Title: Foliation structure in fracton models

Speakers: Xie Chen
Date: March 01, 2021 - 12:30 PM

URL: http://pirsa.org/21030000

Abstract: Fracton models are characterized by an exponentially increasing ground state degeneracy and point excitations with constrained motion. In this talk, I will focus on a prototypical 3D fracton model -- the X-cube model -- and discuss how its ground state degeneracy can be understood from a foliation structure in the model. In particular, we show that there are hidden 2D topological layers in the 3D bulk. To calculate the ground state degeneracy, we can remove the layers until a minimal structure is reached. The ground state degeneracy comes from the combination of the degeneracy of the foliation layers and that associated with the minimal structure. We discuss explicitly how this works for X-cube model with periodic boundary condition, open boundary condition, and even in the presence of screw dislocation defects.

Zhenghan Wang

Wilbur Shirley

Kevin Slagle

Nandagopla Manoj

Foliation structure in fracton models

XIE CHEN, CALTECH

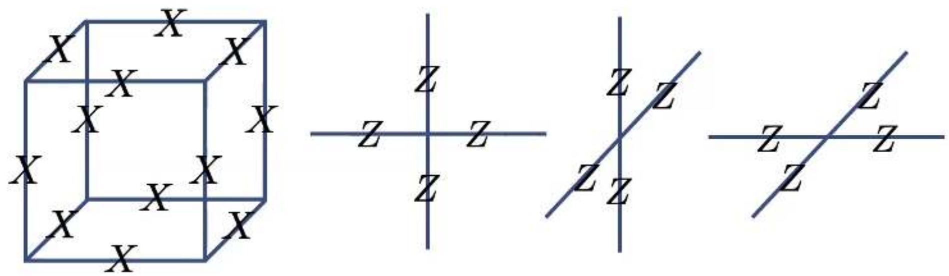
QUANTUM FRONTIER MATTER SEMINARS

MAR. 2021

Simons Collaboration on Ultra-Quantum Matter

X-cube model

Cubic lattice

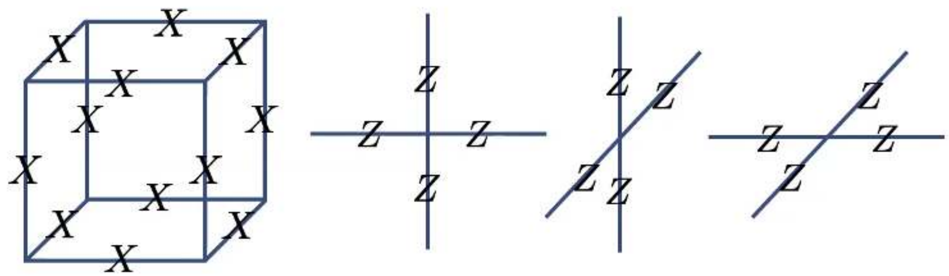


A_c B_v^x B_v^y B_v^z

Vijay, Haah, Fu, 16

X-cube model

Cubic lattice



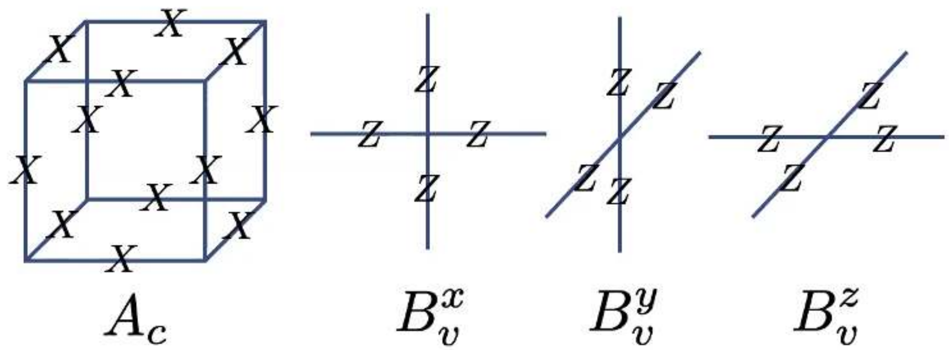
A_c $\quad B_v^x$ $\quad B_v^y$ $\quad B_v^z$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Vijay, Haah, Fu, 16

X-cube model

Cubic lattice



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H = -\sum_c A_c - \sum_v (B_v^x + B_v^y + B_v^z)$$

Vijay, Haah, Fu, 16

X-cube model

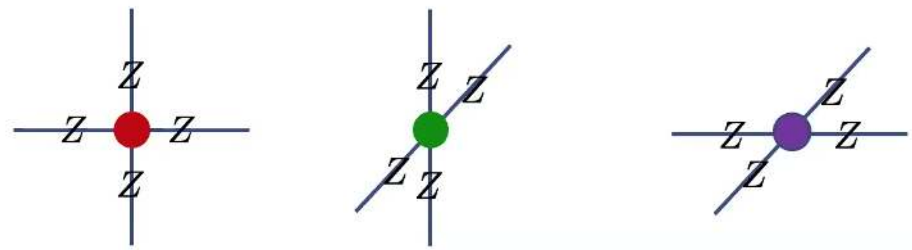
- All Hamiltonian terms commute
- Ground states satisfy

$$A_c = B_v^x = B_v^y = B_v^z = 1$$

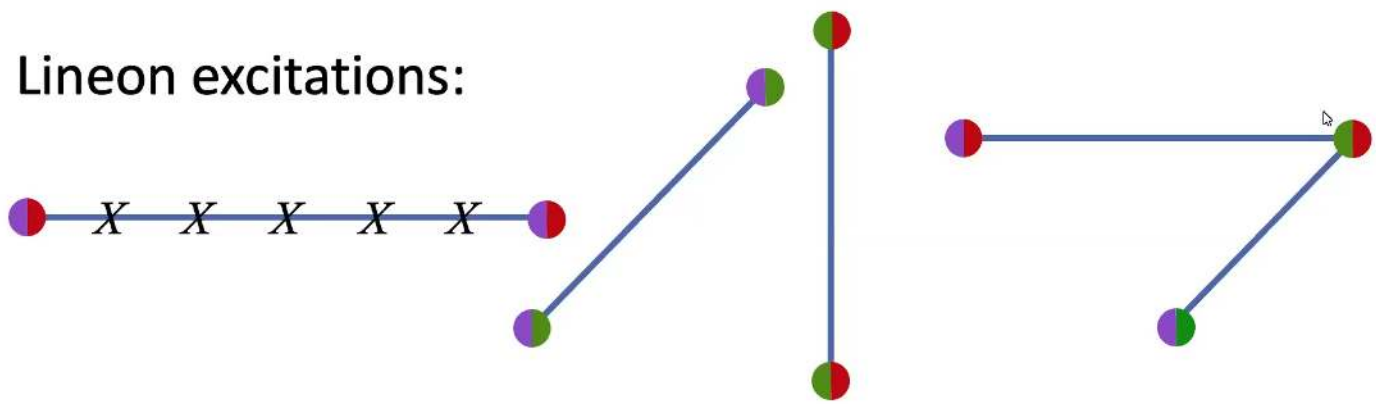
- Gapped
- Periodic boundary condition; Three torus
- Ground State Degeneracy

$$\log_2 D = 2L_x + 2L_y + 2L_z - 3$$

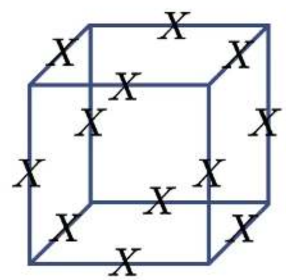
X-cube model: excitation



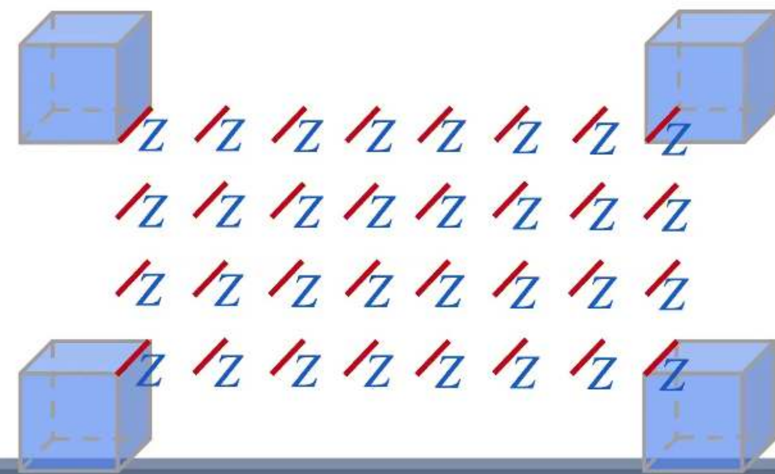
Lineon excitations:



X-cube model: excitation

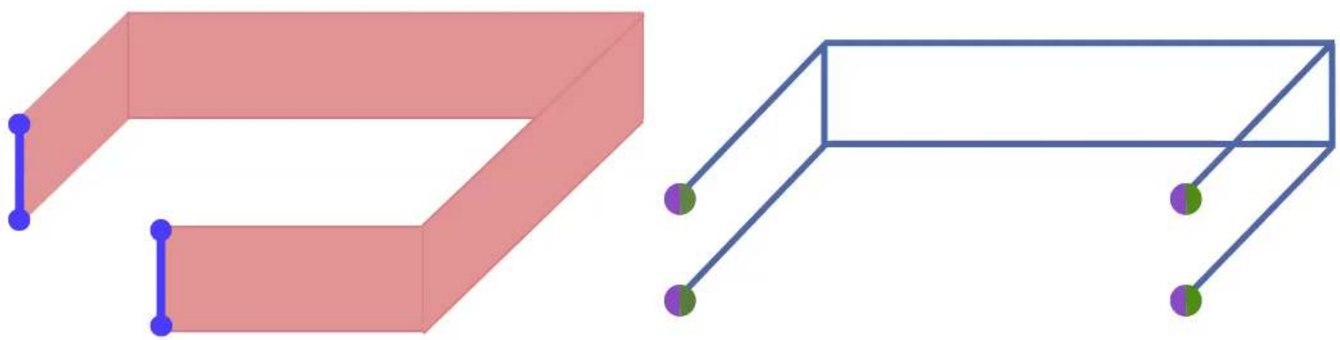


Fracton Excitations



X-cube model: excitation

Planon excitations:

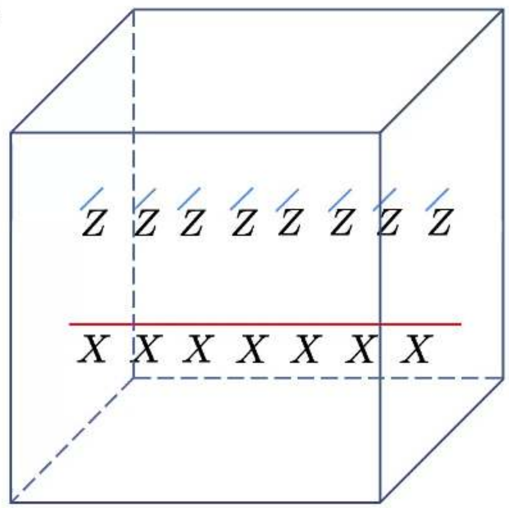


X-cube model

- Exponential ground state degeneracy
- Point excitations with limited motion
- What kind of order?

