

Title: Light echos and coherent autocorrelations in a black hole spacetime

Speakers: Paul Chesler

Series: Strong Gravity

Date: February 25, 2021 - 1:00 PM

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Abstract: Light rays can orbit the photon shell of a black hole many times before escaping to infinity. This means that a distant observer can see a successive series of "echo" images, each separated in time by the photon shell orbital period, and each dimmer than the previous. I will present a study of light echos using coherent autocorrelation functions sourced by fluctuating matter in accretion flows. I will demonstrate that coherent autocorrelation functions are peaked at integer multiples of the photon shell orbital period. Furthermore, I will argue that the power in echos from supermassive black holes is too small to be observed on Earth.

Zoom Link: <https://pitp.zoom.us/j/92751681169?pwd=V0hQeUMwaWtTQjQ3UzdxekJiT0lmQT09>

Light echos and coherent autocorrelations in a black hole spacetime

2012.11778

Work done with Lindy Blackburn, Shep Doeleman, Michael Johnson,
Jim Moran, Ramesh Narayan & Maciek Wielgus.

Paul Chesler



The photon shell in the Kerr spacetime

- There exists bound null orbits in the Kerr spacetime at radii

$$r_-^\gamma \leq r \leq r_+^\gamma,$$

where

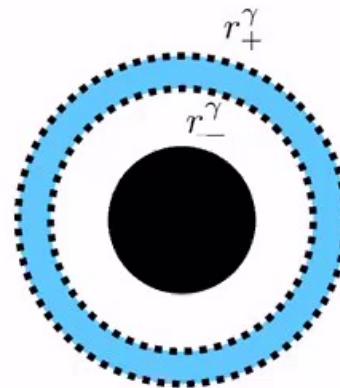
$$r_\pm^\gamma \equiv 2M \left[1 + \cos \left(\frac{2}{3} \arccos \left(\pm \frac{a}{M} \right) \right) \right].$$

- Orbits are unstable,

$$\delta r \sim e^{\gamma(r)n},$$

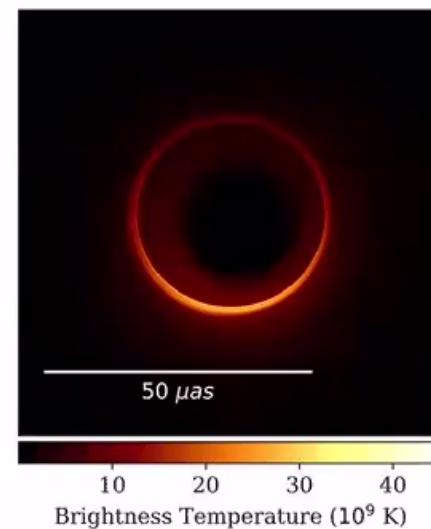
where n = winding number and γ = Lyapunov exponent.

- For Schwarzschild $\gamma(r_\pm^\gamma) = \pi$ and $r_\pm^\gamma = 3M$.



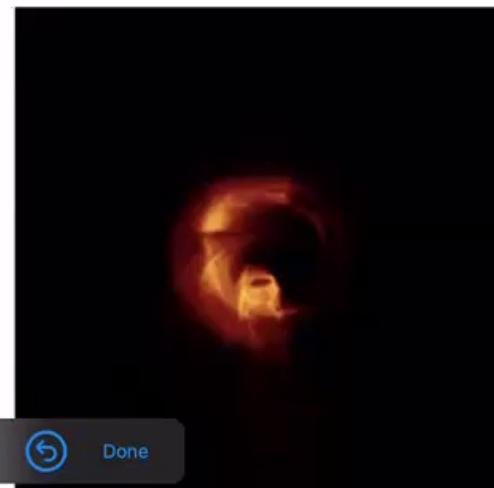
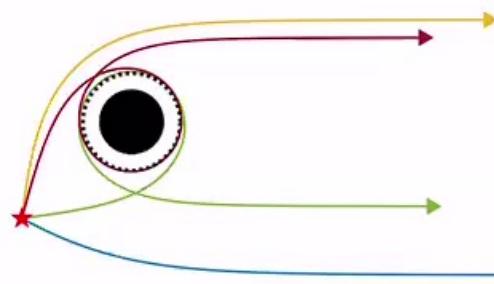
Potentially observable effects of the photon shell

- Longest lived (and high angular momentum) QNMs governed by photon shell physics.
- Photon ring in black hole images encodes photon shell [Johnson et all: 1907.04329].
 - Lots of structure in photon ring from multi-orbit trajectories.



Correlations from multi-path propagation

- Consider light sourced by an accretion flow:
- In a black hole spacetime, light can take infinitely many trajectories to observer.



⇒ Autocorrelations in movies
from multi-path propagation.

[Wong: 2009.06641],

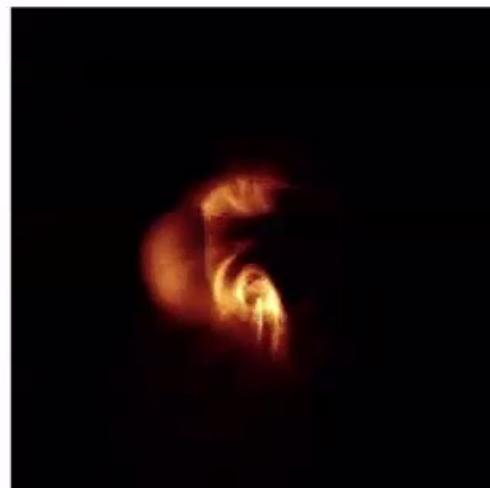
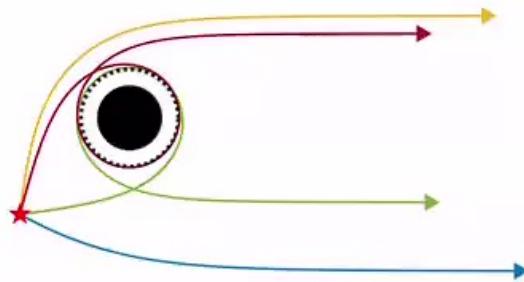
[Hadar et al: 2010.03683]

⇒ Autocorrelations in light
curves from mu



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The challenge of disentangling accretion physics from multi-path propagation effects

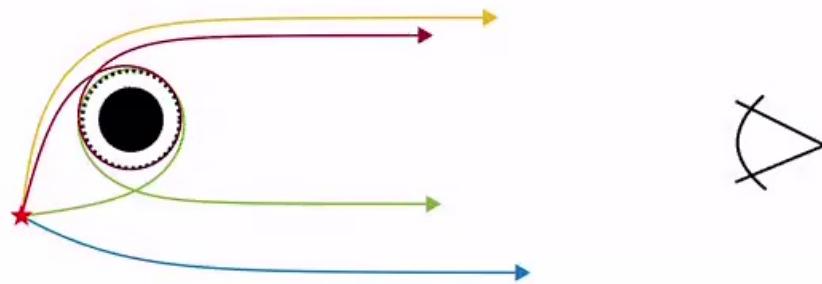
- Accretion physics itself can give rise to correlations too.
- An example is an orbiting hot spot
- Can accretion physics be disentangled from multi-path propagation effects?
- This question can presumably be answered with GRMHD simulations & ray tracing.

