Title: How can statistical mechanics help ecology?

Speakers: Ricardo Garcia

Series: Colloquium

Date: March 31, 2021 - 2:00 PM

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Abstract: Statistical mechanics is the branch of physics that explains how macroscopic properties of matter emerge from the behavior of its microscopic constituents. Population ecology studies how and why populations change over time and space, primarily due to the interaction among individuals and between individuals and the environment where they thrive. Although seemingly very different, both disciplines aim to explain large-scale phenomena based on a description of their underlying drivers, and statistical mechanics tools have been largely used to formalize population ecology. For over 100 years, however, mathematical models in population ecology have relied on very strong and unrealistic assumptions about the way individuals move and get to interact with each other and with the environment. Specifically, they assume that individuals behave like the molecules of an ideal gas: following completely random trajectories through the entire area occupied by the population and only interacting with each other when their trajectories intersect.

In this presentation, I will first discuss why mathematical models are powerful tools to understand ecological processes. Then, I will show how traditional models of population dynamics emerge from ideal gas assumptions for individual movement and briefly touch on our recent efforts to refine those models combining more elaborated tools from statistical physics, random walk theory, and GPS tracking data of natural populations.

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How can statistical mechanics help ecology?

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Perimeter Institute, March 2021

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How can statistical mechanics help ecology?

and the other way around!!

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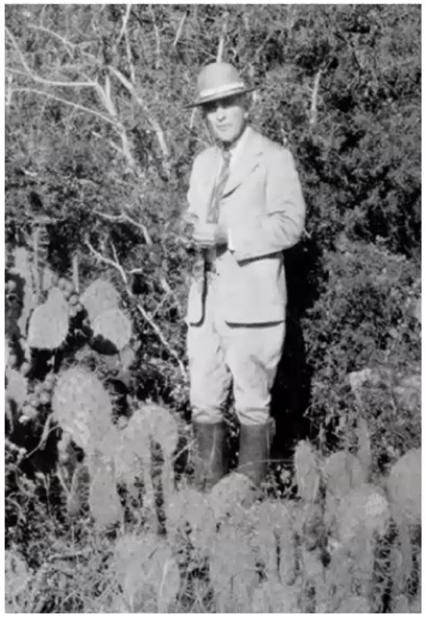
Perimeter Institute, March 2021

Physics and ecology?

