Title: Spinning black holes as cosmic string factories

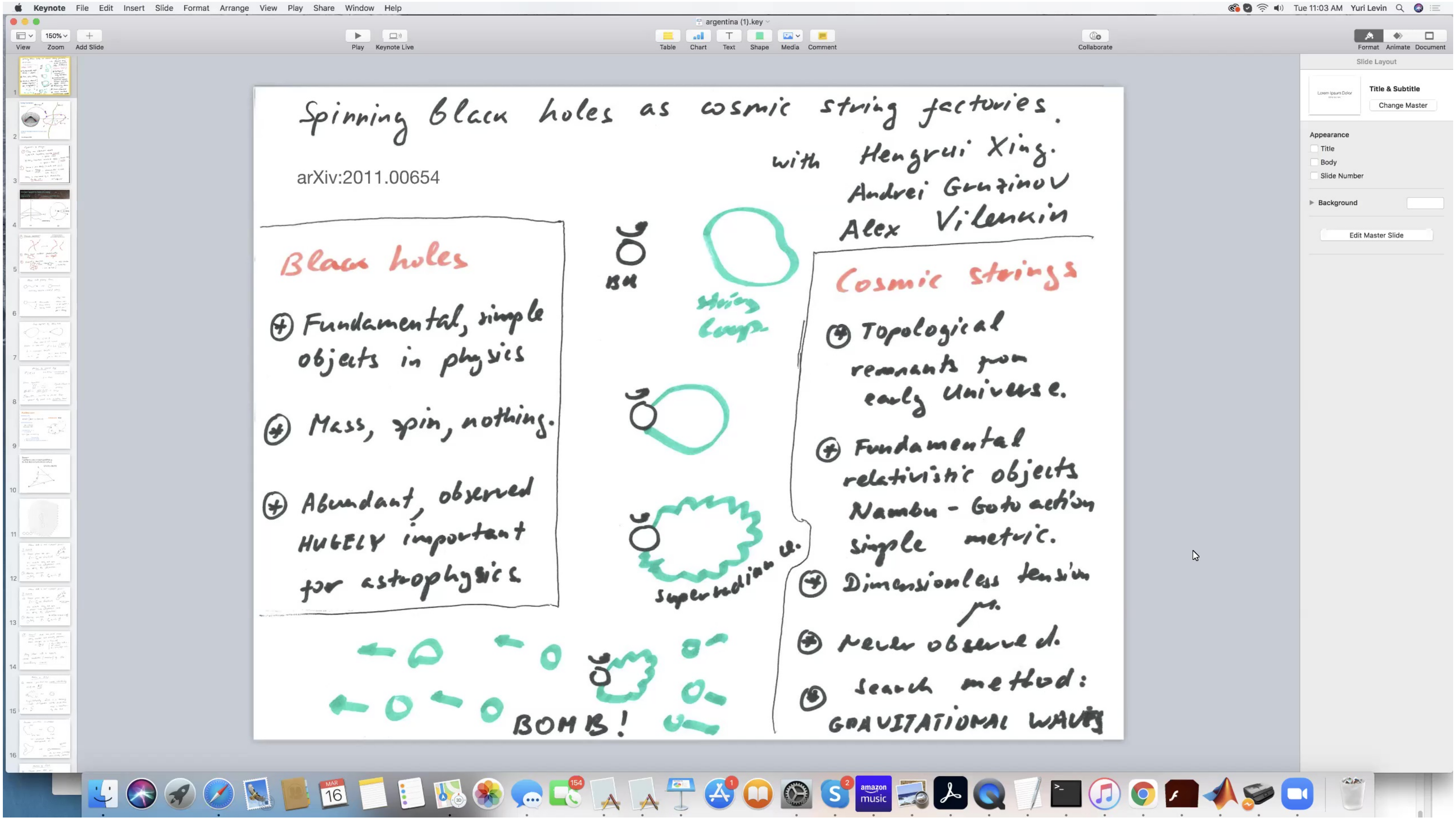
Speakers: Yuri Levin

Series: Cosmology & Gravitation

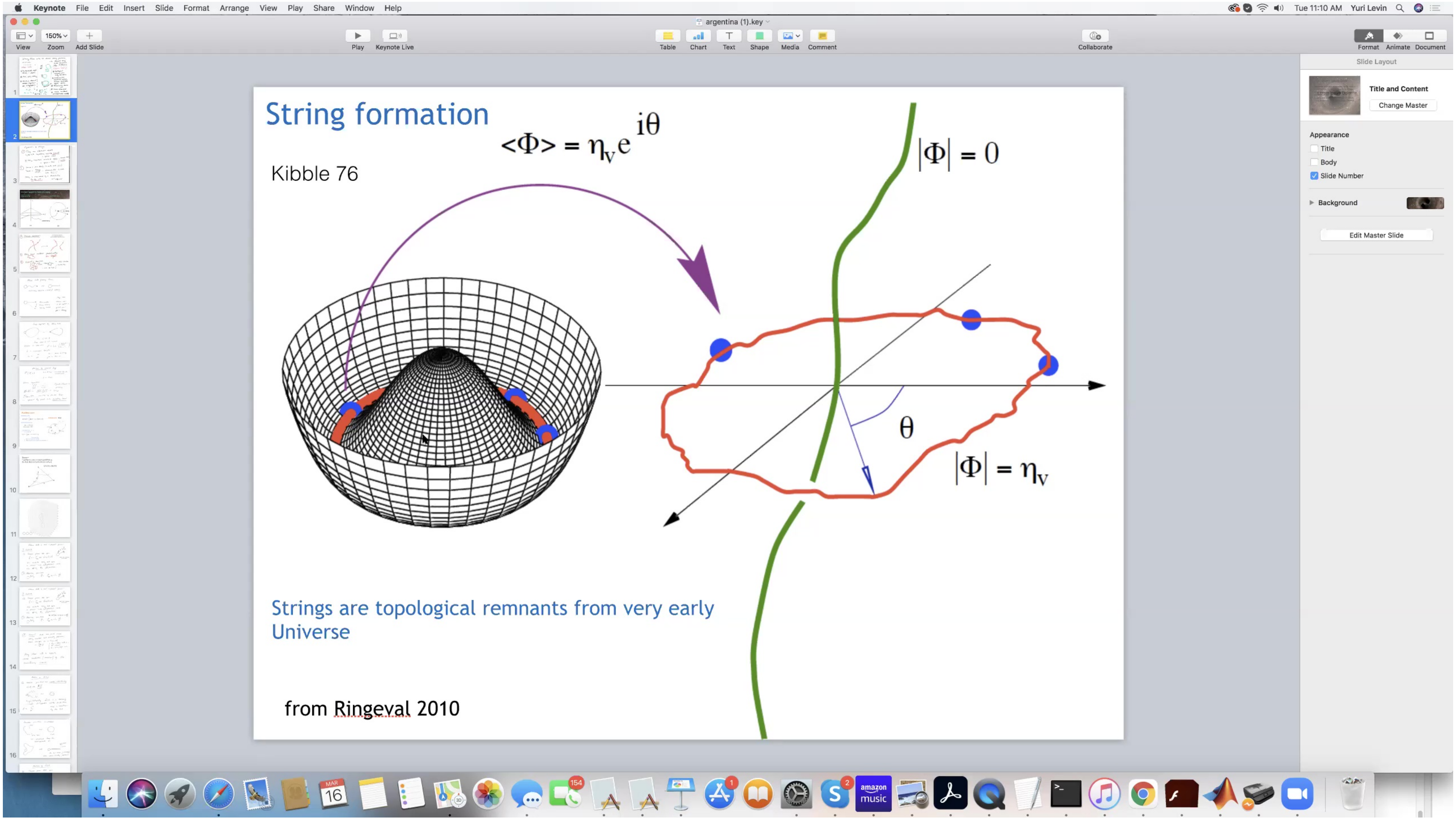
Date: March 16, 2021 - 11:00 AM

URL: http://pirsa.org/21030010

Abstract: I will discuss what happens when a black hole captures a much larger in size cosmic string loop. In some cosmological scenarios, such encounters are not unlikely for supermassive black holes in galactic nuclei, and for primordial black holes. The talk will feature some fun physics and geometry: non-flat quadrilaterals, black-hole superradiance, one-dimensional geometric flows, and persistent ultra-relativistic gravitational-wave whips.