Messy!



The (naive) advantage of coherent autocorrelations

- Light curves & movies contain no phase information about EM waves.
- However, phase information is measurable: e.g. $\langle E(t)E(t') \rangle$.
- **Why is this (naively) useful?**



- Correlation length in plasma is \ll horizon scales.
⇒ $\langle J^\mu(x)J^\nu(y) \rangle$ effectively has delta function support.
⇒ **phase from different points is uncorrelated.**
⇒ **coherent correlation functions are insensitive to complicated accretion physics.**

Today's goals

- Coherent autocorrelations are sensitive to the echo periods and Lyapunov exponents.
- However, at observable wavelengths the power in echos is minuscule, making coherent autocorrelations a bad observable to study echos.



A simple toy problem

Employ the Schwarzschild spacetime:

- Spherical symmetry makes calculations easier.
- Constant photon orbit period $T = 6\pi\sqrt{3}M$.
- Constant Lyapunov exponent $\gamma = \pi$.

A simple toy problem

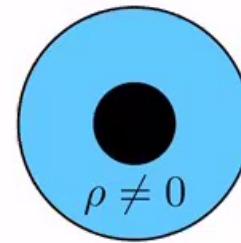
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- } distribution of T and γ for Kerr!

Trade electrodynamics for a scalar field Ψ :

- Equation of motion

$$-\nabla^2\Psi = \rho.$$



- Easy to solve wave equation numerically w/o ray tracing.
- “Matter” source ρ is random variable & localized near the BH.

$$\langle \rho(x)\rho(y) \rangle = \frac{1}{\sqrt{-g}}\chi(r)\delta^4(x-y)$$

- Correlation function $C(t) = \langle \Psi(t)\Psi(0) \rangle$.

Field correlators

- Equation of motion $-\nabla^2 \Psi = \rho$ is solved by

$$\Psi(t, \mathbf{r}) = \int dt' d^3 r' \sqrt{-g} G(t - t', \mathbf{r}, \mathbf{r}') \rho(t', \mathbf{r}'),$$

where G is the Green's function of $-\nabla^2$.

- Using

$$\langle \rho(t, \mathbf{r}) \rho(t', \mathbf{r}') \rangle = \frac{1}{\sqrt{-g}} \chi(r) \delta(t - t') \delta^3(\mathbf{r} - \mathbf{r}'),$$

one has

$$C(t) = \langle \Psi(t, \mathbf{r}) \Psi(0, \mathbf{r}) \rangle = \int dt' d^3 r' \sqrt{-g} G(t - t', \mathbf{r}, \mathbf{r}') G(-t', \mathbf{r}, \mathbf{r}') \chi(r').$$



A simple toy problem

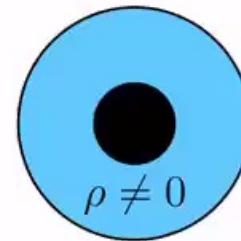
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A few small details

- Fourier transform in time,

$$\tilde{C}(\omega) = \int dt C(t) e^{i\omega t}, \quad \tilde{G}(\omega, \mathbf{r}, \mathbf{r}') = \int dt G(t, \mathbf{r}, \mathbf{r}') e^{i\omega t}.$$

- Expand in spherical harmonics in angles,

$$\tilde{G}(\omega, \mathbf{r}, \mathbf{r}') = \sum_{\ell m} y_{\ell m}(\hat{r}) y_{\ell m}^*(\hat{r}') \mathcal{G}_\ell(r, r'),$$

where $\mathcal{G}_\ell(r, r')$ satisfies

$$\left[\frac{d}{dr} r^2 f \frac{d}{dr} + \frac{r^2 \omega^2 - \ell(\ell+1)f}{f} \right] \mathcal{G}_\ell(r, r') = \delta(r - r').$$

- Correlation function:

$$\tilde{C}(\omega) = \frac{1}{4\pi} \sum_\ell (2\ell + 1) \int r'^2 dr' \chi(r') |\mathcal{G}_\ell(r, r')|^2.$$



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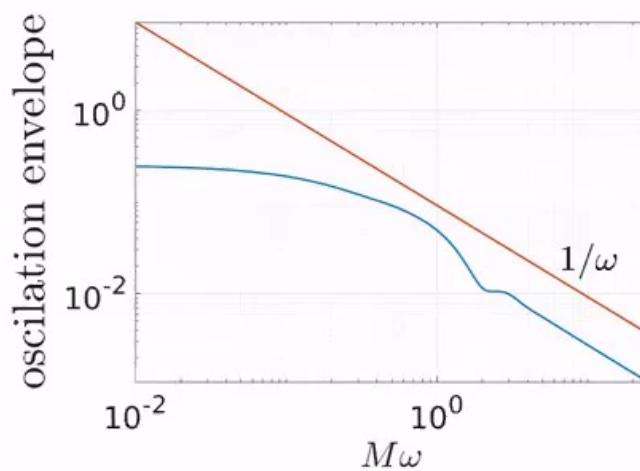
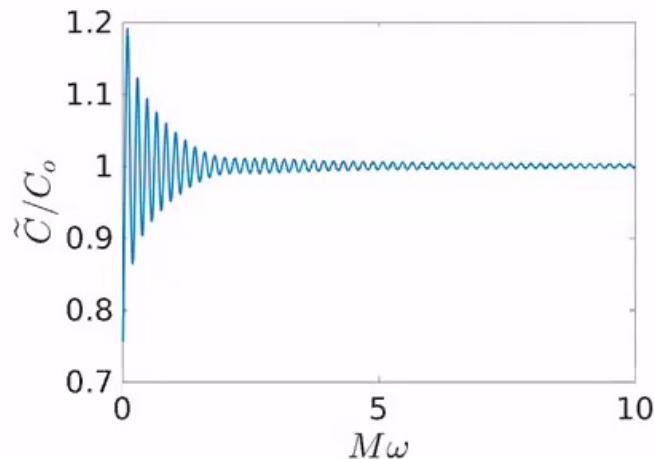
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Must solve a bunch of ODEs. I'll do this numerically.