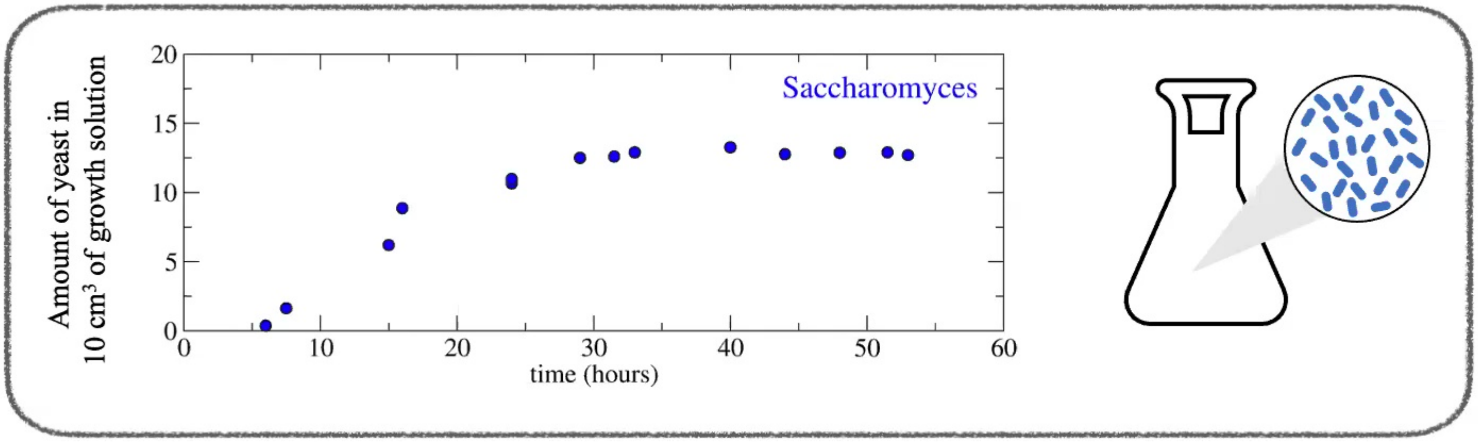
Ecology developed from natural history

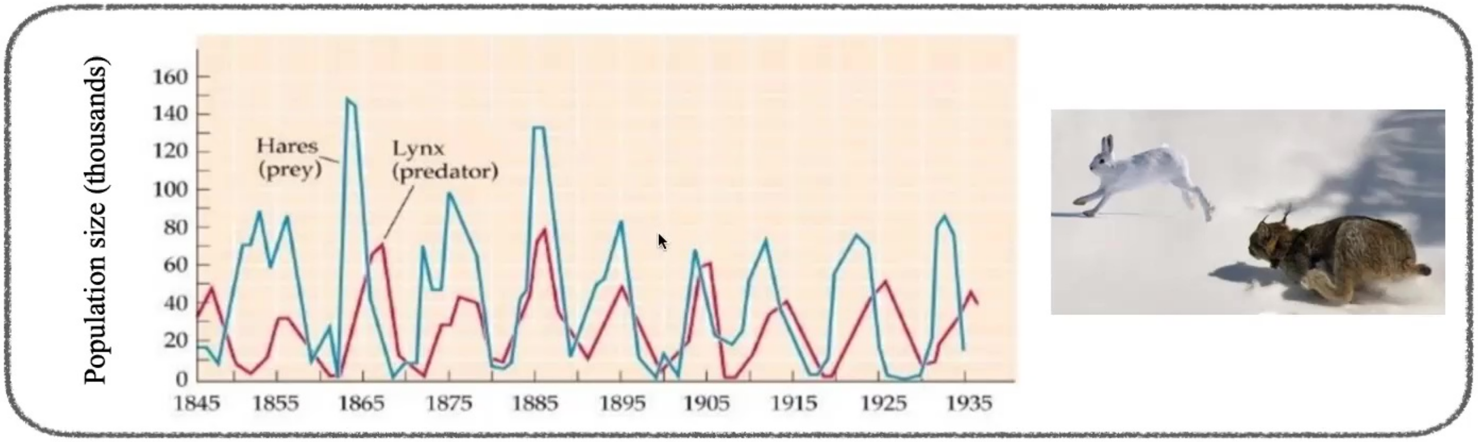
Stephen A. Forbes



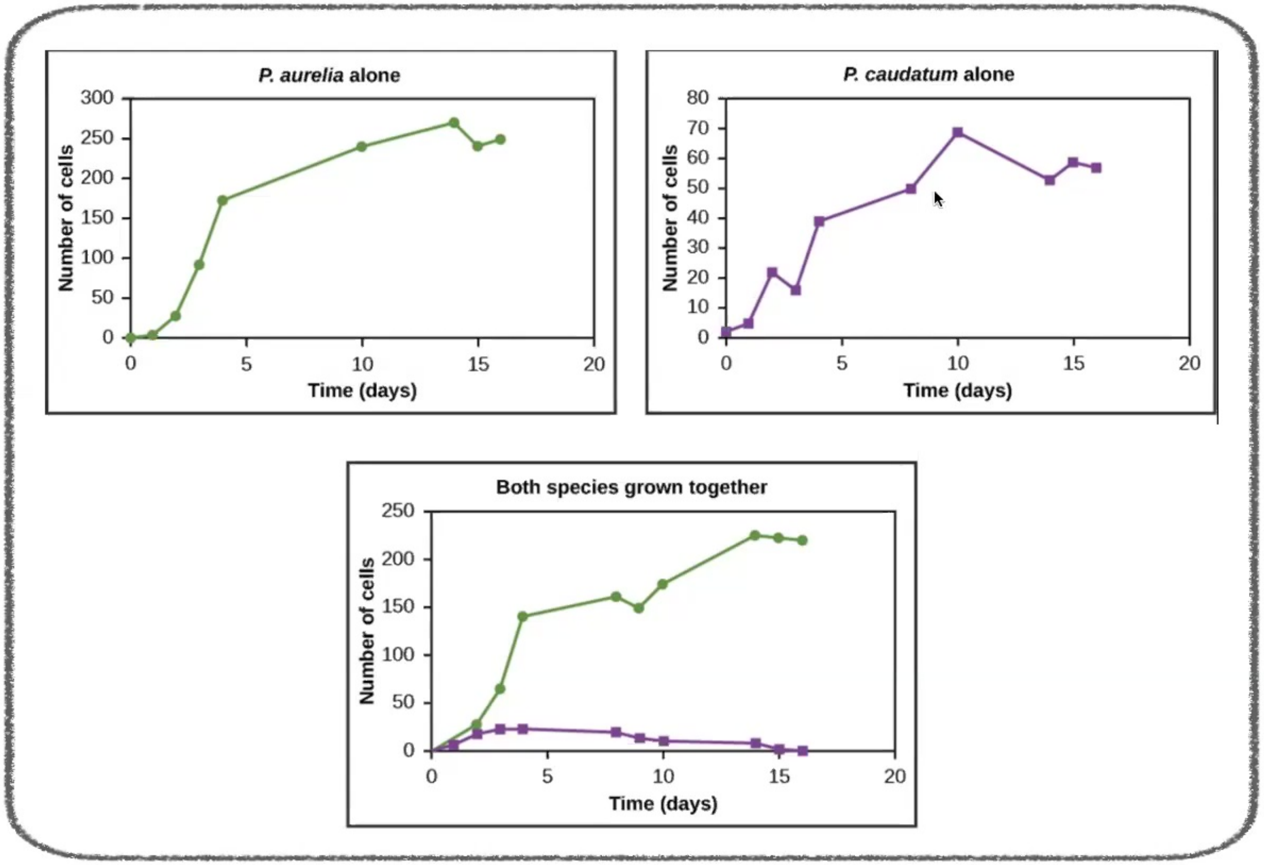
Frederic E. Clements

Ecological pattern → dynamical process

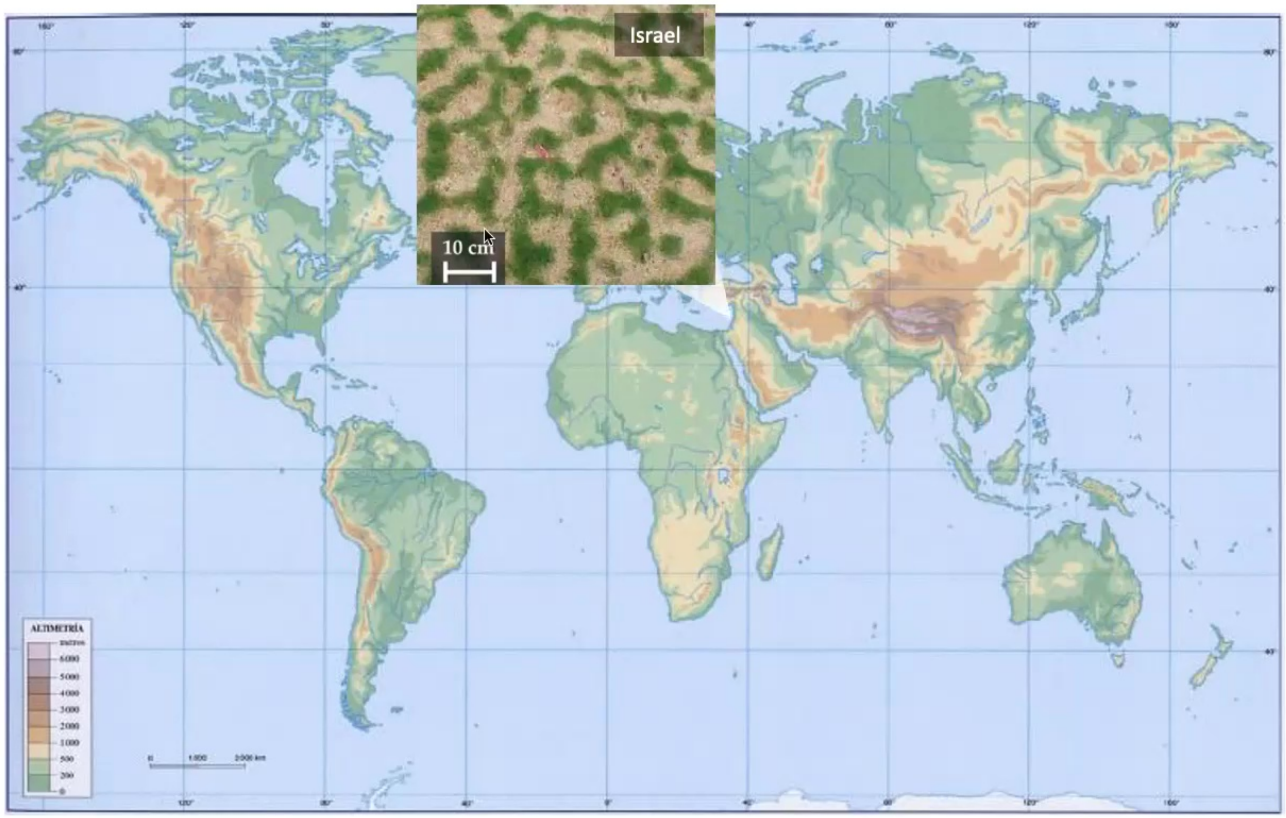




Ecological pattern ⟶ dynamical process

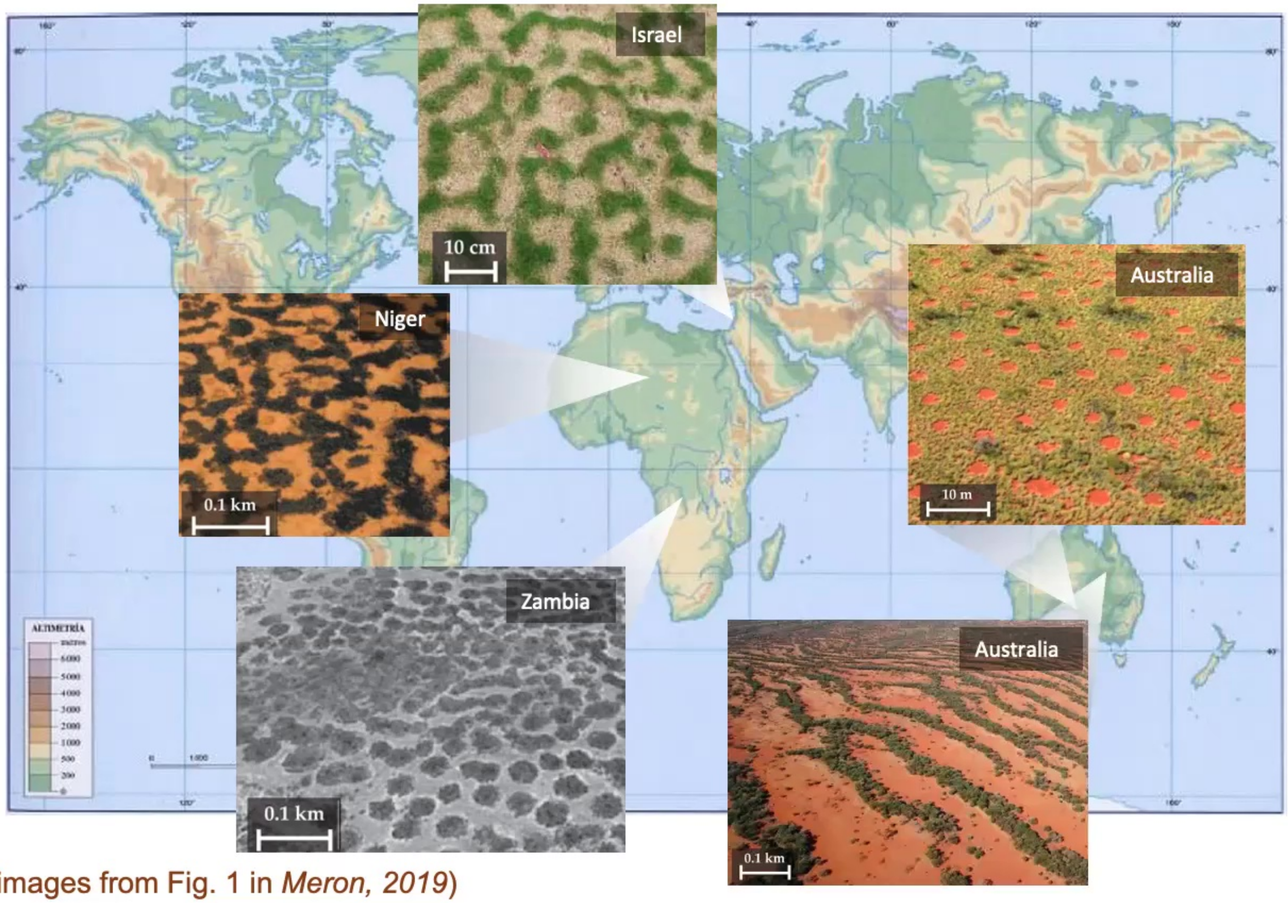


Ecological pattern → dynamical process (also in space)