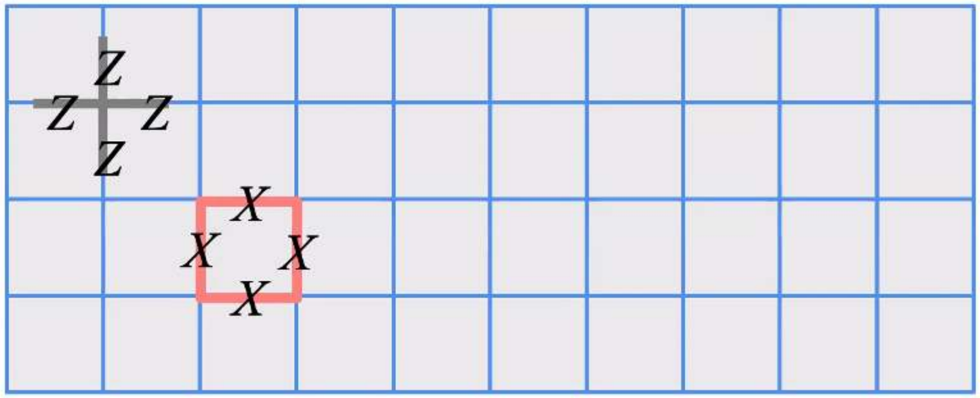
X-cube model

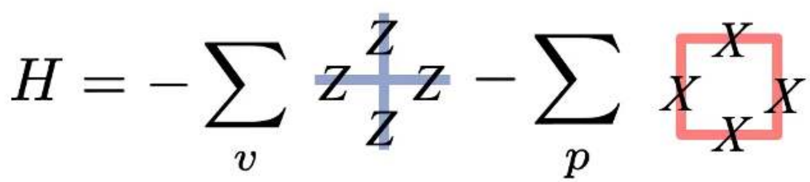
"Topological"



Ground state degeneracy robust against local perturbation

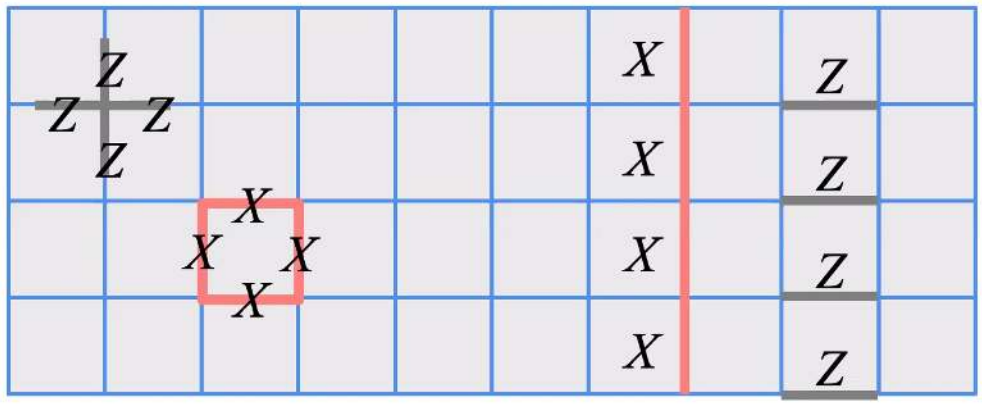
Topological Order – Toric Code





- All terms commute
- Periodic boundary condition, two torus
- Ground State Degeneracy $\log_2 D = 2$

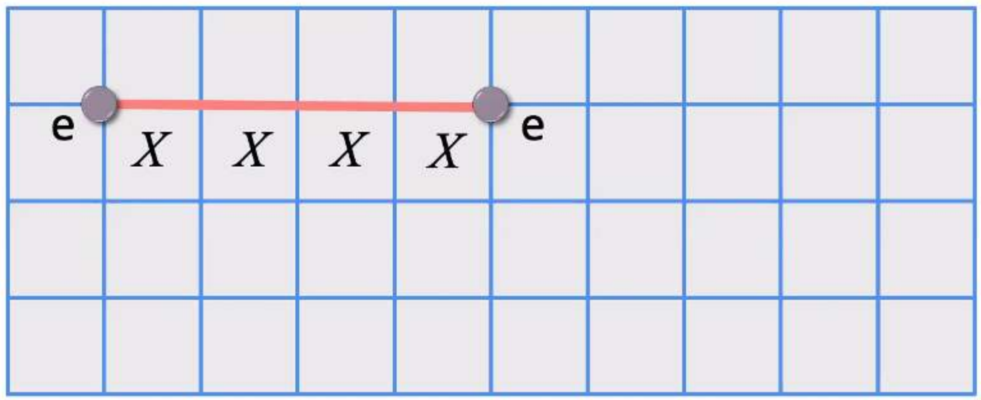
Topological Order – Toric Code



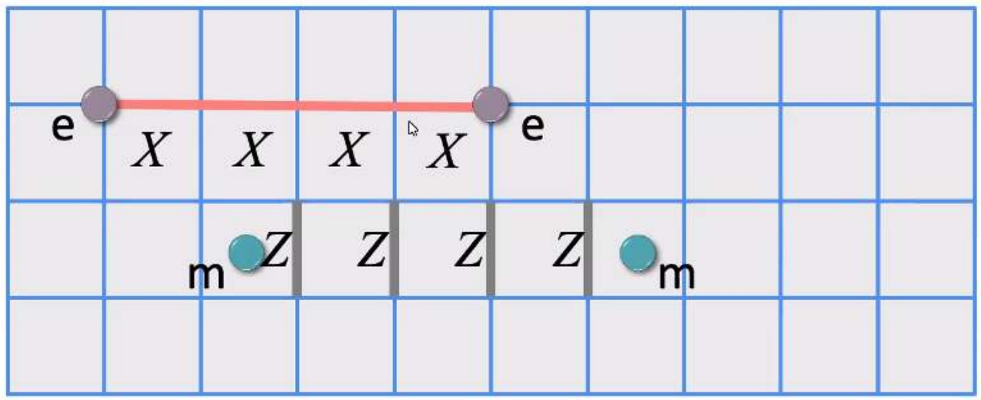
$$H = -\sum_{v} Z\,Z\,Z\,Z - \sum_{p} X\,X\,X\,X$$

- All terms commute
- Periodic boundary condition, two torus
- Ground State Degeneracy $\log_2 D = 2$

Topological Order – Toric Code



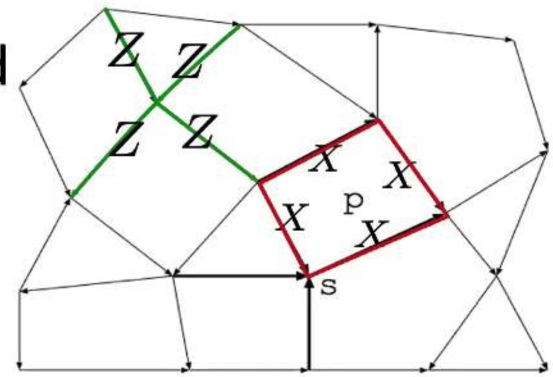
Topological Order – Toric Code



- Fractional excitations: e and m
- Braiding statistics

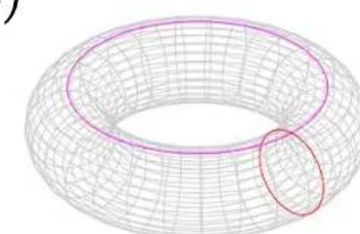
Toric Code

- Hamiltonian can be defined on any 2D lattice
- Ground state degeneracy depends only on the topology of the manifold