Spinning black holes as cosmic string factories.
arXiv:2011.00654
with Hengrui Xing.
Andrei Gruzinov
Alex Vilenkin
Black holes
⊕ Fundamental, simple objects in physics
⊕ Mass, spin, nothing.
⊕ Abundant, observed HUGELY important for astrophysics
BH
String Loops
Superradiance.
BOMB!
Cosmic strings
⊕ Topological remnants from early Universe.
⊕ Fundamental relativistic objects Nambu - Goto action simple metric.
⊕ Dimensionless tension μ
⊕ Never observed.
⊕ Search method: GRAVITATIONAL WAVES


String formation
$\langle\Phi\rangle = \eta_v e^{i\theta}$
Kibble 76
$|\Phi| = 0$
θ
$|\Phi| = \eta_v$
Strings are topological remnants from very early Universe
from Ringeval 2010

Dynamics of strings.

1. They are relativistic objects.

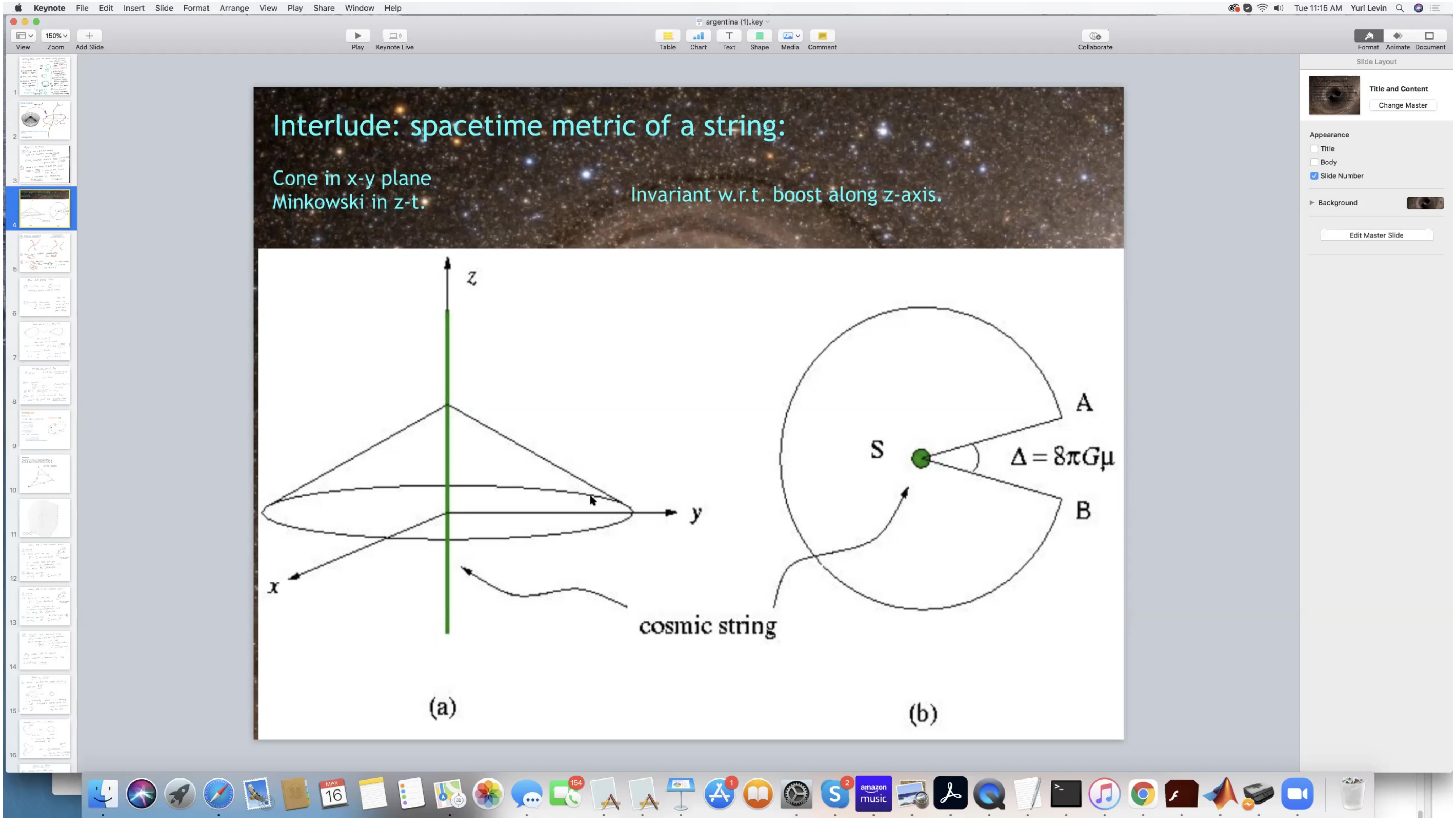
- particle trajectories minimize interval in space-time.
- string trajectories minimize area in space-time. | Nambu-Goto action.

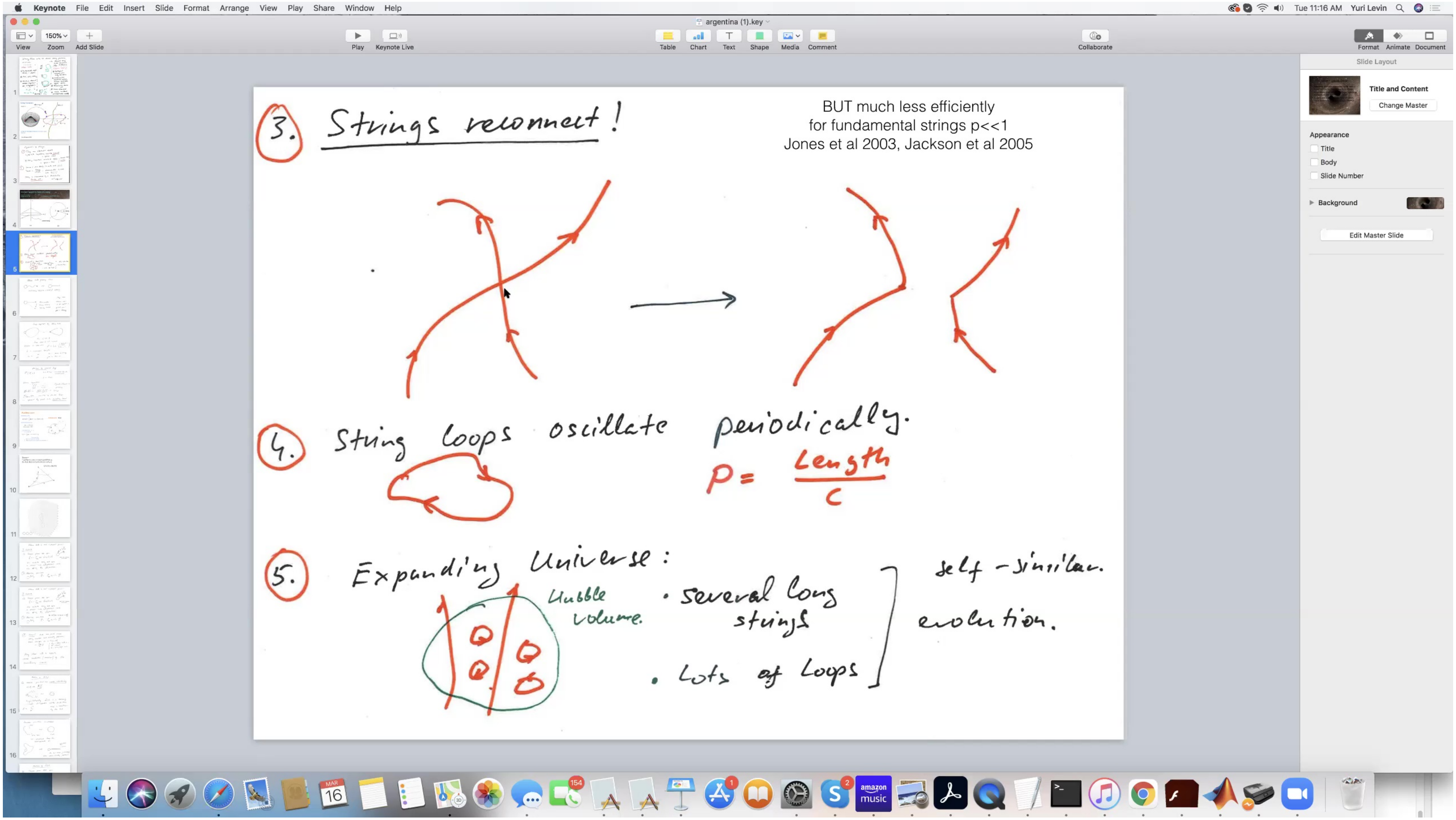
2. Tension = mass density. (in units with $c=1$)

