' a \; P_i'' b + P_i'' b \; (P_i'' a)^\dagger \qquad E_1$$

Multi-action