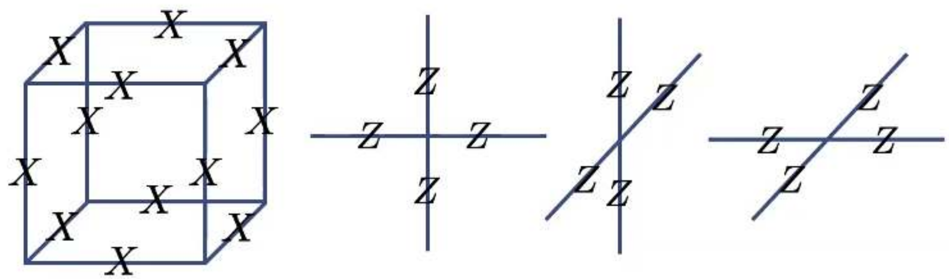


$$\log_2 D = 2(\text{genus of surface})$$

- Topological quantum field theory

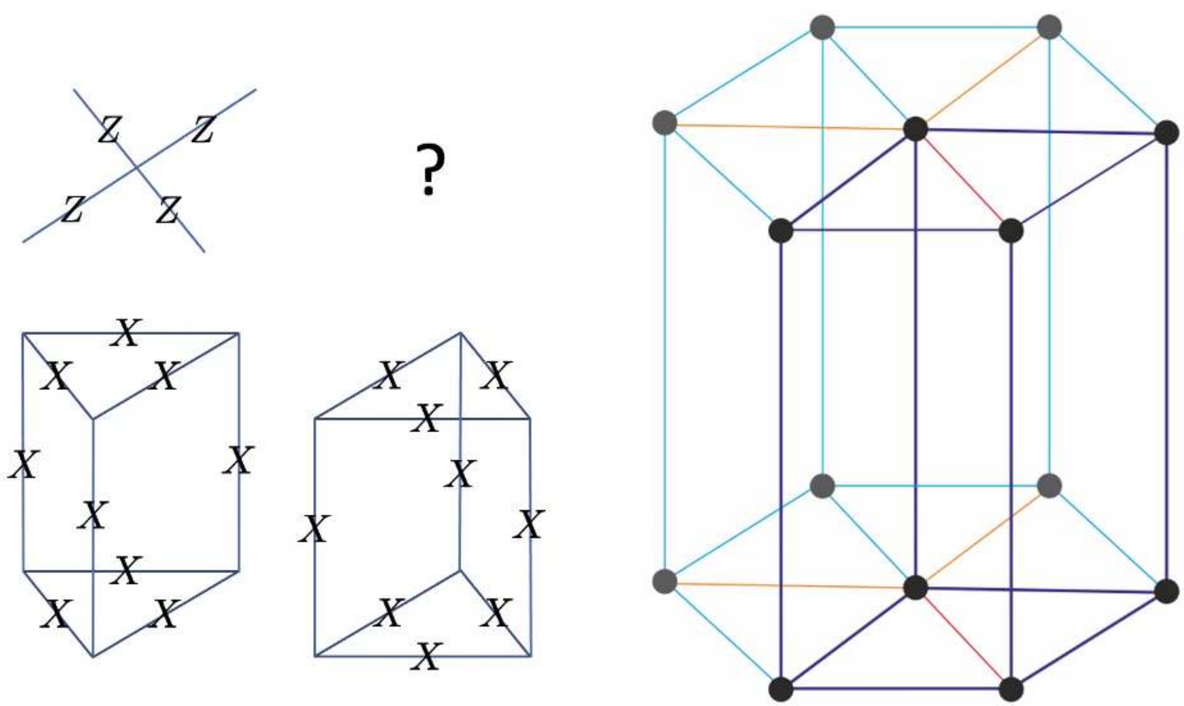


X-cube model

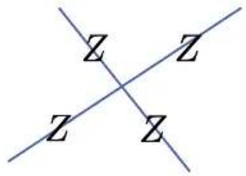


- Different Lattices?
- Different Three Manifolds?

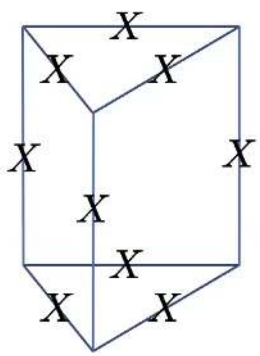
3D lattice

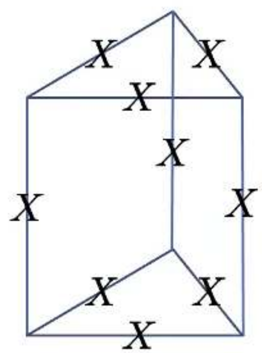


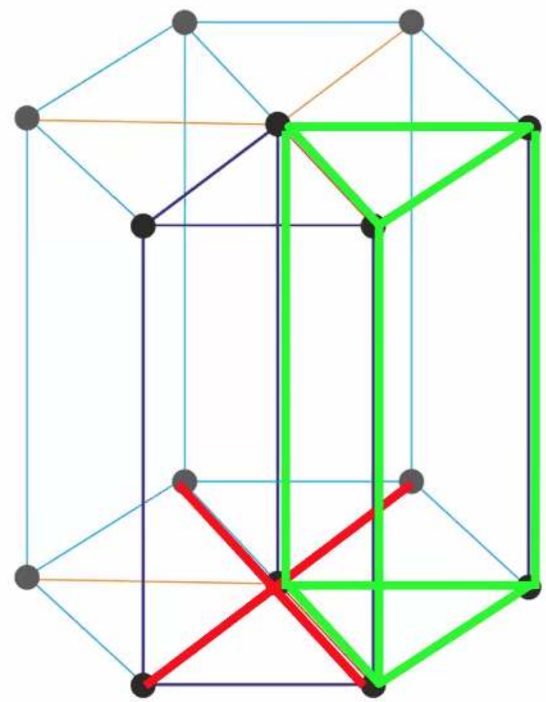
3D lattice



?







Foliation

- Dividing a three dimensional manifold into parallel planes (leaves).
- Example: xy planes (or yz planes, zx planes) in three torus
- Total foliation:
- Three sets of transversely intersecting parallel surfaces
- Example: xy, yz, zx planes in three torus
- And we can do this to other three manifolds as well

- Fracton Models on General Three-Dimensional Manifolds, *Phys. Rev. X* 8, 031051 (2018)

Total Foliation

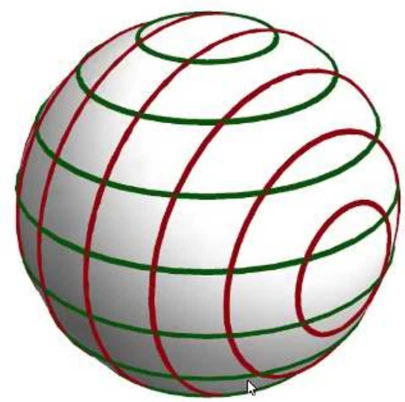
S^3 $\qquad x^2 + y^2 + z^2 + w^2 = 1$

Three sets of leaves

$$x = x_0, \ y = y_0, \ z = z_0$$

Each leaf is a 2-sphere

A $x = x_0$ leaf



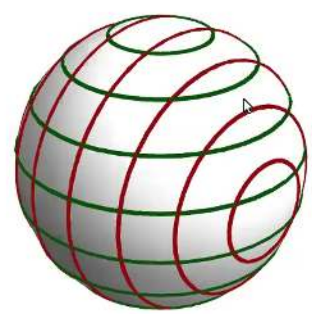
Total Foliation

$$S^2 \times S^1$$
$$x^2 + y^2 + z^2 = 1; 0 \leq w \leq 1, (0 \sim 1)$$

Three sets of leaves

$x = x_0, y = y_0$ 2-Torus leaves

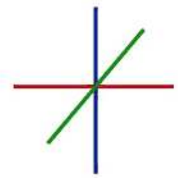
$w = w_0$ 2-Sphere leaves

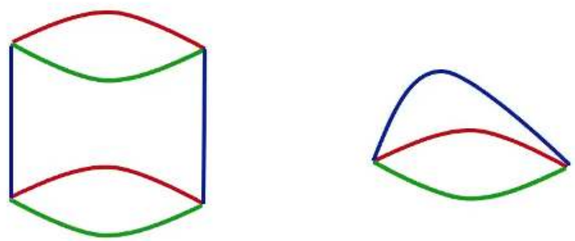


A $w = w_0$ leaf

Total Foliation

- A notion of x, y, z direction
- Each vertex is degree six
- Each body may not be cubes

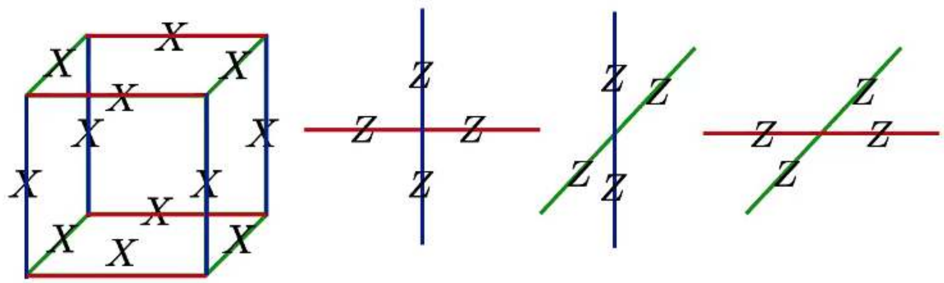




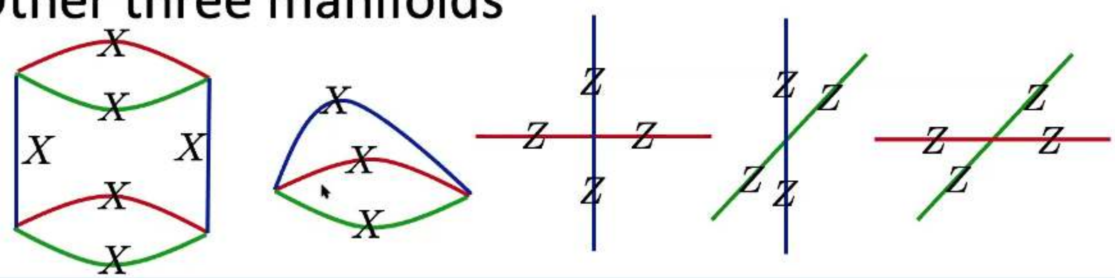
- Compact singular foliation

X-cube on three manifolds

Three torus: cubic lattice

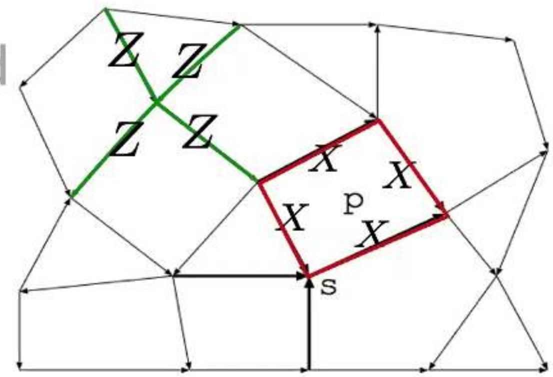


Other three manifolds

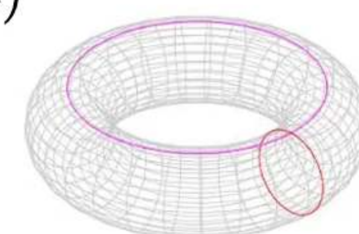


Toric Code

- Hamiltonian can be defined on any 2D lattice
- **Ground state degeneracy depends only on the topology of the manifold**