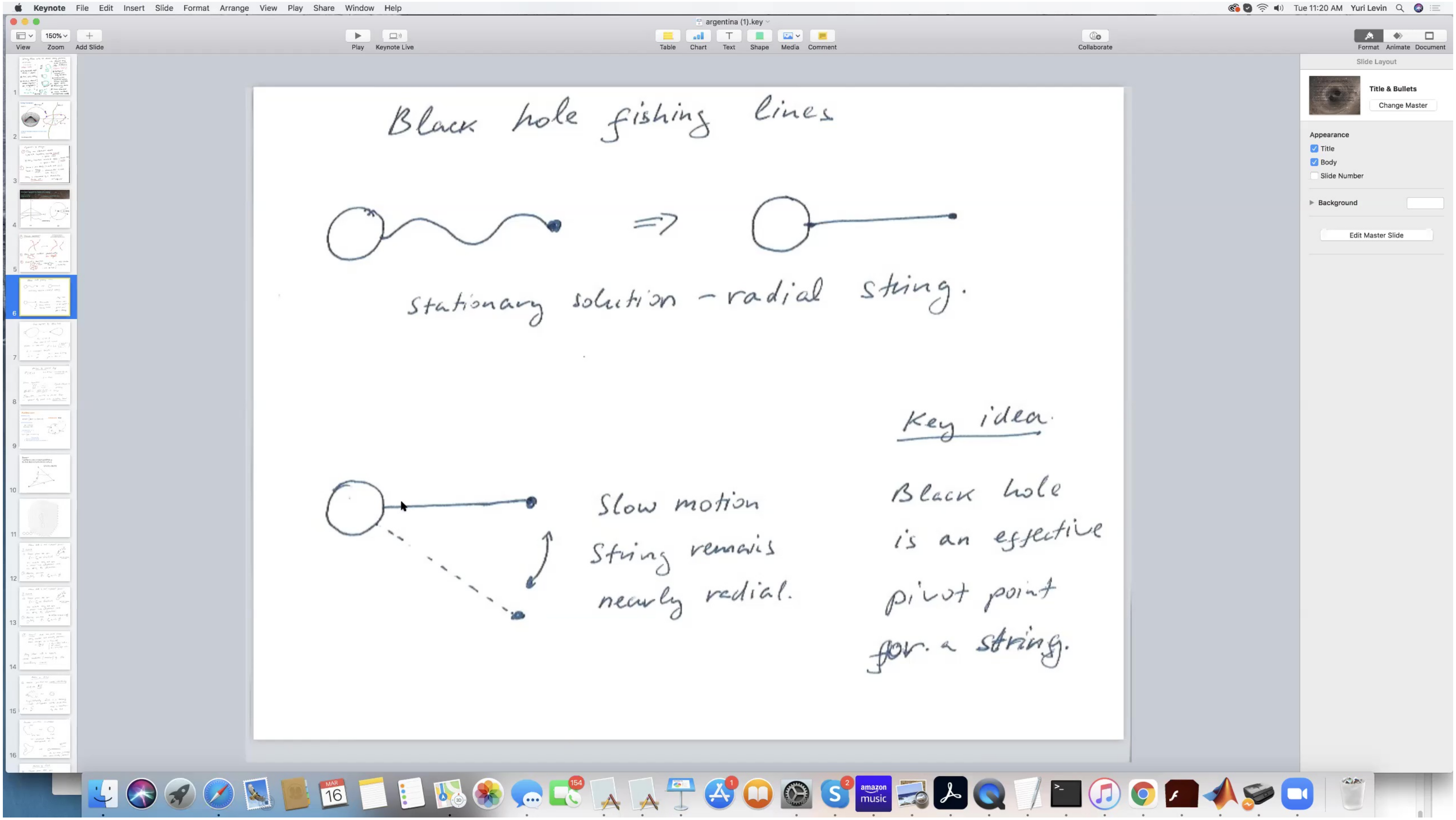
Tension $= \dfrac{\text{Energy}}{\text{Length}}$ – dimensionless in units with $G = c = 1$

String is characterized by a dimensionless tension $\ll 1$

$$10^{-32} < G\mu < 10^{-6}$$


Interlude: spacetime metric of a string:
Cone in x-y plane
Minkowski in z-t.
Invariant w.r.t. boost along z-axis.
z
y
x
cosmic string
(a)
S
A
B
Δ = 8πGμ
(b)


3. Strings reconnect!
BUT much less efficiently
for fundamental strings p<<1
Jones et al 2003, Jackson et al 2005
4. String loops oscillate periodically.
P = Length / c
5. Expanding Universe:
Hubble volume.
• several long strings
• Lots of Loops
self-similar evolution.


Black hole fishing lines
Stationary solution - radial string.
Slow motion
String remains
nearly radial.
Key idea.
Black hole
is an effective
pivot point
for a string.

Motion of pinned loop.

$\vec{r}(\sigma, t)$ $\quad 0 < \sigma < L$ invariant length coordinate.

t – time.

Wave equation.

$$\frac{\partial^2 \vec{r}}{\partial t^2} = \frac{\partial \vec{r}}{\partial \sigma^2}$$

$\vec{r}(0,t) = \vec{r}(L,t) = 0$

pinning.

$$\frac{\partial \vec{r}}{\partial t} \cdot \frac{\partial \vec{r}}{\partial \sigma} = 0 \qquad \left(\frac{\partial \vec{r}}{\partial t}\right)^2 + \left(\frac{\partial \vec{r}}{\partial \sigma}\right)^2 = 1$$

Gauge.

Key idea: motion of pinned loop is specified by fixed 3-D Auxiliary curve.

Motion of pinned loop.

$\vec{r}(\sigma, t)$ $\quad$ $0 < \sigma < L$ invariant length coordinate.

t – time.

Wave equation.

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$$\frac{\partial \vec{r}}{\partial t} \cdot \frac{\partial \vec{r}}{\partial \sigma} = 0 \qquad \left(\frac{\partial \vec{r}}{\partial t}\right)^2 + \left(\frac{\partial \vec{r}}{\partial \sigma}\right)^2 = 1 \quad \text{Gauge.}$$

Key idea: motion of pinned loop is specified by fixed 3-D Auxiliary curve.

Auxiliary curve

General solution

$$\mathbf{r}(\sigma,t) = \frac{1}{2}\left[\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)\right]$$

Boundary conditions

$$\mathbf{a}(\eta) = -\mathbf{b}(-\eta),$$
$$\mathbf{a}(\eta) = \mathbf{a}(\eta + 2L).$$

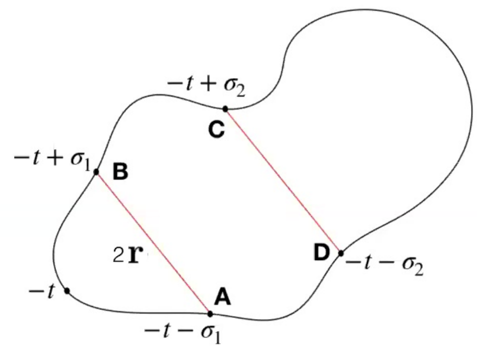
General solution for a pinned loop

$$\mathbf{r}(\sigma,t) = \frac{1}{2}[\mathbf{a}(\sigma - t) - \mathbf{a}(-\sigma - t)]$$

Reconnection:

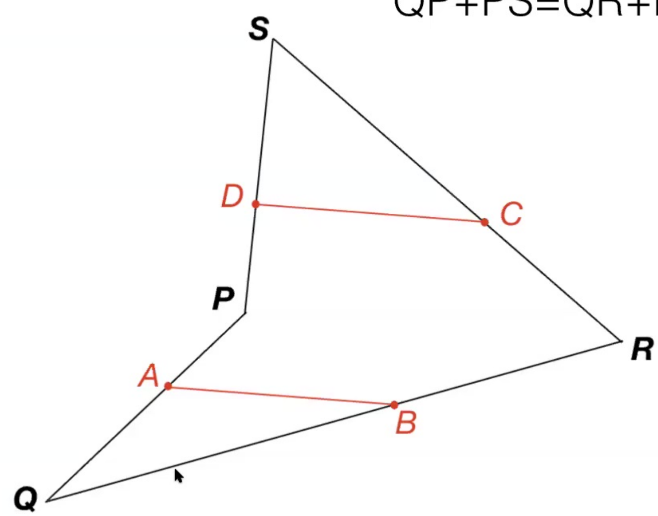
1. Find two identical chords
2. Glue together and chop off the middle

