Title: Novel entanglement phases and phase transitions via spacetime duality

Speakers: Vedika Khemani
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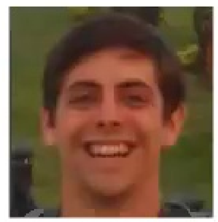
Abstract:

The extension of many-body quantum dynamics to the non-unitary domain has led to a series of exciting developments, including new out-of-equilibrium entanglement phases and phase transitions. We show how a duality transformation between space and time on one hand, and unitarity and non-unitarity on the other, can be used to realize steady state phases of non-unitary dynamics that exhibit a rich variety of behavior in their entanglement scaling with subsystem size --- from logarithmic to extensive to fractal. We show how these outcomes in non-unitary circuits (that are ``spacetime-dual'' to unitary circuits) relate to the growth of entanglement in time in the corresponding unitary circuits, and how they differ, through an exact mapping to a problem of unitary evolution with boundary decoherence, in which information gets ``radiated away'' from one edge of the system. In spacetime-duals of chaotic unitary circuits, this mapping allows us to uncover a non-thermal volume-law entangled phase with a logarithmic correction to the entropy distinct from other known examples. Most notably, we also find novel steady state phases with fractal entanglement scaling, $S(\ell) \sim \ell^{\alpha}$ with tunable $0 < \alpha < 1$ for subsystems of size ℓ in one dimension. These fractally entangled states add a qualitatively new entry to the families of many-body quantum states that have been studied as energy eigenstates or dynamical steady states, whose entropy almost always displays either area-law, volume-law or logarithmic scaling. We also present an experimental protocol for preparing these novel steady states with only a very limited amount of postselection via a type of ``teleportation'' between spacelike and timelike slices of quantum circuits.

Novel Entanglement Phases and Phase Transitions via Spacetime Duality

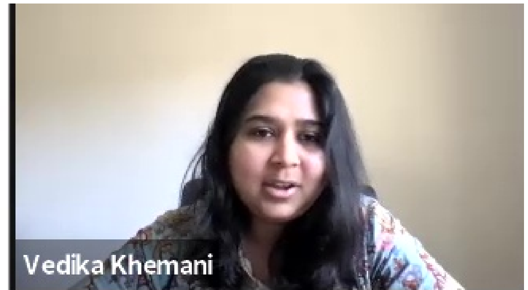
Vedika Khemani
Stanford University

Matteo Ippoliti

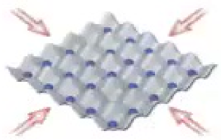


Tibor Rakovszky

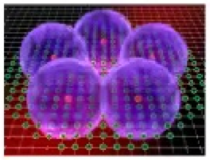
Ippoliti, Rakovszky, VK, arXiv: 2103.06873
Ippoliti, VK, PRL 060501 (2021)



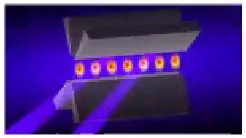
Quantum Simulators: New Platforms for Many-Body Experiments



Ultracold atoms



Rydberg atoms



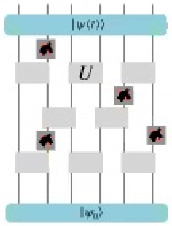
Trapped ions

SC qubits

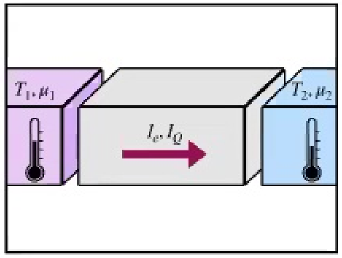
- Quantum simulators with unprecedented control, measurement capabilities
- New focus on out-of-equilibrium dynamics of many-body systems

$$U(t) = \mathcal{T} e^{-i \int H(t) dt}$$

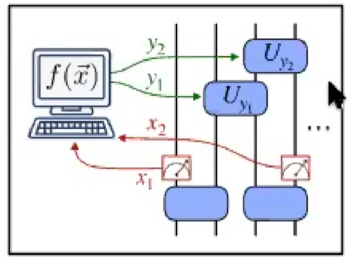
Analog



Digital



Traditional near-equilibrium transport measurement



Quantum circuit experiments

What phenomena can we probe with these new quantum experiments?

- Foundations of quantum statistical mechanics (thermalization, localization, quantum chaos…)
- New dynamical phases and universality classes (time crystals…)?
- New "order parameters" (entanglement, complexity…)

This Talk:
Novel Entanglement Phases and Phase Transitions in Non-Unitary Circuits via Spacetime Duality

- Review: Entanglement dynamics and phase transitions
- Spacetime duality: an experimentally motivated way to make interesting (analytically tractable) non-unitary circuits
- Novel steady state phases: logarithmic, volume-law and *fractal* entanglement scaling
- Preparing these steady states in the lab

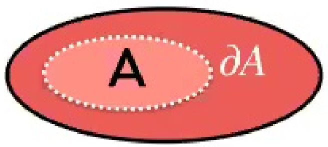
Ippoliti, Rakovszky, VK, arXiv: 2103.06873
Ippoliti, VK, PRL 060501 (2021)

Dynamics of Quantum Entanglement

$$|\psi(t)\rangle = U(t)\,|\psi_0\rangle$$

$$\rho_A(t) = \mathrm{Tr}_B\,|\psi(t)\rangle\langle\psi(t)\,|$$

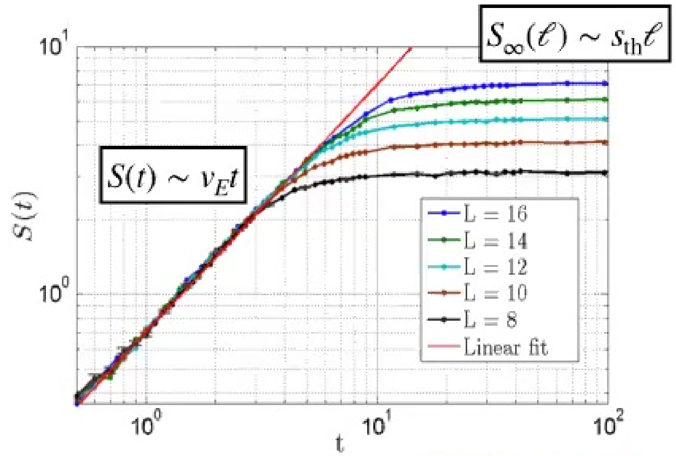
$$S_A^n(t) = \frac{1}{1-n}\log[\mathrm{Tr}(\rho_A(t))^n]$$