$$\log_2 D = 2(\text{genus of surface})$$

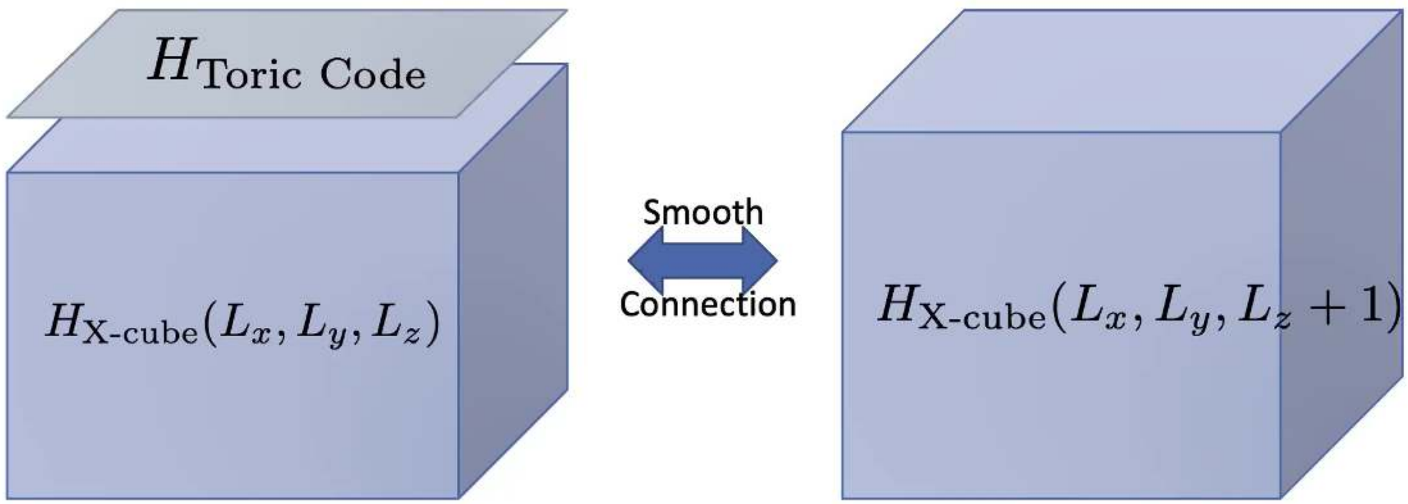


- Topological quantum field theory

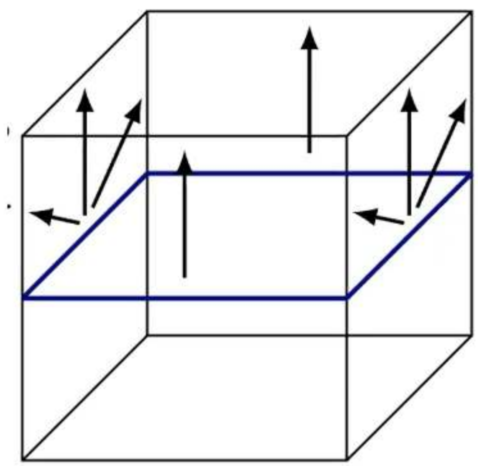
Manifold and degeneracy

Manifold	Foliating Leaves	$\log_2$ Ground State Degeneracy
T^3	T^2, T^2, T^2	$2L_x + 2L_y + xL_z - 3$
S^3	S^2, S^2, S^2	0
$S^2 \times S^1$	T^2, T^2, S^2	$2L_x + 2L_y - 1$

Renormalization Group Trans.



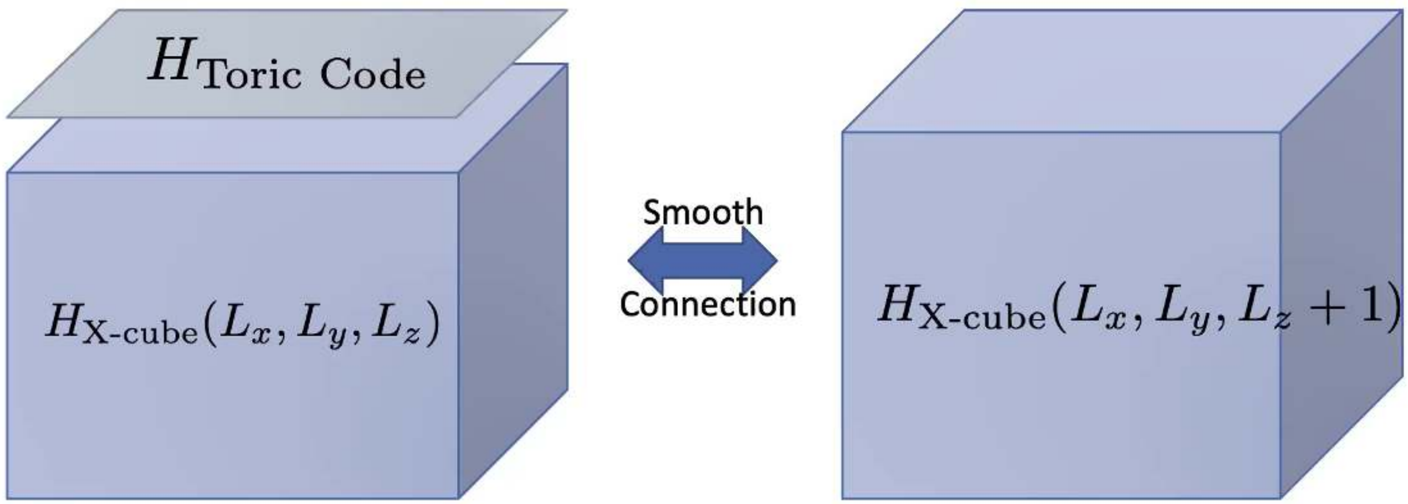
Renormalization Group Trans.



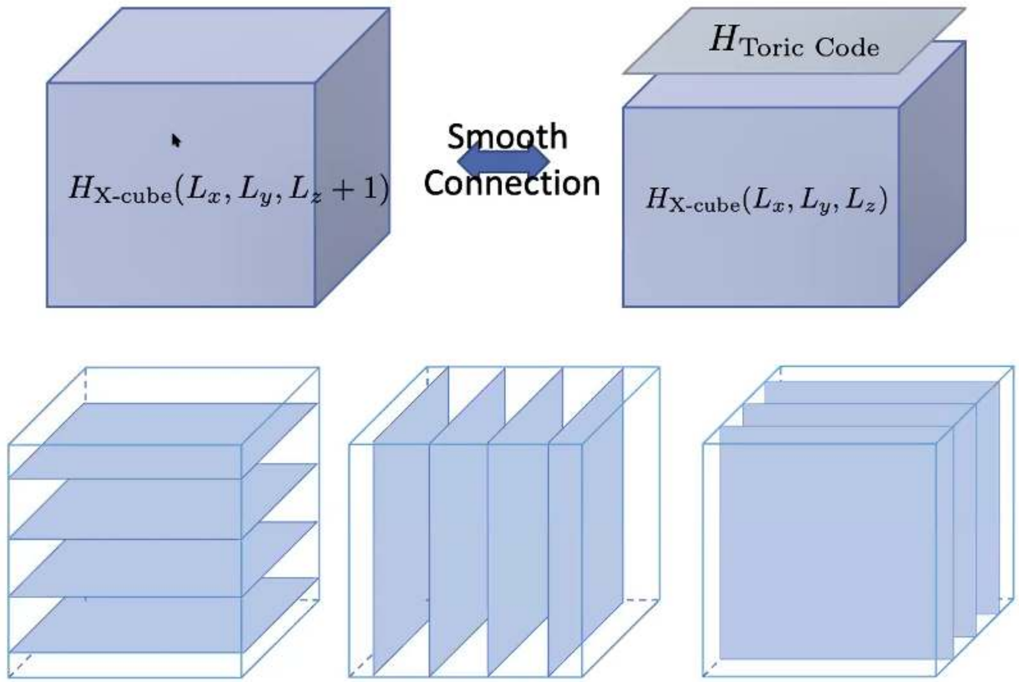
Controlled-Not Gate

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$$

Renormalization Group Trans.



Renormalization Group Trans.

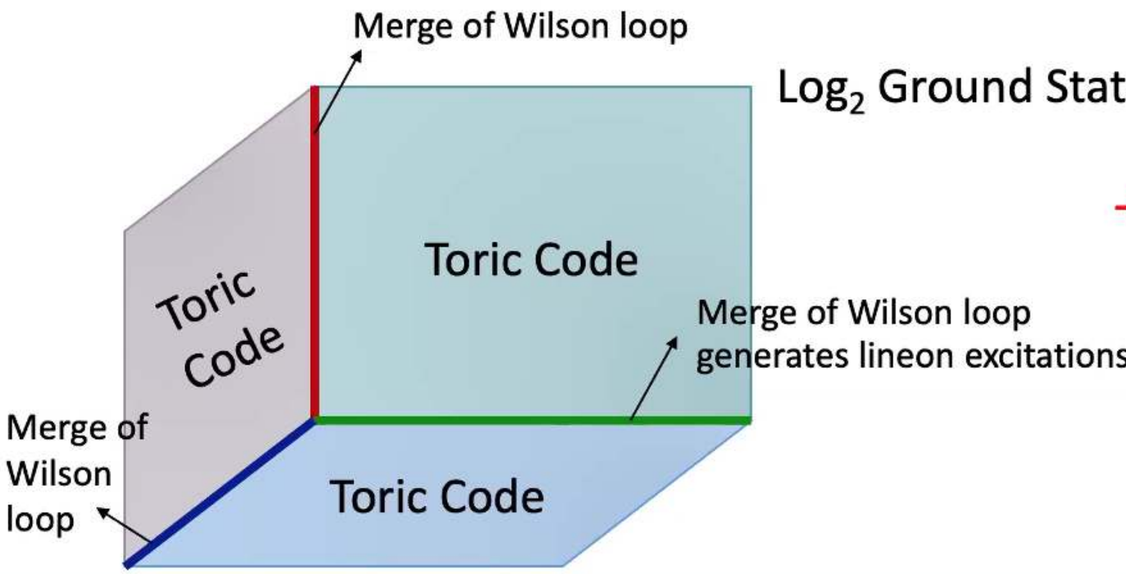


X-cube on three manifolds

Manifold	Foliating Leaves	$\log_2$ Ground State Degeneracy
T^3	T^2, T^2, T^2	$2L_x + 2L_y + 2L_z - 3$
S^3	S^2, S^2, S^2	$0L_x + 0L_y + 0L_z - 0$
$S^2 \times S^1$	T^2, T^2, S^2	$2L_x + 2L_y + 0L_z - 1$