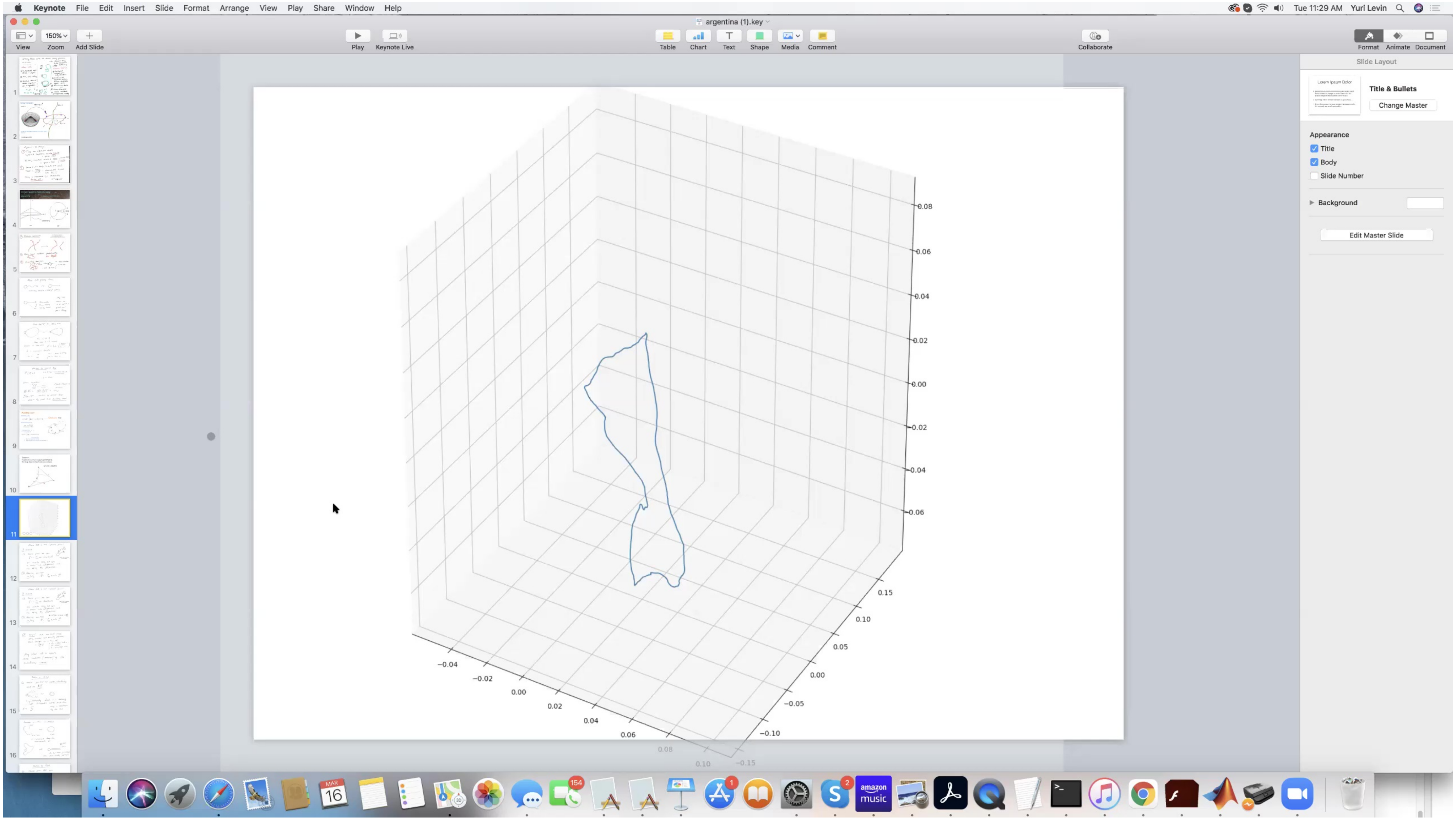
Auxiliary curve $\mathbf{a}(\eta)$



Theorem:
If auxiliary curve is space quadrilateral,
the loop does not self-intersect unless

$$QP+PS=QR+RS$$

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Tue 11:29 AM Yuri Levin
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Appearance
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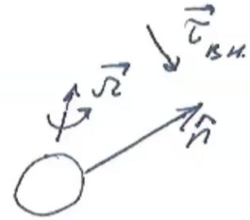
amazon music

Black hole is not a perfect pivot.!

3 effects.



(1) Torque from BH spin.

$$\vec{\tau} = -\vec{\tau}_{BH} \propto \hat{n}\times(\vec{\Omega}\times\vec{n})$$

Frolov, Henry, Larsen 96
Frolov et al 89

For infinite string, BH spin is driven into alignment with the string by projection.

(2) Horizon friction.

$$\vec{\tau} = -\vec{\tau}_{BH} \propto -\hat{n}\times\frac{d\hat{n}}{dt}.$$

Black hole is not a perfect pivot.!

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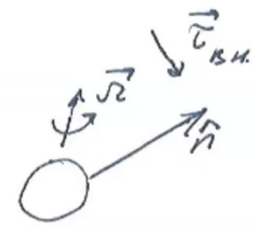

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$$\mathbf{Q_s} = 4\mu R^2 \left[\mathbf{\Omega}_{\rm BH} - (\mathbf{n} \cdot \mathbf{\Omega}_{\rm BH})\, \mathbf{n} - \mathbf{n} \times \frac{d\mathbf{n}}{dt} \right]$$

(2) Horizon friction.

$$\vec{\tau} = -\vec{\tau}_{BH} \propto -\hat{n} \times \frac{d\hat{n}}{dt}.$$

(3) Recoil: BH has finite mass

string motion not strictly periodic.

Orbit changes on a timescal.

$$\sim \frac{M_{BH}}{m} P$$

M_{BH} - black hole mass

P - period.

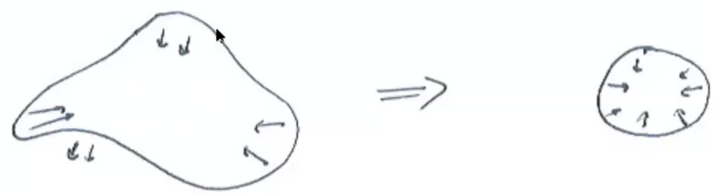
m - string loop mass

Key idea: all 3 effects cause evolution (motion) of the auxiliary curve.

Motion of $\vec{a}(\sigma)$

① Horizon friction. $\Rightarrow$ curve shortening

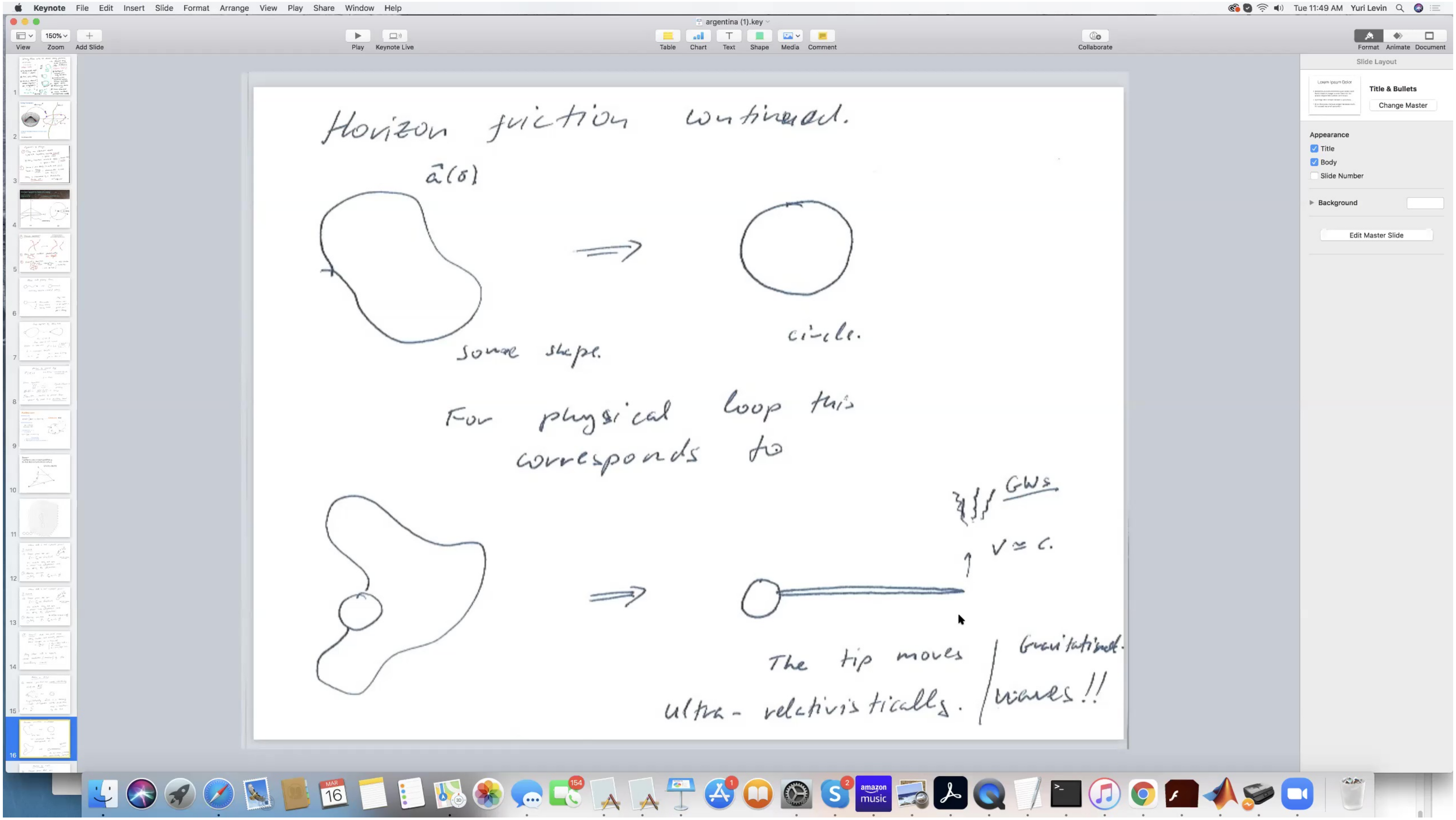
$$V(\sigma) \propto \frac{\partial^2 \vec{a}}{\partial \sigma^2}$$



