Title: Axion Cosmic Strings: Players in the Early Universe?

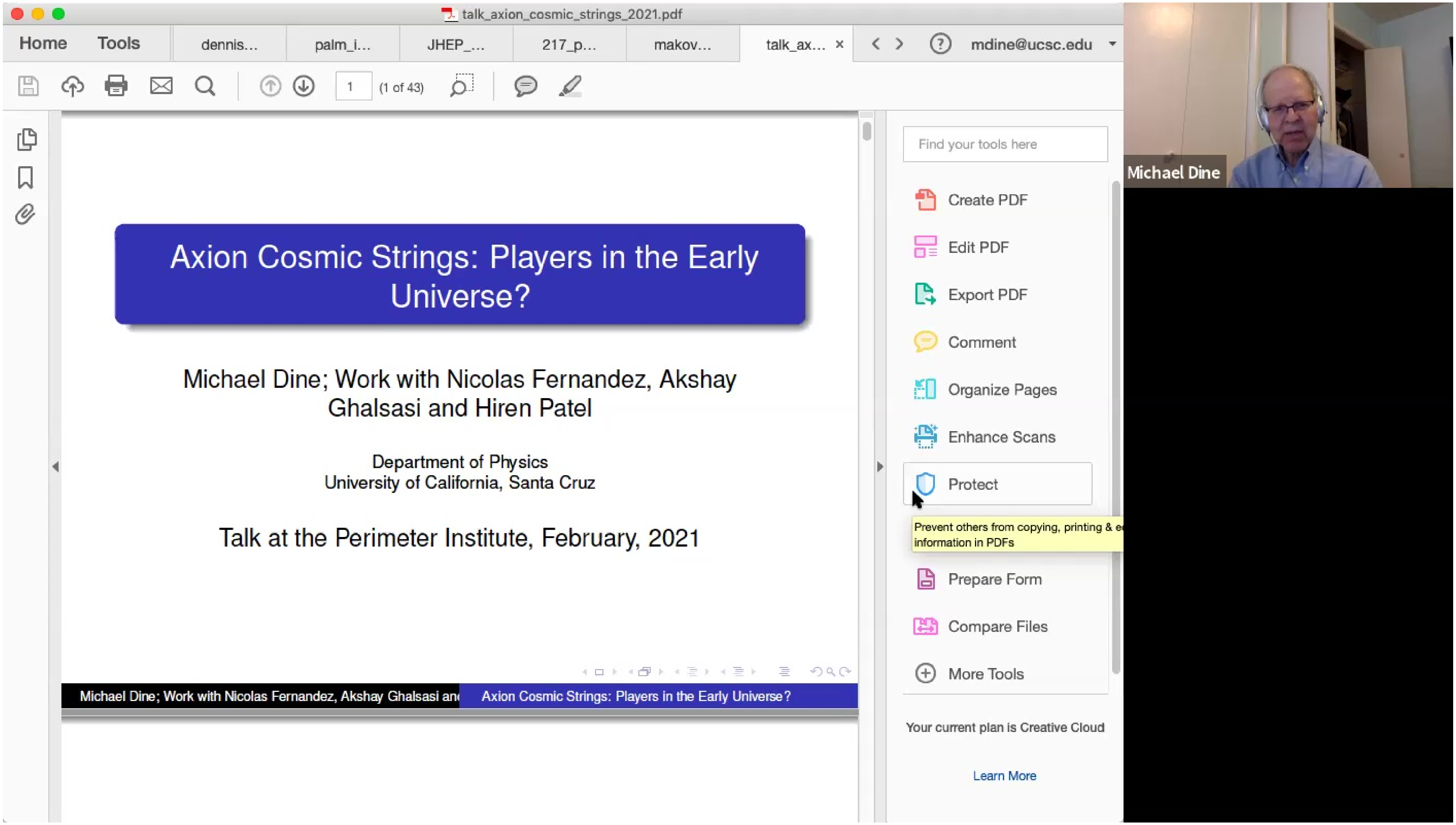
Speakers: Michael Dine

Series: Particle Physics

Date: February 16, 2021 - 1:00 PM

URL: http://pirsa.org/21020049

Abstract: Axion cosmic strings have for some time been considered a potential source of enhancement of axion dark matter production, and have been the subject of extensive simulations (for references, see out in recent years). But axion strings are rather peculiar entities. This talk will explore some aspects of these objects, and suggest that they are not likely to play a distinguished role in early universe cosmology.


talk_axion_cosmic_strings_2021.pdf
Home
Tools
(1 of 43)
mdine@ucsc.edu
Axion Cosmic Strings: Players in the Early Universe?
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi and Hiren Patel
Department of Physics
University of California, Santa Cruz
Talk at the Perimeter Institute, February, 2021
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an
Axion Cosmic Strings: Players in the Early Universe?
Find your tools here
Create PDF
Edit PDF
Export PDF
Comment
Organize Pages
Enhance Scans
Protect
Prepare Form
Compare Files
More Tools
Your current plan is Creative Cloud
Learn More
Michael Dine

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

Axion cosmic strings have for some time been considered a potential source of enhancement of axion dark matter production, and have been the subject of extensive simulations (for references, see out recent paper) in recent years. But axion strings are rather peculiar entities. We explore some aspects of these objects, and suggest that they are not likely to play a distinguished role in early universe cosmology.

Axions have long been considered a promising dark matter candidate. If the Peccei-Quinn transition occurs before inflation, they are produced by the so-called misalignment mechanism, with the result depending on an essentially random parameter, θ_0, the initial value of the angle θ within our present horizon. If the transition occurs after inflation, in a hot universe, then the misalignment mechanism still contributes, but the initial angle varies by amounts of order 2π on Hubble scales. In this case, the dark matter density from misalignment is, in principle, computable, since one averages over angles in different domains of the universe.

Today we will focus on the “Post Inflation" scenario.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?

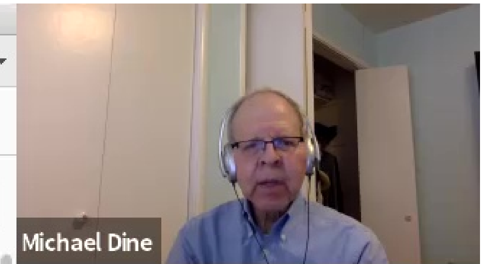
The textbook (e.g. Kolb and Turner) estimate of the axion density assumes that the axion begins to oscillate around the

The textbook (e.g. Kolb and Turner) estimate of the axion density assumes that the axion begins to oscillate around the minimum of its potential once $3H(T_{\rm osc}) = m_a(T_{\rm osc})$. The number density of axions is of order $\rho_a(T_{\rm osc})/m_a(T_{\rm osc})$. So the axion energy density behaves, for small initial misalignment angle, as:

$$\rho_a = \rho_a(T_{\rm osc}) \frac{m_a(T)}{m_a(T_{\rm osc})} \frac{R^3(T_{\rm osc})}{R^3(T)}\,, \qquad (1)$$

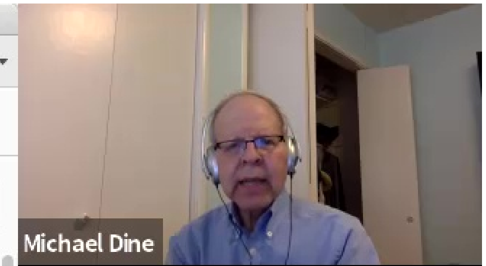
where R is the scale factor. In the case of PQ transition after inflation, one then averages over the initial misalignment angle.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?