Manifold and degeneracy

Remove layers until only one is left in each set, minimal structure



Log_2 Ground State Degeneracy

-3

X-cube on three manifolds

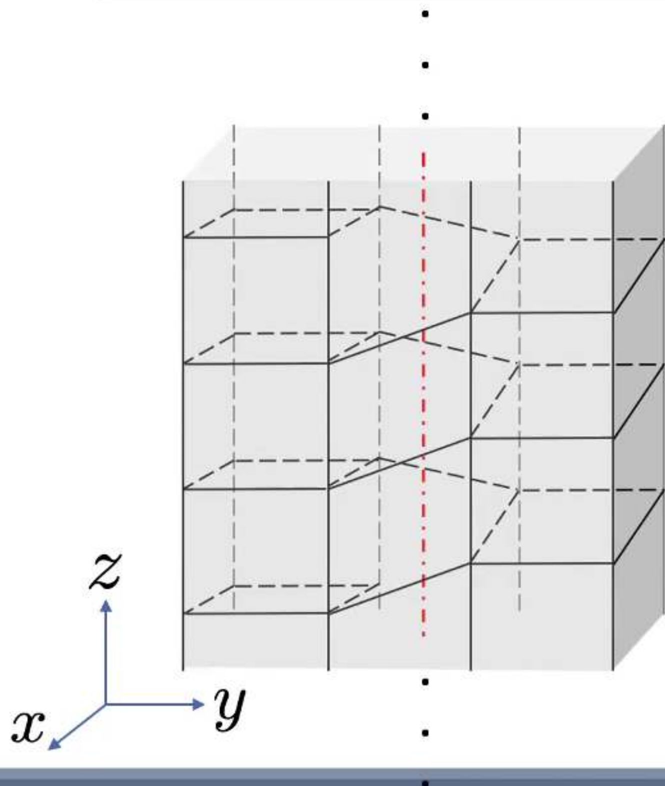
Manifold	Foliating Leaves	$\log_2$ Ground State Degeneracy
T^3	T^2, T^2, T^2	$2L_x + 2L_y + 2L_z - 3$
S^3	S^2, S^2, S^2	$0L_x + 0L_y + 0L_z - 0$
$S^2 \times S^1$	T^2, T^2, S^2	$2L_x + 2L_y + 0L_z - 1$

X-cube on three manifolds

Constant part of degeneracy depends on the topology of the intersection of foliating leaves

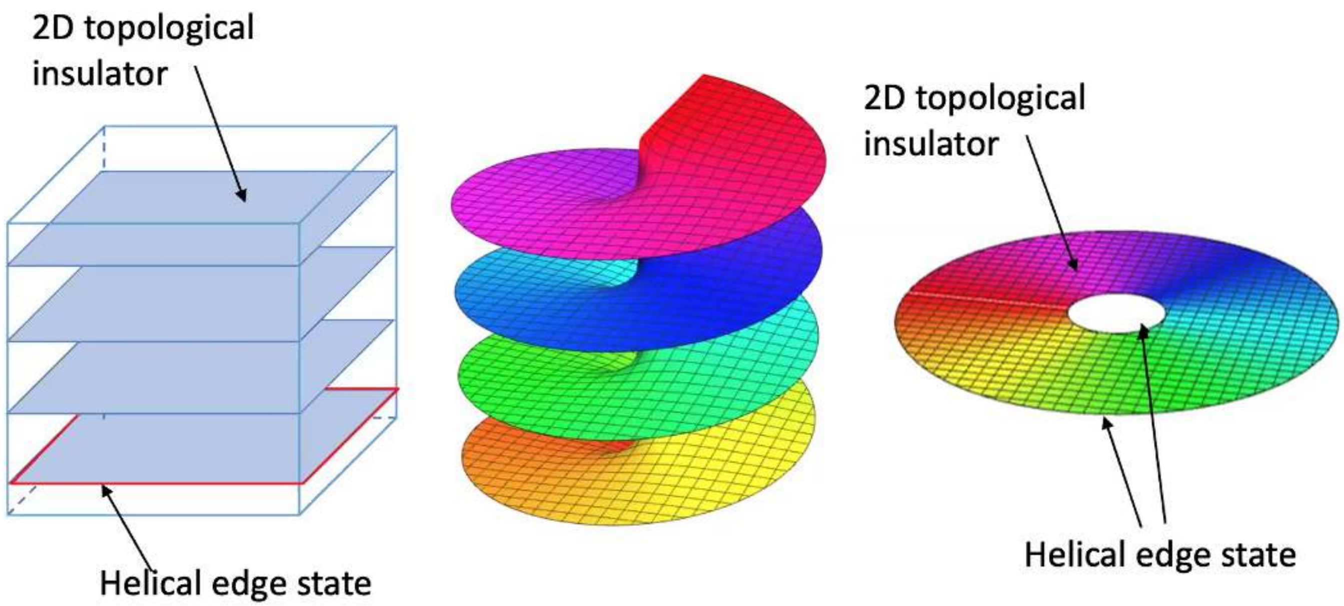
Manifold	Foliating Leaves	Log_2 Ground State Degeneracy
T^3	T^2, T^2, T^2	$2L_x + 2L_y + 2L_z - 3$
S^3	S^2, S^2, S^2	$0L_x + 0L_y + 0L_z - 0$
$S^2 \times S^1$	T^2, T^2, S^2	$2L_x + 2L_y + 0L_z - 1$

Screw defect



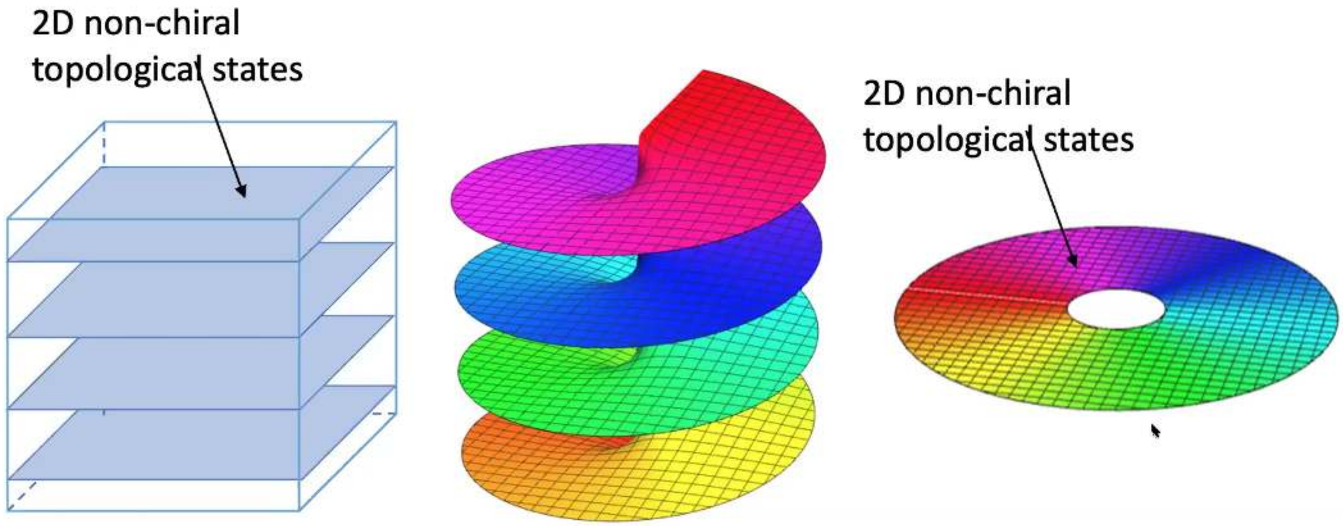
- Screw dislocations in the X-cube fracton model, arXiv:2012.07263