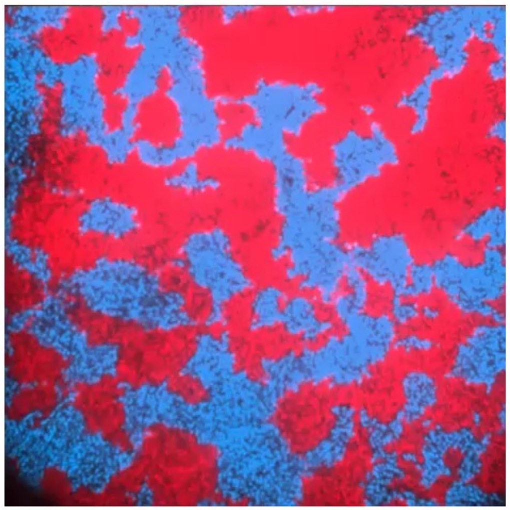
(pattern images from Fig. 1 in *Meron, 2019*)

Ecological pattern → dynamical process (also in space)

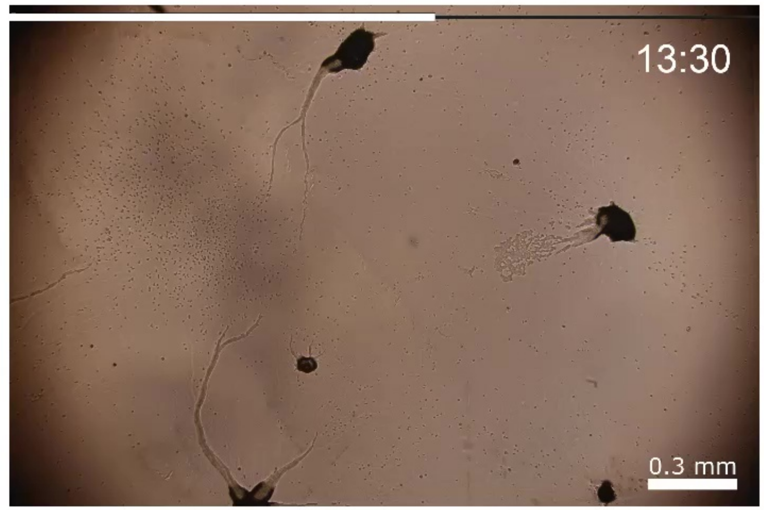


(pattern images from Fig. 1 in *Meron, 2019*)

Ecological pattern → dynamical process (also in space)

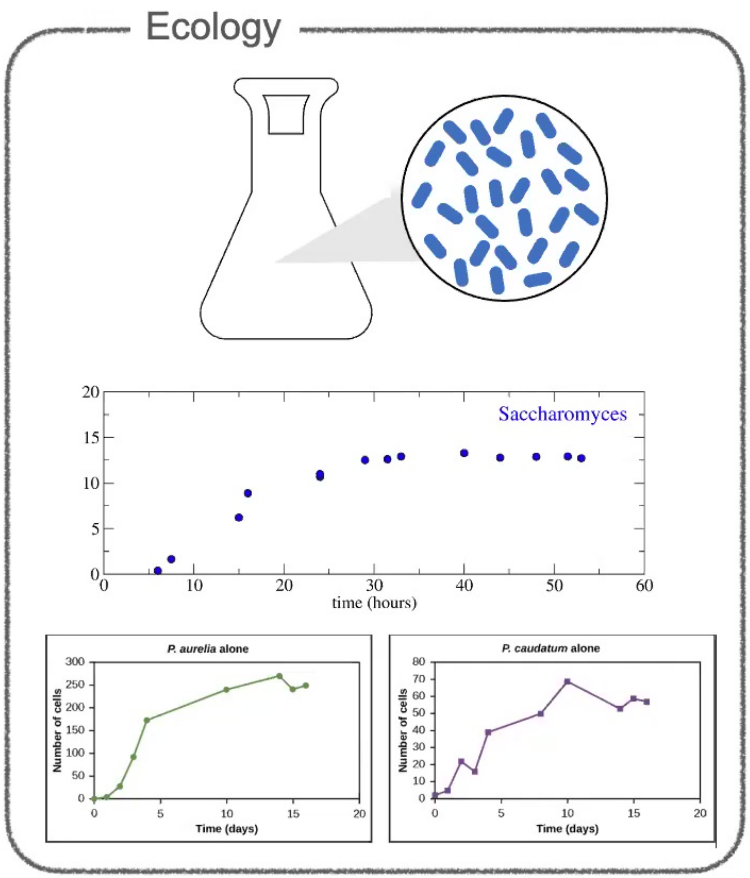


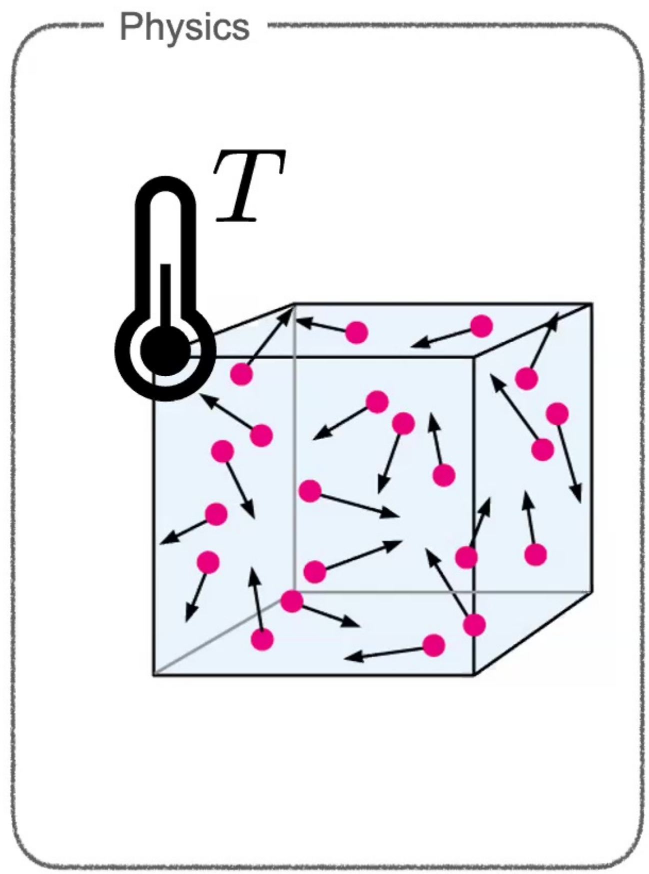
RMG et al. PLOS Comp Biol (2018)