At the order one level there are several sources of uncertainty:

1. Uncertainty as to the topological susceptibility, χ, the second derivative of the free energy at $\theta = 0$: χ is known analytically only at very low and very high temperature. In the intermediate regime, one can estimate χ by interpolating between these results
2. Hubble scale variations of the axion field: These are expected to contribute to the energy density an amount at least of order $f_a^2 H^2$, which is comparable to the assumed zero momentum contribution.
3. Simply averaging the potential, proportional to $\sin^2\theta$, over θ, does not take into account the non-linearity of the axion equations. Here topological objects such as strings and domain walls, might enter. As with the previous item, these are likely to produce at least order one modifications of the axion density.



Might any of these be more than order one effects? There is a large literature exploring this question, including analytic studies and sophisticated simulations (see our recent paper for extensive references). Cosmic strings have been a major focus.

I will argue today that each of these uncertainties translate into order one (but not larger) uncertainties in the axion dark matter density. It would be desirable to reduce them, but their effect on estimates of the axion mass as a function of the dark matter density would seem modest. Our principle focus today will be on the last item, and the possible role of cosmic strings (and at the final stages, domain walls).

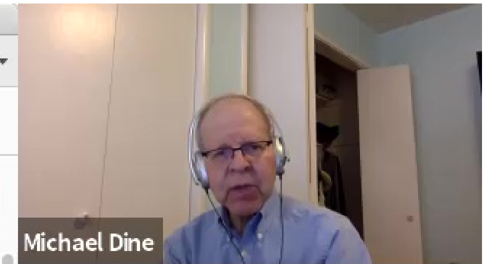
In a post-inflationary scenario, a linearized treatment of the axion field is not reliable, as a/f_a varies by 2π or more on Hubble scales. Allowing such variation, one expects to encounter Hubble-scale closed loops for which

$$\oint \frac{a}{f_A}\, dl = 2\pi n \qquad (2)$$

for some integer n. Shrinking the loop to smaller and smaller size, we must find regions in which the modulus of the underlying complex scalar field vanishes: cosmic strings.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?

We can consider a theory with a complex field Φ transforming



talk_axion_cosmic_strings_2021.pdf

We can consider a theory with a complex field, Φ, transforming under a spontaneously broken global $U(1)$ symmetry as $\Phi \to e^{i\alpha}\Phi$, spontaneously broken:

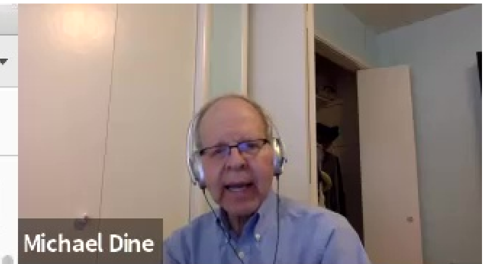
$$\Phi = (f_a + \sigma(x))\, e^{ia(x)/f_a}. \tag{3}$$

Such theories admit cosmic string solutions:

$$\Phi_{cl} = f(\rho) e^{in\phi}. \tag{4}$$

$f(\rho) \to 0$ as $\rho \to 0$.
$f \to f_a$ as $\rho \to \infty$. Away from the string core:

$$\vec{\nabla}\Phi = i\frac{n}{\rho} f_a e^{in\phi}\hat{\phi}. \tag{5}$$



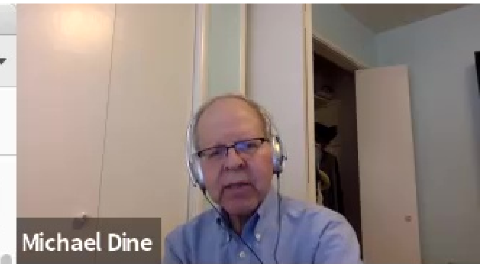
$f \to f_a$ as $\rho \to \infty$. Away from the string core:

$$\vec{\nabla}\Phi = i\frac{n}{\rho} f_a e^{in\phi}\hat{\phi}. \tag{5}$$

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?

The energy stored in the string configuration per unit length is logarithmically divergent The UV cutoff of the logarithmic divergence comes from the size of the string core. The IR cutoff comes from physical considerations. E.g. for closed, roughly circular, strings, the IR cutoff would be provided by the string circumference.; for a long string-antistring, the string separation. In a cosmic string network with $O(1)$ string per Hubble volume, we would expect the cutoff to be of order H^{-1}, and the effective string tension to be of order:

$$T = 2\pi f_a^2 \log(f_a/H) \equiv 2\pi f_a^2 \xi \tag{6}$$



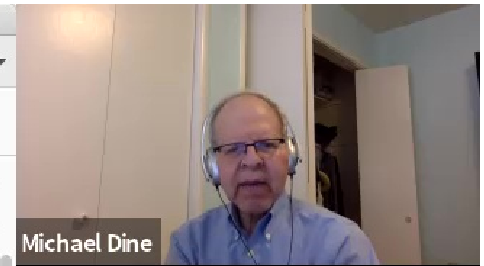
separation. In a cosmic string network with $O(1)$ string per Hubble volume, we would expect the cutoff to be of order H^{-1}, and the effective string tension to be of order:

$$T = 2\pi f_a^2 \log(f_a/H) \equiv 2\pi f_a^2 \xi \qquad (6)$$

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi and | Axion Cosmic Strings: Players in the Early Universe?

Prospects for Density Enhancement

The parameter $\xi = \log(f_a/H) \sim 70$ at the QCD phase transition, if $f_a \sim 10^{12}$ GeV. It is the possibility of enhancement of the axion density by powers of ξ which gives rise to the interest in cosmic axion strings (and domain walls) in the early universe.



talk_axion_cosmic_strings_2021.pdf

Home Tools dennis… palm_i… JHEP_… 217_p… makov… talk_ax… mdine@ucsc.edu

11 (11 of 43)

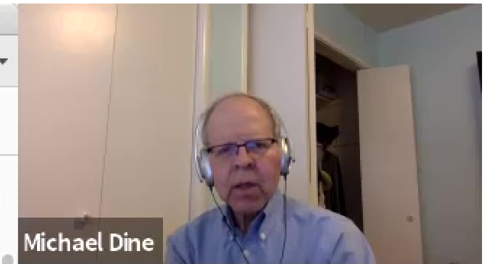
Skepticism as to an enhancement by powers of $\log(f_a/H)$

- Due to the the infrared divergences, one must consider an effective action containing both string core collective coordinates and axions with momenta less than the inverse core size, where the cutoff length defining the core is chosen arbitrarily, but is short compared to the actual infrared cutoff on the system. We will see the matching of the expected axion distributions at this cutoff.
- The logarithmic enhancement of the axion energy density from high momentum axions is indeed what is expected from considerations of Hubble scale variations of the phase of the Peccei-Quinn field, and their subsequent cosmic evolution. In this language, cosmic strings are just a piece of this set of variations.