Screw defect in 3d weak topological insulator

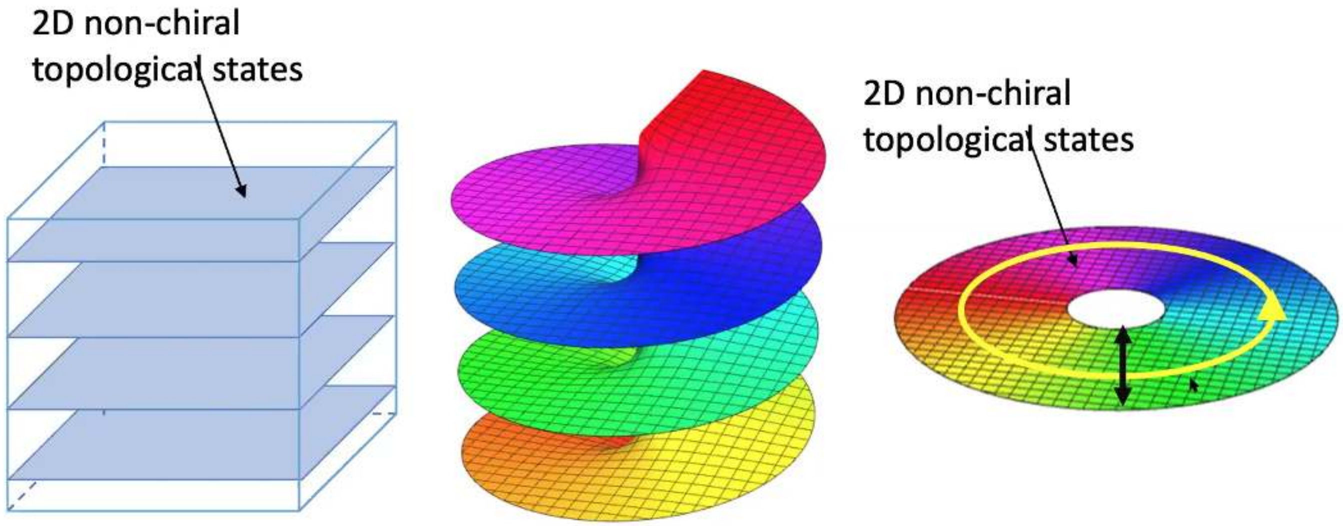


Y. Ran, Y. Zhang and A. Vishwanath, 2009

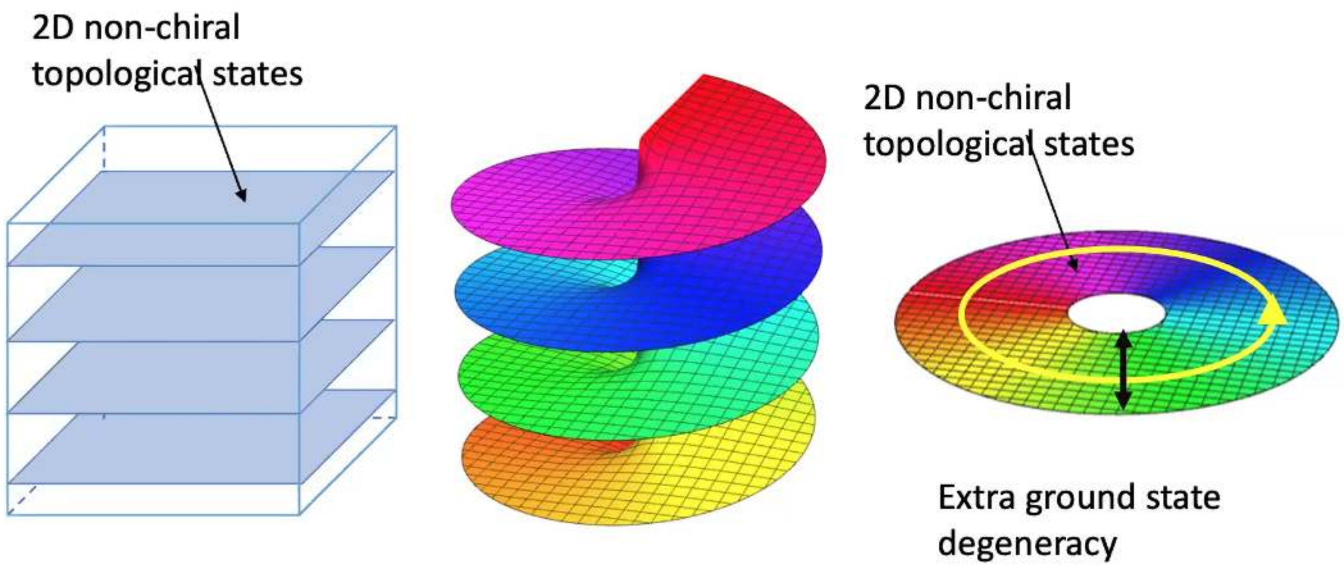
Screw defect in a stack of 2D non-chiral topological order



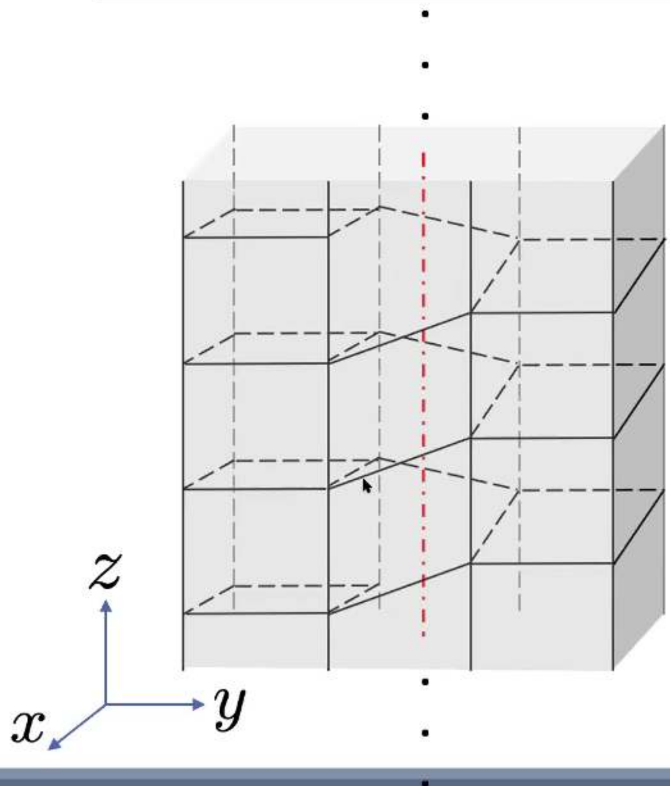
Screw defect in a stack of 2D non-chiral topological order



Screw defect in a stack of 2D non-chiral topological order

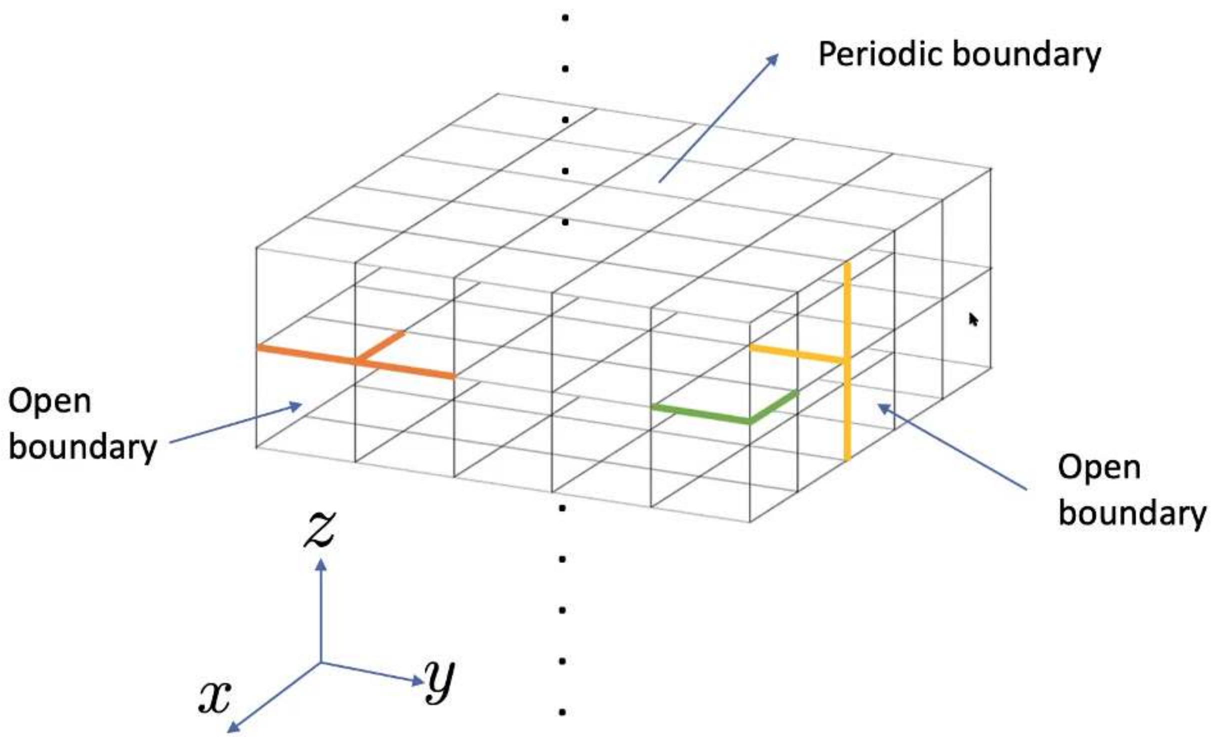


Screw defect

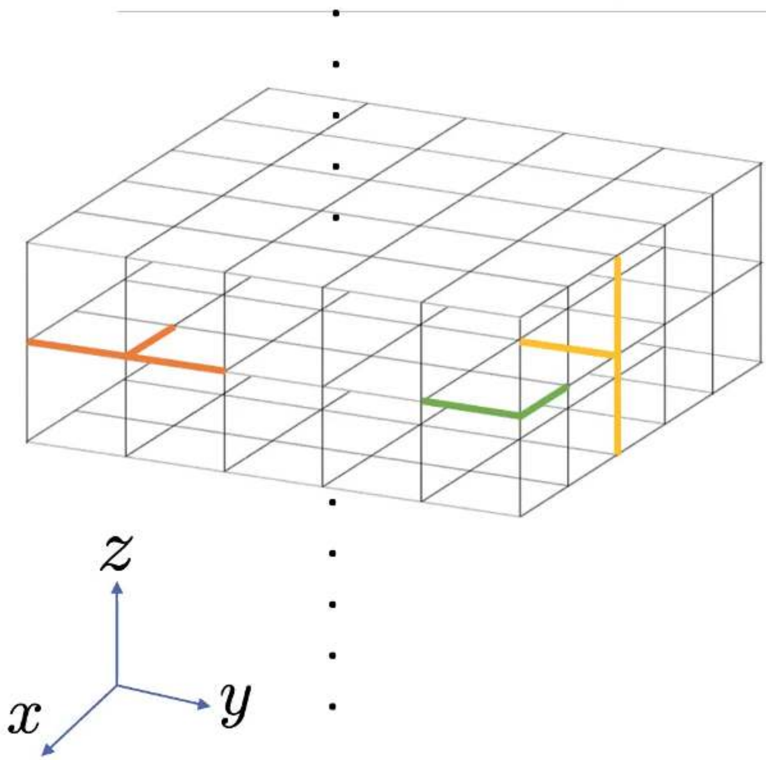


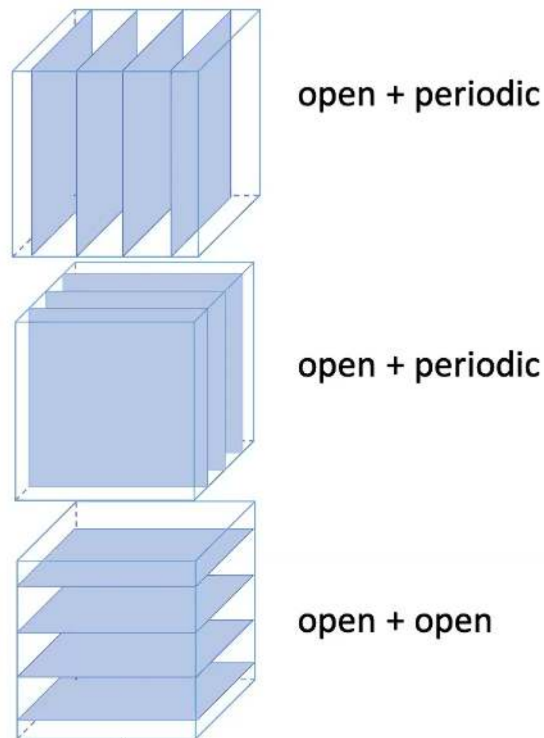
- Extra ground state degeneracy?
- Foliation
- Minimal structure