Asymptotically $\vec{a}(\sigma)$ is a shrinking circle. disappears after finite time.

$$t \sim \frac{L^3}{R^2}$$

Loop is swallowed by the BH.



Horizon friction continued.
$\vec{a}(\sigma)$
some shape.
circle.
For physical loop this corresponds to
GWs
$v \simeq c$.
The tip moves ultra-relativistically.
Gravitational waves!!

Motion of $\vec{a}(\sigma)$

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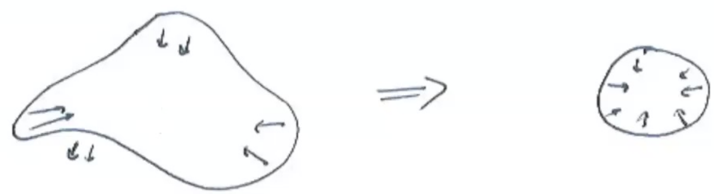

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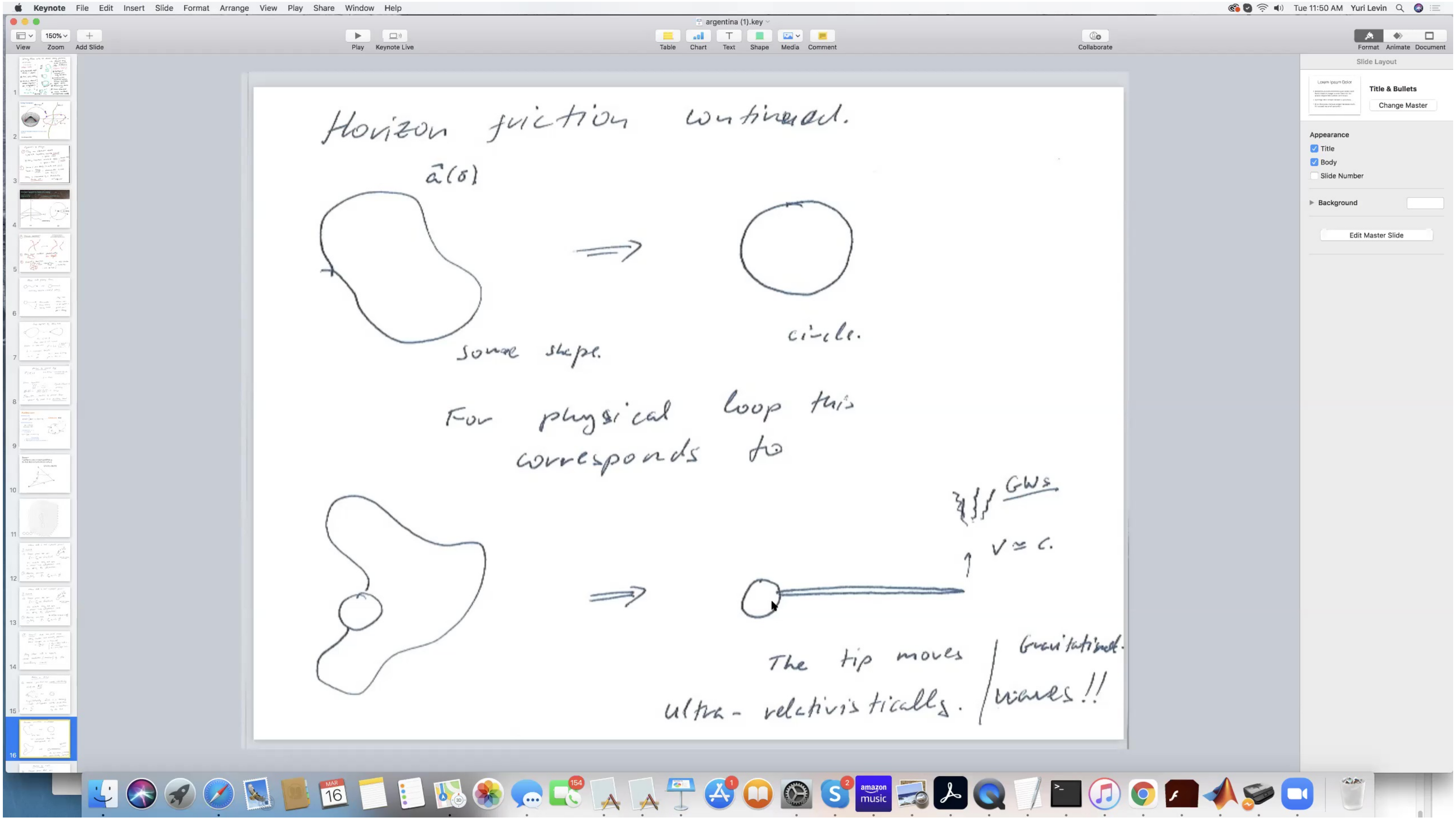
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Motion of $\vec{a}(\sigma)$

② Torque from BH spin.

$$\vec{V}(\sigma) \propto \frac{\partial \vec{a}}{\partial \sigma} \times \vec{\Omega}_{BH.}$$

- typically leads to loop lenghening
- amplifies short-wavelength helical wiggles.

$\Rightarrow$ Superradince!!

$\Rightarrow$ Black-hole bomb!!

Motion of $\vec{a}(\sigma)$

(2) Torque from BH spin.

$$\vec{V}(\sigma) \propto \frac{\partial \vec{a}}{\partial \sigma} \times \vec{\Omega}_{BH}.$$

- typically leads to loop lengthening
- amplifies short-wavelength helical wiggles.

$\Rightarrow$ superradiance!!