$S_A \sim |A|$ Volume Law

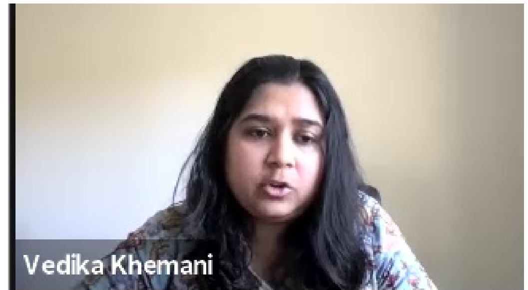
$S_A \sim |\partial A|$ Area Law

$S_A \sim \log(|A|)$ Log Law

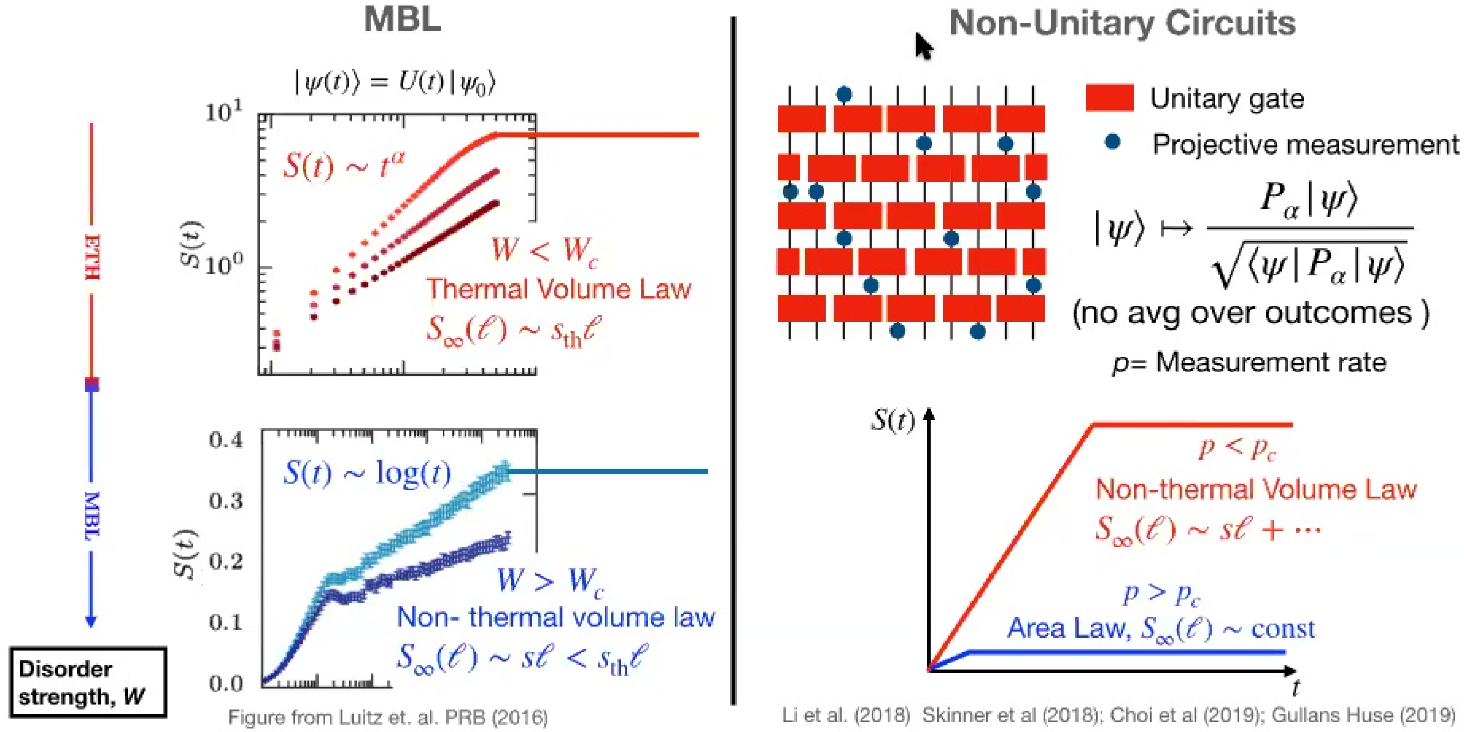


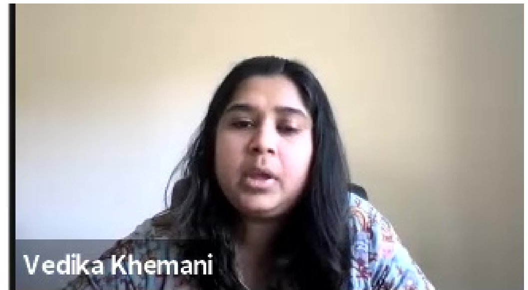
Kim-Huse (2013)

Generically:
- Ballistic growth of entanglement
- Saturation to thermal volume-law



Entanglement phase transitions in dynamics





Entanglement phase transitions in dynamics

Unitary dynamics with an MBL transition

ETH —●— MBL → W

Subballistic entanglement growth
$S(t) \sim t^{\alpha},\ 0 < \alpha \leq 1$

Logarithmic entanglement growth
$S(t) \sim \log(t)$

Thermal Volume Law
$S_{\infty}(\ell) \sim s_{\text{th}}\ell$

Non- thermal volume law
$S_{\infty}(\ell) \sim s\ell < s_{\text{th}}\ell$

Non-Unitary Circuits

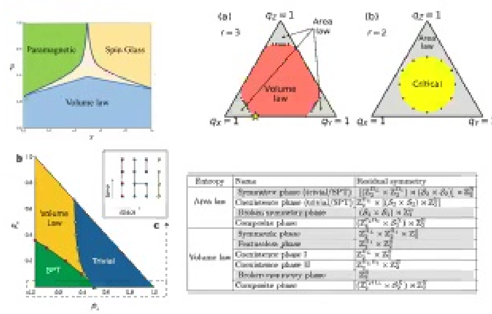
Entangling —●— Disentangling → P

Non-thermal Volume Law
$S_{\infty}(\ell) \sim s\ell + \cdots$

Log Law
$S_{\infty}(\ell) \sim \log(\ell)$

Area Law
$S_{\infty}(\ell) \sim \text{const}$

Li Chen Fisher (2018) Skinner Ruhman Nahum (2018)



Sang Hsieh (20), Ippoliti…VK (20), Lavasani et al (20), Bao et al (21)

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Novel Entanglement Phases and Phase Transitions in Non-Unitary Circuits via Spacetime Duality

- Review: Entanglement dynamics and phase transitions
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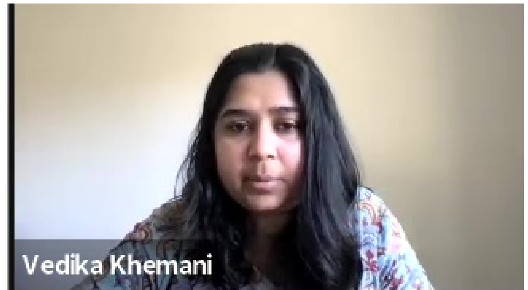
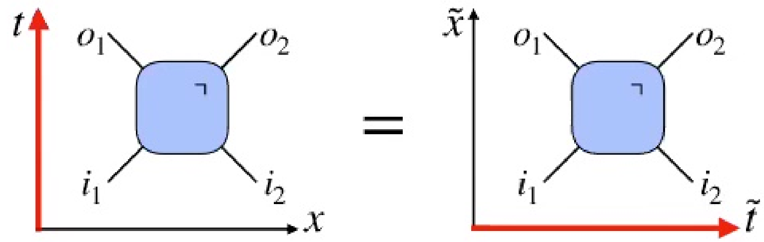


Spacetime duality

$$U^{o_1 o_2}_{i_1 i_2} =$$

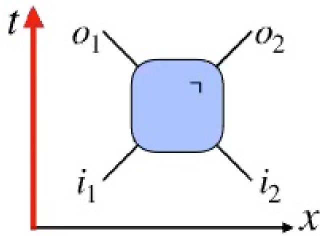


$$= \tilde{U}^{i_2 o_2}_{i_1 o_1}$$

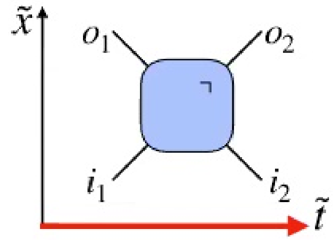
Time ⟷ Space

Spacetime duality

$$U^{o_1 o_2}_{i_1 i_2} =$$



$$=$$

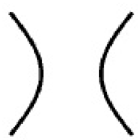


$$= \tilde{U}^{i_2 o_2}_{i_1 o_1}$$

Time ⟷ Space
Unitary ⟷ Non-unitary

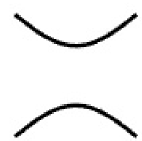
$$U = I$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\tilde{U} = 2\,|B^+\rangle\langle B^+|$$

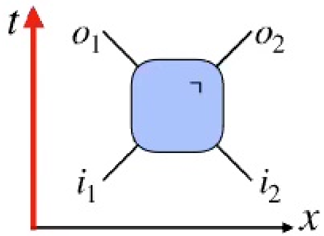
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$



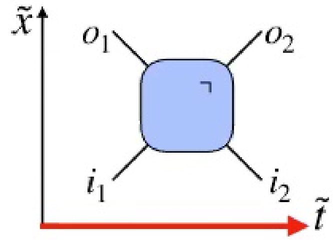
$$|B^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Spacetime duality