Open boundary condition

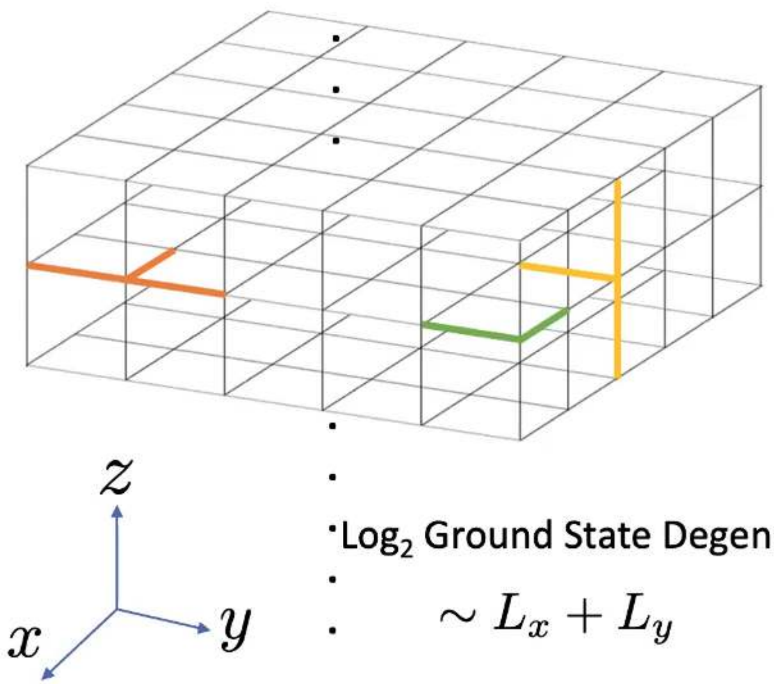


Open boundary condition: foliation



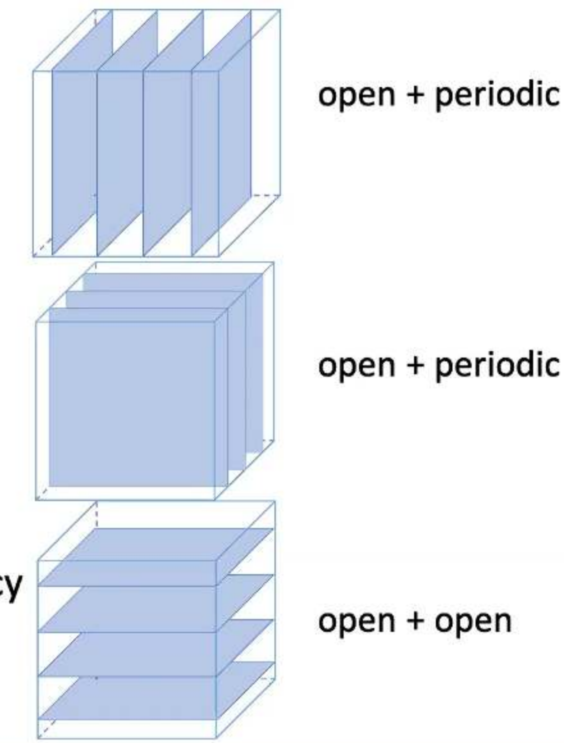


Open boundary condition: foliation

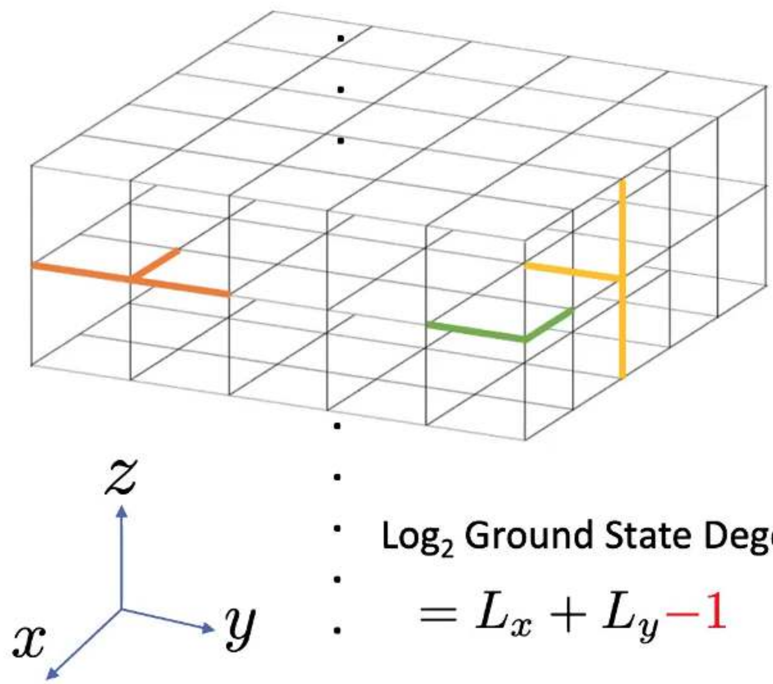


Log$_2$ Ground State Degeneracy

$$\sim L_x + L_y$$

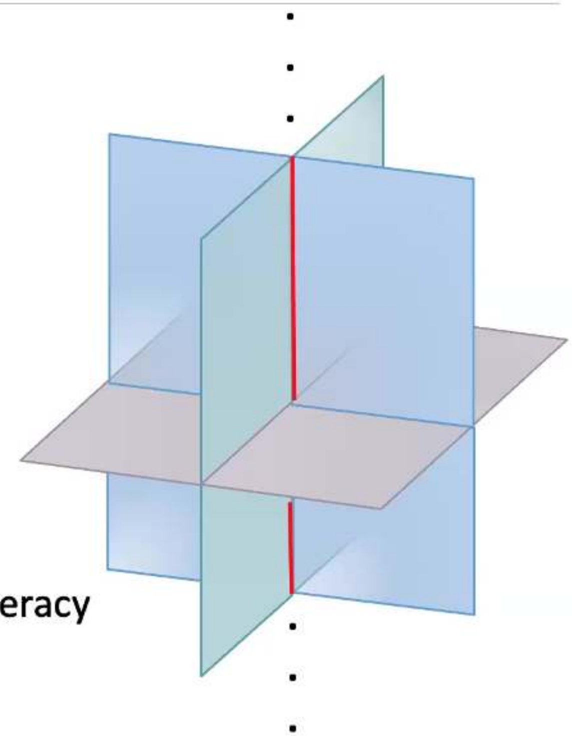


Open boundary condition: minimal structure



Log_2 Ground State Degeneracy

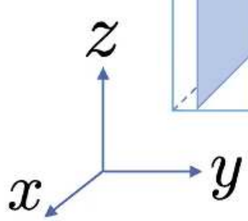
$$= L_x + L_y - 1$$

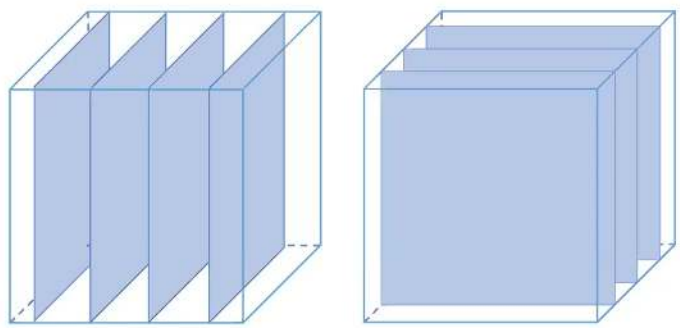


Screw defect: foliation

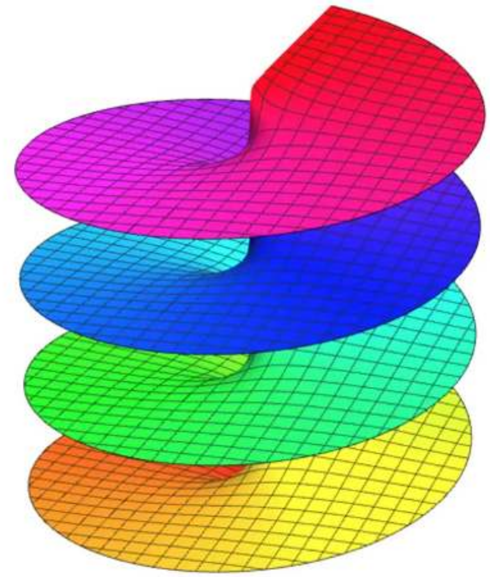
Log$_2$ Ground State Degeneracy

$$\sim L_x + L_y$$



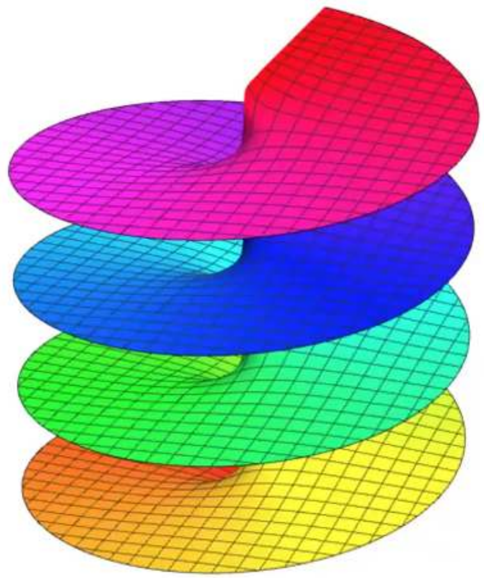


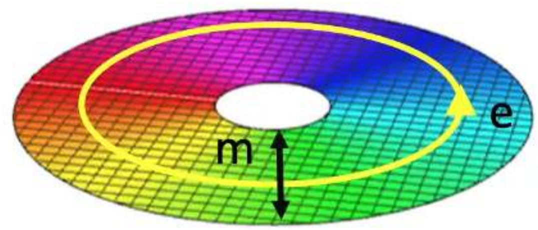
open + periodic



open

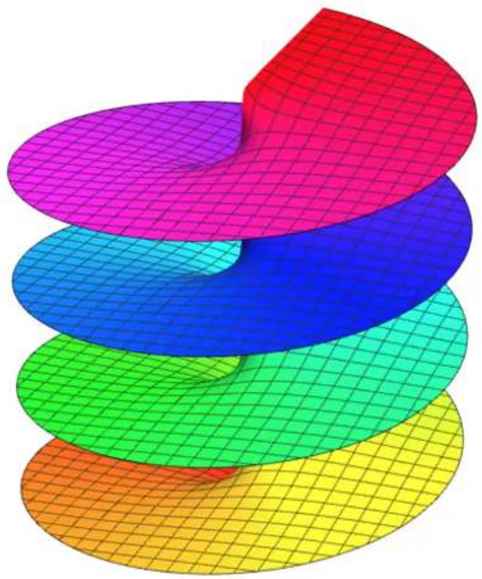
Screw defect: spiral leaf

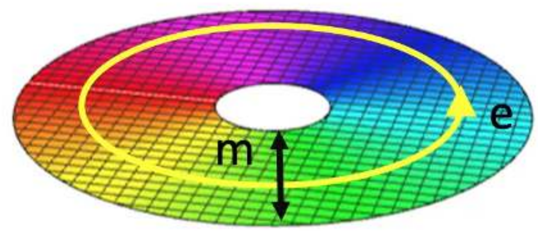




Log_2 Ground State Degeneracy
=1

Screw defect: spiral leaf

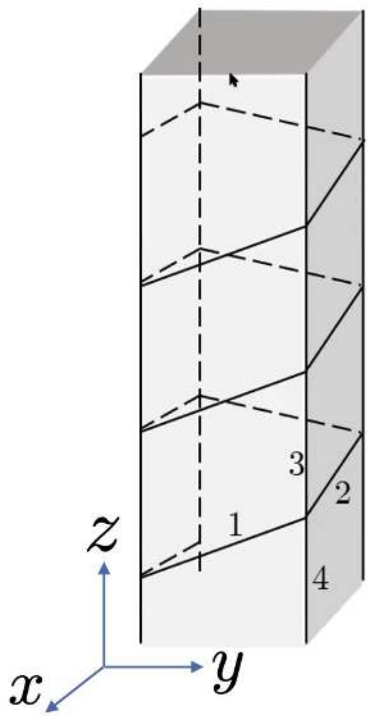




Log_2 Ground State Degeneracy
=1

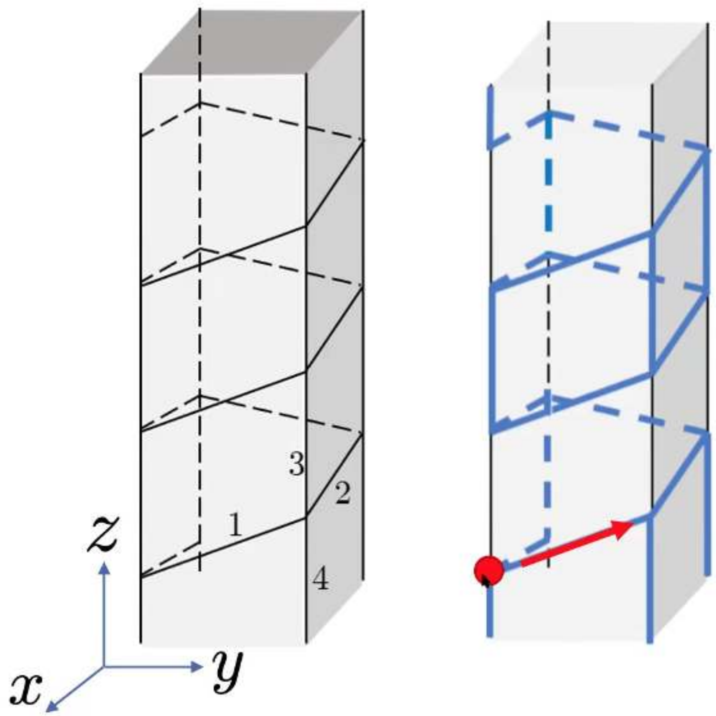