Results for the spectral density $\tilde{C}(\omega)$



Large frequency behavior of numerics consistent with:

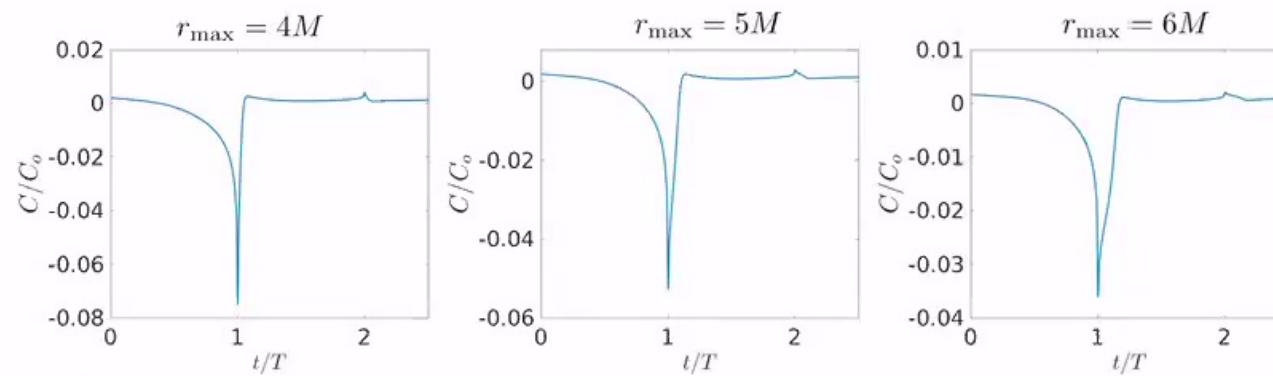
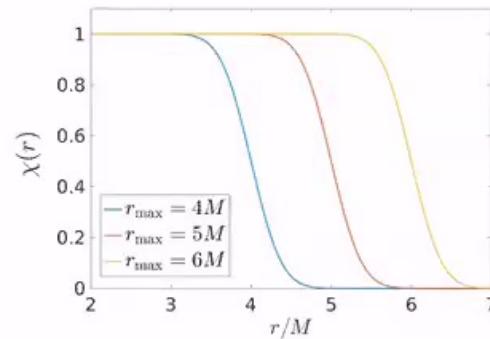
$$\tilde{C}(\omega) = C_o + \frac{1}{M\omega} \sum_{n=1} A_n \sin(\omega nT + \delta_n).$$



Results of numerics

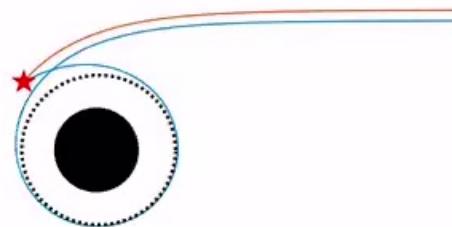
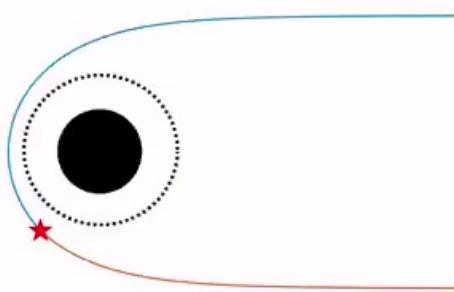
Please note:

- Plots made with ∞ bandwidth.
- There is a δ -function at $t = 0$ with unit amplitude.



- $C(t)$ nonzero at all times, but peaked at $t = T, 2T, \dots$ with amp $\sim e^{-\pi t/T}$.
- Width of peak increases as r_{\max} increases.

Why are there peaks at $t = T, 2T, \dots$?



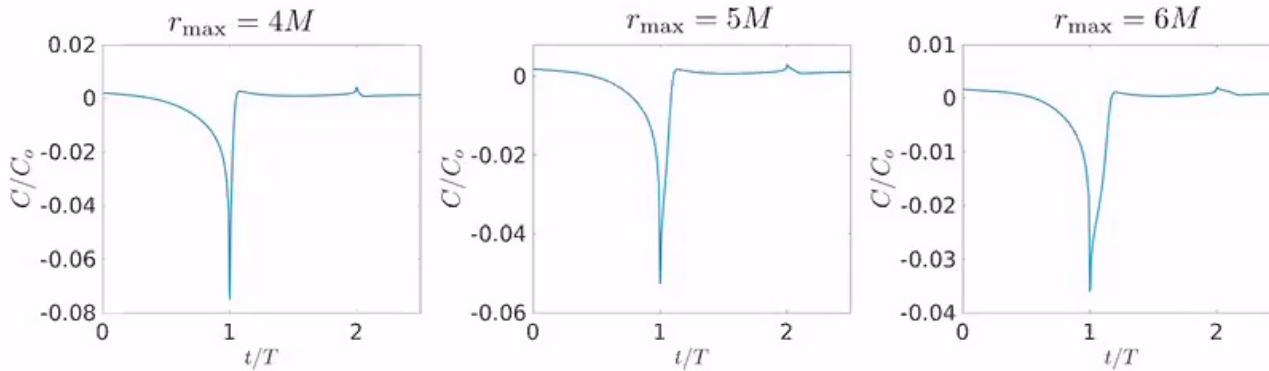
- Each point x has a different lag time τ .

⇒ **small neighborhood of point contributes.**

- All points x have nearly the same lag time $\tau \approx T$.
⇒ **entire volume contributes.**
- Distribution of τ becomes broader as r_{\max} increases.



Why do the peaks alternate in sign?



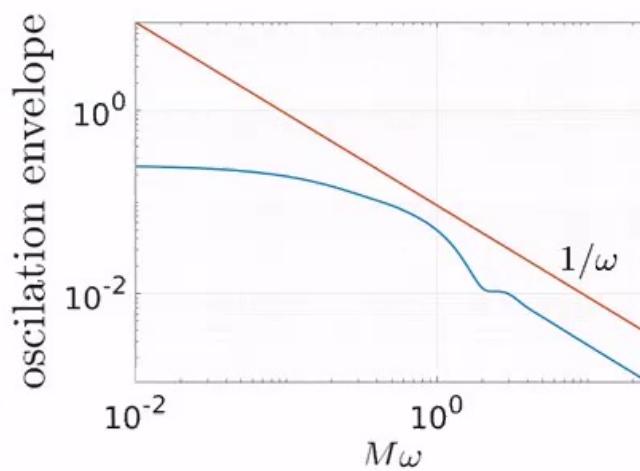
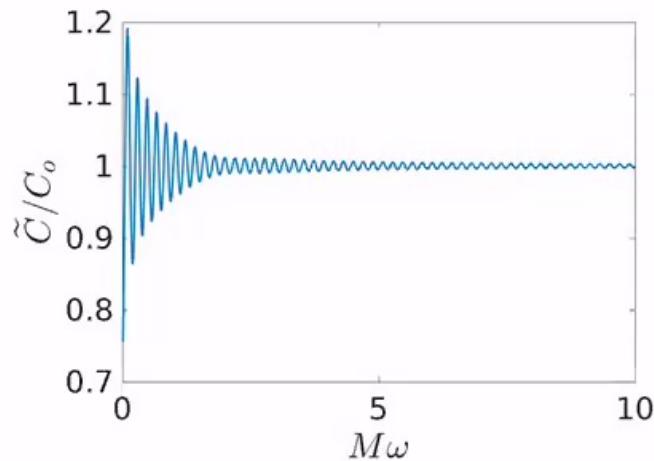
- Longest lived QNMs (at high ℓ):

$$\omega_{\text{QNM}} = \pm \frac{2\pi}{T} \left(\ell + \frac{1}{2} \right) - \frac{i\pi}{T}.$$