$U^{o_1 o_2}_{i_1 i_2} =$



$=$



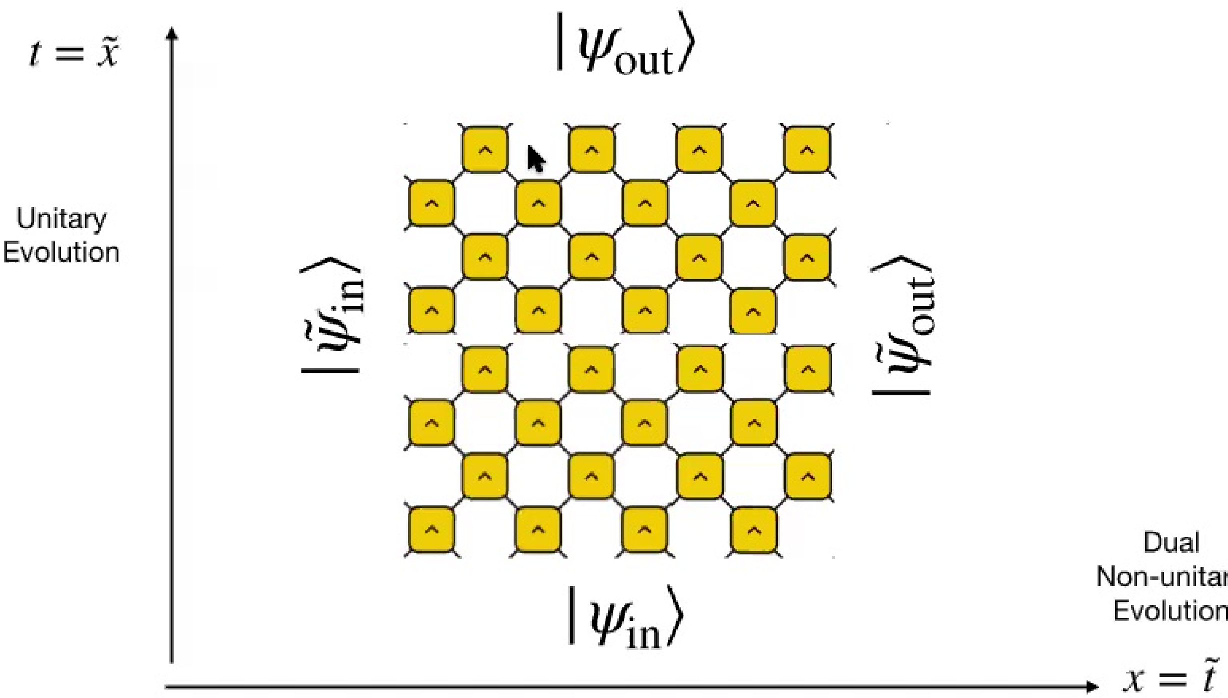
$= \tilde{U}^{i_2 o_2}_{i_1 o_1}$

Time ⟷ Space
Unitary ⟷ Non-unitary

- Dualized gates need not be unitary! Generally, a **weak measurement**
- **Forced** measurement - always same outcome! ($\tilde{U}$ is deterministic like U)
- Actual measurements are costly (**postselection**)
  - Can we use this to engineer "cheaper" coherent, non-unitary evolution?
  - Cut down on / eliminate actual measurements?

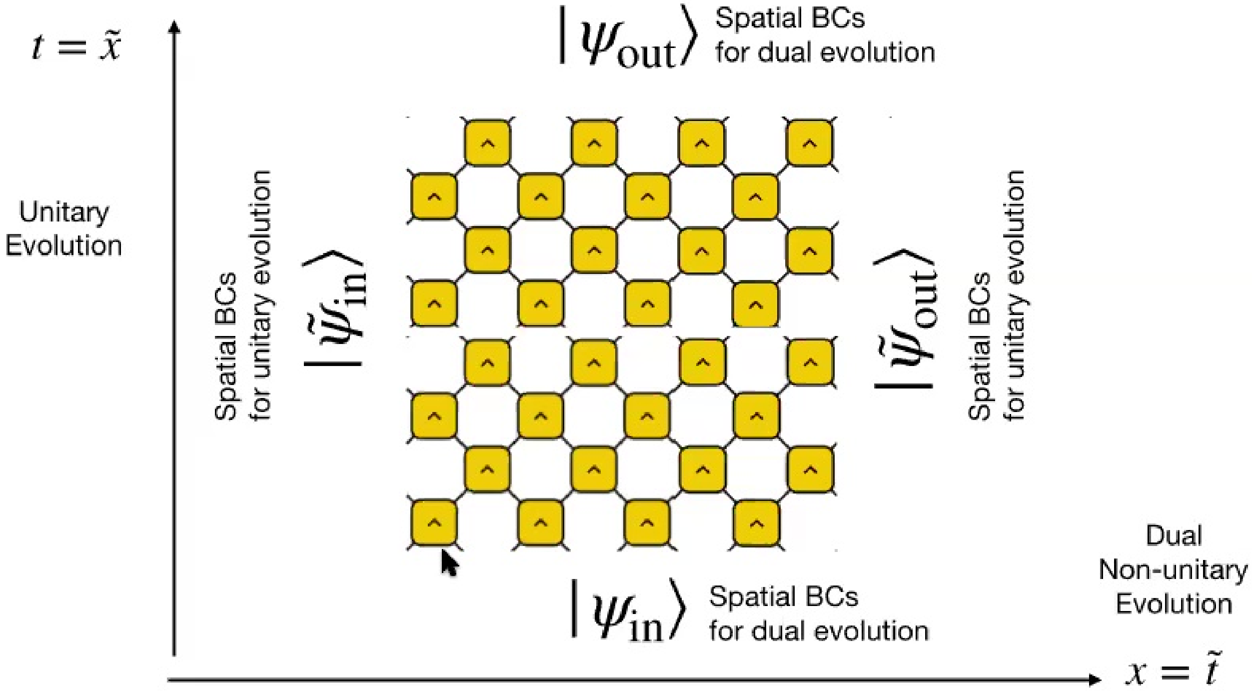
Spacetime dual circuits

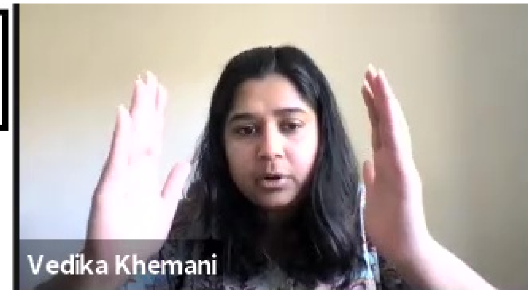
Time ⟷ Space
Unitary ⟷ Non-unitary



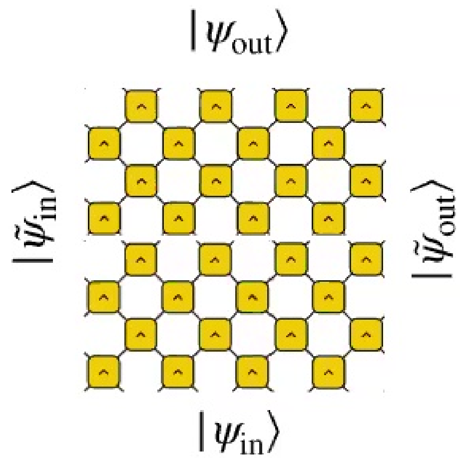
Spacetime dual circuits

Time ⟷ Space
Unitary ⟷ Non-unitary



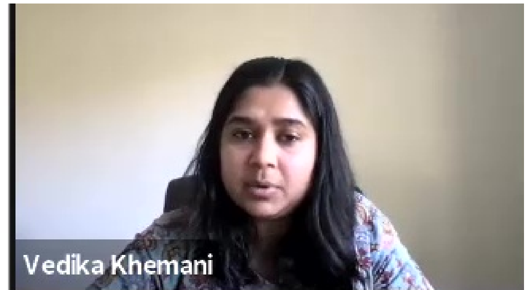


Spacetime dual circuits

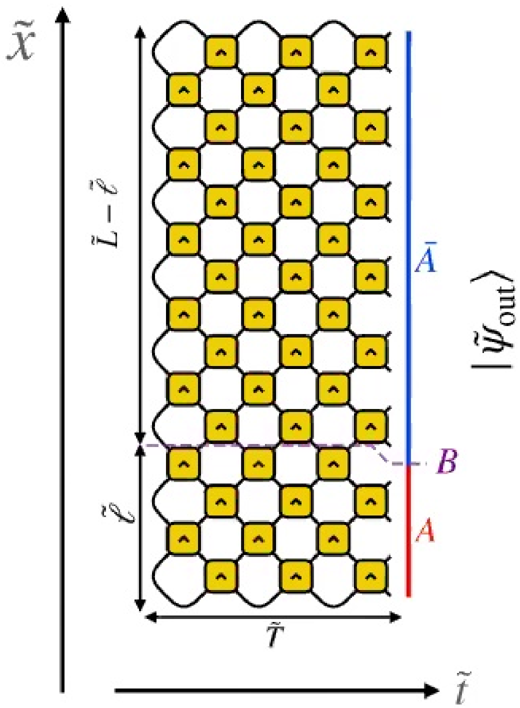


Time ⟷ Space
Unitary ⟷ Non-unitary

Search for new non-unitary phenomena via spacetime duality!