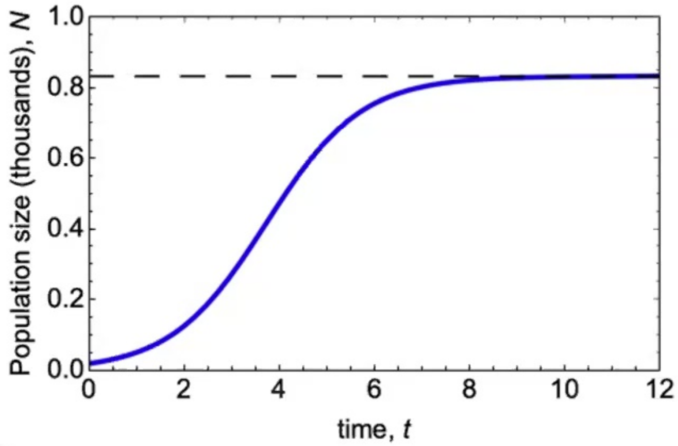
Rossine, RMG et al. PLOS Biol (2020)

Macroscopic patterns are independent of **microscopic** details

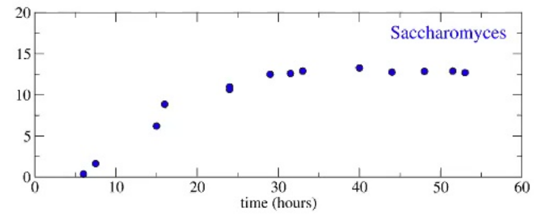


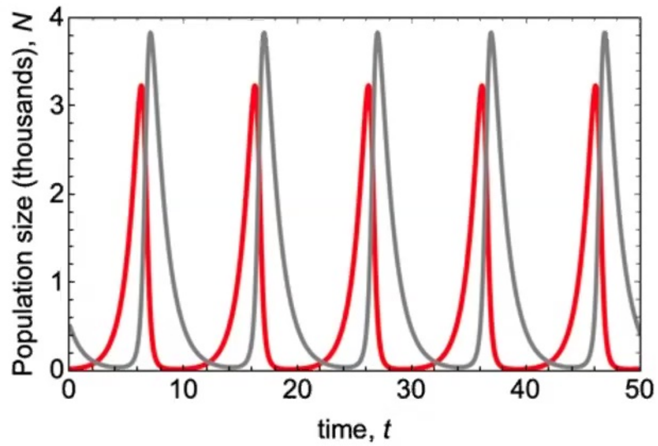


Quantitative relations to predict patterns



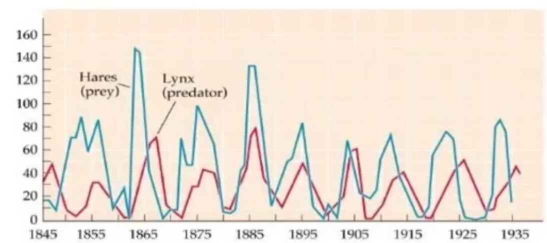
$$\dot{N}(t) = rN(t) - dN(t)^2$$



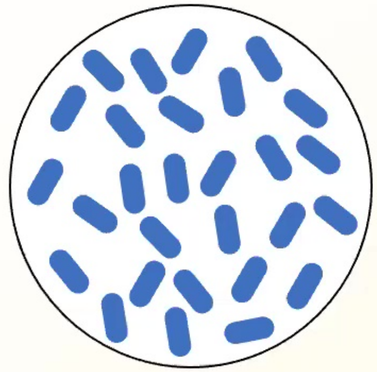


$$\dot{N}_1(t) = r_1 N_1(t) - d_{12} N_1(t) N_2(t)$$

$$\dot{N}_2(t) = -r_2 N_2(t) + d_{21} N_1(t) N_2(t)$$

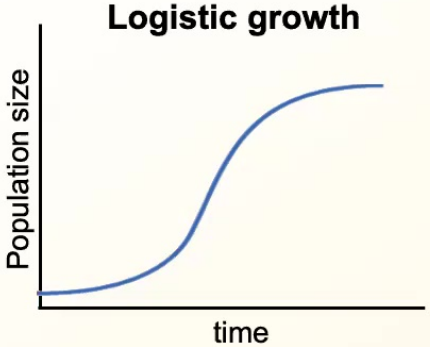


Population dynamics is an emergent phenomena



Macroscopic pattern

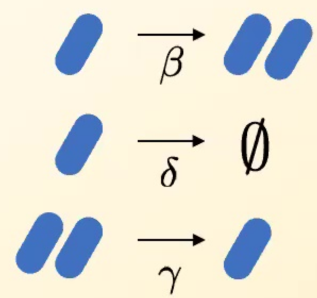
Logistic growth



$$\dot{N}(t) = (\beta - \delta)N(t) - \gamma\, N^2(t)$$

Microscopic processes

Birth, death & competition



Well-mixed model: “law of mass action”

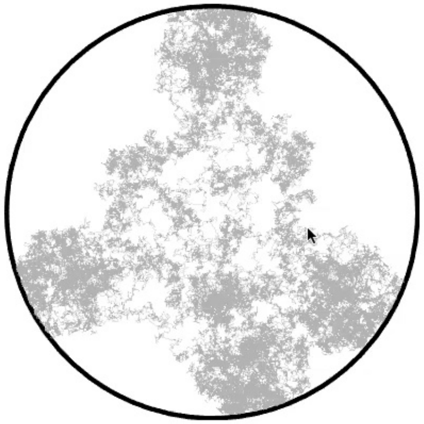
Assumptions: *cells are ideal gas particles*.

Ballistic or Reflected Brownian Motion within a ”solid” container

$$\dot{\mathbf{z}} = g\,\boldsymbol{\xi}(t)$$

Steady-state PDF individual position: **UNIFORM**

$$f(\boldsymbol{z}, t \to \infty) = \begin{cases} \frac{1}{\pi \mathcal{R}^2} & \text{inside } \mathcal{R} \\ 0 & \text{outside } \mathcal{R} \end{cases}$$

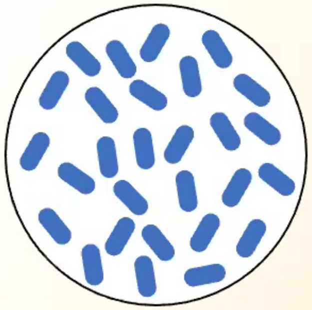


Hutchinson & Wasser 2007 Biological Reviews

Q: How do complex microscopic features translate to macroscopic dynamics?