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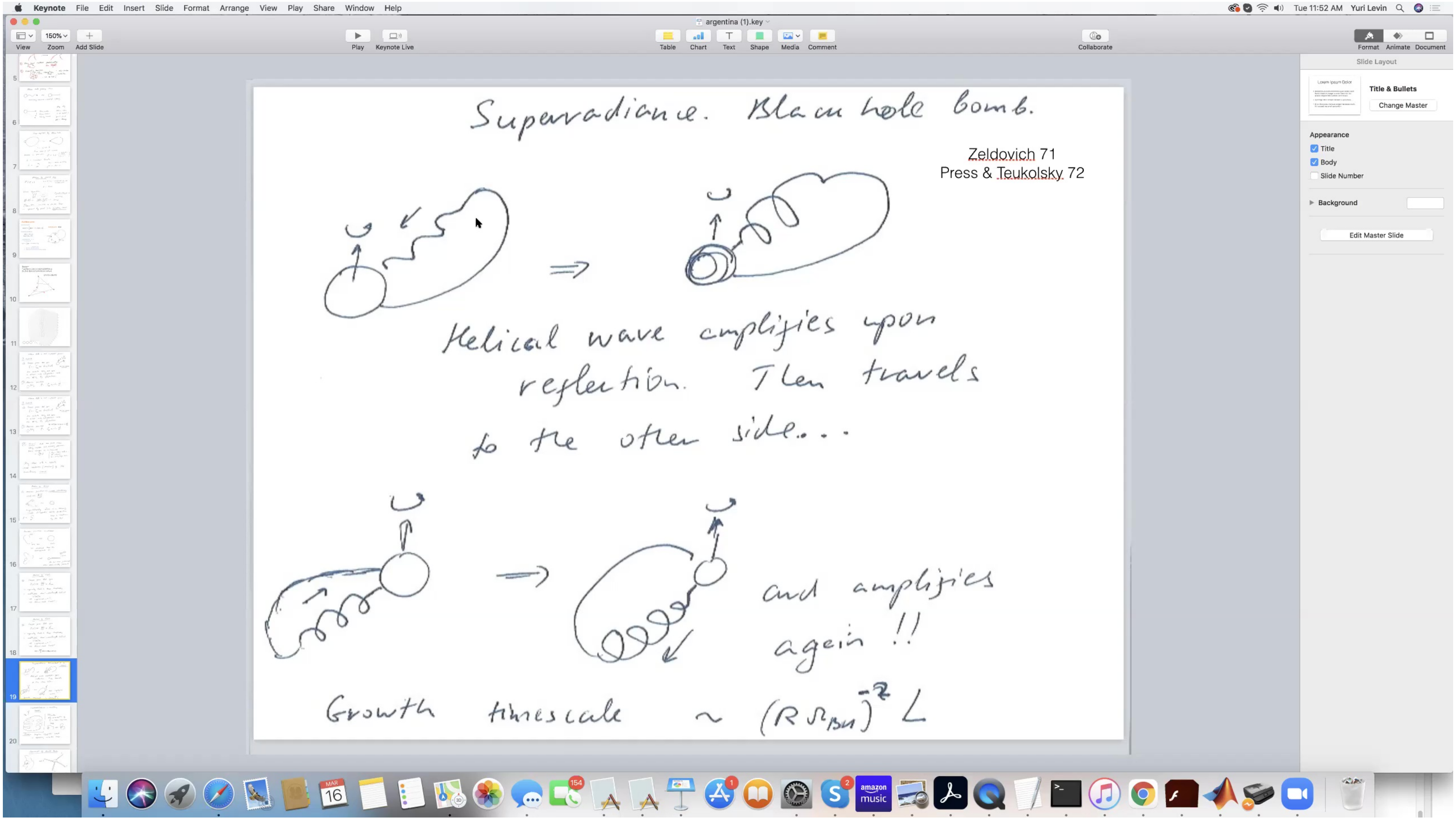
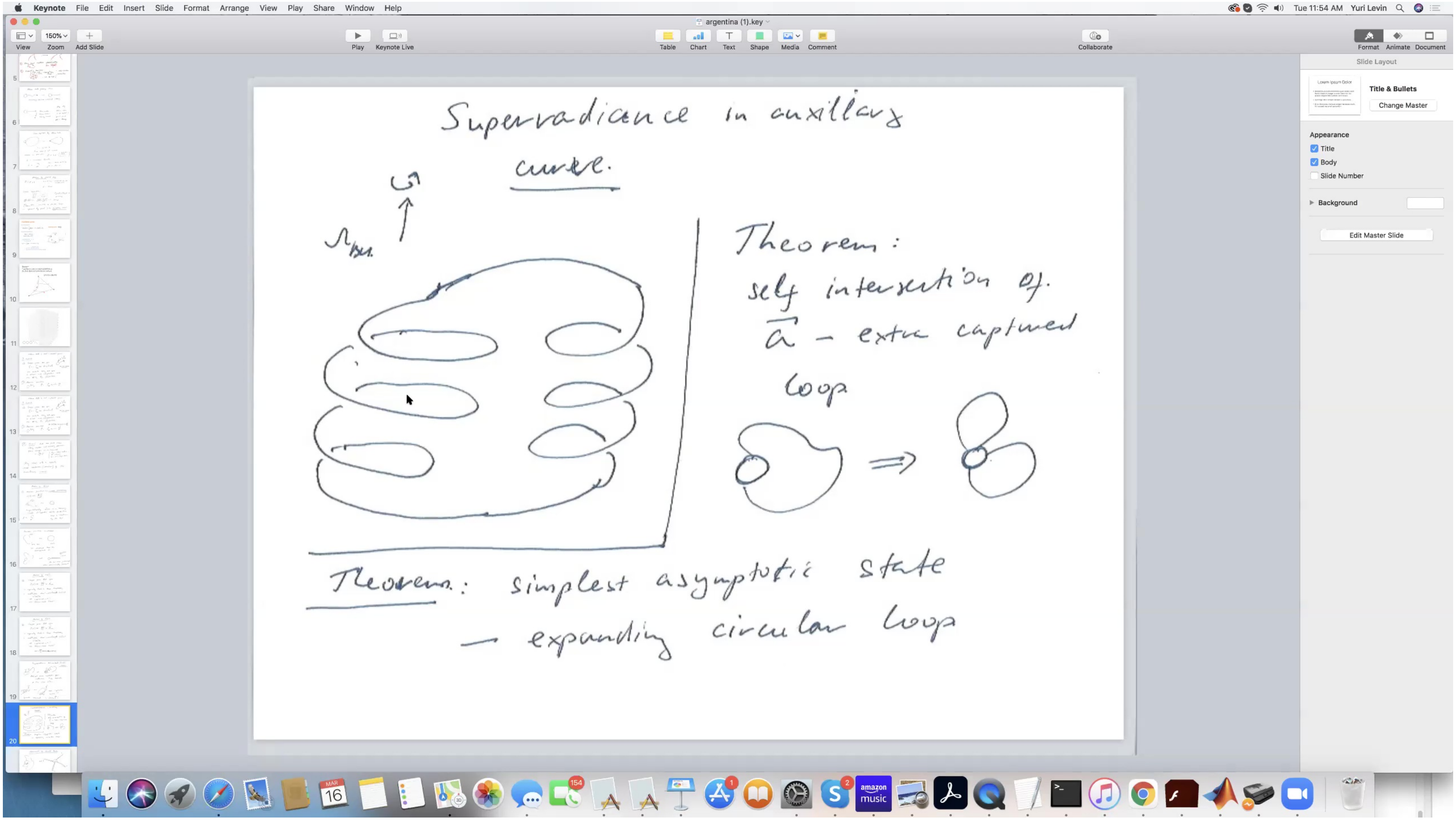
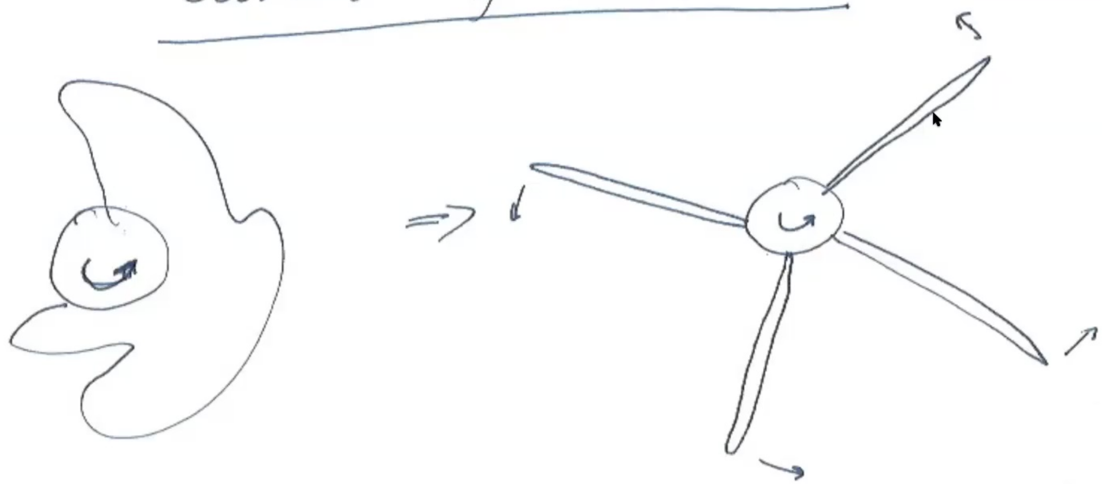
$$\mathbf{v}(\sigma) = \frac{8R^2}{L}\left[\mathbf{a}'(\sigma) \times \mathbf{\Omega}_{\rm BH} + \mathbf{a}''(\sigma)\right].$$


Superradiance. Black hole bomb.
Zeldovich 71
Press & Teukolsky 72
Helical wave amplifies upon reflection. Then travels to the other side...
and amplifies again !!
Growth timescale ~ $(R\Omega_{BH})^{-2} L$


Superradiance in auxillary curve.
Λ_{ext}
Theorem:
self intersection of
$\bar{a}$ – extra captured
loop
Theorem: simplest asymptotic state
– expanding circular loop

Carousel of double lines.