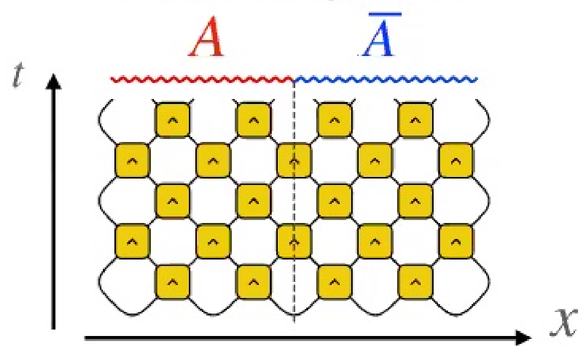


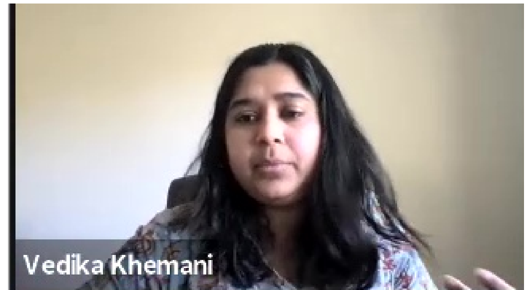
Entanglement in spacetime-dual circuits



Saturation value at late time ($\tilde{T} \gg \tilde{\ell}_A$)
Scaling with subsystem size $|A| \equiv \tilde{\ell}_A$

cf. “usual” entanglement cut

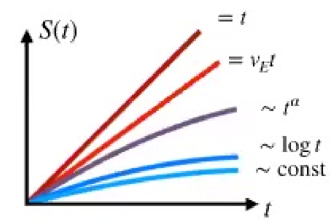


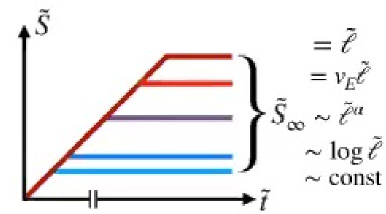


Entanglement in spacetime dual circuits

Summary of results

	Anderson localized	MBL	Thermalizing Griffiths	Chaotic	Dual-unitary
$S(t)$ unitary	$\sim t^0$	$\sim \log t$	$\sim t^\alpha + \ldots$	$= v_E t$	$= t$
$\tilde{S}_\infty(\tilde{\ell})$ non-unitary late-time $\lvert A \rvert \sim \tilde{\ell}$	$\sim \log \tilde{\ell}$	$\sim \log \tilde{\ell}$	$\sim \tilde{\ell}^\alpha + \ldots$	$= v_E \tilde{\ell} + \frac{1}{2} \log \tilde{\ell}$	$= \tilde{\ell}$





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New types of late-time state entanglement!

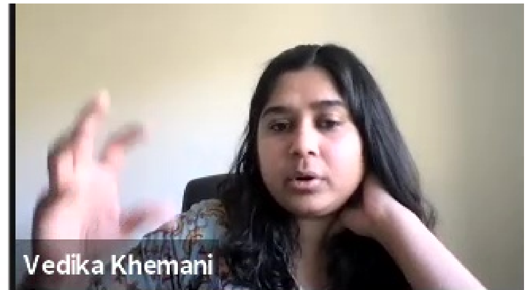
Intermediate between logarithmic & extensive: ***fractal***

Important **logarithmic corrections**

Growth of entanglement in time in unitary *U(t)*

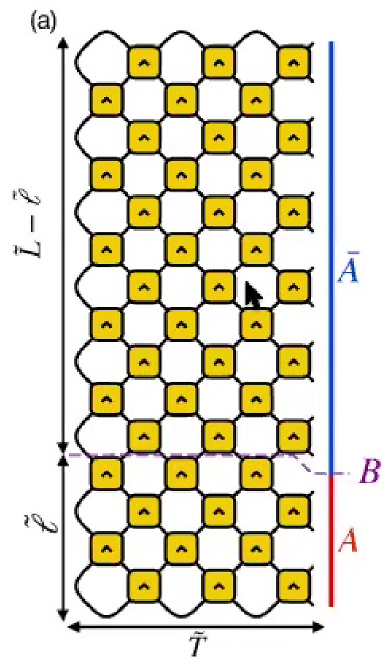
↕

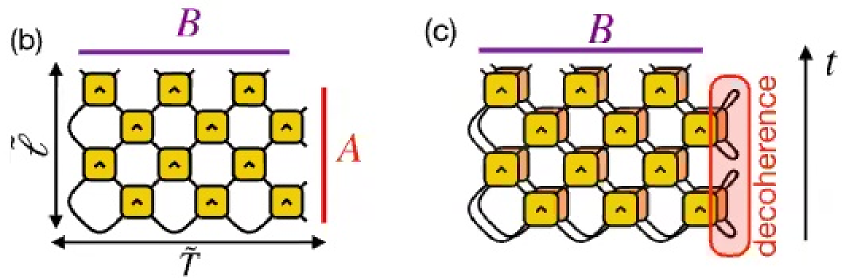
Spatial scaling of steady-state entanglement in time in non-unitary $\tilde{U}$



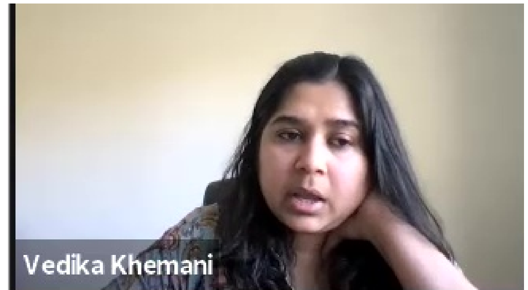
Entanglement in spacetime dual circuits

Mapping to edge decoherence





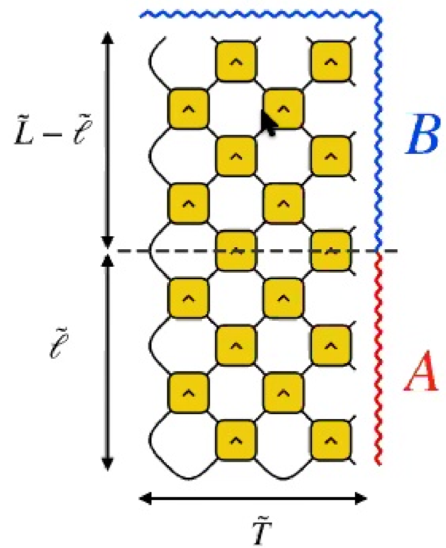
$$\lim_{\tilde{L}\to\infty} \tilde{S}(\tilde{T},\tilde{\ell}) = S_{\mathrm{dec}}(t=\tilde{\ell}, L=\tilde{T})$$



Entanglement in "spacetime-flipped" circuits

Mapping to edge decoherence

Simplification — becomes exact for $\tilde{L} \to \infty$



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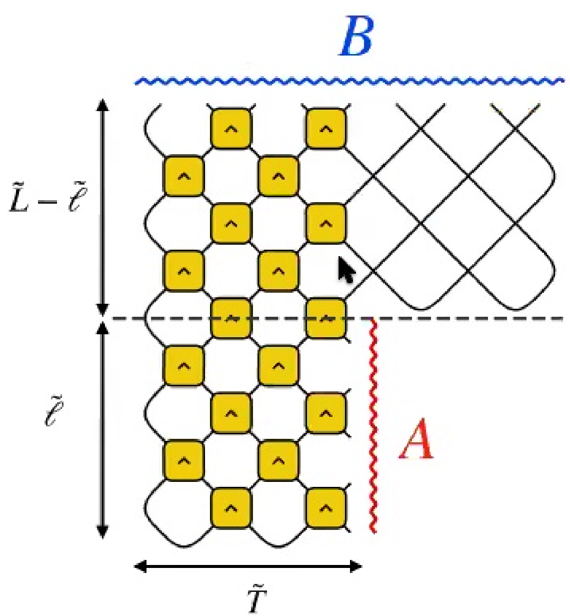
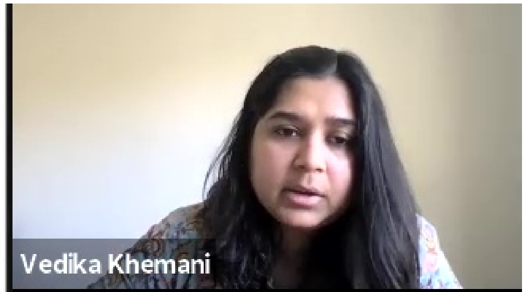
Mapping to edge decoherence





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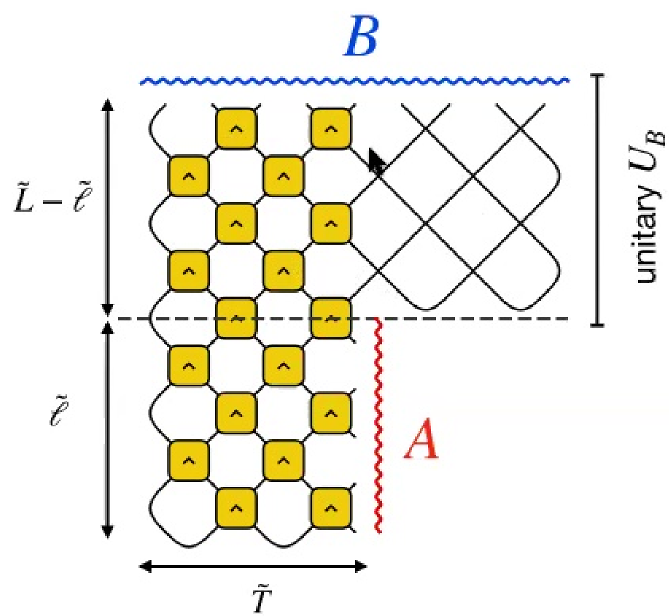
Mapping to edge decoherence



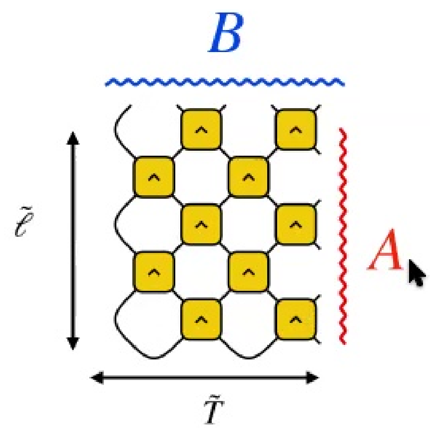
$$\rho_B = \text{Tr}_A \, |\psi_{AB}\rangle\langle\psi_{AB}|$$

$$S(\rho_B) = S(U_B^\dagger \rho_B U_B)$$

$\Longrightarrow$ can remove the $\tilde{L} - \tilde{\ell}$ segment of the circuit!

