Title: Spinning black holes as cosmic string factories

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Abstract: I will discuss what happens when a black hole captures a much larger in size cosmic string loop. In some cosmological scenarios, such encounters are not unlikely for supermassive black holes in galactic nuclei, and for primordial black holes. The talk will feature some fun physics and geometry: non-flat quadrilaterals, black-hole superradiance, one-dimensional geometric flows, and persistent ultra-relativistic gravitational-wave whips.
Spinning black holes as cosmic string factories.

with Hengrui Xing
Andrei Gutsanov
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Black holes

1. Fundamental, simple objects in physics
3. Abundant, observed, hugely important for astrophysics

Cosmic strings

1. Topological remnants from early Universe.
2. Fundamental relativistic objects Nambu-Goto action single metric.
3. Dimensionless tension μ.
4. Never observed.
5. Search method: gravitational waves

BOMB!
String formation

\[ \langle \Phi \rangle = \eta_v e^{i\theta} \]

|\Phi| = 0

\[ |\Phi| = \eta_v \]

Strings are topological remnants from very early Universe

from Ringeval 2010
Dynamics of strings.

1. They are relativistic objects.
   - Particle trajectories minimize interval in space-time.
   - String trajectories minimize area in space-time.

2. Tension = mass density. (in units with \( c = 1 \))
   \[
   \text{Tension} = \frac{\text{Energy}}{\text{Length}} \quad \text{dimension less in units with} \quad G = c = 1
   \]

String is characterized by a dimensionless tension \( \ll 1 \) \( \quad 10^{-12} < \mu < 10^{-6} \)
Interlude: spacetime metric of a string:

Cone in x-y plane
Minkowski in z-t.

Invariant w.r.t. boost along z-axis.

(a) Cosmic string

\( \Delta = 8\pi G \mu \)
3. Strings reconnect! BUT much less efficiently for fundamental strings $p < 1$
Jones et al 2003, Jackson et al 2005

4. String loops oscillate periodically.
$p = \frac{\text{length}}{c}$

5. Expanding Universe: unable volume, several long strings, self-similar evolution.
Lots of loops
Black hole fishing lines

\[ \text{stationary solution - radial string.} \]

Slow motion
String remains
nearby radial.

Black hole
is an effective
pivot point
for a string.

Key idea.
Motion of pinned loop.

\[ \mathbf{r}(\sigma, t) \quad 0 < \sigma < L \quad \text{invariant length coordinate.} \]

\[ t - \text{time.} \]

Wave equation.

\[ \frac{\partial^2 \mathbf{r}}{\partial t^2} = \frac{\partial^2 \mathbf{r}}{\partial \sigma^2} \]

\[ \mathbf{r}(0, t) = \mathbf{r}(L, t) = 0 \quad \text{pinning.} \]

\[ \frac{\partial \mathbf{r}}{\partial t} \cdot \frac{\partial \mathbf{r}}{\partial \sigma} = 0 \quad \left( \frac{\partial \mathbf{r}}{\partial t} \right)^2 + \left( \frac{\partial \mathbf{r}}{\partial \sigma} \right)^2 = 1 \quad \text{Gauge.} \]

Key idea: motion of pinned loop is specified by fixed 3-D Auxiliary curve.
Motion of pinned loop.

\[ \mathbf{r} (\sigma, t) \quad 0 < \sigma < L \quad \text{invariant length coordinate.} \]

\[ t - \text{time}. \]

Wave equation.

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**Key idea:** motion of pinned loop is specified by fixed 3-D auxiliary curve.
**Auxiliary curve**

General solution

$$\mathbf{r}(\sigma, t) = \frac{1}{2} [\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)]$$

Boundary conditions

$$\mathbf{a}(\eta) = -\mathbf{b}(-\eta),$$
$$\mathbf{a}(\eta) = \mathbf{a}(\eta + 2L)$$

General solution for a pinned loop

$$\mathbf{r}(\sigma, t) = \frac{1}{2} [\mathbf{a}(\sigma - t) - \mathbf{a}(-\sigma - t)]$$

Reconnection:

1. Find two identical chords
2. Glue together and chop off the middle
Theorem:
If auxiliary curve is space quadrilateral, the loop does not self-intersect unless

\[ QP + PS = QR + RS \]
Blow hole is not a project pivot!

3 effects.

1. Torque from BH spin.
   \[ \vec{t} = - \vec{t}_{BH} \propto \hat{n} \times (\vec{a} \times \hat{n}) \]
   Frolov, Henry, Larsen 96
   Frolov et al 89

2. Horizon friction.
   \[ \vec{t} = - \vec{t}_{BH} \propto \hat{n} \times \frac{\partial}{\partial t} \hat{n} \]
Blow hole is not a perfect pivot!

3 effects.