Find your tools here
Create PDF
Edit PDF
Export PDF
Comment
Organize Pages
Enhance Scans
Protect
Fill & Sign
Prepare Form
Compare Files
More Tools

Your current plan is Creative Cloud

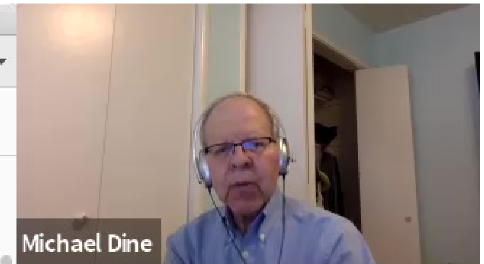
Learn More



talk_axion_cosmic_strings_2021.pdf

a piece of this set of variations.

- For non-relativistic motion of the string, it is straightforward to derive an effective action for the string core collective coordinates and the axion. This action exhibits the cutoff-dependent tension. It also yields an axion-string coupling which is not localized on this string. As we will see, treated carefully is like that for critical strings (Kalb-Ramond action; Vilenkin-Vachaspati, Dabholkar et al).
- The Axion is a compact field, $0 < \frac{a}{f_a} < 2\pi$. This suggests that there is a limit to how much of the low momentum axion there can be.

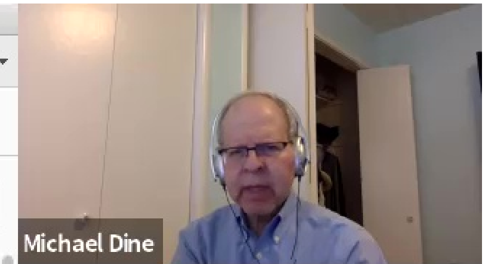


Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an[d] | Axion Cosmic Strings: Players in the Early Universe?

Two limiting cases and expectations for axion radiation

In simple situations, one can guess the expected axion spectrum:

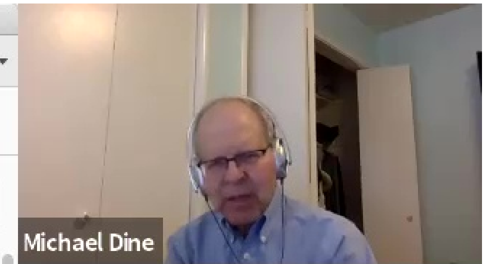
- **Adiabatic Approximation**: For long, parallel string and antistring, one can think of the separation of the strings as a dynamical variable, $b(t)$, and obtain an action for b. One can demonstrate that, starting from rest and well separated, the system initially evolves in an adiabatic fashion. This means that the potential energy of the separated strings is largely converted to kinetic energy of b; little is converted into low momentum axions at this early stage.

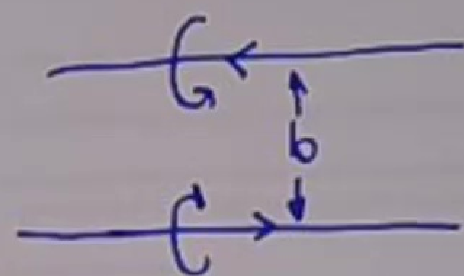


Two limiting cases and expectations for axion radiation

In simple situations, one can guess the expected axion spectrum:

- **Adiabatic Approximation**: For long, parallel string and antistring, one can think of the separation of the strings as a dynamical variable, $b(t)$, and obtain an action for b. One can demonstrate that, starting from rest and well separated, the system initially evolves in an adiabatic fashion. This means that the potential energy of the separated strings is largely converted to kinetic energy of b; little is converted into low momentum axions at this early stage.





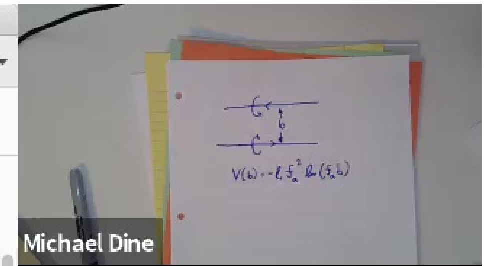
$$V(b) = -\ell f_a^2 \ln(f_a b)$$

Michael Dine

talk_axion_cosmic_strings_2021.pdf

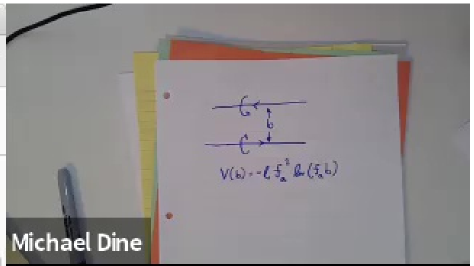
- **Sudden Approximation**: As the strings approach each other, a “sudden" description becomes more appropriate, and the axion field of the system on scale $b(t)$ is converted into axions of wavelength b^{-1}, with energy $f_a^2 \ell$ (ℓ is the string length) per change of b by a factor of e (the base of natural logarithms). These cases suggest no accumulation of axions at small k.

Collective Coordinates and Low Momentum



Collective Coordinates and Low Momentum Axions

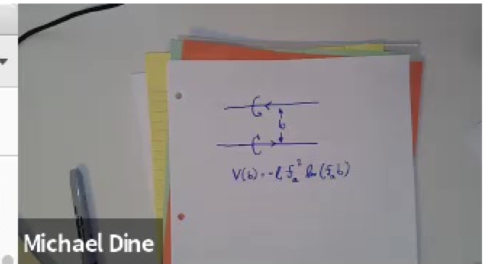
Consider a long string in the abelian Higgs model. Call z the string direction; denote the transverse directions by x and y. Take the classical configuration to be $\Phi_{cl}(x, y)$ (constant in z, t). Now allow for a slow variation of the transverse coordinates with z and t. In other words, consider displacements, $(X(t, z),\ Y(t, z)) \equiv \vec{X}_{\perp}(t, z)$, which vary slowly on the underlying scale of the theory (the masses of the charged scalar and gauge bosons).



talk_axion_cosmic_strings_2021.pdf

Home | Tools | dennis... | palm_i... | JHEP_... | 217_p... | makov... | talk_ax... | mdine@ucsc.edu

16 (16 of 43)



Substituting

$$\Phi(x,y,x,t) = \Phi_{cl}(x - X(z,t), y - Y(x,t)) \qquad (7)$$

in the action, one obtains an action for the fields X_0, Y_0, from the ∂_t, ∂_z terms in the action. This is,

$$\int dtdz \left(\left(\frac{\partial \vec{X}_\perp}{\partial t} \right)^2 - \left(\frac{\partial \vec{X}_\perp}{\partial z} \right)^2 \right) \int d^2x_\perp (\vec{\nabla}_\perp \Phi_{cl})^2 . \qquad (8)$$

The integral over the transverse coordinates yields the string tension. This is the standard action for a non-relativistic string.