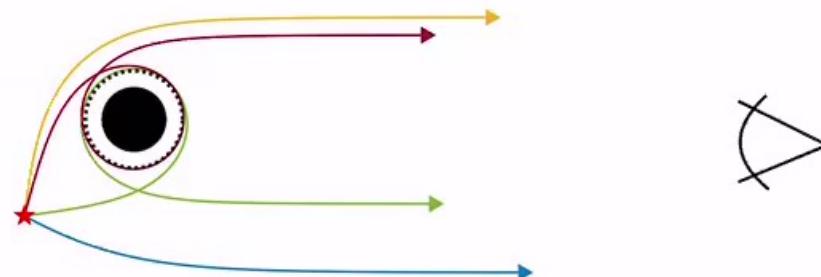
[Schutz & Will: 1985]

- This is just the dispersion relation of a damped 2D wave equation on a sphere!
- $e^{-i\omega_{QNM}(t+T)} = -e^{-\pi} e^{-i\omega_{QNM}t}$.

Why do oscillations in the spectral density $\tilde{C}(\omega) \sim \frac{1}{\omega}$?



Fall-off due to
cancelations from
different emission
points.

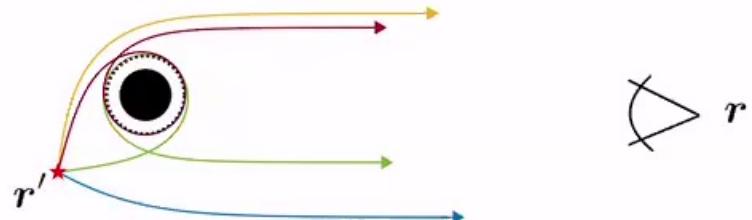


Geometric optics analysis

- Use geometric optics when $\omega M \gg 1$:

$$\tilde{G}(\omega, \mathbf{r}, \mathbf{r}') = \sum_p L_p(\mathbf{r}, \mathbf{r}') e^{i\omega\tau_p(\mathbf{r}, \mathbf{r}')},$$

where sum over p is over trajectories with τ_p = retarded time.



- This means $\tilde{C}(\omega) = \tilde{C}_{\text{direct}}(\omega) + \tilde{C}_{\text{multi-path}}(\omega)$ where,

$$\tilde{C}_{\text{direct}}(\omega) = \sum_p \int \sqrt{-g} d^3 r' \chi(r') L_p(\mathbf{r}, \mathbf{r}')^2,$$

$$\tilde{C}_{\text{multi-path}}(\omega) = \sum_{p \neq p'} \int \sqrt{-g} d^3 r' \chi(r') L_p(\mathbf{r}, \mathbf{r}') L_{p'}(\mathbf{r}, \mathbf{r}') e^{i\omega\Delta\tau_{pp'}(\mathbf{r}, \mathbf{r}')},$$

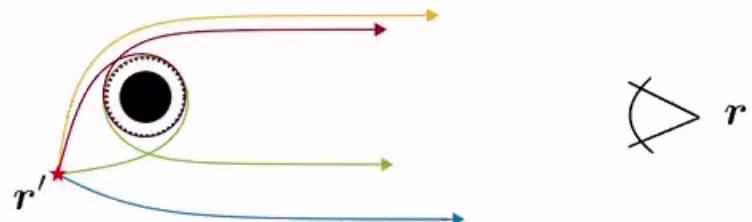
with $\Delta\tau_{pp'} \equiv \tau_p - \tau'_{p'}$.

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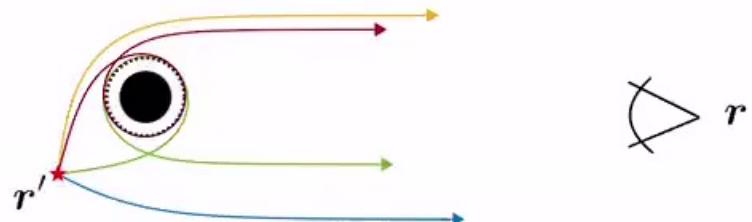


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with $\Delta\tau_{pp'} \equiv \tau_p - \tau'_{p'}$.

saddle point analysis

Saddle point analysis

- Consider the $\omega \rightarrow \infty$ limit of $I(\omega) = \int dz f(z) e^{i\omega g(z)}$.
- Integral dominated by points z_s where $g'(z_s) = 0$.
- Expand

$$\begin{aligned}f(z) &= f(z_s) + f'(z_s)(z - z_s) + \frac{1}{2}f''(z_s)(z - z_s)^2 + \dots, \\g(z) &= g(z_s) + \frac{1}{2}g''(z_s)(z - z_s)^2 + \dots,\end{aligned}$$

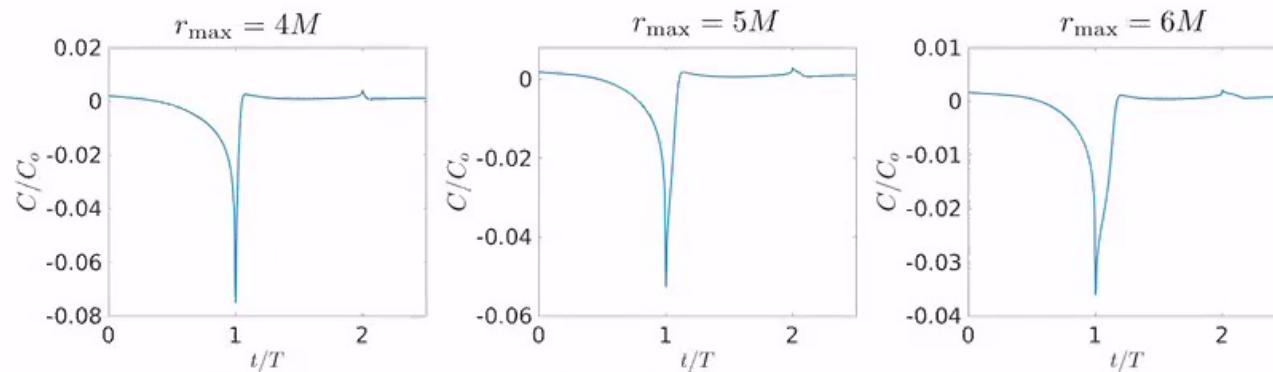
- Gaussian integration yields

$$I(\omega) = \sum_s \sqrt{\frac{\pi}{-i\omega g''(z_s)}} f(z_s) + O(1/\omega).$$

- Cylindrical symmetry therefore means

$$\begin{aligned}\tilde{C}_{\text{multi-path}}(\omega) &= \sum_{p \neq p'} \int \sqrt{-g} d^3 r' \chi(r') L_p(\mathbf{r}, \mathbf{r}') L_{p'}(\mathbf{r}, \mathbf{r}') e^{i\omega \Delta \tau_{pp'}(\mathbf{r}, \mathbf{r}')}, \\ &\sim \frac{1}{\omega}.\end{aligned}$$

In practice, are echos observable in coherent autocorrelation functions?