1. Torque from BH spin.
   \[ \mathbf{\tau} = - \mathbf{\omega}_{BH} \times \mathbf{\hat{n}} \times (\mathbf{\hat{n}} \times \mathbf{\hat{n}}) \]

For infinite string BH spin is driven into alignment with the string by projection.

\[ Q_s = 4\mu \frac{L^2}{q} \left[ \Omega_{BH} - (n \cdot \Omega_{BH}) \mathbf{n} - \mathbf{n} \times \frac{dn}{dt} \right] \]

2. Horizon friction.
   \[ \mathbf{\tau} = - \mathbf{\omega}_{BH} \times \mathbf{\hat{n}} \times \frac{d\mathbf{\hat{n}}}{dt}. \]

Frolov, Henry, Larsen 96
Frolov et al 89
Recoil: BH has finite mass, string motion not strictly periodic. Orbit changes on a timescale.
\[ \sim \frac{M_{BH}}{m} P \]
\[ M_{BH} \text{ - black hole mass} \]
\[ P \text{ - period} \]
\[ m \text{ - string loop mass} \]

Key idea: all 3 effects cause evolution (motion) of the auxiliary curve.
Motion of $A(t)$

1. Horizon function $\Rightarrow$ curve shortening

$V(t) \propto \frac{d^2 \Delta}{dt^2}$

Asymptotically, $\Delta(t)$ is a shrinking circle. It disappears after finite time.

$t \sim \frac{L^2}{R^2}$

Loop is swallowed by the BH.
Horizon friction continued.

\[ a(t) \quad \Rightarrow \quad \text{circle.} \]

Some shape.

For physical loop this corresponds to

\[ \gamma \text{ GWs} \]

\[ v = c. \]

The tip moves ultrarelativistically! Waves!!
Motion of $\alpha(t)$

1. Horizon function: $\Rightarrow$ curve shortening

$$V(t) \propto \frac{d^2 \alpha}{dt^2}$$

Asymptotically, $\alpha(t)$ is a shrinking circle. disappears after finite time.

$$t \sim \frac{L^2}{R^2}$$

Loop is swallowed by the BH.
Horizon friction continued.

\[\text{circle.}\]

For physical loop this corresponds to

\[\text{ultra-relativistic waves}!!\]

\[\begin{array}{c}
\text{some shape.} \\
\end{array}\]
Motion of $\vec{a}(\delta)$

1. Torque from BH spin.
   $$\vec{\tau}(\delta) \propto \frac{\partial \vec{a}}{\partial \delta} \times \vec{J} \text{BH}.$$  
   - typically leads to loop lengthening
   - amplifies short-wavelength helical wigglies.

$\Rightarrow$ superradiance!!
$\Rightarrow$ Black-hole Bomb!!
Motion of $\vec{a}(\sigma)$

2. Torque from BH spin.

$\vec{T}(\sigma) \propto \vec{a} \times R \vec{\Omega}_{BH}$

- Typically leads to loop lengthening
- Amplifies short-wavelength helical wiggles.

$\Rightarrow$ Superradiance!!

$\Rightarrow$ Black-hole bomb!!

$v(\sigma) = \frac{8R^2}{L} [a' \sigma \times \Omega_{BH} + a''(\sigma)]$. 
Superradiance. Black hole bomb.

Zeldovich 71
Press & Teukolsky 72

Helical wave amplifies upon reflection. Then travels to the other side...

and amplifies again!!

Growth timescale \( \sim \left( R / R_{\text{BH}} \right)^2 \tau \)
Supervariance in auxiliary curve.

Theorem: self-intersection of \( \overline{\mathcal{L}} \) - extra captured loop

Theorem: simplest asymptotic state - expanding circular loop
Carousel of double lines.

OK, but what about reconnections?

Theorem: non-intersecting asymptotic double lines exist:

Are they common? Are they even reached? we don't know.
Ok. What about real world?

Small-tension loops can get captured by forming halos.

Growing black holes increase density of loops near them.

Captures are likely for sufficiently small $P$ or reconnection probability $P$.

If loops survive they can tap spin energy of SMBH.
Numbers.

Energy BH spins \( \sim 10^{-7} \left( \frac{\text{BH mass}}{\text{M}_\odot} \right)^2 \).

Energy of the universe huge!!

LISA will detect stochastic background.

\[ \frac{\text{Energy GW}}{\text{Energy of the Universe}} \sim 10^{-12.5} \]

SMBH spin down on timescale.

\[ t_{\text{spin down}} \sim \frac{R}{\mu} = 10^{10} \text{ yr} \cdot \left( \frac{\text{BH mass}}{10^8 \text{M}_\odot} \right) \left( \frac{10^{-15}}{\mu} \right) \]
Take away messages

- Nice geometry and physics of loop + BU interaction.
  - 1-d geometric flows
  - superconductors.
  - ultra-relativistic whips.
  - etc.

- GWs would make it practically interesting.

- Much work to do: reconstructions, etc.
Superradiance. Black hole bomb.

Zel'dovich 71
Press & Teukolsky 72

Helical wave amplifies upon reflection. Then travels to the other side...

Growth timescale \( \sim (R_{\text{Schw}})^2 L \)

and amplifies again!!
Supervariance in auxiliary curve.

Theorem:
self intersection of \( \Gamma \) — extra captured loop

Theorem: simplest asymptotic state — expanding circular loop.
3. Strings reconnect!

BUT much less efficiently for fundamental strings $p < 1$
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4. String loops oscillate periodically:

$$p = \frac{\text{length}}{c}$$

5. Expanding Universe:

- Volume
- Several long strings
- Lots of loops

self-similar evolution.