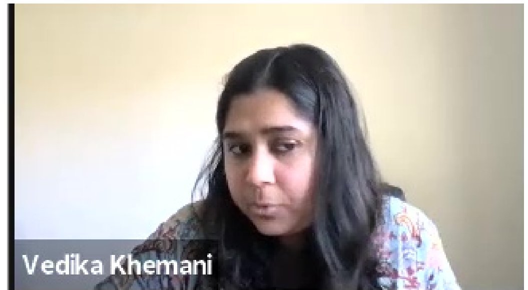
Entanglement in "spacetime-flipped" circuits

Mapping to edge decoherence



$$S(\rho_A) = S(\rho_B)$$

$$\rho_B = \mathrm{Tr}_A \rho$$



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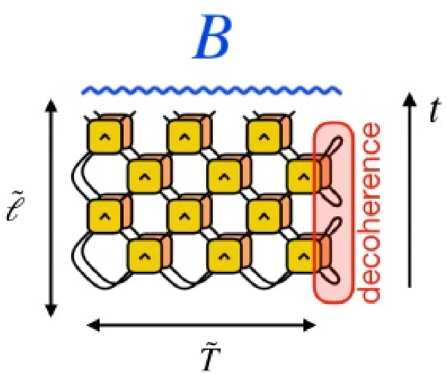

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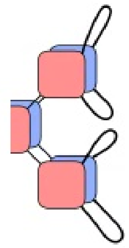


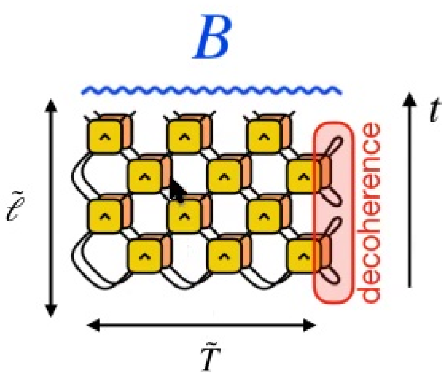
At every cycle, the edge qubit gets fully **depolarized**:

$$\rho \mapsto \mathrm{Tr}_{\mathrm{edge}}(\rho) \otimes \frac{I_{\mathrm{edge}}}{2}$$

Decoherence at the edge propagates into the bulk and produces a mixed density matrix ρ_B at *B*. $S(\rho_B)$ is what we’re after

Entanglement in "spacetime-flipped" circuits

Mapping to edge decoherence



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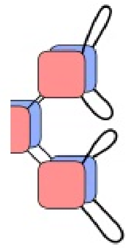

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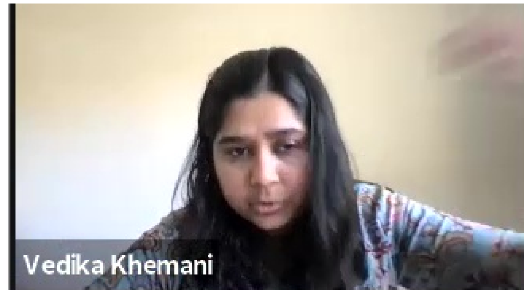
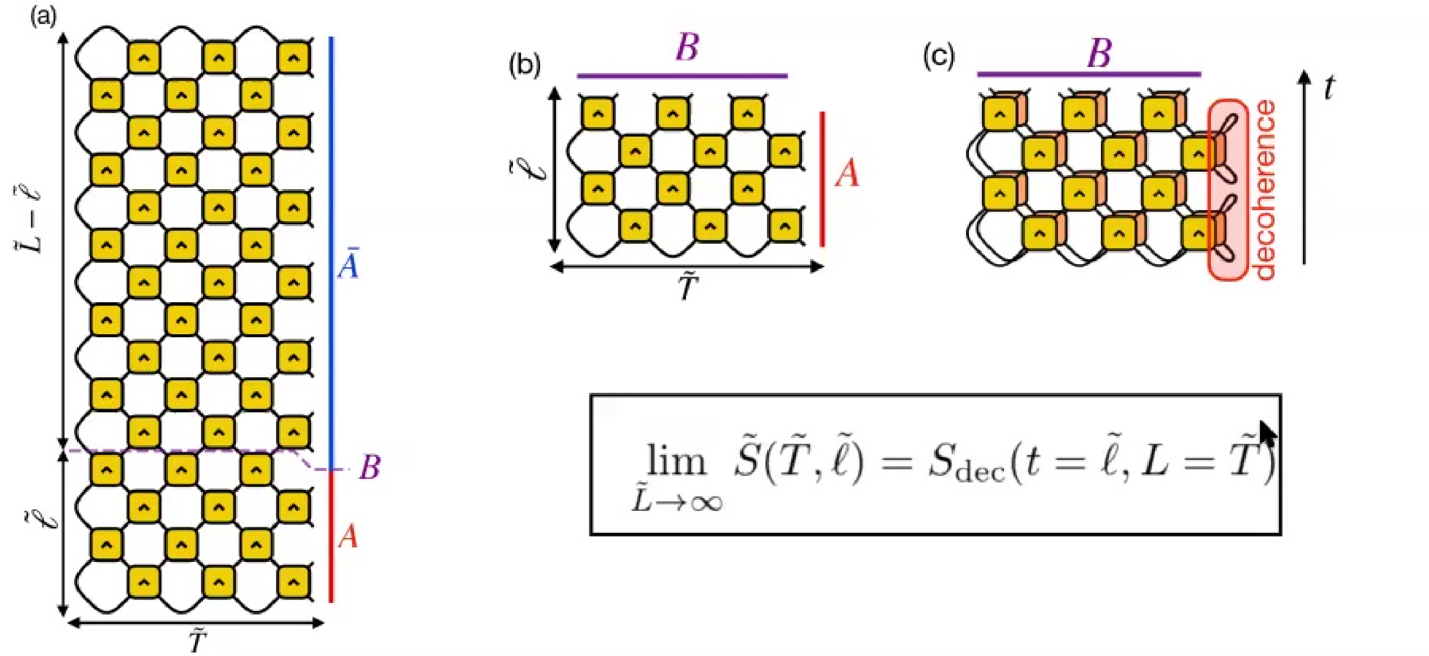


Entanglement in spacetime dual circuits

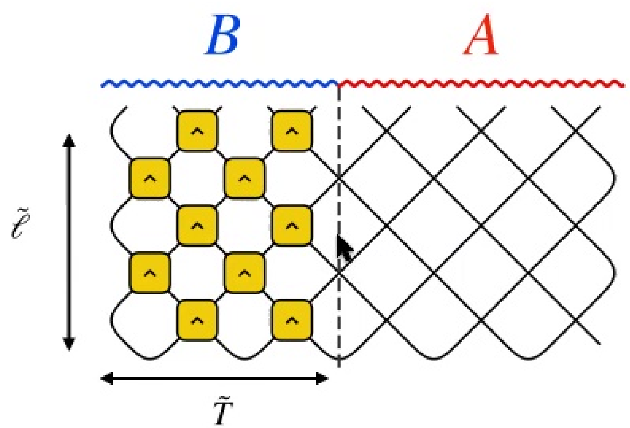
Mapping to edge decoherence



$$\lim_{\tilde{L}\to\infty} \tilde{S}(\tilde{T},\tilde{\ell}) = S_{\text{dec}}(t=\tilde{\ell}, L=\tilde{T})$$

Entanglement in “spacetime-flipped” circuits
Mapping to edge decoherence

Alternative perspective: conventional entanglement cut in a **bipartite circuit**,
* **Nontrivial gates in B**
* **SWAPs in A**