Find your tools here

Create PDF

Edit PDF

Export PDF

Comment

Organize Pages

Enhance Scans

Protect

Fill & Sign

Prepare Form

Compare Files

More Tools

Your current plan is Creative Cloud

Learn More

Contrast this with the axion string. The integral for the tension is now infrared divergent. The problem is one of "slow variation" of the axion field. This is a signal that we cannot treat the string, *by itself*, as an independent entity. We need to consider the string including axion excitations of momenta above some (Wilsonian) cutoff Λ, and the axion *field* with momenta below this cutoff.

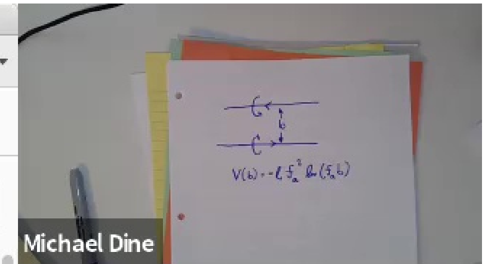
So our low energy action should include both the excitations of the long string and axions with momenta below some cutoff Λ_w. We should choose

$$H \ll \Lambda_w \ll f_{pq} \tag{9}$$

For such a description to be sensible, the evolution of the axion distribution, $\langle a(k)a(-k)\rangle$ should match above and beyond the cutoff.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?

Naive Expectations of the Axion Number Density from Hubble Scale Variations



Naive Expectations of the Axion Number Density from Hubble Scale Variations

The scale factor satisfies, in a radiation dominated universe:

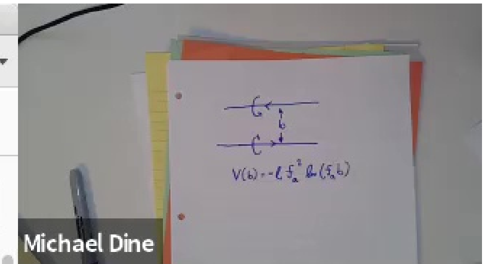
$$R(t) = \sqrt{\frac{t}{t_0}} R_0. \tag{10}$$

If we define the conformal time,

$$\tau = 2\sqrt{t t_0} R_0 = 2 t_0 R(\tau) \tag{11}$$

then the metric is:

$$ds^2 = R(\tau)^2 (d\tau^2 - d\vec{x}^2). \tag{12}$$



We can expand the axion field $a(x,\tau)$ field

$$a(x,\tau) = \int \frac{d^3k}{(2\pi)^3}\phi(k,\tau)e^{i\vec{k}\cdot\vec{x}}. \tag{13}$$

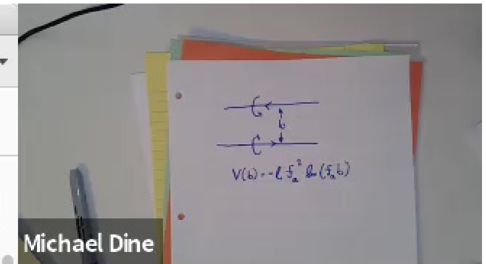
In the conformal frame, if we can neglect he potential:

$$\ddot{\phi}(k,\tau) + \frac{2}{\tau}\dot{\phi} + k^2\phi = 0. \tag{14}$$

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

So, for $k \ll \tau^{-1}$, $\phi(k,\tau)$ i.e. modes outside Hubble horizon, $\phi(k,\tau)$ is essentially constant; for $k \gg \tau^{-1}$,

(15)



So, for $k \ll \tau^{-1}$, $\phi(k,\tau)$ i.e. modes outside Hubble horizon, $\phi(k,\tau)$ is essentially constant; for $k \gg \tau^{-1}$,

$$\phi(k,\tau) = \phi(k,\tau_0)\frac{\tau_0}{\tau}\cos(k\tau). \tag{15}$$

To compute the energy density we need to make some assumption about the mean value of $\phi(k,\tau_0)$. Take:

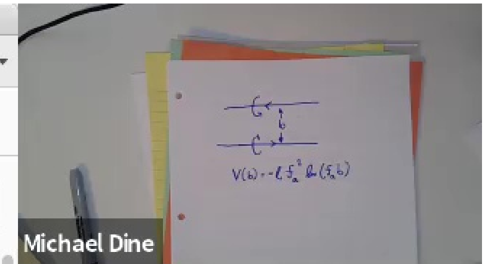
$$\langle \phi(k,\tau_0)\phi^\dagger(k',\tau_0)\rangle = \mathcal{J}(\vec{k},\vec{k}')\delta(\vec{k}-\vec{k}') = \frac{f_a^2}{k^3}\delta(\vec{k}-\vec{k}') \tag{16}$$

Then

$$\langle \partial_i\phi\partial_j\phi g^{ij}\rangle_{PQ} = \frac{1}{R^2(t)}\int d^3k \frac{k^2}{(2\pi)^3}\frac{f_a^2}{k^3}\theta(1/\tau_{PQ}-k) \tag{17}$$

$$= \frac{f_a^2 t_0^2}{\tau^4} \propto f_a^2 H_{PQ}^2 \tag{18}$$

This is exactly as we expect.



talk_axion_cosmic_strings_2021.pdf

$$= \frac{f_a^2 t_0^2}{\tau^4} \propto f_a^2 H_{PQ}^2 \qquad (18)$$

This is exactly as we expect.

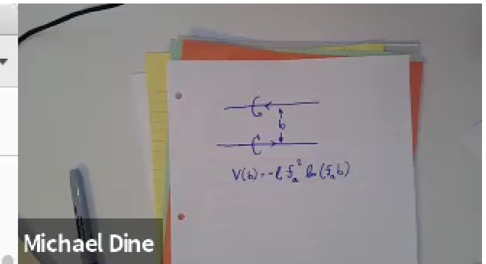
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?

Now consider the contribution to the energy density from larger momenta, $k > \tau$. These modes start to oscillate for $k = \tau(k)$, and damp as $\tau(k)^2/\tau^2 = \frac{1}{k^2}/\tau^2$. So the corresponding contribution is:

$$\rho = \left(\frac{1}{R(t)}\right)^2 \int d^3k \frac{k}{(2\pi)^3}\frac{f_a^2}{k^2}\frac{1}{\tau^2 k^2}\theta(k - 1/\tau) \qquad (19)$$

$$= \frac{f_a^2}{\tau^2 R(t)^2}\log(\Lambda\tau) = f_a^2 H^2 \log(\Lambda/H) \qquad (20)$$

for some ultraviolet cutoff Λ.



talk_axion_cosmic_strings_2021.pdf

Home Tools dennis… palm_i… JHEP_… 217_p… makov… talk_ax… mdine@ucsc.edu

22 (22 of 43)

If the bulk of this energy were converted into low momentum axions, it is conceivable that these could be the dominant source of axion dark matter.