Vivian Dornelas

Well-mixed

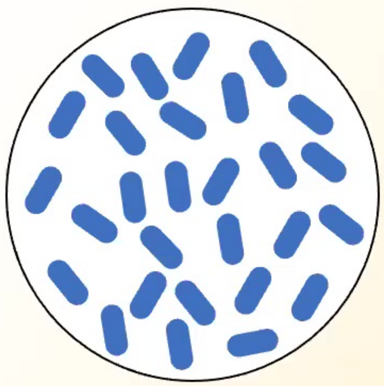


$$\dot{N}(t) = (\beta - \delta)N(t) - \gamma\, N(t)$$

Interactions with complex environments

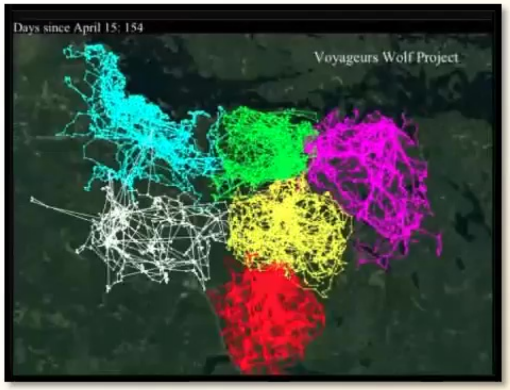
Q: How do complex microscopic features translate to macroscopic dynamics?

Well-mixed



$$\dot{N}(t) = (\beta - \delta)N(t) - \gamma N(t)$$

Range residency / territoriality

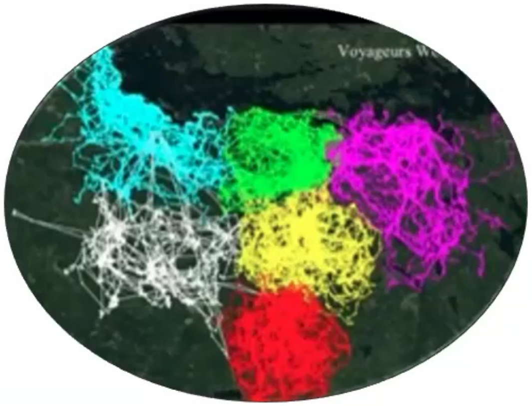


?

(video from @BoixRichter)

RMG et al. Journal of Theoretical Biology (2020)

Q: How do complex microscopic features translate to macroscopic dynamics?



Let's be physicists:

- *N* = 2
- Harmonic oscillator

How do **two** individuals encounter with each other if they are not well-mixed?

After that...more complex models and larger population sizes
STATISTICAL PHYSICS

Encounter rates link movement with population-level processes

Not so mixed: beyond the "law of mass action"

Assumptions: '*animals are trapped in harmonic potentials*'.

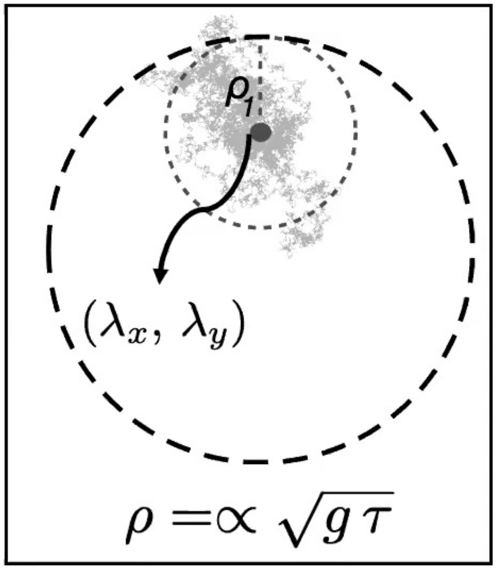
Ornstein-Uhlenbeck models, no need of boundaries.

$$\dot{\mathbf{z}} = -\frac{1}{\tau}\left[\mathbf{z} - \boldsymbol{\lambda}\right] + g\,\boldsymbol{\xi}(t)$$

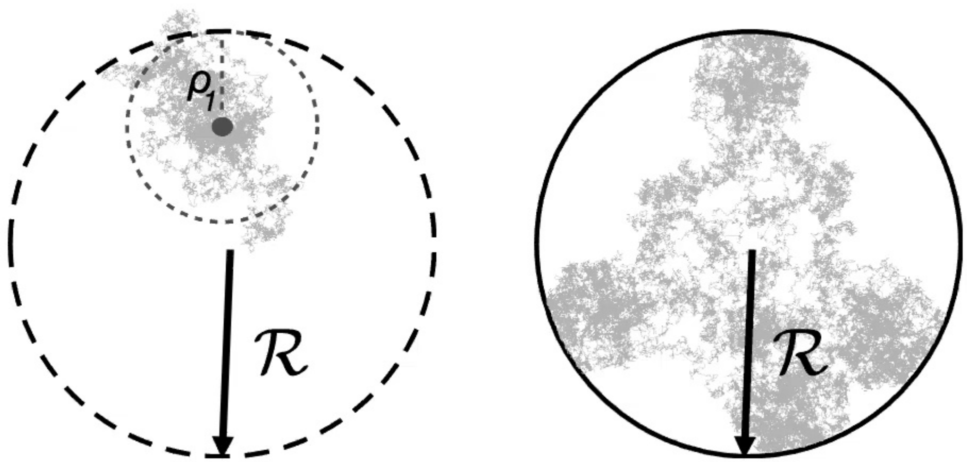
Steady-state position PDF: **NORMAL**

Mean position: $\boldsymbol{\mu} = (\lambda_x,\, \lambda_y)$

Covariance matrix: $\Sigma = \begin{pmatrix} \frac{g\tau}{2} & 0 \\ 0 & \frac{g\tau}{2} \end{pmatrix}$



OU vs RBM

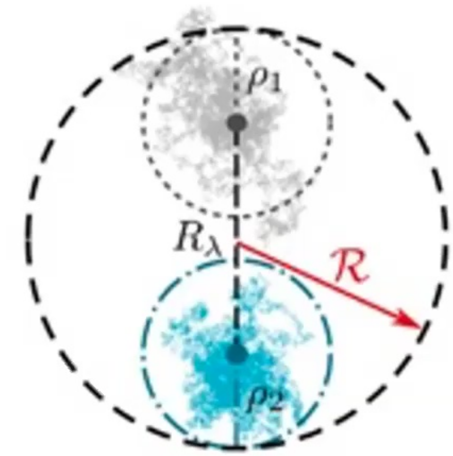


OU, due to trapping in the harmonic potential, features a spatial scale for interactions at the individual level, $\rho \propto \sqrt{g\tau}$, that is much smaller than the whole habitat occupied by the population.

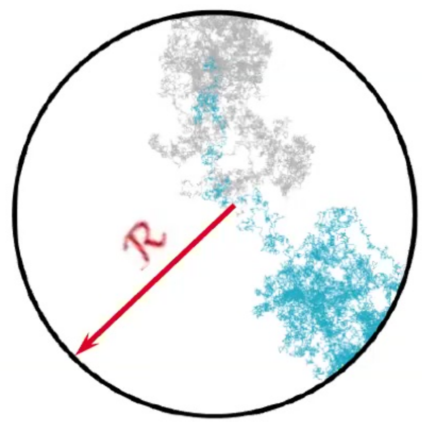
How does the existence of such scale influence encounters?