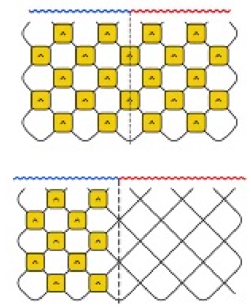


Information that crosses the cut gets “radiated” away to infinity, never comes back

Entanglement in spacetime dual circuits

Summary of results

	Anderson localized	MBL	Thermalizing Griffiths	Chaotic	Dual-unitary
$S(t)$ unitary	$\sim t^0$	$\sim \log t$	$\sim t^\alpha + \ldots$	$= v_E t$	$= t$
$\tilde{S}_\infty(\tilde{\ell})$ non-unitary late-time $\lvert A \rvert \sim \tilde{\ell}$	$\sim \log \tilde{\ell}$	$\sim \log \tilde{\ell}$	$\sim \tilde{\ell}^\alpha + \ldots$	$= v_E \tilde{\ell} + \frac{1}{2} \log \tilde{\ell}$	$= \tilde{\ell}$



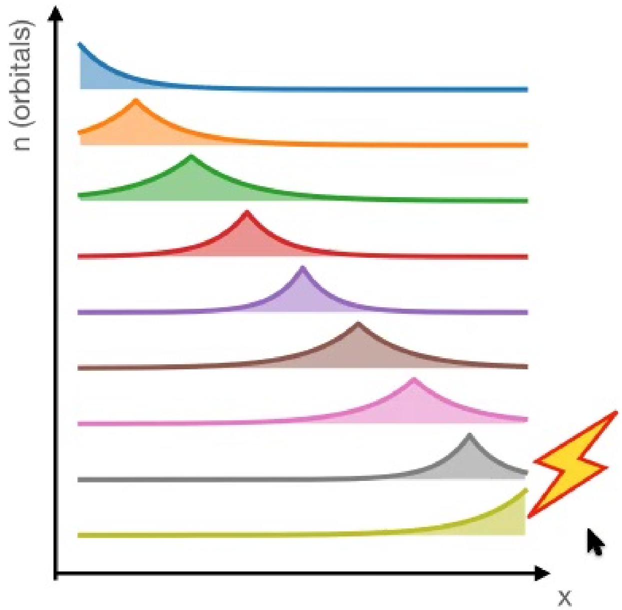
Growth of entanglement in time in unitary *U(t)*

↕

Spatial scaling of steady-state entanglement in time in non-unitary $\tilde{U}$

Floquet-Anderson localized circuits

Logarithmic entanglement from edge decoherence



Floquet-Anderson circuit:

Exponentially localized orbitals, $\psi_n(x) \sim e^{-|x-n|/\xi}$

Unitary time evolution: **area-law entanglement**

Edge decoherence affects all orbitals!

ψ_n decoheres in time $\tau_n \sim e^{n/\xi}$

Each orbital contributes 1 bit of mixed-state entropy

$\implies S(t) \sim \xi \log(t)$

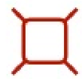
Spacetime-dual steady state: $S_A \sim \xi \log |A|$

[Area-law provably impossible in almost all of these circuits]

Haar-random circuits

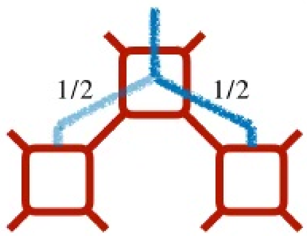
Random-walk picture

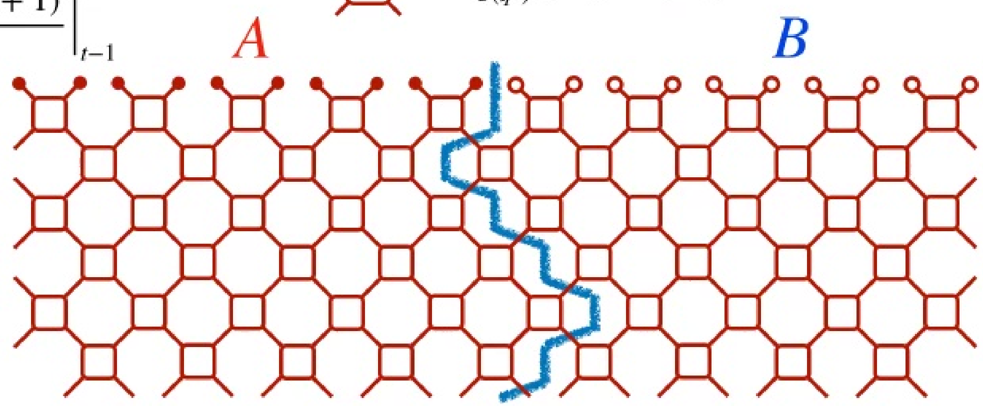
- Averaged purity $P \equiv \mathbb{E}[\mathrm{Tr}(\rho^2)]$ (& "annealed average" of S_2) exactly tractable via mapping to **random walk**



$= \mathbb{E}_{\mathrm{U}(q^2)}[(U \otimes U^*)^{\otimes 2}]$

$$P(x)\,|_t = e^{-v_E} \cdot \frac{P(x-1) + P(x+1)}{2}\Big|_{t-1}$$



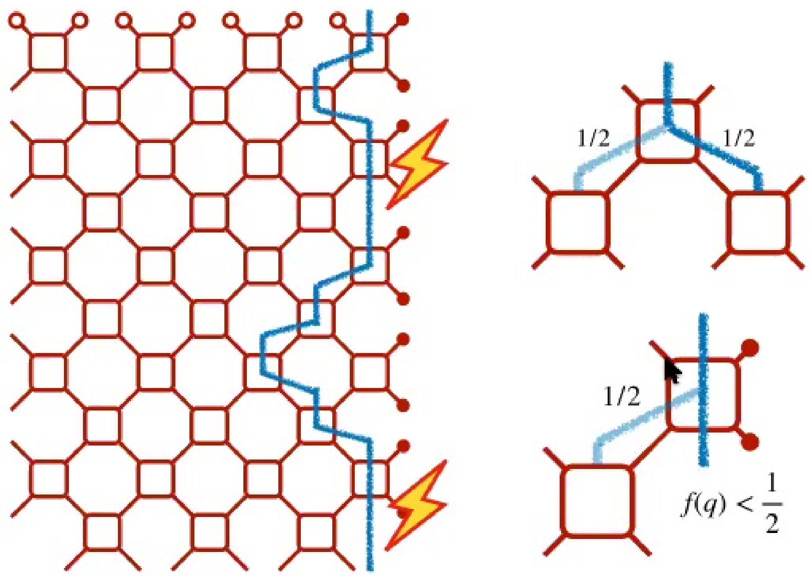


$$P(x)\,|_t = e^{-v_E t} \implies S(t) = v_E t$$

[Nahum, Vijay, Haah PRX 2018]

Haar-random circuits

Random-walk picture with edge decoherence



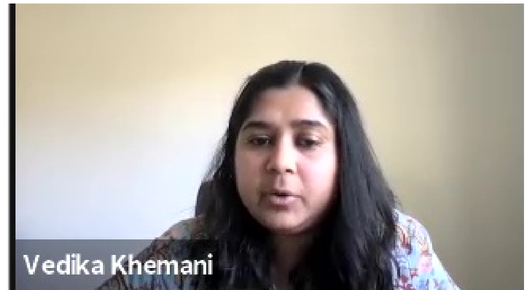
Probability loss due to absorbing wall:

$$\mathbb{E}_{\mathrm{Haar}}[\mathrm{Tr}(\rho^2)] \sim \frac{e^{-v_E \tilde{\ell}}}{\sqrt{\tilde{\ell}}}$$

Universal logarithmic correction to (annealed-averaged) entropy:

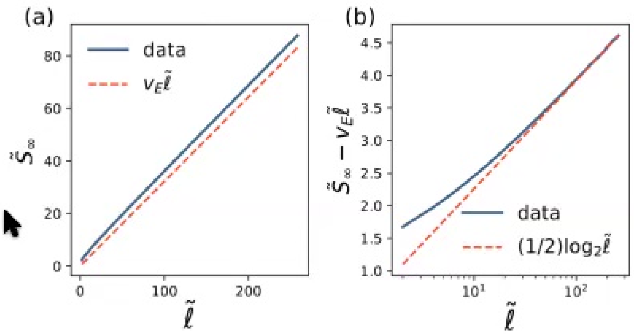
$$S_2(A) = v_E|A| + (1/2)\log|A|$$

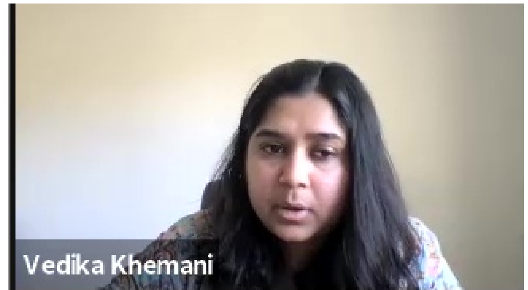
Partially absorbing wall
from edge decoherence