Ok, but what about reconnections?

Theorem: non-intersecting asymptotic double lines exist.

Are they common? Are they ever reached? We don't know.

Ok. What about real world?

Chernoff 09
Jain & Vilenkin 20

- Small-tension loops can get captured by forming halos.
- Growing black holes increase density of loops near them.
- Captures are likely for sufficiently small μ or reconnection probability p.
- If loops survive they can tap spin energy of SMBH.

Numbers.

$$\frac{\text{Energy BH spins}}{\text{Energy of the Universe}} \sim 10^{-7} (\nu_{BH}\,\kappa)^2$$

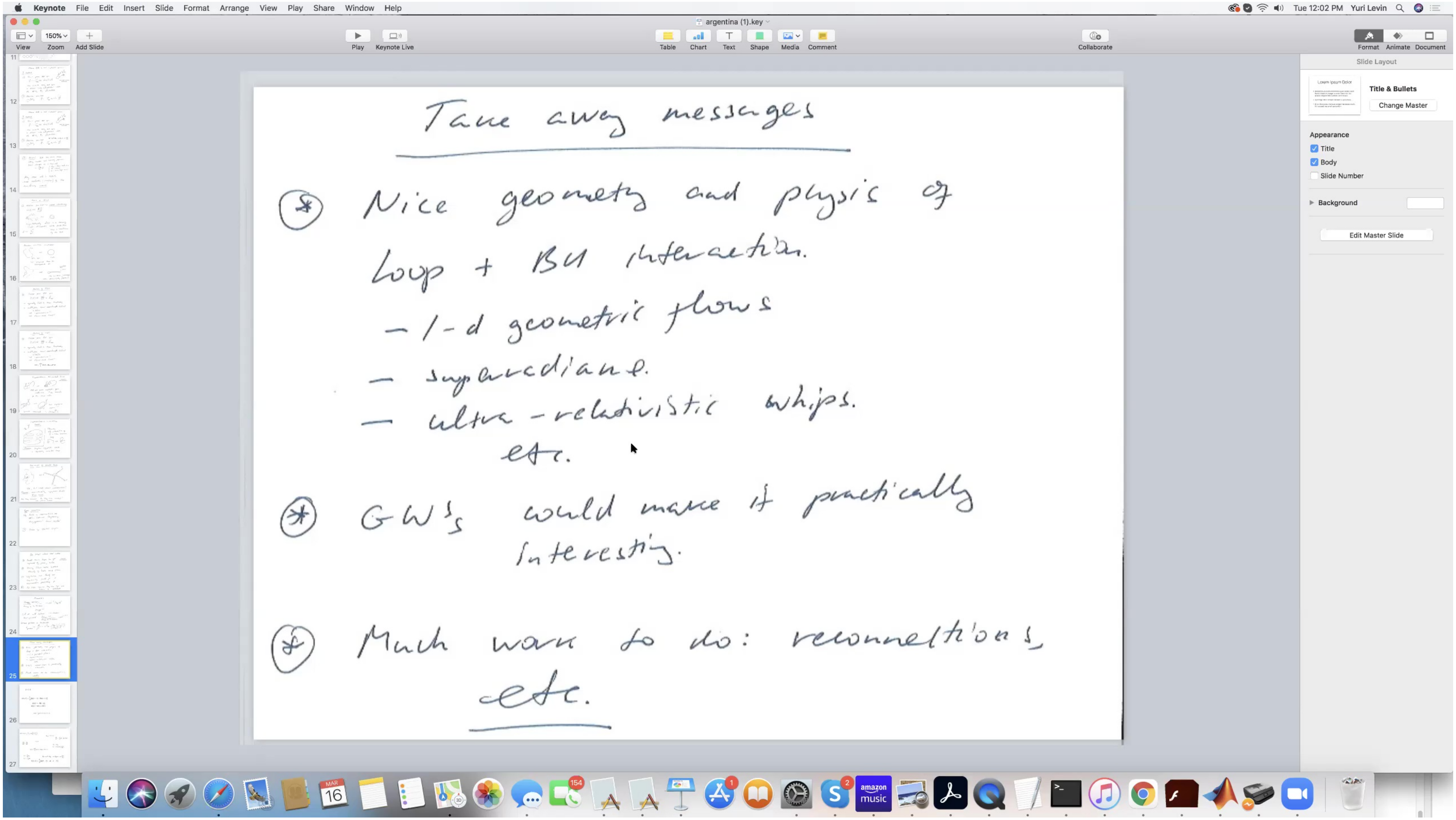
Huge!!

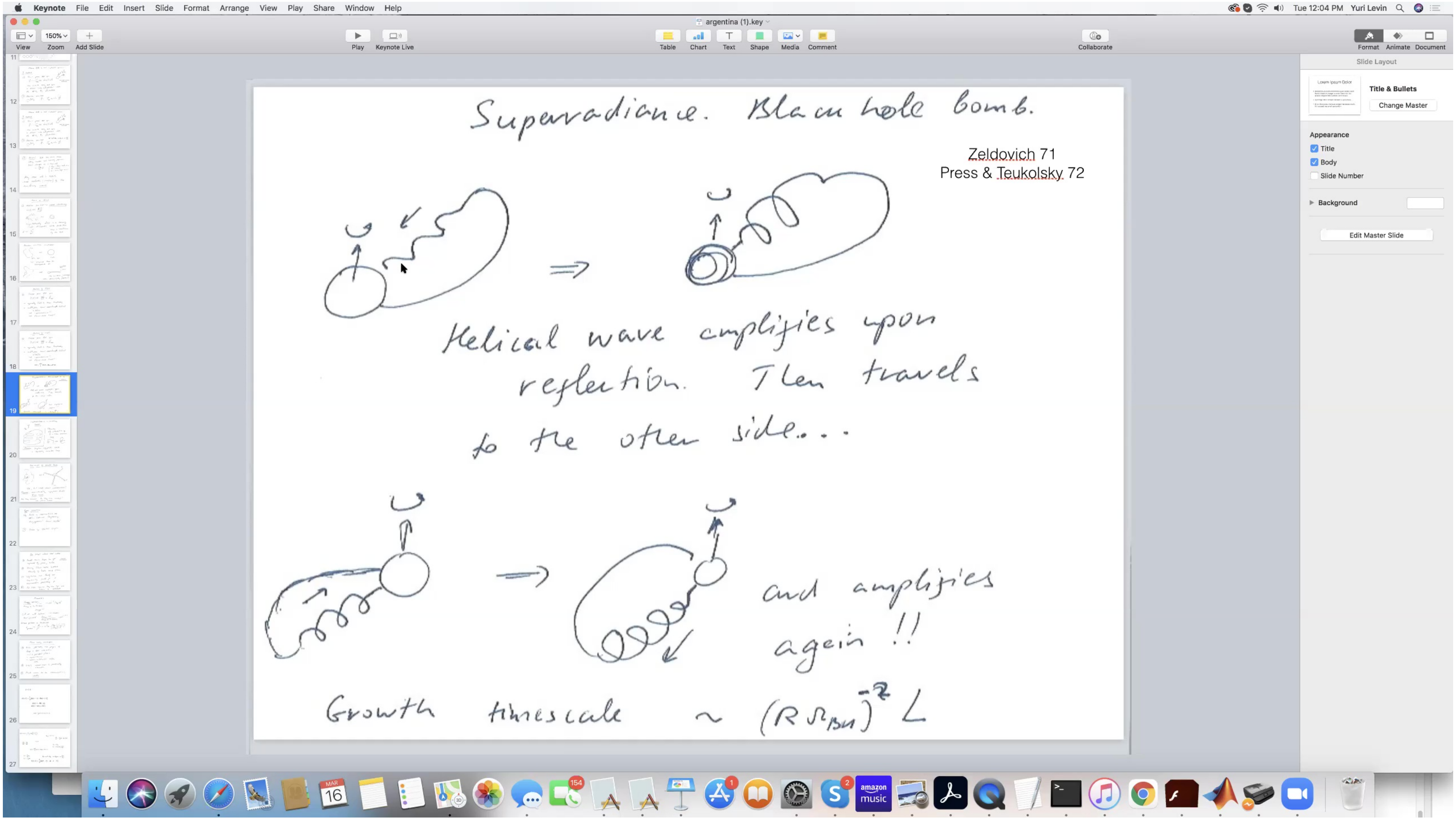
LISA will detect stochastic background.