Plots made with infinite bandwidth!

- **Key point:** Observations are made with finite bandwidth.
 - For EHT, $\omega \sim$ hundreds of GHz, $\Delta\omega \sim 1$ GHz.
- **Key question:** For a given bandwidth, how much power is in echos?



Relationship between power and $\tilde{C}(\omega)$

- Consider the windowed Fourier Transform,

$$\hat{\Psi}(\omega) = \frac{1}{\sqrt{t_{\text{win}}}} \int_{-t_{\text{win}}/2}^{-t_{\text{win}}/2} \Psi(t) e^{i\omega t}.$$

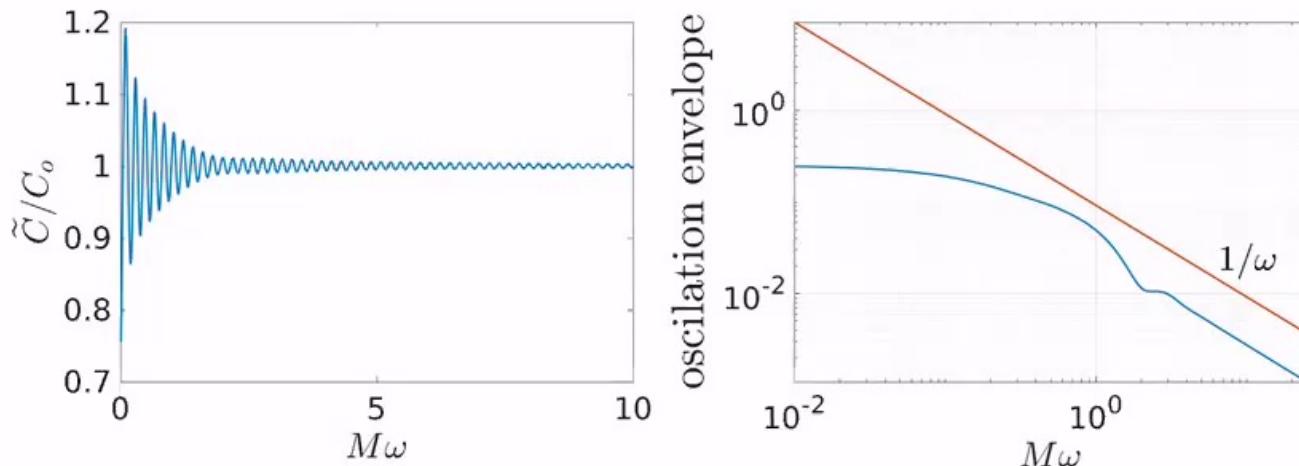
- Time translation invariance then implies

$$\begin{aligned}\langle |\hat{\Psi}(\omega)|^2 \rangle &= \int_{-t_{\text{win}}/2}^{-t_{\text{win}}/2} dt \langle \Psi(t) \Psi(0) \rangle e^{i\omega t}, \\ &= \tilde{C}(\omega).\end{aligned}$$

- Therefore, up to normalization $\tilde{C}(\omega)$ is the power in mode ω .



The hopelessness of coherent correlation functions



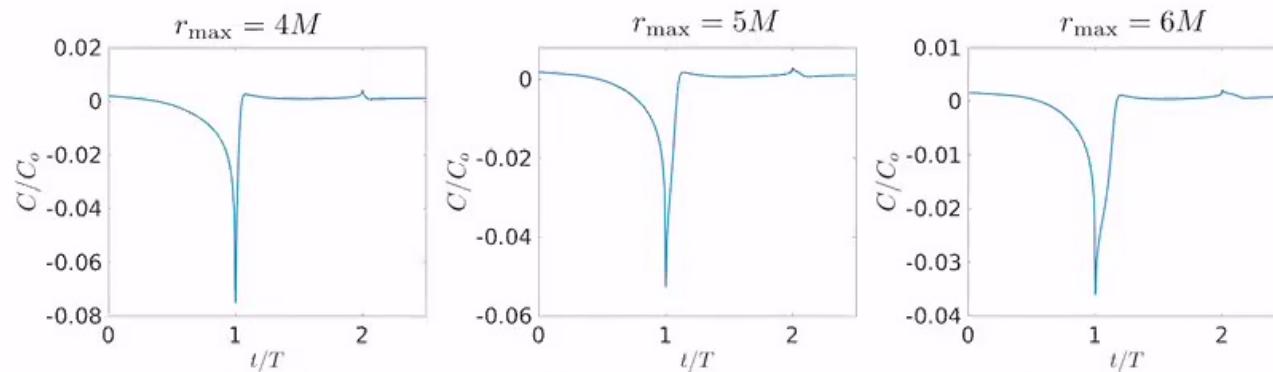
- Power in echos suppressed relative to direct light by $\frac{1}{M\omega}$.
- For $\omega \sim$ hundreds of GHz with $\Delta\omega \sim 1$ GHz, and with $M \sim 10^6 M_\odot$,

$$\frac{1}{M\omega} \sim 10^{-14}.$$

- This is hopelessly small!