Haar-random circuits

A novel non-thermal volume-law entangled phase



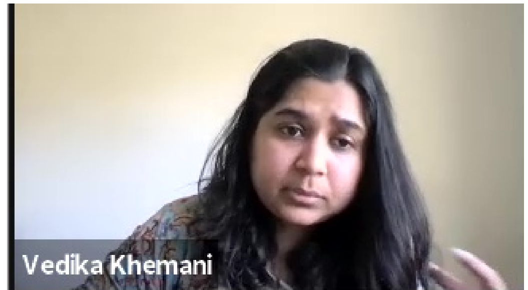


- $S(\tilde{\ell}) = v_E\tilde{\ell} + (1/2)\log\tilde{\ell}$
- **Entanglement velocity ↔ entropy density**

Haar-random circuits

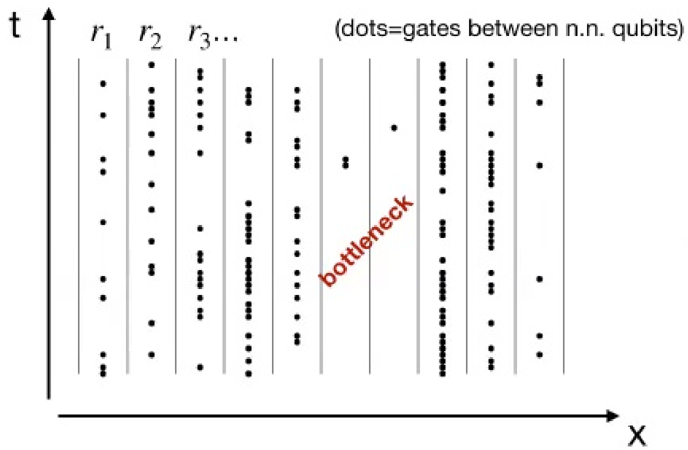
A novel non-thermal volume-law entangled phase

- $S(\tilde{\ell}) = v_E \tilde{\ell} + (1/2) \log \tilde{\ell}$
- **Entanglement velocity** ↔ **entropy density** (order parameter)
- Subleading log correction is **different** from universal value in "hybrid" unitary-measurement circuits, $(3/2)\log(\ell)$ [Fan, Vijay, Vishwanath, You arXiv:2002.12385]
- 3/2 coefficient derived on very general grounds from stat-mech picture as **domain-wall free energy** [Li, Fisher arxiv:2007.03822]
    - Not applicable to this (highly anisotropic) case



Fractal entanglement in "Griffiths model"

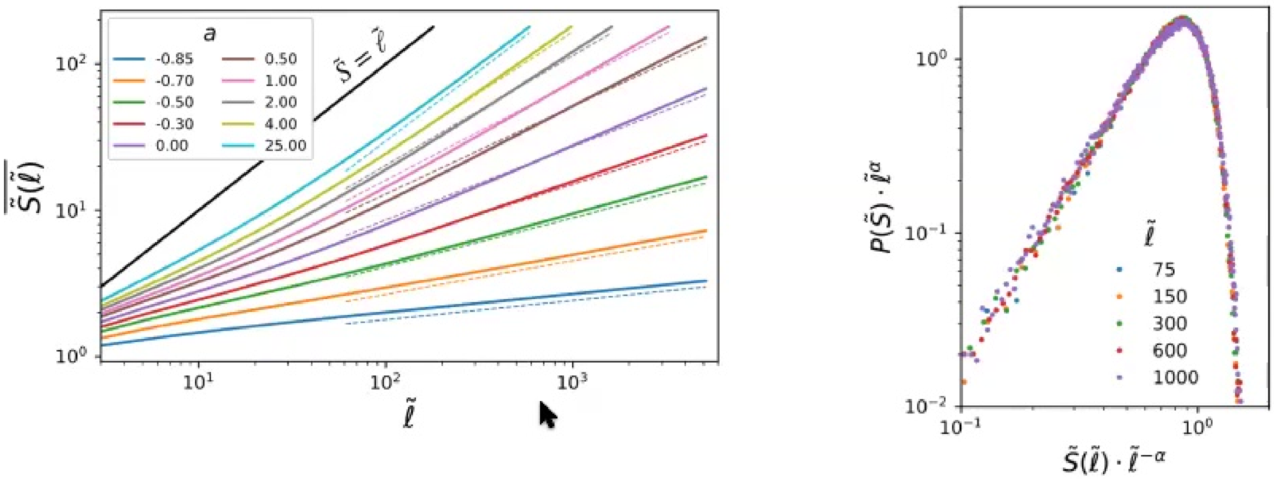
- Coarse-grained model of entanglement dynamics in random systems with "weak links" (Griffiths effects)
- Each bond x has a rate $r_x \in [0,1]$ drawn from $P(r) \sim r^a$

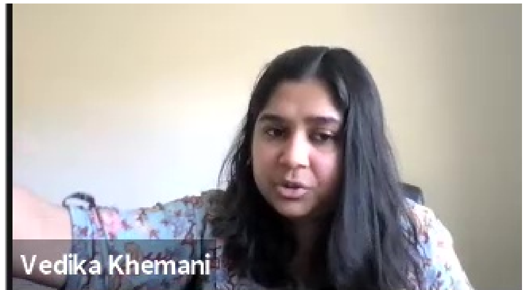


- Entanglement grows in time as $S(t) \sim t^\alpha$, $\alpha = (a+1)/(a+2)$

[Nahum, Ruhman, Huse, PRB 98, 035118 (2018)]

Fractal entanglement in "Griffiths model"





- At late times, $S_A \sim |A|^{\beta}$, $0 < \beta < 1$: **fractal entanglement!**

Fractal entanglement in "Griffiths model"

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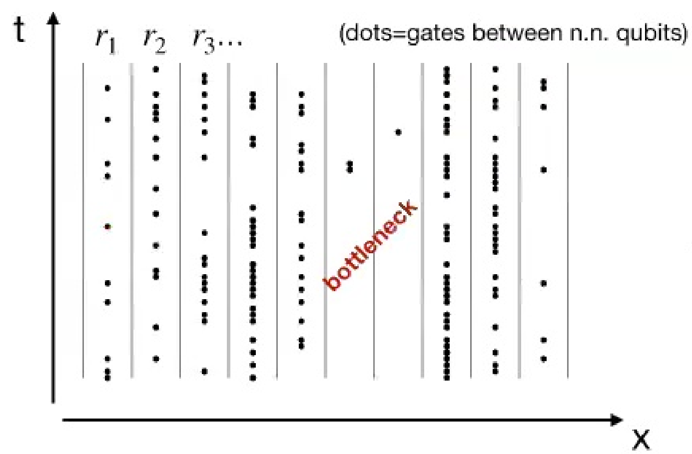

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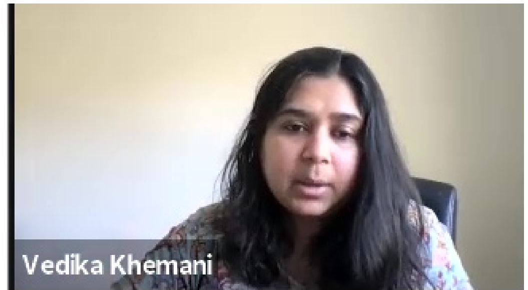
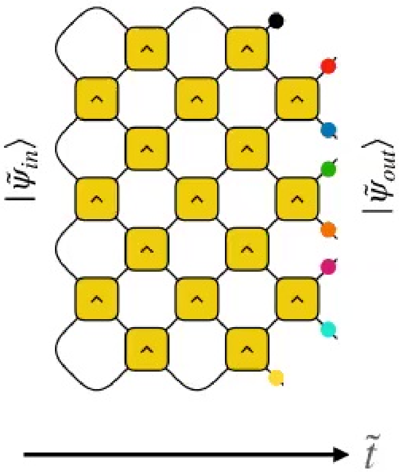
This Talk:
Novel Entanglement Phases and Phase Transitions in Non-Unitary Circuits via Spacetime Duality

- Review: Entanglement dynamics and phase transitions
- Spacetime duality: an experimentally motivated way to make interesting (analytically tractable) non-unitary circuits
- Novel steady state phases: logarithmic, volume-law and *fractal* entanglement scaling
- Preparing these steady states in the lab

Ippoliti, Rakovszky, VK, arXiv: 2103.06873
Ippoliti, VK, PRL 060501 (2021)