Encounters between two individuals

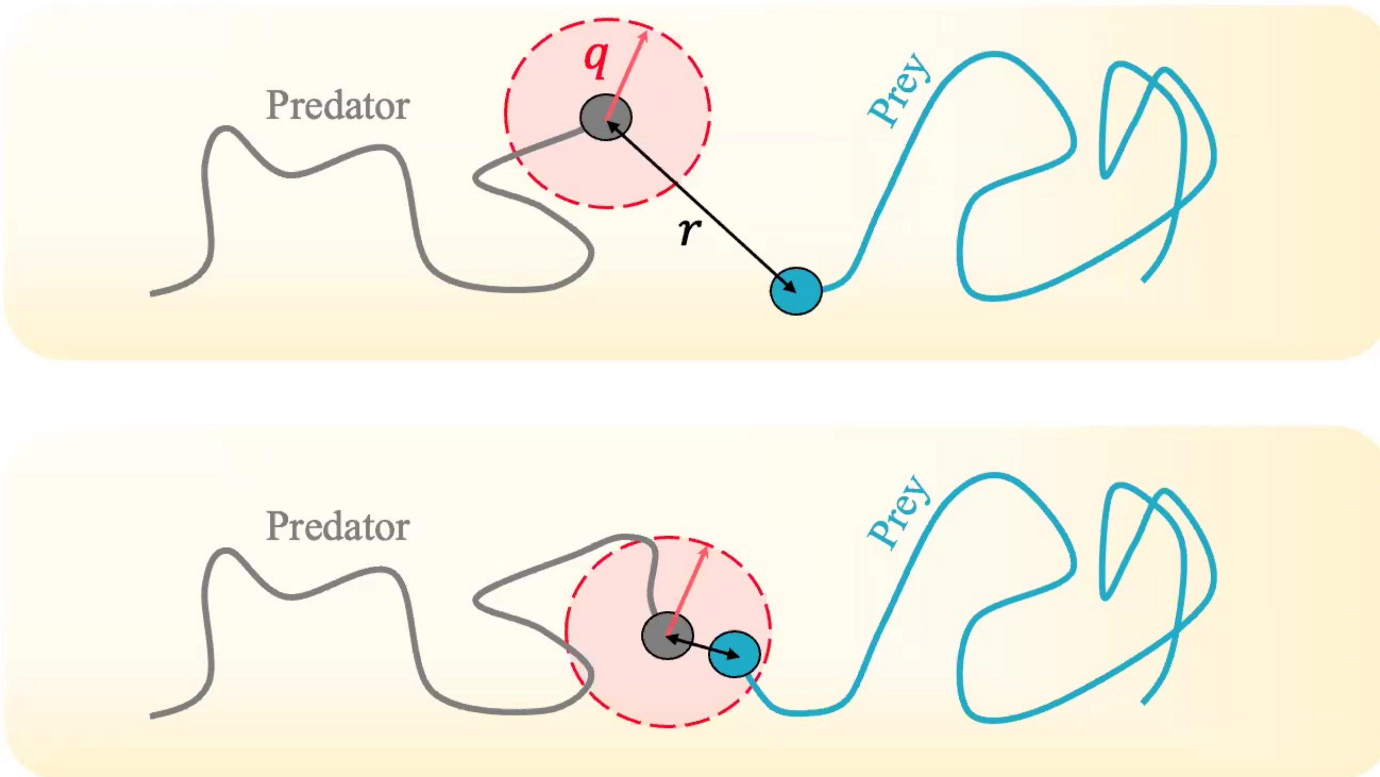
Ornstein-Uhlenbeck



Reflected Brownian motion



Defining the encounter



Key for the calculation:

obtain the distribution for the distance between individuals

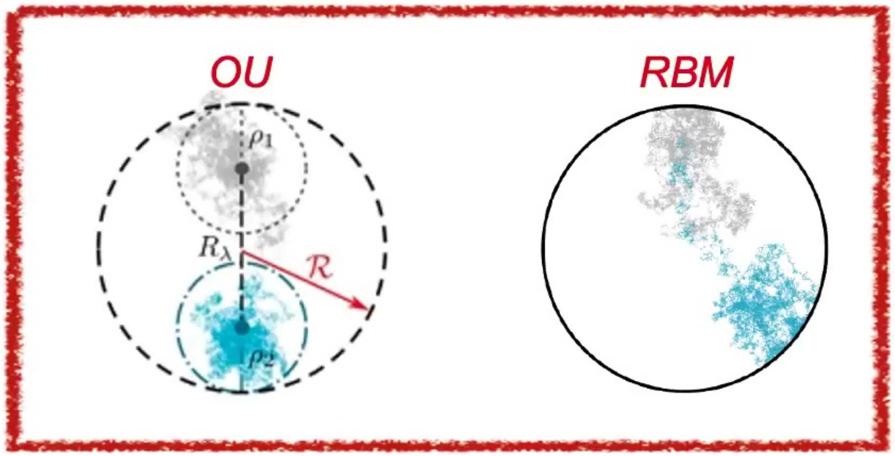
The steady-state mean encounter rate

Ornstein–Uhlenbeck

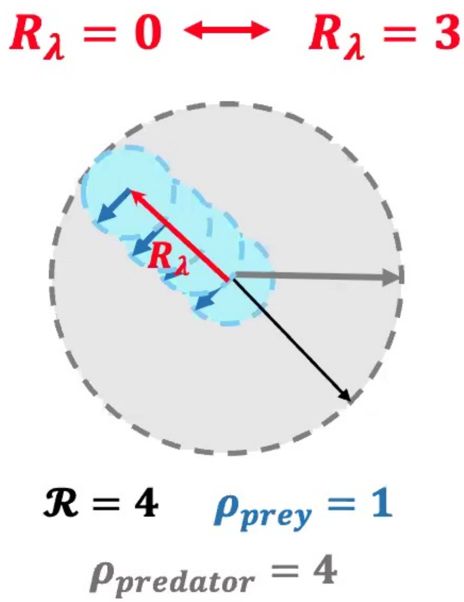
$$\tilde{\mathcal{E}}_{OU}(q) \approx \frac{\gamma}{2\pi\sigma_r^2(\boldsymbol{\rho_1}, \boldsymbol{\rho_2}, \boldsymbol{R_\lambda})} \exp\left(\frac{-\boldsymbol{R_\lambda^2}}{2\sigma_r^2(\boldsymbol{\rho_1}, \boldsymbol{\rho_2}, \boldsymbol{R_\lambda})}\right)$$

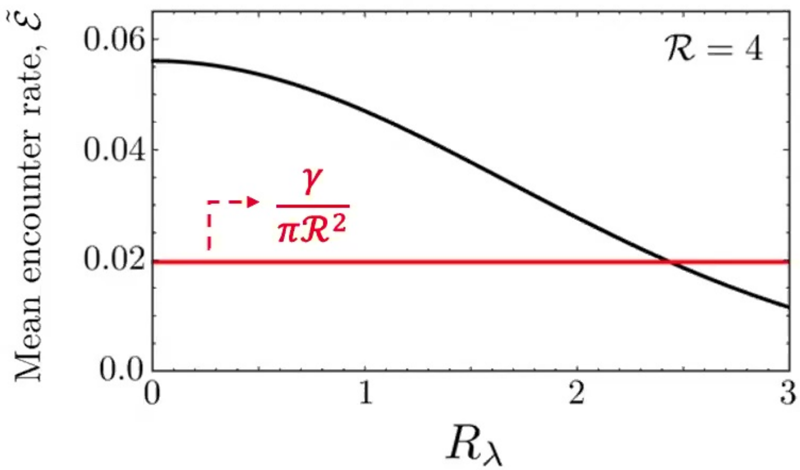
Reflected Brownian Motion

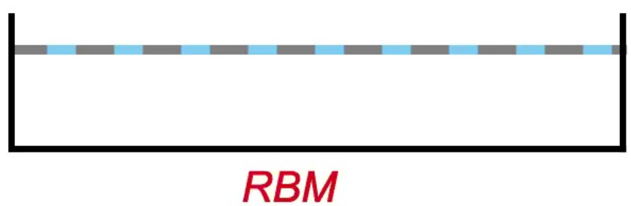
$$\tilde{\mathcal{E}}_{RBM}(q) \approx \frac{\gamma}{\pi\boldsymbol{\mathcal{R}^2}}$$