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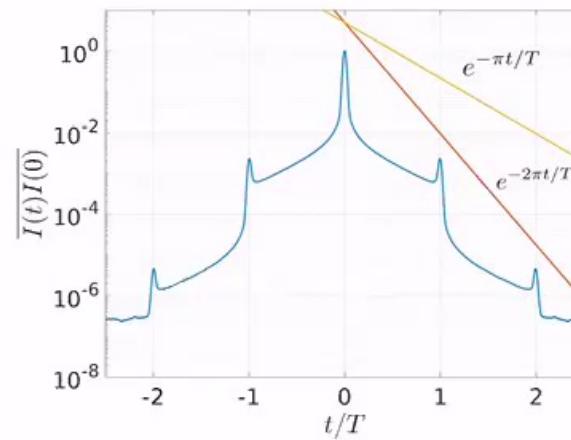
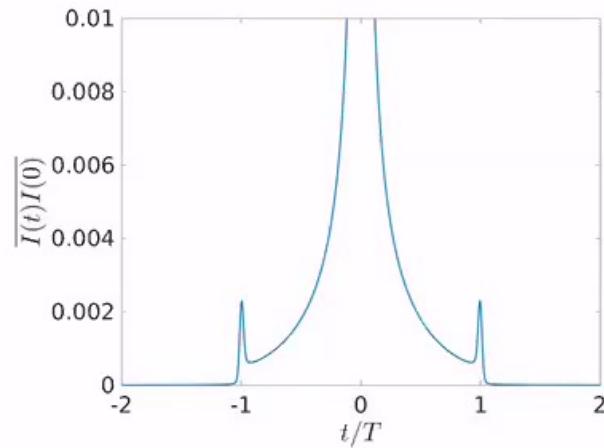
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Light curve autocorrelations

Naive assumption:

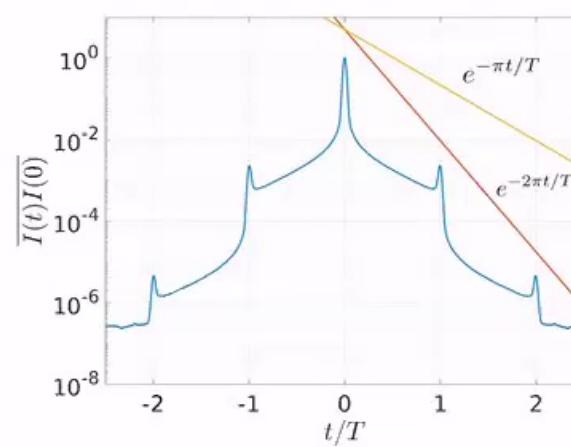
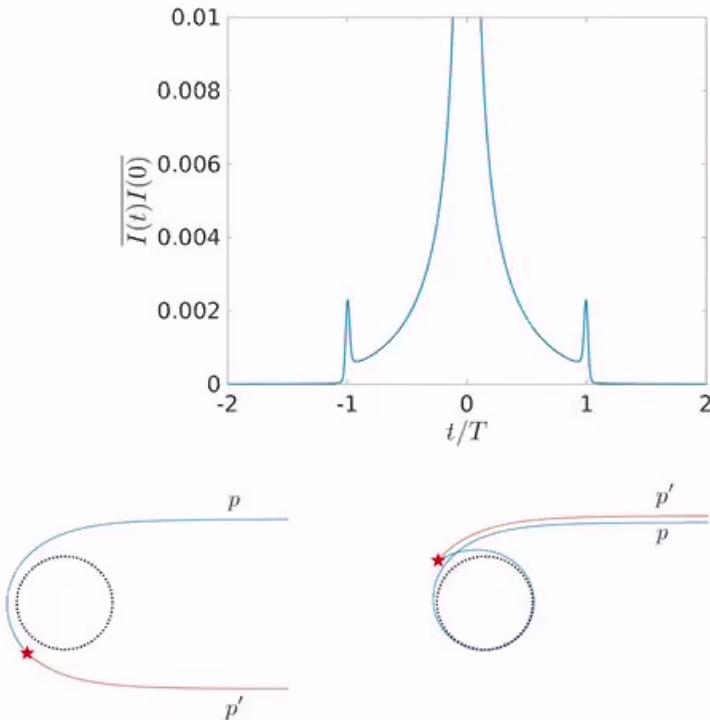
- For simplicity assume emissivity correlation length $\ll M$
- Results:



Light curve autocorrelations

Naive assumption:

- For simplicity assume emissivity correlation length $\ll M$
- Results:



- Not large effect!
- Easy to imagine accretion physics pollution!

What needs to be done

To ensure accretion physics can be disentangled from echos:

- Compute emissivity correlators $\langle \varepsilon(t, \mathbf{r})\varepsilon(t', \mathbf{r}') \rangle$ for a wide class of simulations.

– Includes transient effects, such as hot spots.

- Ray trace to compute light curve correlator,

$$\langle I(t)I(t') \rangle = \int G(t - t', \mathbf{r}, \mathbf{r}') G(t' - t'', \mathbf{r}, \mathbf{r}'') \langle \varepsilon(t'', \mathbf{r}'') \varepsilon(t''', \mathbf{r}''' \rangle).$$

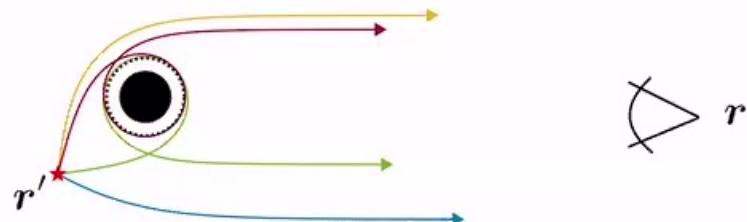
- Develop methods to differentiate echo peaks from accretion physics.

Geometric optics analysis

- Use geometric optics when $\omega M \gg 1$:

$$\tilde{G}(\omega, \mathbf{r}, \mathbf{r}') = \sum_p L_p(\mathbf{r}, \mathbf{r}') e^{i\omega\tau_p(\mathbf{r}, \mathbf{r}')},$$

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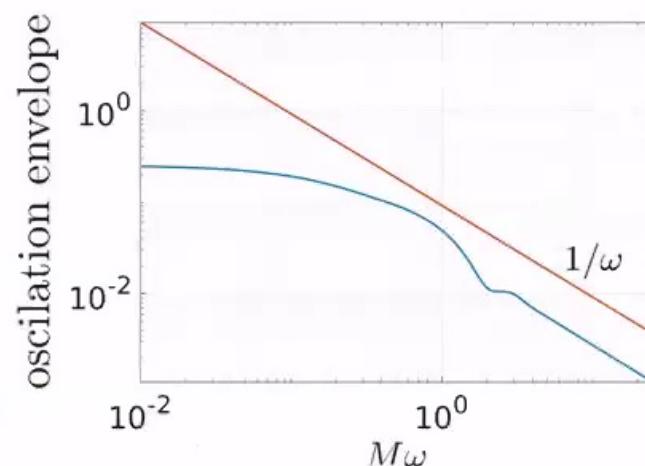
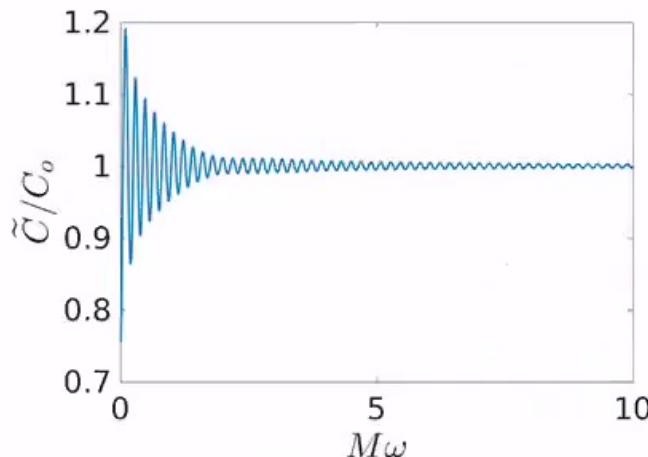
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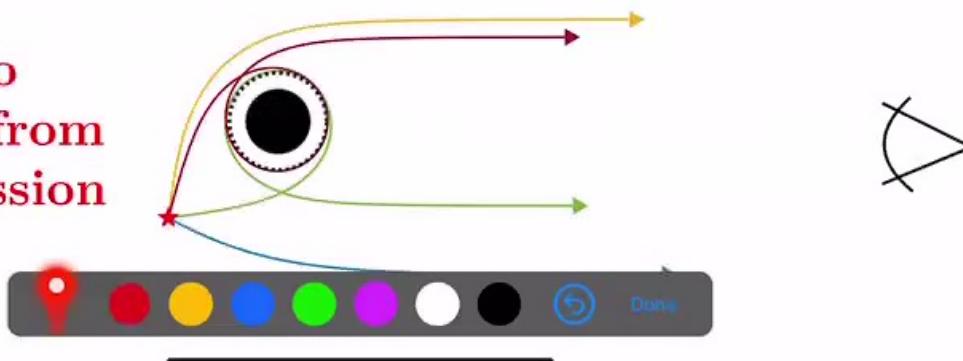
with $\Delta\tau_{pp'} \equiv \tau_p - \tau_{p'}$