Total Log_2 Ground State Degeneracy $= L_x + L_y - 1 + 1?$

Screw defect: minimal structure



2 xz leaves,
2 yz leaves,
1 xy spiral leaf

Screw defect: minimal structure

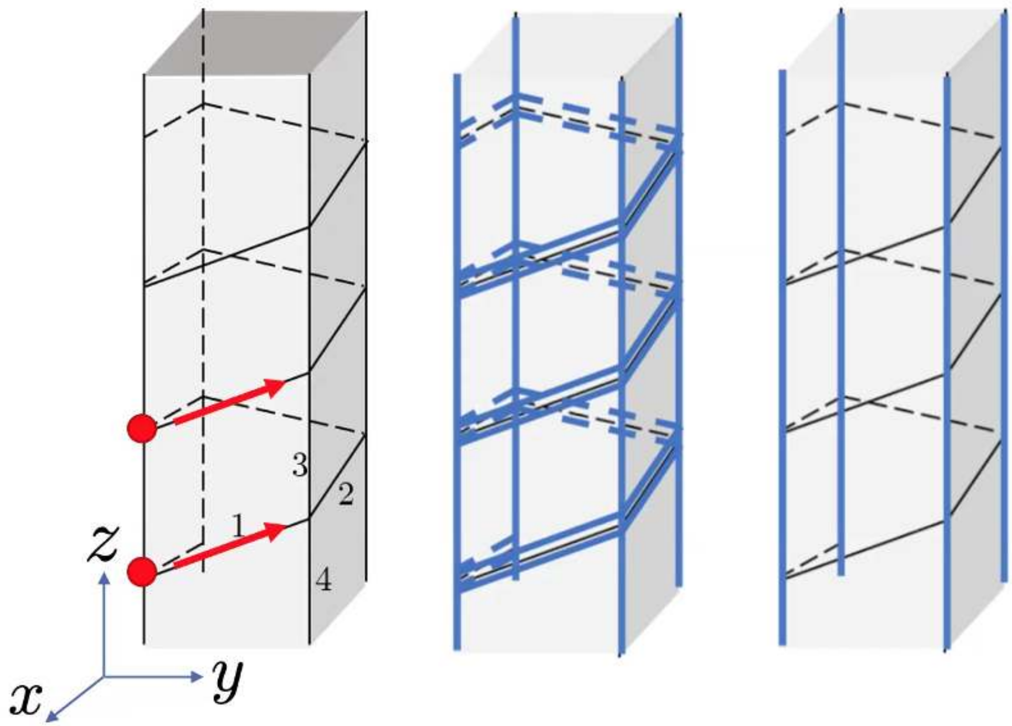


Winding of lineon around defect

Even Lz, Log_2 GSD +1

Odd Lz, Log_2 GSD +0

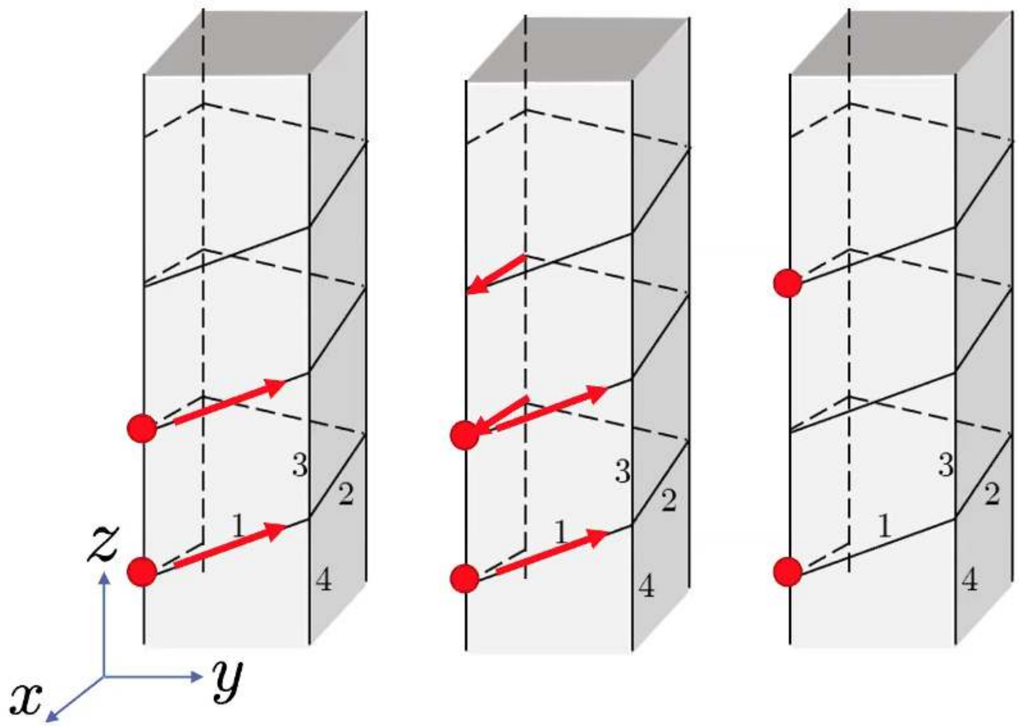
Screw defect: minimal structure



Log_2 GSD +1

For all Lz

Screw defect: minimal structure

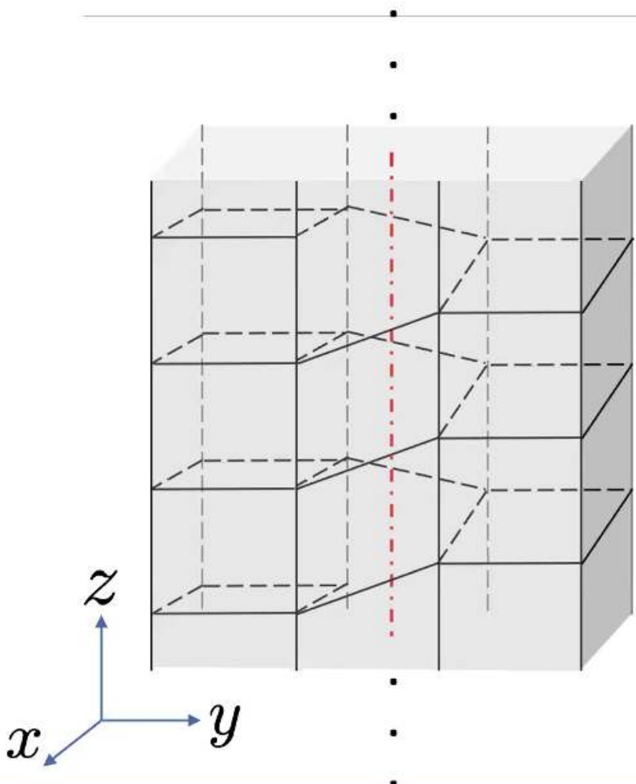


Tunneling of lineon around the defect

Log_2 GSD +1

For all Lz

Screw defect



- Winding of lineon around defect
- Even Lz, Log_2 GSD +1
- Odd Lz, Log_2 GSD +0

- Tunneling of lineon around defect
- Log_2 GSD +1
- For all Lz

Total Log_2 Ground State Degeneracy

$$= L_x + L_y - 1 + 1(+1)$$

Summary

- X-cube model
- Different manifold, boundary condition, screw defect
- GSD from foliation + coupled layer minimal structure

Open question

- other fracton models
- non-exactly solvable models
- with disorder
- field theory description