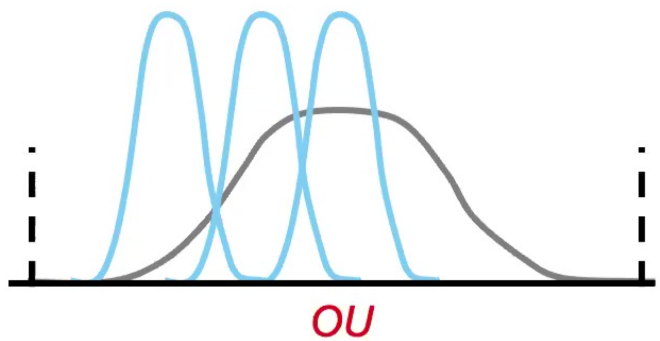


Assuming well-mixing biases A LOT encounter results

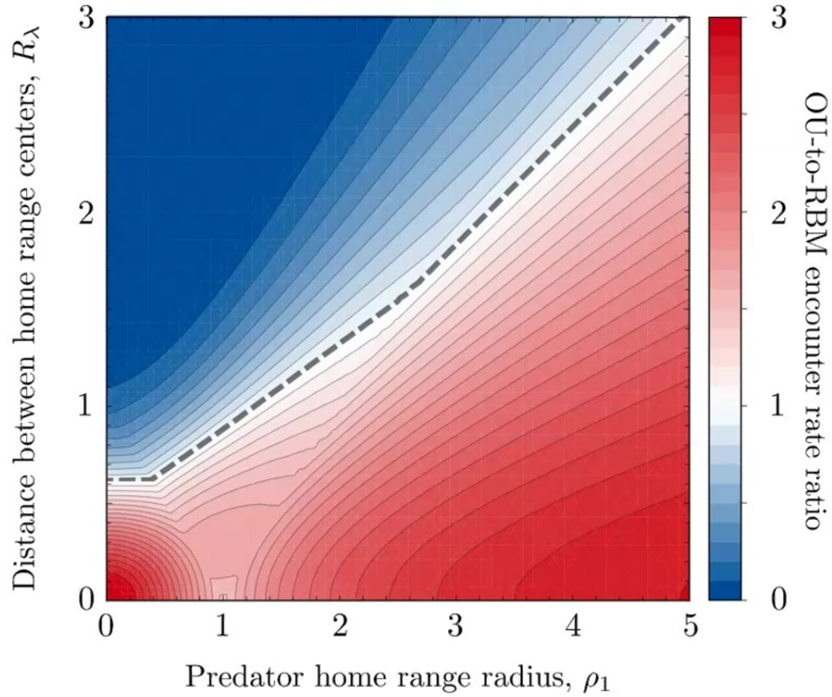








Assuming well-mixing biases A LOT encounter results



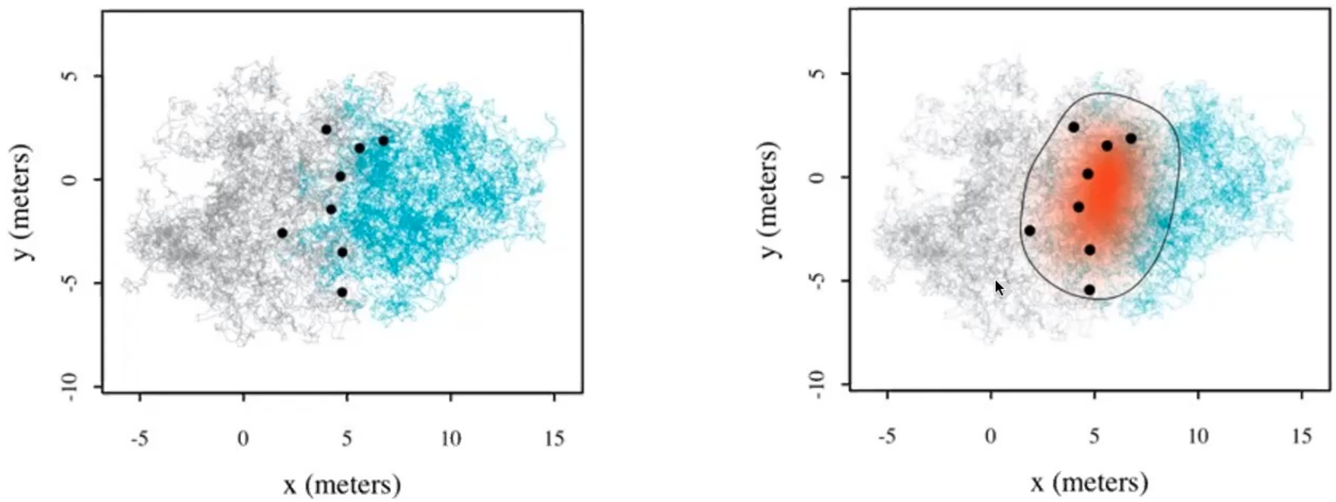
Refined encounter rates could either be higher or lower relative to mass action.

Context dependence makes it very hard to know even in which direction the predictions of well-mixed models will be wrong.

Realistic movement also changes **WHERE** encounters occur

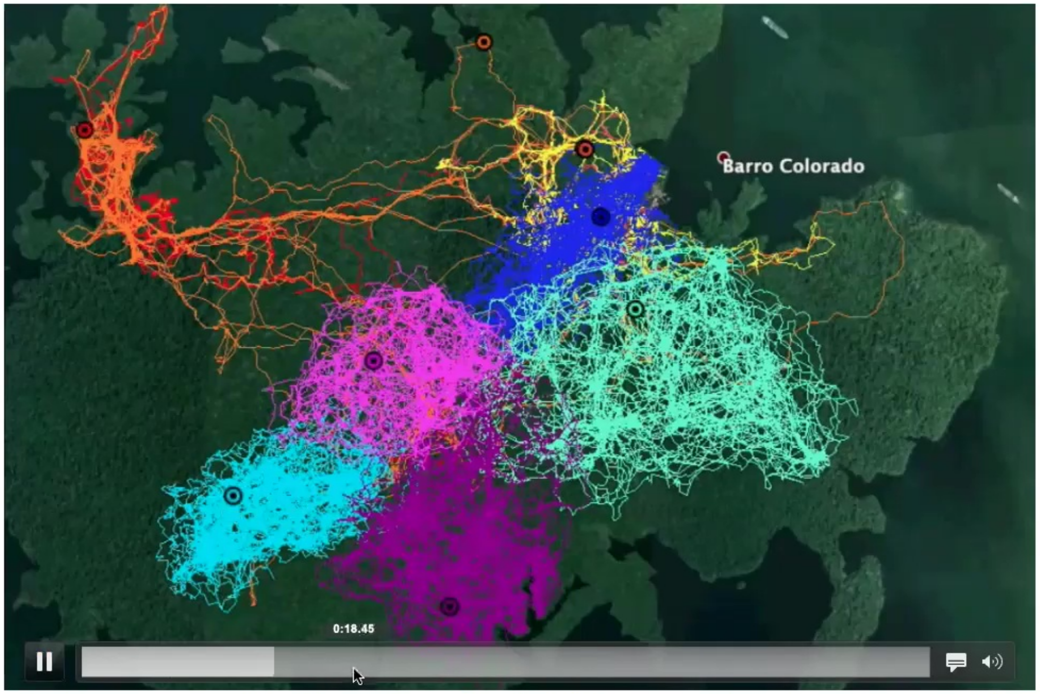
Probability that two animals encounter at a location, (x, y):

$$f_E(x,y) \propto f_1(x,y)\, f_2(x,y)$$



Noonan, RMG et al., (2021). Methods in Ecology and Evolution.