$$\frac{\text{Energy GW}}{\text{Energy of the Universe}} \sim 10^{-13.5}$$

SMBH spindown on timescale.

$$t_{\text{spindown}} \sim \frac{R}{\mu} = 10^{10}\,\text{yr} \cdot \left(\frac{M_{BH}}{10^8 M_\odot}\right)\left(\frac{10^{-15}}{\mu}\right)$$


Take away messages
(*) Nice geometry and physics of
Loop + BH interaction.
– 1-d geometric flows
– superradiance.
– ultra-relativistic whips.
etc.
(*) GW's would make it practically interesting.
(*) Much work to do: reconnections,
etc.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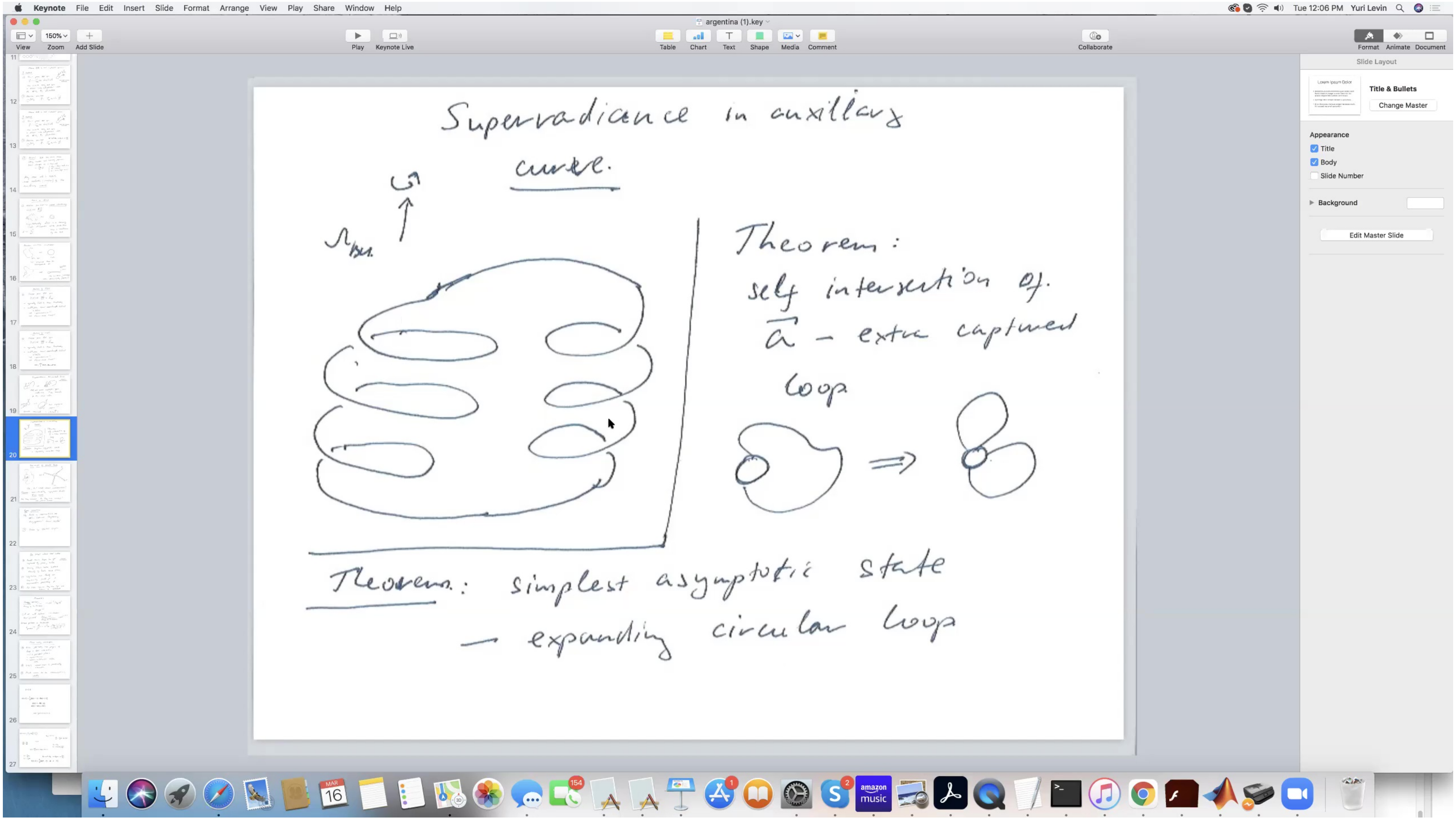
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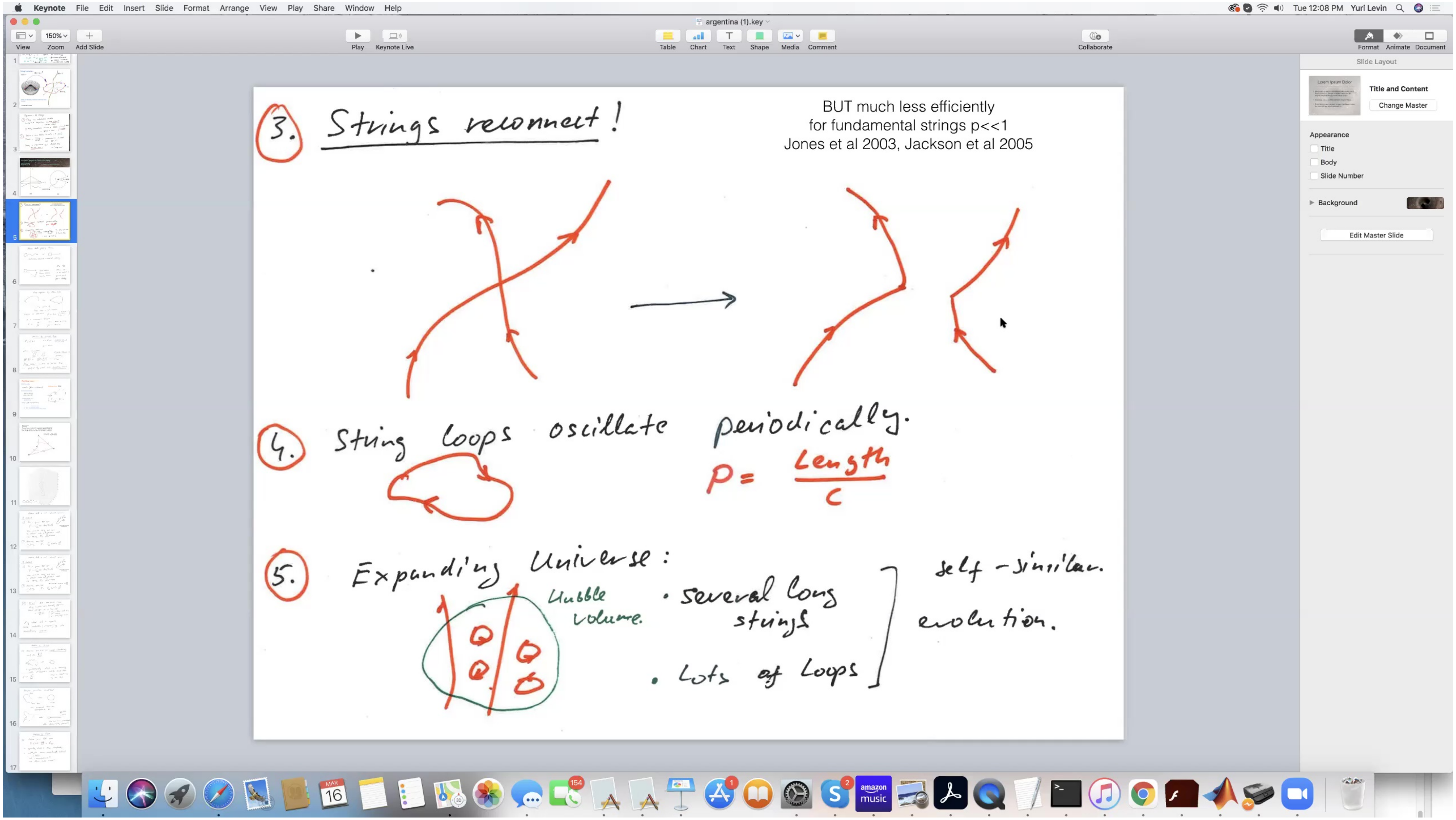
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