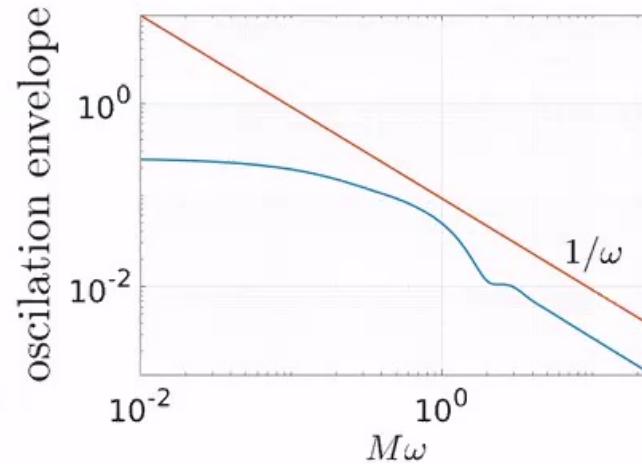
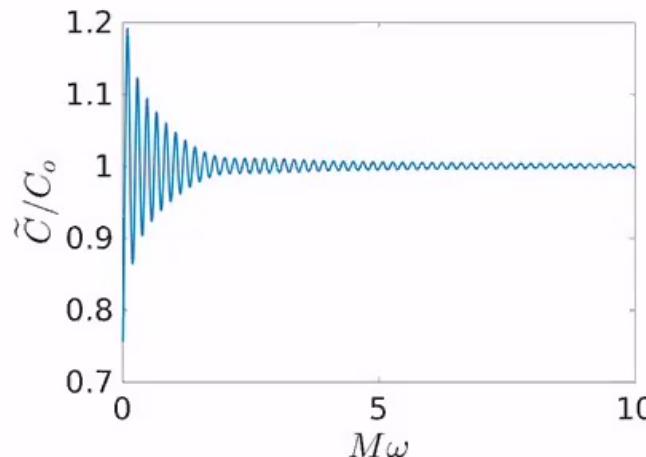
Why do oscillations in the spectral density $\tilde{C}(\omega) \sim \frac{1}{\omega}$?



Fall-off due to
cancelations from
different emission
points.



Results for the spectral density $\tilde{C}(\omega)$

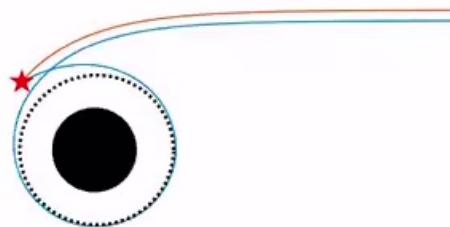
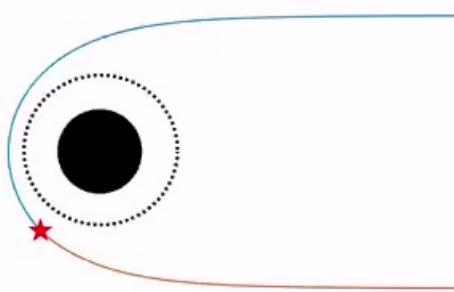


Large frequency behavior of numerics consistent with:

$$\tilde{C}(\omega) = C_o + \frac{1}{M\omega} \sum_{n=1} A_n \sin(\omega nT + \delta_n).$$



Why are there peaks at $t = T, 2T, \dots$?



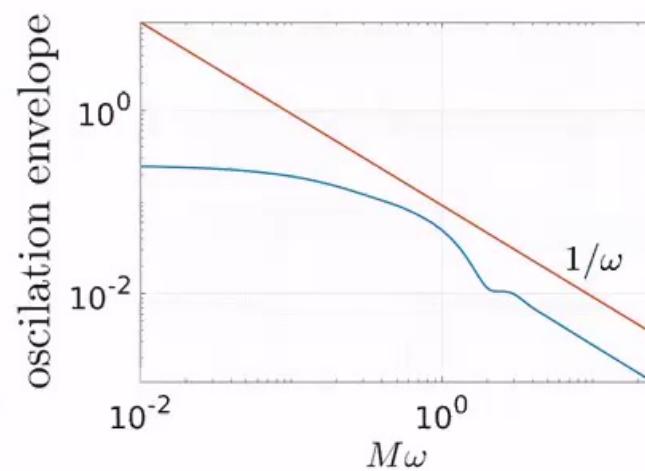
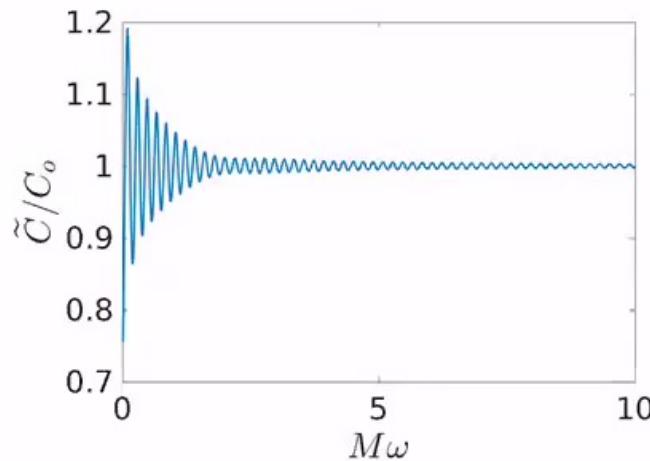
- Each point x has a different lag time τ .