If we write the correlator of two axion fields, in momentum space, as

$$\langle a(\vec{k})a(-\vec{k})\rangle = \frac{f_a^2 H^{q+1}}{k^{4+q}} \equiv \Delta(\vec{k}),\ |k| \geq H. \tag{21}$$

Then the axion energy density is:

$$\rho = \int \frac{dk}{2\pi^2}\frac{d\rho(k)}{dk} \equiv \int dk \frac{f_a^2 H^2 H^{q-1}}{k^q}. \tag{22}$$

Find your tools here

Create PDF

Edit PDF

Export PDF

Comment

Organize Pages

Enhance Scans

Protect

Fill & Sign

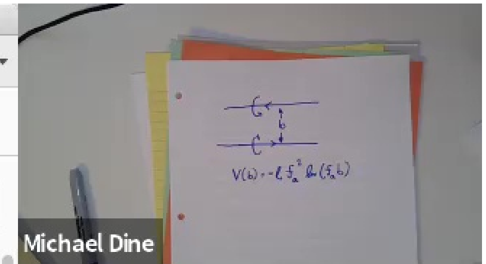
Prepare Form

Compare Files

More Tools

Your current plan is Creative Cloud

Learn More



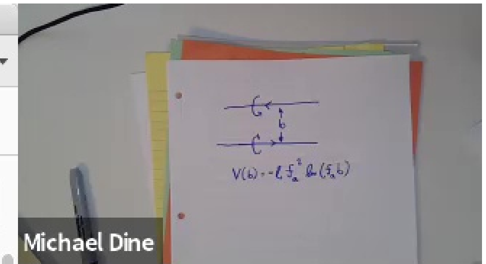
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

The corresponding number density is:

$$n_{QCD} = \int \frac{dk}{2\pi^2} \frac{d\rho(k)}{dk} = \int dk \frac{C}{k^{q+1}} \tag{23}$$

Then if $q > 1$, there is the potential for a significant contribution to the density at low momentum

Our studies of adiabatic and sudden approximations suggest that $q < 1$ at low momenta.

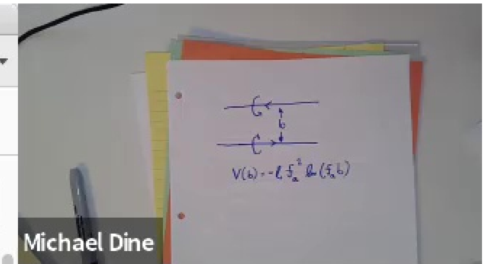


Axion as a Compact Field

The fact that the axion is a compact field suggests that there is a limit to the density of low momentum axions. With Φ the PQ field, outside of the string core we have

$$\Phi(x) = f_a e^{i\frac{a(x)}{f_a}} \equiv f_a e^{i\theta} \tag{24}$$

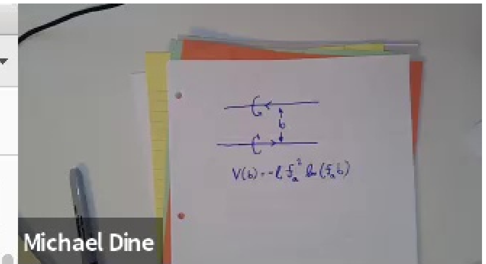
If there is of order one string per Hubble volume, then, almost everywhere, Φ takes this form.



In other words

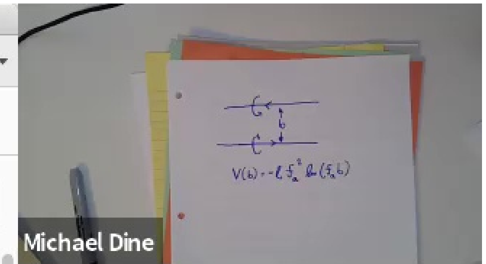
$$\langle \partial_i \Phi \partial_j \Phi \rangle \sim f_a^2 k^2 \quad (25)$$

where k is a momentum typical of the distribution. $k \sim H$.



In a recent paper of Gorghetto, Hardy, and Nicolaescu this observation is criticized with the comment:
"Recently it has been claimed that this cannot be true because the compactness of the axion feld bounds the energy that can be stored in low momentum modes. In fact, the periodicity of the axion only affects the zero-mode: all the other modes can be populated by arbitrarily large amplitudes."

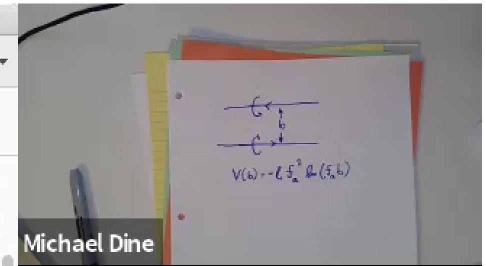
So it is perhaps worth elaborating on our argument.



In other words

$$\langle \partial_i \Phi \partial_j \Phi \rangle \sim f_a^2 k^2 \tag{25}$$

where k is a momentum typical of the distribution. $k \sim H$.



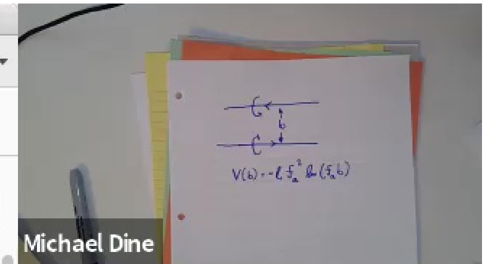
talk_axion_cosmic_strings_2021.pdf

Suppose

$$\langle a(k)a(-k)\rangle = f_a^2 H^2 (q-1)\,\xi \frac{H^{q-1}}{k^{4+q}} \equiv \Delta(k) \quad q \neq 1 \qquad (26)$$

Here the normalization is fixed by the requirement that

$$\rho = \int_H^{f_a} \frac{d^3k}{(2\pi)^3} k^2 \Delta(k) = f_a^2 H^2 \log(f_a^2/H^2) \equiv f_a^2 H^2 \xi. \qquad (27)$$



talk_axion_cosmic_strings_2021.pdf

Assuming a Gaussian distribution for the axion field:

$$\int [da(k)] e^{-\int d^3k a(k)\Delta(k)a(k) + \int d^3k J(k) a(-k)}. \tag{28}$$