And we can learn a lot of ecology from it!

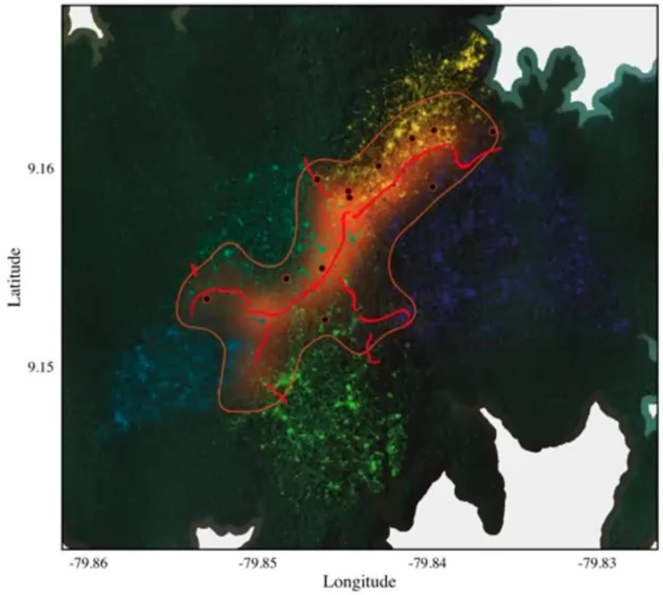


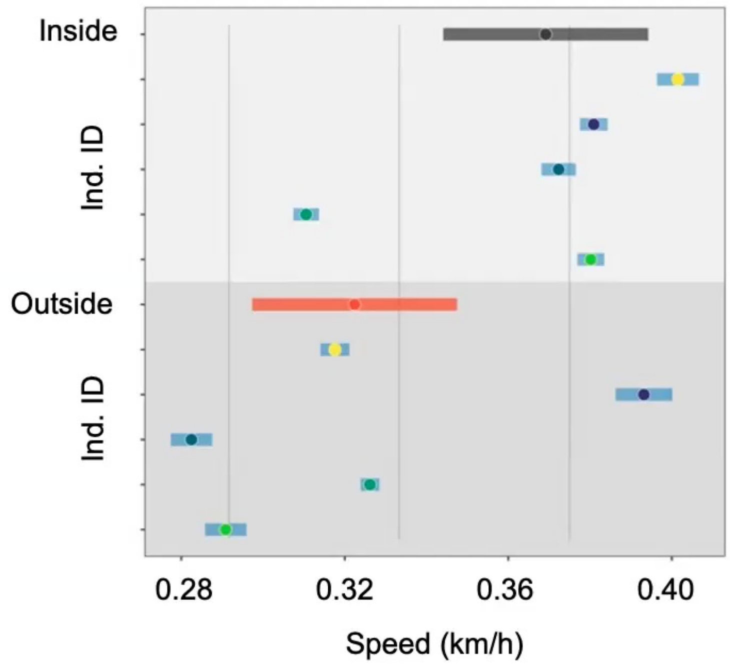
Video credits: Smithsonian Tropical Research Institute (Panama)

Data collection: Grace Davis, Meg Croofot

(UC Davis & Smithsonian Tropical Researh Institute, Panama)

And we can learn a lot of ecology from it!

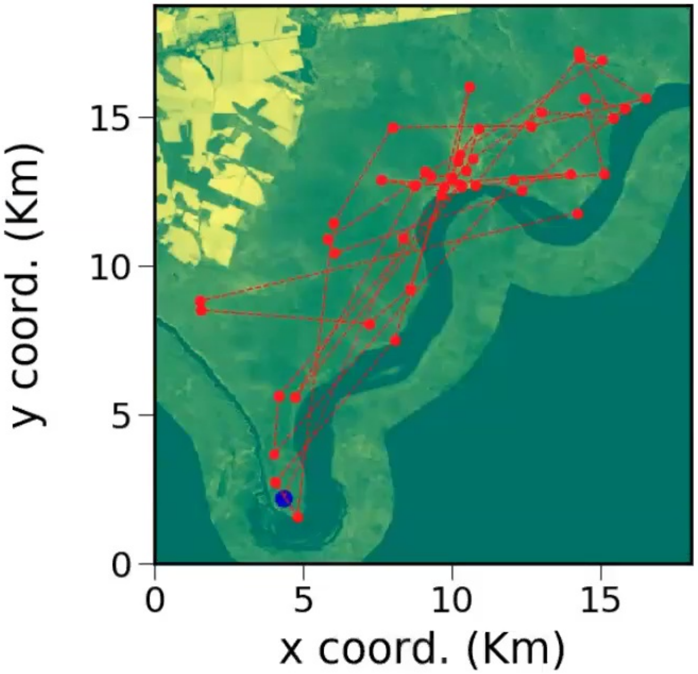




Noonan, RMG et al., (2021). Methods in Ecology and Evolution.

Extensions: effect of landscape heterogeneity

$$\dot{\mathbf{z}} = -\frac{1}{\tau}\left[\mathbf{z} - \boldsymbol{\lambda}\right] + g\,\boldsymbol{\xi}(t) - \mu\nabla\phi(\mathbf{r}, t)$$



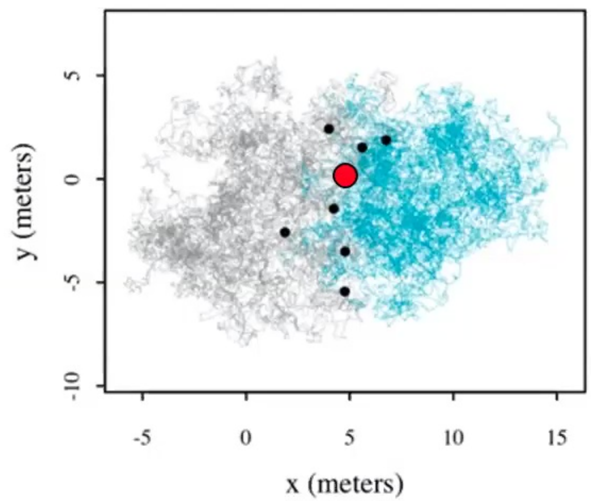
Gabriel Andreguetto

Ronaldo Morato
(Instituto Chico Mendes Conservação da Biodiversidade)

Extensions: other encounter metrics.

- When and where will two animals encounter for the first time?
- What is the probability that a predator encounters a prey within the next 3 days?
- What is the probability that a female and a male encounter each other within a mating season?

First encounter statistics and survival analysis



Benjamin Garcia de Figueiredo

Extensions: long-range (but finite) interactions.

$$\dot{\mathbf{z}}_p = -\frac{1}{\tau_p}\left[\mathbf{z}_p - \boldsymbol{\lambda}_p\right] - \nabla V_{pv}(|\mathbf{z}_p - \mathbf{z}_v|) + g_p\, \boldsymbol{\xi}_p(t)$$

$$\dot{\mathbf{z}}_v = -\frac{1}{\tau_v}\left[\mathbf{z}_v - \boldsymbol{\lambda}_v\right] + g_v\, \boldsymbol{\xi}_v(t)$$

- How does this change the frequency and location of encounters?
- Can we account for behavioral state of the animals?
- What makes the pairwise interaction potential a realistic one?
- Can we account for prey avoidance too?

Acknowledgments

FAPESP

SIMONS FOUNDATION

serrapilheira