⇒ small neighborhood of point contributes.

- All points x have nearly the same lag time $\tau \approx T$.
⇒ entire volume contributes.
- Distribution of τ becomes broader as r_{\max} increases.



Why do oscillations in the spectral density $\tilde{C}(\omega) \sim \frac{1}{\omega}$?

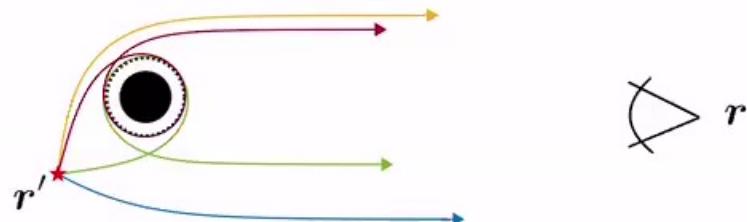


Geometric optics analysis

- Use geometric optics when $\omega M \gg 1$:

$$\tilde{G}(\omega, \mathbf{r}, \mathbf{r}') = \sum_p L_p(\mathbf{r}, \mathbf{r}') e^{i\omega\tau_p(\mathbf{r}, \mathbf{r}')},$$

where sum over p is over trajectories with τ_p = retarded time.



- This means $\tilde{C}(\omega) = \tilde{C}_{\text{direct}}(\omega) + \tilde{C}_{\text{multi-path}}(\omega)$ where,

$$\tilde{C}_{\text{direct}}(\omega) = \sum_p \int \sqrt{-g} d^3 r' \chi(r') L_p(\mathbf{r}, \mathbf{r}')^2, \quad \Leftarrow \omega \text{ independent}$$

$$\tilde{C}_{\text{multi-path}}(\omega) = \sum_{p \neq p'} \int \sqrt{-g} d^3 r' \chi(r') L_p(\mathbf{r}, \mathbf{r}') L_{p'}(\mathbf{r}, \mathbf{r}') e^{i\omega\Delta\tau_{pp'}(\mathbf{r}, \mathbf{r}')},$$

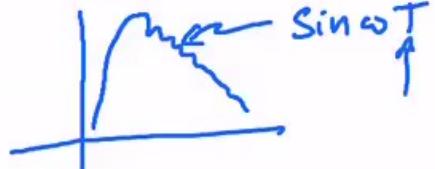
with $\Delta\tau_{pp'} \equiv \tau_p - \tau_{p'}$



Correlations in Hawking radiation

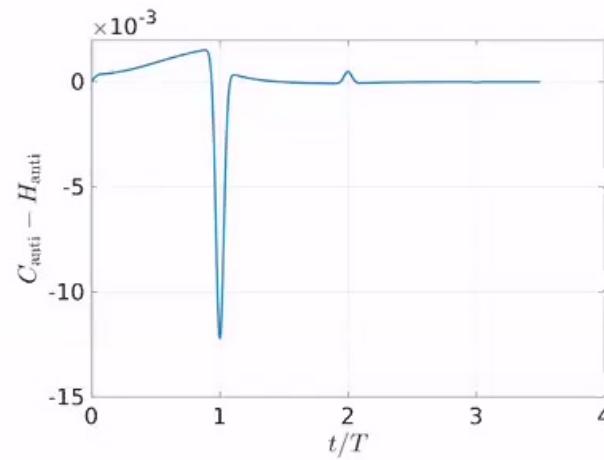
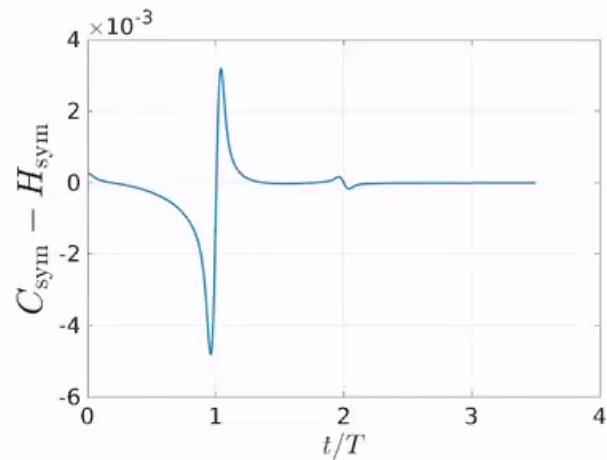
- Photon shell also imprints itself in Hawking radiation emission spectrum.

[Decanini et al: 1101.0781]



- Quantum mechanical correlators:

$$C_{\text{sym}}(t) = \langle \Psi(t)\Psi(0) + \Psi(0)\Psi(t) \rangle, \quad C_{\text{anti}}(t) = \langle \Psi(t)\Psi(0) - \Psi(0)\Psi(t) \rangle.$$



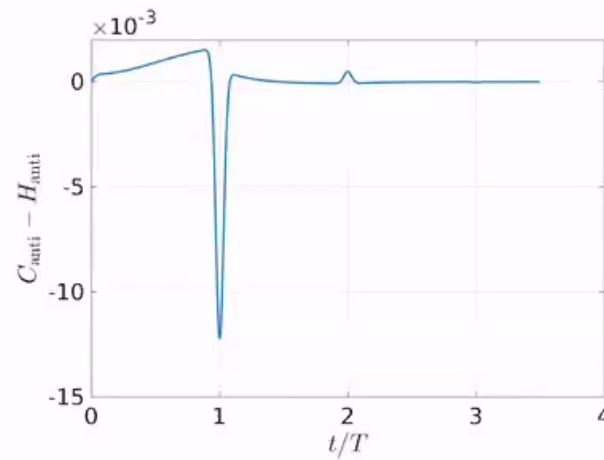
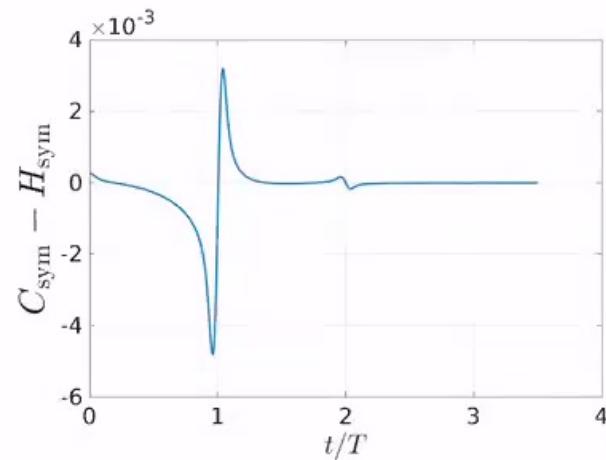
- Same geometric



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- Same geometric optics analysis applies.



Thank you!