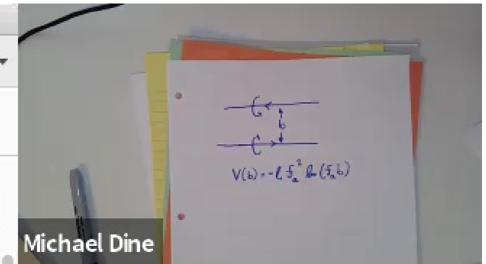
we can compute:

$$\mathcal{M}(x) = \langle \Phi(x)\Phi(0)\rangle = f_a^2 \langle e^{\frac{ia(x)}{f_a}} e^{\frac{ia(0)}{f_a}} \rangle.$$

by taking $J(x') = i\delta(x - x') - i\delta(x')$. With

$$\Delta(x) = \int \frac{d^3k}{(2\pi)^3} \Delta(k) e^{i\vec{k}\cdot\vec{x}} \sim \xi(q-1) H^{1+q} f_a^2 |x|^{q+1} \quad q \neq 1 \tag{29}$$

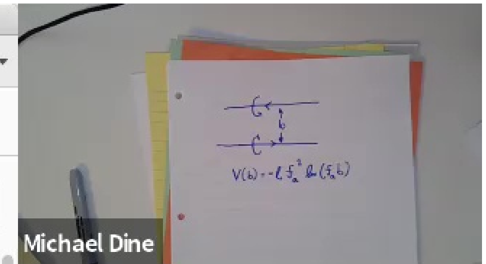
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?



$$\langle \Phi(x)\Phi(0)\rangle = f_a^2 e^{(q-1)\xi H^{q+1}|x|^{q+1}} \quad q \neq 1. \qquad (30)$$

For $q > 1$, Φ changes by e^{ξ}, when $|x|$ changes by amounts of order H^{-1}. Derivatives of Φ are enhanced by ξ. This would be consistent with a number of strings per unit volume scaling like a power of ξ, but, as I will explain shortly, this doesn't translate into enhanced low momentum axions. In other words, this sort of high momentum occupation number for low momentum axions corresponds to very rapid variation in Φ, which seems unlikely.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

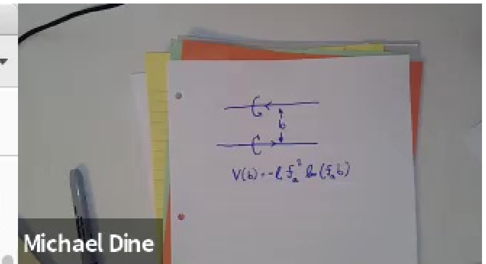


Radiation from Strings: Coupling of String Collective Modes to Axions

We have seen that the system may be described by an effective theory consisting of strings and axions, where the string is essentially the core, out to some distance Λ_w^{-1}, and the axion field includes only momenta below a matching scale, Λ_w.

We can derive an expression for the coupling of the string collective modes to the axion field, Take

$$\Phi(z,t,\vec{x}_\perp) = \Phi_0(\vec{x}_\perp - X_{0\perp})e^{ia(z,t,\vec{x}_\perp)/f_a}; \quad \vec{x}_\perp = (x,y) = (\rho,\phi). \tag{31}$$



talk_axion_cosmic_strings_2021.pdf

Home Tools dennis... palm_i... JHEP_... 217_p... makov... talk_ax... mdine@ucsc.edu

32 (32 of 43)

$$\Phi(z,t,\vec{x}_\perp) = \Phi_0(\vec{x}_\perp - X_{0\perp})e^{ia(z,t,x_\perp)/f_a}; \quad \vec{x}_\perp = (x,y) = (\rho,\phi). \quad (31)$$

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an | Axion Cosmic Strings: Players in the Early Universe?

Plugging this into the action

$$S = \int dzdtd^2x_\perp \partial_\mu \Phi^* \partial^\mu \Phi \,. \quad (32)$$

From the time derivative term:

$$\int d^2x_\perp dzdt \; \Phi^* \; \partial_i \Phi \; \dot{X}^i_\perp \; \partial_0 a/f_a + \text{c.c.}; \; i = 1,2. \quad (33)$$

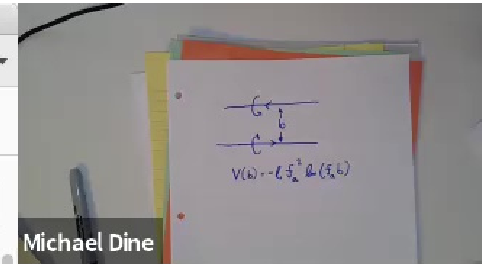
140%

Find your tools here

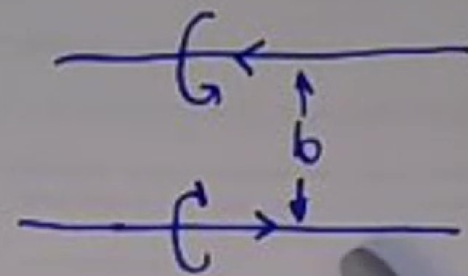
Create PDF
Edit PDF
Export PDF
Comment
Organize Pages
Enhance Scans
Protect
Fill & Sign
Prepare Form
Compare Files
More Tools

Your current plan is Creative Cloud

Learn More



Michael Dine



$$V(b) = -\ell f_a \quad (f_a b)$$

$$\partial_0 \Phi = \partial_0 f_a e^{i\phi}$$

ϕ: depends on X_i

$$\partial_0 \phi = \partial_i \phi \, \dot{X}_i$$

$$\partial_i \phi = \epsilon_{ij} \frac{x_j}{|x_\perp|^2}$$

talk_axion_cosmic_strings_2021.pdf

At large distances, far from the string core

$$\vec{\nabla}_\perp \Phi_0(\vec{x}_\perp) = f_a \vec{\nabla}_\perp e^{i\phi} = i\frac{f_a\, \epsilon_{ij} x_j}{|\vec{x}_\perp|^2} \qquad (34)$$

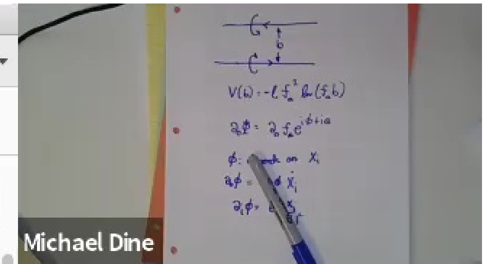
so the large distance part of the coupling is:

$$\int d^2x_\perp dz dt \frac{f_a\, \epsilon_{ij} x_j}{|\vec{x}_\perp|^2} \partial_\alpha X^i_\perp \partial^\alpha a + \mathrm{c.c.}:\ \alpha = 0, 3. \qquad (35)$$

Because of the infrared behavior of this system, not surprisingly, the string is not simply a localized source for the axion.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

Description in Terms of an antisymmetric Tensor



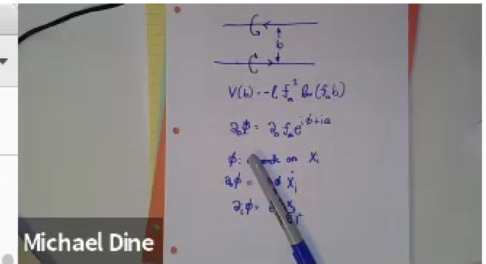
Description in Terms of an antisymmetric Tensor Field

D1 branes provide an interesting model for axion strings. In particular, in Type IIB superstring theory, such strings are sources for a Ramond-Ramond two form gauge field– an axion. Many of the features discussed here in field theory are reproduced in the D1 brane case. Indeed, one can make parallel analyses in field theory and string theory. Here, we'll just take advantage of the fact that some features of the axion are more simply understood in terms of the behavior of an antisymmetric tensor field, $B_{\mu\nu}$, to which it is equivalent.