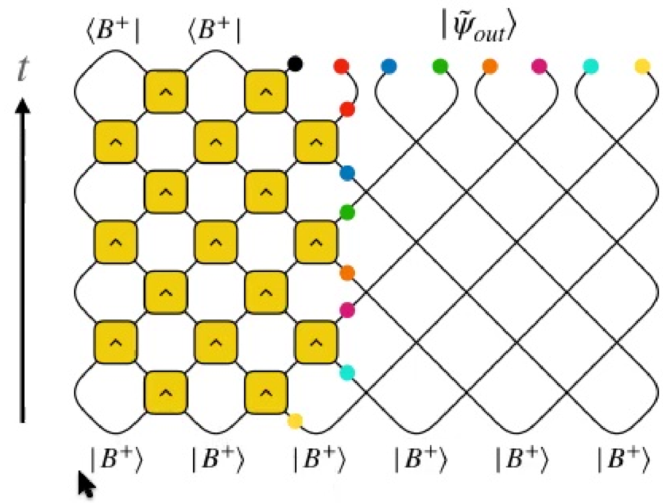


State preparation



* Input state $|\tilde{\psi}_{in}\rangle$ as Bell-pair states from open unitary BC
* Traverse the circuit left to right
* Produce $|\tilde{\psi}_{out}\rangle$ on a **time-like surface**

State preparation via "teleportation"

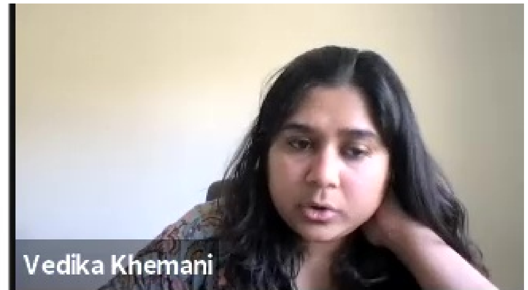


$$|B^+\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
(one of 4 Bell states)

* Initialize Bell pair state on system + ancillas
* Traverse the circuit with unitary gates from top to bottom
* "Teleport" $|\tilde{\psi}_{out}\rangle$ it to a spacelike-surface with ancillas & SWAPs
* "Forced" (**postselected**) Bell measurements on system qubits at final time-step

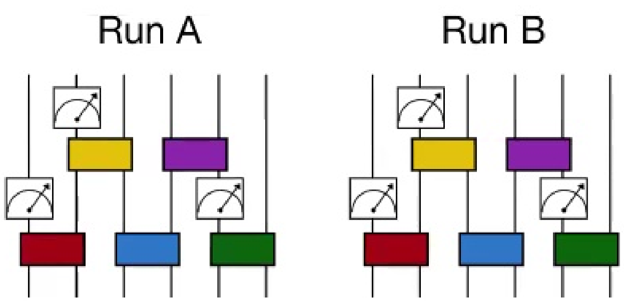
N_{meas} ~ |circuit's **boundary**| ~ L

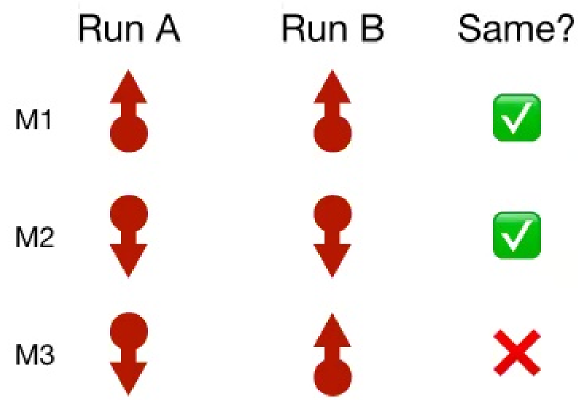
cf. hybrid unitary-measurement circuit: N_{meas} ~ p |circuit's **spacetime volume**| ~ pLT



The problem of postselection

- Measuring entropy: $\mathrm{Tr}(\rho \otimes \rho \cdot \mathrm{SWAP}) = \mathrm{Tr}(\rho^2)$ [Greiner et al 15]
- Producing two identical copies of the state is hard!
  - Quantum randomness of measurement outcomes requires **postselection**
  - Exponential overhead





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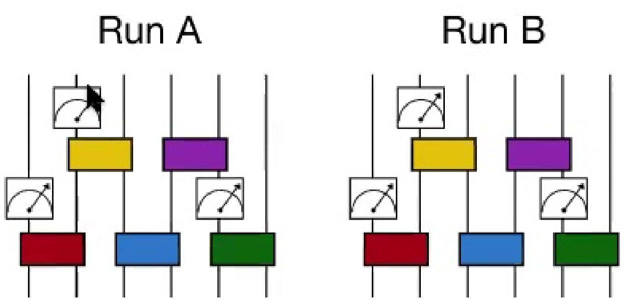
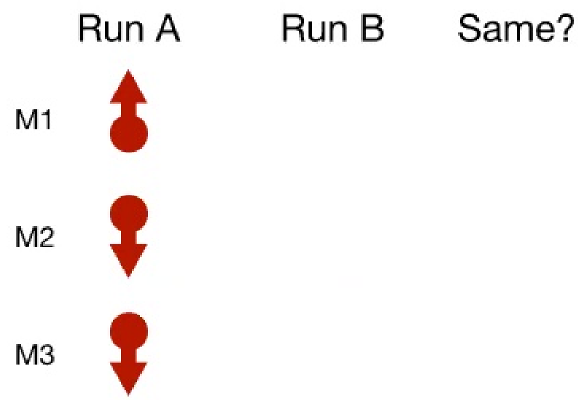
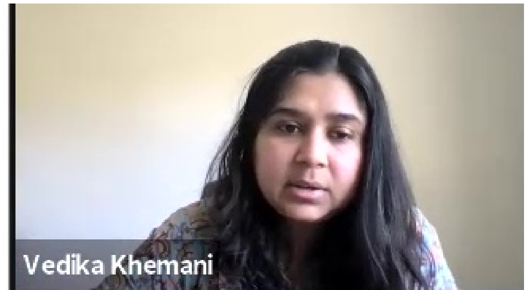
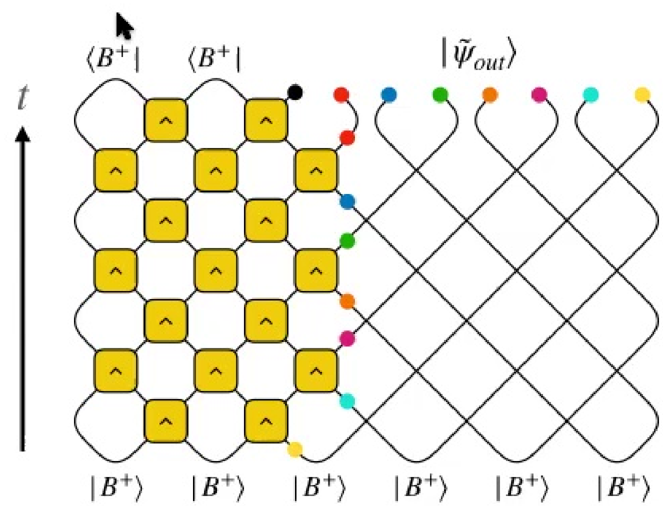






State preparation via "teleportation"



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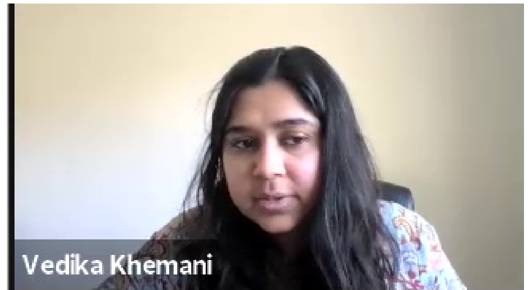

Spacetime duality helps with postselection

We can prepare output states of **purification dynamics** in spacetime-duals of unitary circuits with **no postselection** on measurement outcomes!

Ippoliti, VK, PRL **126**, 060501 (2021)

We can prepare output states of **entanglement dynamics** in spacetime-duals of unitary circuits with **limited postselection (scaling as boundary of spacetime)** on measurement outcomes.

Ippoliti, Rakovszky, VK, arXiv: 2103.06873

Summary

- **Spacetime duality:** Physically motivated route to interesting non-unitary circuits with analytic tractability
- A zoo of out-of-equilibrium late-time states with all kinds of entanglement scaling — from logarithmic to extensive through **fractal**
- Nature of new non-thermal volume-law phase?
- Duals of other classes of "interesting" unitary circuits?
- Obtaining fractal steady states as ground states of generalized Hamiltonians?

Ippoliti, Rakovszky, VK, arXiv: 2103.06873; Ippoliti, VK, PRL 060501 (2021)

Thanks to my collaborators

Matteo Ippoliti

Tibor Rakovszky

Ippoliti, Rakovszky, VK, arXiv: 2103.06873;
Ippoliti, VK, PRL 060501 (2021)