(In progress with Shijun Sun)

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

Duality between axion and three form field



talk_axion_cosmic_strings_2021.pdf

Home Tools dennis... palm_l... JHEP_... 217_p... makov... talk_ax... mdine@ucsc.edu

35 (35 of 43)

Duality between axion and three form field strength:

$$H_{\mu\nu\rho} = \frac{1}{3!}\Big(\sum_{\text{perms}} (-1)^P \partial_\mu B_{\nu\rho}\Big) \tag{36}$$

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma}\partial^\sigma a. \tag{37}$$

The three index field strength, satisfies, in presence of string:

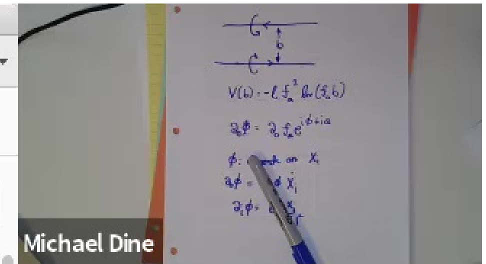
$$\partial^\mu H_{\mu\nu\rho} = J_{\nu\rho}; \quad J_{\nu\rho} = \delta(\vec{x}_\perp)\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}\epsilon^{\alpha\beta}. \tag{38}$$

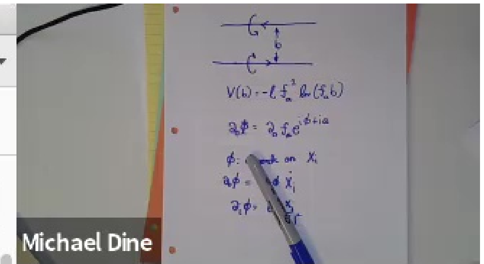
Find your tools here

Create PDF
Edit PDF
Export PDF
Comment
Organize Pages
Enhance Scans
Protect
Fill & Sign
Prepare Form
Compare Files
More Tools

Your current plan is Creative Cloud

Learn More





For a long straight string, can take $X^3 = z$, $X^0 = t$, and $\sigma^0 = t$, $\sigma^1 = z$, then

$$\partial_i H^{i0\,z} = \delta^2(\vec{x}_\perp) \quad i = 1, 2. \tag{39}$$

Now

$$\partial_i H_{i0\,z} = \epsilon_{ij0z} \partial_i \partial_j a = \delta^2(\vec{x}_\perp). \tag{40}$$

This is, indeed, solved by $a = f_a \theta$ (θ is the angle in cylindrical coordinates). This can be seen by applying Stoke's theorem:

$$\int d^2\sigma \hat{z} \cdot (\vec{\nabla} \times \vec{a}) = \int d^2\sigma \epsilon_{3ij} \partial_i \partial_j a = \oint d\vec{\ell} \cdot \vec{\nabla} a = \int d\theta \frac{da}{d\theta}. \tag{41}$$

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

Understanding the Infrared Divergence:

Understanding the Infrared Divergence: Thickening the String

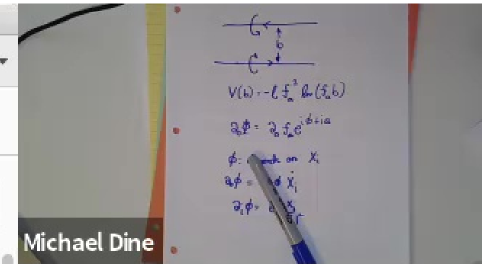
For a long straight string along the z axis, we can consider small fluctuations in the transverse directions, X^i. We now have

$$B^{0i}(t,z;\vec{x}_\perp) \sim \log(|\vec{x}_\perp|)\frac{\partial X^i}{\partial t};\ B^{zi}(t,z;\vec{x}_\perp) \sim \log(|\vec{x}_\perp|)\frac{\partial X^i}{\partial z} \quad (42)$$

Computing H_{0ij}, etc., and substituting back into the action,

$$S = \int d^2x_\perp \frac{1}{x_\perp^2}\int dt\, dz\left(\left(\frac{\partial X^i}{\partial t}\right)^2 - \left(\frac{\partial X^i}{\partial z}\right)^2\right) \quad (43)$$

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?



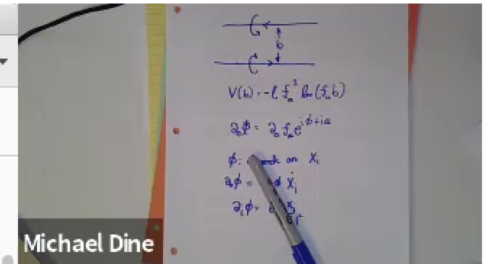
Comparison with Radiation In terms of Antisymmetric Tensor Field, $B_{\mu\nu}$ (Kalb-Ramond)

Consider the term in the action

$$\int d^4x \mathcal{L} \sim \int d^4x \left(H_{zij}^2 - H_{0ij}^2 \right). \tag{44}$$

Noting, for example, that

$$H_{zij}(t, z, \vec{x}_\perp) = \partial_0 a + \epsilon_{ij} \partial_i G^{(2)}(\vec{x}_\perp) \frac{\partial X^i}{\partial t} \tag{45}$$



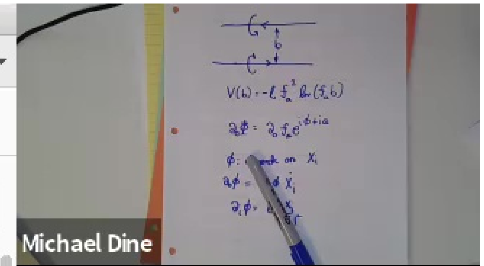
The cross terms yield an axion string coupling identical to that we found in the collective coordinate analysis:

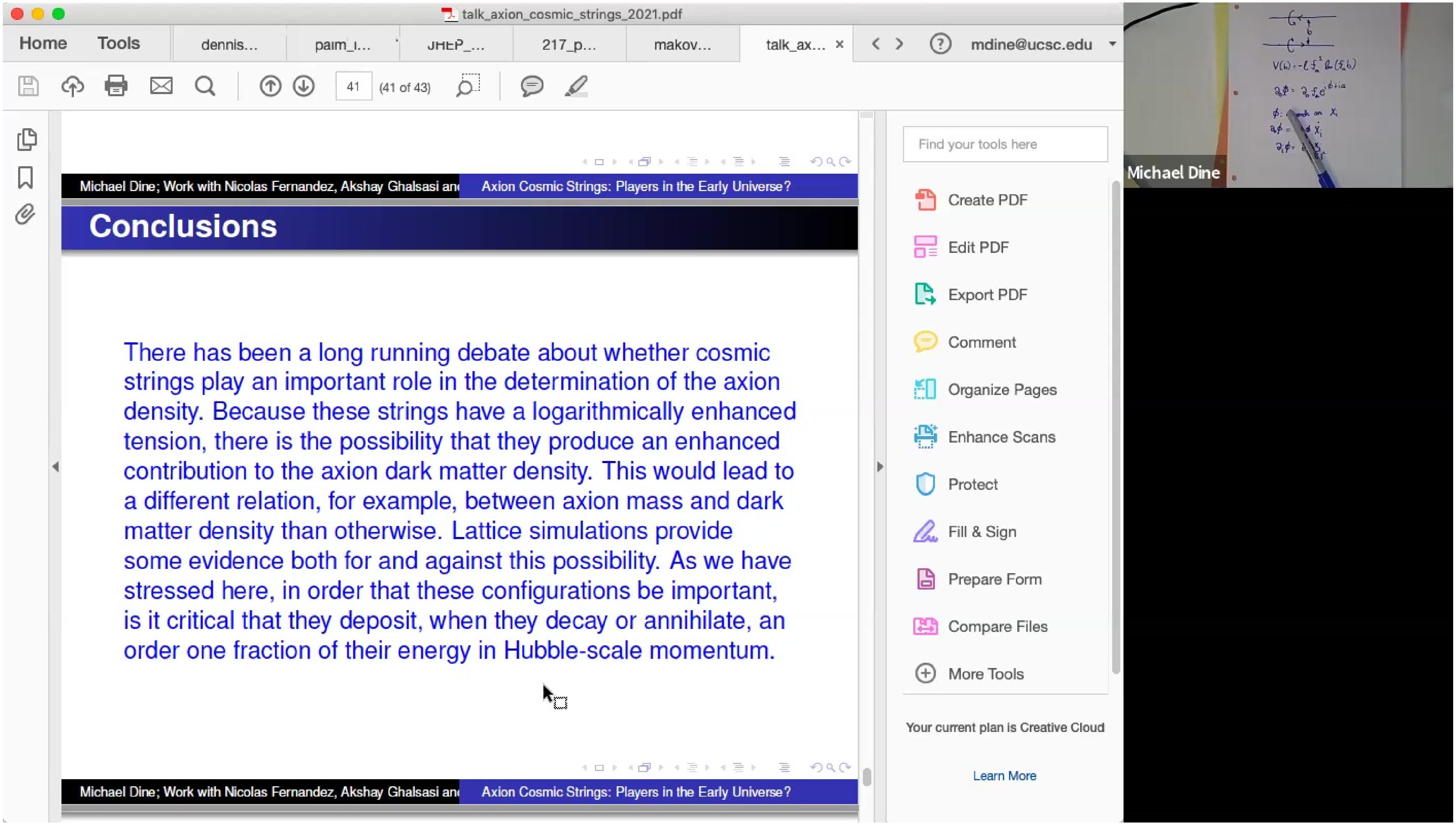
$$\mathcal{L}_{a-X^i} = \int dzdt\, d^2x_\perp \left(\partial_0 a \frac{dX^i}{dt} - \partial_z a \frac{dX^i}{dz}\right) \epsilon_{ij} \frac{x^j}{x_\perp^2}. \qquad (46)$$

Again, we see that it is necessary to consider the dynamical axion ($H_{\mu\nu\rho}$) at the same time, and solve its equations in the presence of the string. The axion is sourced throughout space. More precisely, if we have, say, a parallel string and antistring, the relevant bulk fields lie in the region near the strings.

Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi an Axion Cosmic Strings: Players in the Early Universe?

Conclusi

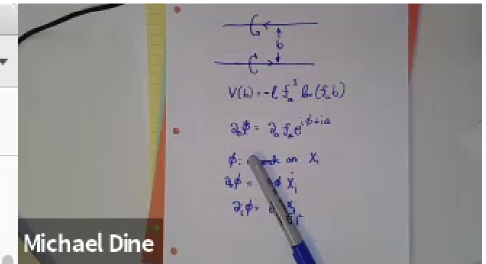



talk_axion_cosmic_strings_2021.pdf
Home
Tools
dennis...
palm_l...
JHEP_...
217_p...
makov...
talk_ax...
mdine@ucsc.edu
41
(41 of 43)
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi and
Axion Cosmic Strings: Players in the Early Universe?
Conclusions
There has been a long running debate about whether cosmic strings play an important role in the determination of the axion density. Because these strings have a logarithmically enhanced tension, there is the possibility that they produce an enhanced contribution to the axion dark matter density. This would lead to a different relation, for example, between axion mass and dark matter density than otherwise. Lattice simulations provide some evidence both for and against this possibility. As we have stressed here, in order that these configurations be important, is it critical that they deposit, when they decay or annihilate, an order one fraction of their energy in Hubble-scale momentum.
Michael Dine; Work with Nicolas Fernandez, Akshay Ghalsasi and
Axion Cosmic Strings: Players in the Early Universe?
Find your tools here
Create PDF
Edit PDF
Export PDF
Comment
Organize Pages
Enhance Scans
Protect
Fill & Sign
Prepare Form
Compare Files
More Tools
Your current plan is Creative Cloud
Learn More
Michael Dine

We have argued that the compactness of the axion field bounds the density of Hubble-scale axions by $f_a^2 H^2$. By itself, this makes it hard to understand an enhancement of the axion density over the standard computation which yields an axion density enhanced by a power of ξ.

Axion strings, due to their infrared divergent tension, need to be carefully defined. In particular, we have focused on a low energy effective action consisting of axions with momenta below some cutoff, k_0, and string collective coordinates for strings defined by a cutoff in space of order k_0^{-1}. We have seen in a sharp sense that the string is a thickened object, with thickness of this order. Axion radiation is emitted from throughout the thickened string.

By studying the low momentum axion system in the conformal frame, we have seen that there is a natural matching of the low energy axion energy density to the string.



energy axion energy density to the string.

Considerations of two extremes, adiabatic and sudden motion of the strings, indicates that one does not expect an appreciable enhancement of low momentum axion production.

This leaves us, then, with several sources of order one uncertainty in the axion dark matter density. These include

1. Limited knowledge of the QCD free energy in the relevant region.
2. Imperfect knowledge of the low momentum axion distribution.

The first item requires improved lattice simulations of the topological susceptibility in finite temperature QCD; the latter of the universe evolution near the QCD phase transition.